Problem 1

$J(\theta_0) = \theta_0^2$, $\alpha = 0.7$ initial $\theta_0 = 4$, 6 iterations

$$\frac{\partial J}{\partial \theta_0} = 2\theta_0 \qquad \theta_0 = \theta_0 - \alpha \frac{\partial J}{\partial \theta_0}$$

1. $\left.\frac{\partial J}{\partial \theta_0}\right|_{\theta_0 = 4} = 8 \qquad \theta_0 = 4 - 0.7 \times 8 = -1.6$

2. $\left.\frac{\partial J}{\partial \theta_0}\right|_{\theta_0 = -1.6} = -3.2 \qquad \theta_0 = -1.6 - 0.7 \times (-3.2) = 0.64$

3. $\left.\frac{\partial J}{\partial \theta_0}\right|_{\theta_0 = 0.64} = 1.28 \qquad \theta_0 = 0.64 - 0.7 \times (1.28) = -0.256$

4. $\left.\frac{\partial J}{\partial \theta_0}\right|_{\theta_0 = -0.256} = -0.512 \qquad \theta_0 = -0.256 - 0.7 \times (-0.512) = 0.1024$

5. $\left.\frac{\partial J}{\partial \theta_0}\right|_{\theta_0 = 0.1024} = 0.2048 \qquad \theta_0 = 0.1024 - 0.7(0.2048) = -0.04096$

6. $\left.\frac{\partial J}{\partial \theta_0}\right|_{\theta_0 = -0.04096} = -0.08192 \qquad \theta_0 = -0.04096 - 0.7(-0.08192)$

   $= 0.016384$

It appears that θ_0 covers a certain number, namely 0, and setting θ_0 to 0 can minimize

```
initial θ: 4.0000
j(θ): 8.0000
1θ: -1.6000
1j(θ): -3.2000
2θ: 0.6400
2j(θ): 1.2800
3θ: -0.2560
3j(θ): -0.5120
4θ: 0.1024
4j(θ): 0.2048
5θ: -0.0410
5j(θ): -0.0819
6θ: 0.0164
6j(θ): 0.0328
```

Problem 2

$J(\theta_0) = \theta_0^2$, $\alpha = 1.2$ initial $\theta_0 = 4$, 6 iterations

$$\frac{\partial J}{\partial \theta_0} = 2\theta_0 \qquad \theta_0 = \theta_0 - \alpha \frac{\partial J}{\partial \theta_0}$$

1. $\left.\frac{\partial J}{\partial \theta_0}\right|_{\theta_0 = 4} = 8 \qquad \theta_0 = 4 - 1.2 \times 8 = -5.6$

2. $\left.\frac{\partial J}{\partial \theta_0}\right|_{\theta_0 = -5.6} = -11.2 \qquad \theta_0 = -5.6 - 1.2 \times (-11.2) = 7.84$

3. $\left.\frac{\partial J}{\partial \theta_0}\right|_{\theta_0 = 7.84} = 15.68 \qquad \theta_0 = 7.84 - 1.2 \times (15.68) = -10.976$

4. $\left.\frac{\partial J}{\partial \theta_0}\right|_{\theta_0 = -10.976} = -21.952 \qquad \theta_0 = -10.976 - 1.2 \times (-21.952) = 15.366$

5. $\left.\frac{\partial J}{\partial \theta_0}\right|_{\theta_0 = 15.3664} = 30.7327 \qquad \theta_0 = 15.366 - 1.2(30.7327) = -21.51296$

6. $\left.\frac{\partial J}{\partial \theta_0}\right|_{\theta_0 = -21.51296} = -43.02592 \qquad \theta_0 = -21.51296 - 1.2(-43.02592) = 30.118144$

No, it doesn't coverage the certain number

To address the issue and complete coverage, it is necessary to adjust α value from 0 to 1.

```
initial θ: 4.0000
j(θ): 8.0000
1 θ: -5.6000
1 j(θ): -11.2000
2 θ: 7.8400
2 j(θ): 15.6800
3 θ: -10.9760
3 j(θ): -21.9520
4 θ: 15.3664
4 j(θ): 30.7328
5 θ: -21.5130
5 j(θ): -43.0259
6 θ: 30.1181
6 j(θ): 60.2363
```

Problem 3

$J(\theta_0) = \frac{1}{3}\theta_0^3 + \frac{1}{2}\theta_0^2 - 2\theta_0$, $\alpha = 0.15$ initial $\theta_0 = 3$

6 iterations

$\frac{\partial J}{\partial \theta_0} = \theta_0^2 + \theta_0 - 2$ $\theta_0 = \theta_0 - \alpha \frac{\partial J}{\partial \theta_0}$

1. $\left.\frac{\partial J}{\partial \theta_0}\right|_{\theta_0 = 3} = 10$ $\theta_0 = 3 - 0.15(10) = 1.5$

2. $\left.\frac{\partial J}{\partial \theta_0}\right|_{\theta_0 = 1.5} = 1.75$ $\theta_0 = 1.5 - 0.15(1.75) = 1.2375$

3. $\left.\frac{\partial J}{\partial \theta_0}\right|_{\theta_0 = 1.2375} = 0.769$ $\theta_0 = 1.2375 - 0.15(0.769) =$ 1.122

4. $\left.\frac{\partial J}{\partial \theta_0}\right|_{\theta_0 = 1.122} = 0.381$ $\theta_0 = 0.9806 - 0.15(0.381)$ $= 1.065$

5. $\left.\frac{\partial J}{\partial \theta_0}\right|_{\theta_0 = 1.065} = 0.199$ $\theta_0 = 0.98927 - 0.15(0.199)$ $= 1.035$

6. $\left.\frac{\partial J}{\partial \theta_0}\right|_{\theta_0 = 1.035} = 0.107$ $\theta_0 = 0.99407 - 0.15(0.107)$ $= 1.019$

It appears that θ_0 covers a certain number, namely 1, and setting θ_0 to 1 can minimize $J(\theta_0)$.

```
initial θ: 3.0000
j(θ): 10.0000
1 θ: 1.5000
1 j(θ): 1.7500
2 θ: 1.2375
2 j(θ): 0.7689
3 θ: 1.1222
3 j(θ): 0.3814
4 θ: 1.0650
4 j(θ): 0.1991
5 θ: 1.0351
5 j(θ): 0.1065
6 θ: 1.0191
6 j(θ): 0.0577
```

Problem 4

$$J(\theta_0) = \frac{1}{3}\theta_0^3 + \frac{1}{2}\theta_0^2 - 2\theta_0, \quad \alpha = 0.15 \quad \text{initial } \theta_0 = -3$$

6 iterations

$$\frac{\partial J}{\partial \theta_0} = \theta_0^2 + \theta_0 - 2 \qquad \theta_0 = \theta_0 - \alpha \frac{\partial J}{\partial \theta_0}$$

1. $\left.\frac{\partial J}{\partial \theta_0}\right|_{\theta_0=-3} = 4 \qquad \theta_0 = -3 - 0.15(4) = -3.6$

2. $\left.\frac{\partial J}{\partial \theta_0}\right|_{\theta_0=-3.6} = 7.36 \quad \theta_0 = -3.6 - 0.15(7.36) = -4.704$

3. $\left.\frac{\partial J}{\partial \theta_0}\right|_{\theta_0=-4.704} = 15.424 \quad \theta_0 = -4.704 - 0.15(15.424) =$
$= -7.0176$

4. $\left.\frac{\partial J}{\partial \theta_0}\right|_{\theta_0=-7.0176} = 40.228 \quad \theta_0 = -7.0176 - 0.15(40.936)$
$= -13.052$

5. $\left.\frac{\partial J}{\partial \theta_0}\right|_{\theta_0=-13.158} = 155.298 \quad \theta_0 = -13.158 - 0.15(157.975)$
$= -36.346$

6. $\left.\frac{\partial J}{\partial \theta_0}\right|_{\theta_0=-36.8543} = 1282.717 \quad \theta_0 = -36.8543 - 0.15(1282.717)$
$= -228.754$

No, it doesn't coverage the certain number

Because the minimum number is negative infinite.

```
initial θ: -3.0000
j(θ): 4.0000
1 θ: -3.6000
1 j(θ): 7.3600
2 θ: -4.7040
2 j(θ): 15.4236
3 θ: -7.0175
3 j(θ): 40.2284
4 θ: -13.0518
4 j(θ): 155.2976
5 θ: -36.3464
5 j(θ): 1282.7169
6 θ: -228.7540
6 j(θ): 52097.6227
```

```python
def equation (a,b,c, initial,iteration,alpha):
    x = initial
    y = a *pow(x,2) + b * x + c
    print('initial θ: {:.4f}'.format(x))
    print('j(θ): {:.4f}'.format(y))
    x = x - alpha * y
    for i in range(1,iteration+1):
        y = a *pow(x,2) + b * x + c
        print('{} θ: {:.4f}'.format(i,x))
        print('{} j(θ): {:.4f}'.format(i,y))
        x = x - alpha * y
```