Solution for assignment 10:

$$\theta^{(1)} = \begin{bmatrix} \theta_{10}^{(1)} = -3 & \theta_{11}^{(1)} = 0 & \theta_{12}^{(1)} = 1 \\ \theta_{20}^{(1)} = 2 & \theta_{21}^{(1)} = -3 & \theta_{22}^{(1)} = 1 \\ \theta_{30}^{(1)} = -2 & \theta_{31}^{(1)} = -2 & \theta_{32}^{(1)} = 1 \end{bmatrix}_{3\times 3}$$

$$\theta^{(2)} = \begin{bmatrix} \theta_{10}^{(2)} = 1 & \theta_{11}^{(2)} = -2 & \theta_{12}^{(2)} = 3 & \theta_{13}^{(2)} = -4 \end{bmatrix}_{1\times 4}$$

$$X = \begin{bmatrix} x_1^{(1)} = 1 \\ x_2^{(1)} = 2 \end{bmatrix}_{2\times 1}.$$

After adding $x_0 = 1$, $X = \begin{bmatrix} x_0 = 1 \\ x_1 = 1 \\ x_2 = 2 \end{bmatrix}_{3\times 1}$

propagating from 1_{st} layer to 2_{nd} layer:

Let $Z_1^{(2)}$ be the value that goes into the first unit of the 2_{nd} layer

Let $Z_2^{(2)}$ be the value that goes into the 2nd unit of the 2nd layer

Let $Z_3^{(2)}$ be the value that goes into the 3rd unit of the 2nd layer

$$\underset{3\times1}{\begin{bmatrix} Z_1^{(2)} \\ Z_2^{(2)} \\ Z_3^{(2)} \end{bmatrix}} = \Theta^{(1)} * X = \underset{3\times3}{\begin{bmatrix} -3 & 0 & 1 \\ 2 & -3 & 1 \\ -2 & -2 & 1 \end{bmatrix}} * \underset{3\times1}{\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}} = \underset{3\times1}{\begin{bmatrix} (-3)1+(0)1+(1)2 \\ (2)1+(-3)1+(1)2 \\ (-2)1+(-2)1+(1)2 \end{bmatrix}} = \underset{3\times1}{\begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix}}$$

$$\underset{3\times1}{\begin{bmatrix} a_1^{(2)} \\ a_2^{(2)} \\ a_3^{(2)} \end{bmatrix}} = \underset{3\times1}{\begin{bmatrix} g(Z_1^{(2)}) \\ g(Z_2^{(2)}) \\ g(Z_3^{(2)}) \end{bmatrix}} = \underset{3\times1}{\begin{bmatrix} \frac{1}{1+e^{-(-1)}} \\ \frac{1}{1+e^{-(1)}} \\ \frac{1}{1+e^{-(-2)}} \end{bmatrix}} = \underset{3\times1}{\begin{bmatrix} 0.27 \\ 0.73 \\ 0.12 \end{bmatrix}}$$

Now, we can add $a_0^{(2)}=1$ and get $\begin{bmatrix} a_0^{(2)}=1 \\ a_1^{(2)}=0.27 \\ a_2^{(2)}=0.73 \\ a_3^{(2)}=0.12 \end{bmatrix}$

So, as $X=\begin{bmatrix} x_0=1 \\ x_1=1 \\ x_2=2 \end{bmatrix}$ propagates from 1st layer to 2nd layer, it will become $a^{(2)} = \begin{bmatrix} a_0^{(2)}=1 \\ a_1^{(2)}=0.27 \\ a_2^{(2)}=0.73 \\ a_3^{(2)}=0.12 \end{bmatrix}$

propagating from 2nd layer to 3rd layer:

Let $Z_1^{(3)}$ be the value that goes into the 1st unit of the 3rd layer

$$\left[Z_1^{(3)}\right] = \theta^{(2)} * a^{(2)} = \underset{1\times4}{\begin{bmatrix} 1 & -2 & 3 & -4 \end{bmatrix}} * \underset{4\times1}{\begin{bmatrix} 1 \\ 0.27 \\ 0.73 \\ 0.12 \end{bmatrix}}$$

$$= \left[(1)(1) + (-2)(0.27) + (3)(0.73) + (-4)(0.12)\right] = [2.17]$$

$$\left[a_1^{(3)}\right] = \left[g\left(Z_1^{(3)}\right)\right] = \left[\frac{1}{1+e^{-(2.17)}}\right] = 0.90$$

This means $h(x_1=1, x_2=2) = 0.90$

This means $P(y=1) = 0.90$ for input point $\begin{bmatrix} x_1 = 1 \\ x_2 = 2 \end{bmatrix}$

This means input point $\begin{bmatrix} x_1 = 1 \\ x_2 = 2 \end{bmatrix}$ should be classified as belonging to class 1.