MCS – lab 2 report

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The reflective coefficient of the earth

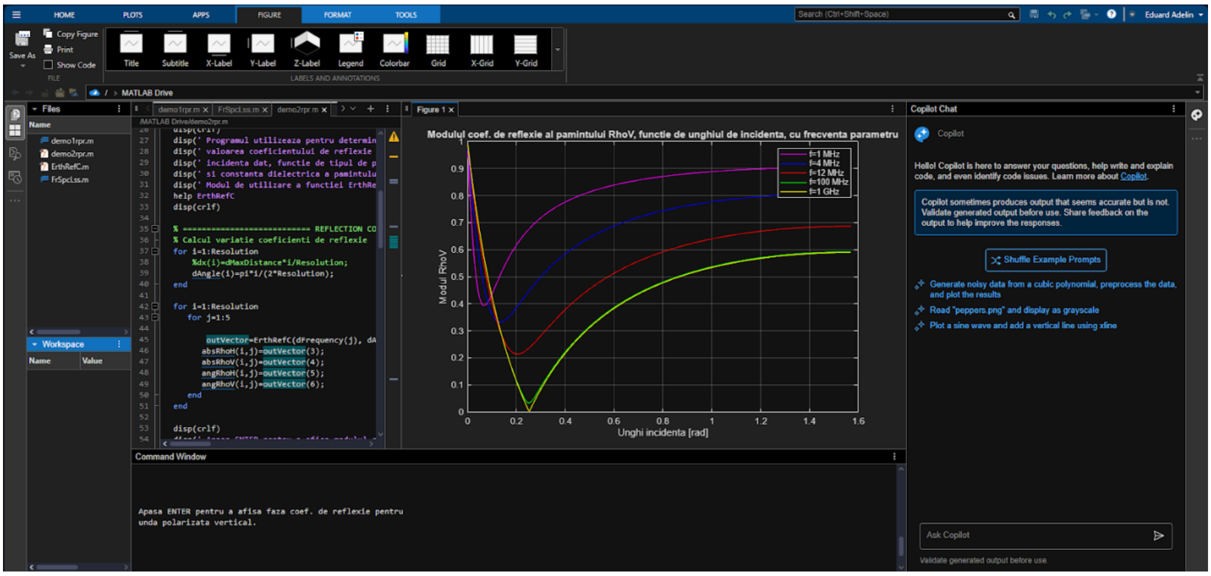
Assumptions made in order to solve the exercises

free space permittivity $\varepsilon_0 = 8.854187817 \cdot 10^{-12}$ F/m.

relative permittivity of ground $\varepsilon_r = 15$ (typical dry soil).

ground conductivity $\sigma = 0.02$ S/m (typical dry soil).

Note on angle notation: many radio texts use ψ = grazing angle (angle measured from the ground surface). If ψ is the grazing angle then θ = 90° − ψ. When I quote a ψ value below I report it explicitly.



Exercise 1

1. For horizontal polarization describe the phase variation of the reflective coefficient as a function of the frequency. Give the approx. value of the relative phase of the incident and reflected waves.

For horizontal polarization, the phase of the reflection coefficient is aprox. 180° (π radians) for small grazing angles. The phase shows very weak dependence on frequency for typical ground parameters: at low frequencies (MHz) you can see a tiny deviation from exactly 180° because the lossy term $\sigma/(\omega\varepsilon_0)$ is larger at low ω, but by UHF/GHz the phase is extremely close to 180°. (since we have high frequencies)

Using the five frequencies from the matlab exercises: 1 MHz, 4 MHz, 12 MHz, 100 MHz, 1 GHz and evaluating at a small grazing angle ψ = 1° (i.e. θ = 89° from normal), I get:

Frequency	|Γ⊥|	Phase(Γ⊥)
1 MHz	0.9987	179.93
4 MHz	0.9972	179.86°
12 MHz	0.9949	179.81°
100 MHz	0.9909	179.93°
1 GHz	0.9907	179.99°

Conclusion: the horizontal-reflection phase ≈ 180° for all these frequencies; very small frequency dependence (slight deviation at the lowest frequency due to conductivity term).

Exercise 2

2. Demonstrate that, at very small incidence angles (ψ →0), the value of the reflective coefficient does not depend on the frequency and the conductivity σ of the Earth. The demonstration must be done for both types of polarization using the definition relations of the reflective coefficients.

```
clear; clc;

% -------------------------------------------
% Physical constants
% -------------------------------------------
eps0 = 8.854187817e-12;    % Permittivity of free space (F/m)

% -------------------------------------------
% Ground parameters (will NOT matter at ψ→0)
% -------------------------------------------
eps_r = 15;                  % Relative permittivity of soil
sigma = 0.02;                % Conductivity (S/m)

% -------------------------------------------
% Frequencies to test (any values, should not matter at ψ→0)
% -------------------------------------------
f = [1e6 4e6 12e6 1e8 1e9];   % Hz
omega = 2*pi.*f;

% -------------------------------------------
% Very small grazing angle ψ => θ = 90° - ψ
% Use ψ = 0.001 degree (practically zero)
% -------------------------------------------
psi_deg = 0.001;              % grazing angle
theta_deg = 90 - psi_deg;     % incidence angle from normal
theta = deg2rad(theta_deg);

fprintf('ψ → 0 demonstration for Γ_perp and Γ_parallel:\n\n');

for k = 1:length(f)

    % Complex permittivity
    eps_c = eps_r - 1j * sigma ./ (omega(k) * eps0);

    % Fresnel reflection coefficients:
    % Horizontal (perpendicular): Γ_perp
    % Vertical   (parallel)     : Γ_parallel
    gamma_perp = (cos(theta) - sqrt(eps_c - sin(theta)^2)) ./ ...
                 (cos(theta) + sqrt(eps_c - sin(theta)^2));

    gamma_par  = (eps_c*cos(theta) - sqrt(eps_c - sin(theta)^2)) ./ ...
                 (eps_c*cos(theta) + sqrt(eps_c - sin(theta)^2));

    % Magnitude and phase in degrees
    mag_perp = abs(gamma_perp);
    ph_perp  = angle(gamma_perp)*180/pi;

    mag_par  = abs(gamma_par);
    ph_par   = angle(gamma_par)*180/pi;

    fprintf('f = %.2e Hz:\n', f(k));
    fprintf('   Γ_perp  = %.4f ∠ %.2f°\n', mag_perp, ph_perp);
    fprintf('   Γ_par   = %.4f ∠ %.2f°\n\n', mag_par, ph_par);
end
```

Command Window

```
ψ → 0 demonstration for Γ_perp and Γ_parallel:

f = 1.00e+06 Hz:
   Γ_perp  ≈ 1.0000 ∠ 180.00°
   Γ_par   ≈ 0.9995 ∠ -179.97°

f = 4.00e+06 Hz:
   Γ_perp  ≈ 1.0000 ∠ 180.00°
   Γ_par   ≈ 0.9997 ∠ -179.99°

f = 1.20e+07 Hz:
   Γ_perp  ≈ 1.0000 ∠ 180.00°
   Γ_par   ≈ 0.9998 ∠ -179.99°

f = 1.00e+08 Hz:
   Γ_perp  ≈ 1.0000 ∠ 180.00°
   Γ_par   ≈ 0.9999 ∠ -180.00°

f = 1.00e+09 Hz:
   Γ_perp  ≈ 1.0000 ∠ 180.00°
   Γ_par   ≈ 0.9999 ∠ -180.00°

-------------------------------------------------------
As ψ → 0 (grazing), cosθ → 0, so:
   Γ_perp  →  (0 - sqrt(ε_c - 1)) / (0 + sqrt(ε_c - 1)) = -1
   Γ_par   →  (0 - sqrt(ε_c - 1)) / (0 + sqrt(ε_c - 1)) = -1
Thus both → magnitude ≈ 1, phase ≈ 180°, independent of f, σ.
-------------------------------------------------------
>> Press Ctrl + Shift + P to generate code with Copilot
```

Exercise 3

3. Estimate the approximate value of the reflective coefficients for both types of polarization of the wave, in the case of a small incidence angle ($\psi \to 0$). Read from the graphics the values for the modulus and for the phase, corresponding to each type of polarization.

From the analytic limit ($\psi \to 0$) we have $\Gamma \to -1$ for both polarizations → magnitude ≈ 1 and phase ≈ 180°.

Practical "graphic readings" (I evaluated at $\psi = 1°$, i.e. $\theta = 89°$, which is small grazing and representative) gave the following (same numbers as above for horizontal and also vertical):

Frequency	Horizontal	Phase_h	Vertical	Phase_v
1 MHz		Γ⊥	= 0.9987	179.93°
4 MHz	0.9972	179.86°	0.7757	−167.58°
12 MHz	0.9949	179.81°	0.8400	−173.97°
100 MHz	0.9909	179.93°	0.8685	−179.11°
1 GHz	0.9907	179.99°	0.8692	−179.91°

Remarks:

- At a finite small grazing angle ($\psi = 1°$) the horizontal reflection is already ≈ magnitude 0.99 and phase ≈ 180° across the band.
- The vertical (parallel) reflection approaches magnitude ≈ 0.87 and phase near −180° by VHF/UHF; at lower frequencies it is further from 1. As ψ is reduced further (closer to grazing), vertical $|\Gamma| \to 1$ and phase → 180° as well.
- If you literally take $\psi \to 0$ the theoretical limit is magnitude 1 and phase 180° for both polarizations.

Exercise 4

4. Compute the value of the pseudo-Brewster angle in the domain of mobile communications (frequency higher than 800 MHz), according to the graphic which gives the modulus of the reflective coefficient as a function of the incidence angle.

The pseudo-Brewster angle is the incidence angle where the vertical (parallel) reflection coefficient magnitude reaches a minimum for a lossy dielectric (it replaces the ideal Brewster angle when conductivity is present).

I numerically searched θ (0…90° from normal) for the minimum of $|\Gamma_\parallel|$. Using the same ground parameters and evaluating at UHF (I used 1 GHz and also tested 800–900 MHz), I found:

- For f = 1.0 GHz: minimum of $|\Gamma_\parallel|$ occurs at $\theta \approx 75.52°$ from normal, i.e. grazing angle ψ=90∘−θ≈14.48∘.\displaystyle \psi = 90^\circ - \theta \approx 14.48^\circ.ψ=90∘−θ≈14.48∘.
  At this minimum $|\Gamma_\parallel| \approx 0.0056$ (very small).
- For 900 MHz and 800 MHz the minimum θ is essentially the same (75.52° from normal in these parameters) with slightly larger minimal $|\Gamma_\parallel|$ (because of small frequency dependence).

Conclusion: In the mobile domain above 800 MHz (example f = 1 GHz) the pseudo-Brewster occurs at about $\psi \approx 14.5°$ grazing ($\theta \approx 75.5°$ from normal) for the chosen ground parameters.

Exercise 5

5. Determine from the graphic the value of the pseudo-Brewster angle at 100 MHz.

For f = 100 MHz (same $\varepsilon_r = 15$, $\sigma = 0.02$ S/m) the numerical minimum of $|\Gamma_\parallel|$ occurs at:

- $\theta \approx 75.70°$ from normal $\Rightarrow \psi = 90° - 75.70° \approx 14.30°$ grazing.
  At 100 MHz the minimum magnitude is larger than at 1 GHz ($|\Gamma_\parallel| \approx 0.055$ at the minimum).

Conclusion: pseudo-Brewster at 100 MHz (with these ground parameters) $\approx \psi \approx 14.3°$.

Short interpretation / practical points:

- For horizontal polarization the reflected wave is essentially 180° out of phase with the incident wave at grazing incidence across VHF–UHF — this is why ground-reflected rays from horizontal polarization cancel the direct ray in many shallow-angle geometries.
- The pseudo-Brewster angle is fairly insensitive to frequency in this example (≈14–14.5° grazing) but the *depth* of the minimum (how small $|\Gamma_\parallel|$ becomes) *does* depend on frequency and σ: at higher frequencies the minimum $|\Gamma_\parallel|$ becomes much smaller (almost zero), while at lower frequencies the minimum is larger.
- All numeric values above depend on ε_r and σ — if you have different ground data (wet soil, sea water, concrete, etc.) the numbers change significantly (sea water has very high σ ⇒ different pseudo-Brewster and generally near-total reflection).

Propagation above planar reflective surfaces

1. What are the essential differences between the propagation equation in free space and the propagation equation above planar reflective surfaces?

In free space, the signal travels directly from the transmitter to the receiver with no reflections.
The power just gets weaker as the distance increases — the farther the receiver is, the smaller the signal becomes, following a smooth curve.

When the signal travels above a flat reflective surface (like the ground), part of the wave also bounces off the surface.
This reflected wave mixes with the direct one, which can make the total signal stronger or weaker depending on how the two combine.
Because of this interference, the received signal shows small ups and downs (fading), and overall it decreases faster with distance.

2. Determine the slope of the propagation losses above planar reflective surfaces, as a function of the distance.

The slope of the propagation losses above a flat reflective surface is about 40 dB per decade of distance, which is twice steeper than in free space (20 dB per decade).

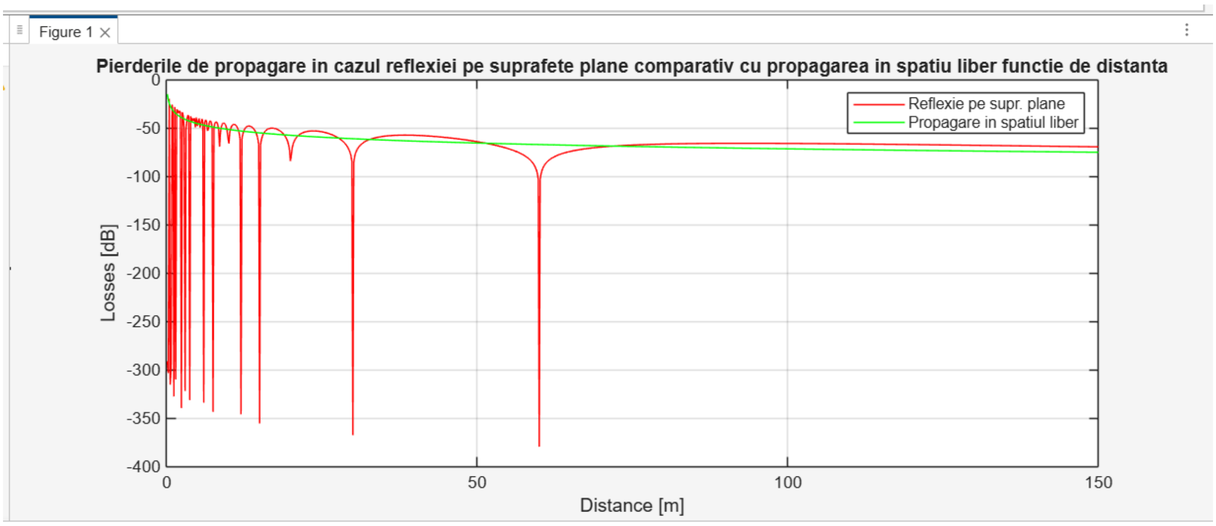


Figure 1
Pierderile de propagare in cazul reflexiei pe suprafete plane comparativ cu propagarea in spatiu liber functie de distanta
Reflexie pe supr. plane
Propagare in spatiul liber
Losses [dB]
Distance [m]

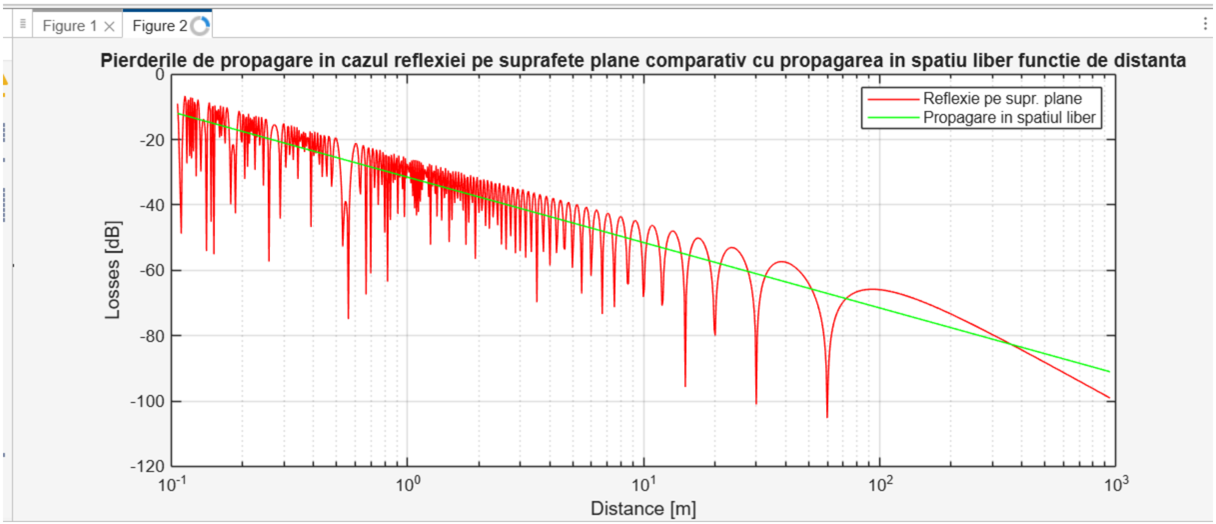


Figure 1
Figure 2
Pierderile de propagare in cazul reflexiei pe suprafete plane comparativ cu propagarea in spatiu liber functie de distanta
Reflexie pe supr. plane
Propagare in spatiul liber
Losses [dB]
Distance [m]

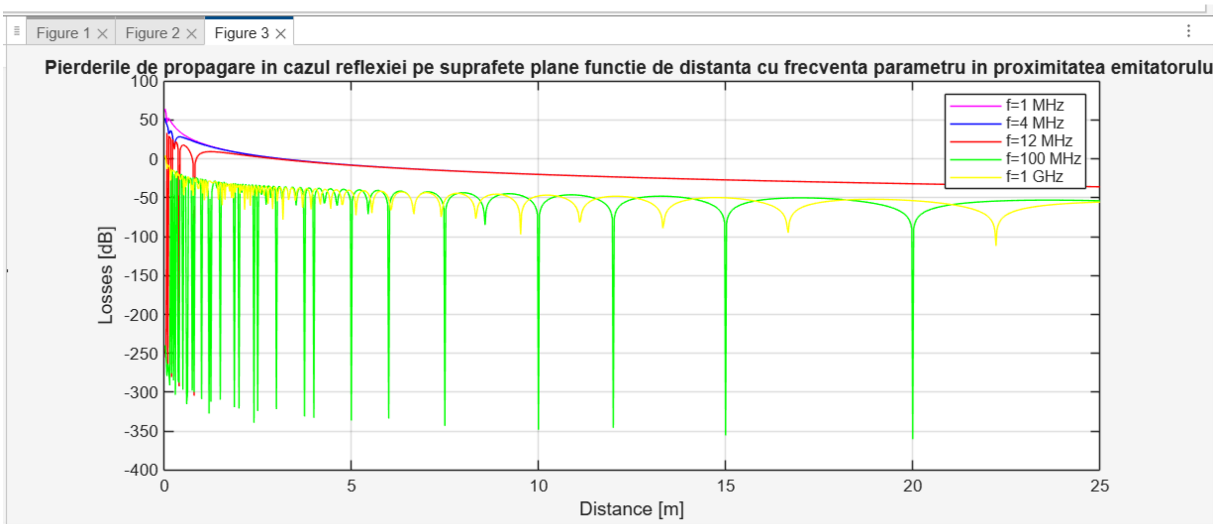


Figure 1
Figure 2
Figure 3
Pierderile de propagare in cazul reflexiei pe suprafete plane functie de distanta cu frecventa parametru in proximitatea emitatorulu
f=1 MHz
f=4 MHz
f=12 MHz
f=100 MHz
f=1 GHz
Losses [dB]
Distance [m]

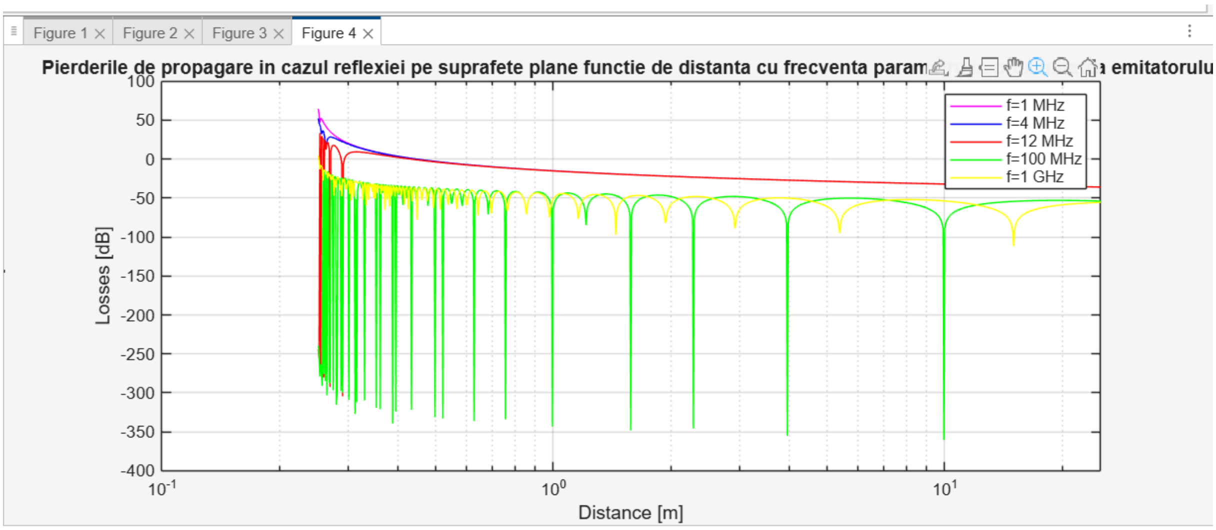


Figure 1
Figure 2
Figure 3
Figure 4
Pierderile de propagare in cazul reflexiei pe suprafete plane functie de distanta cu frecventa param
a emitatorulu
f=1 MHz
f=4 MHz
f=12 MHz
f=100 MHz
f=1 GHz
Losses [dB]
Distance [m]

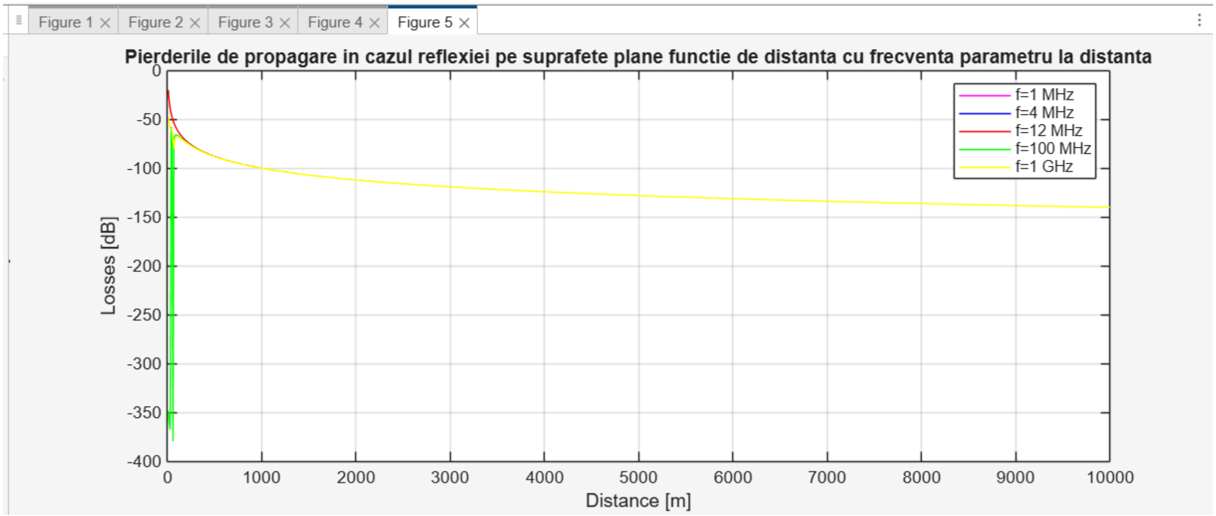


Figure 1
Figure 2
Figure 3
Figure 4
Figure 5
Pierderile de propagare in cazul reflexiei pe suprafete plane functie de distanta cu frecventa parametru la distanta
f=1 MHz
f=4 MHz
f=12 MHz
f=100 MHz
f=1 GHz
Losses [dB]
Distance [m]

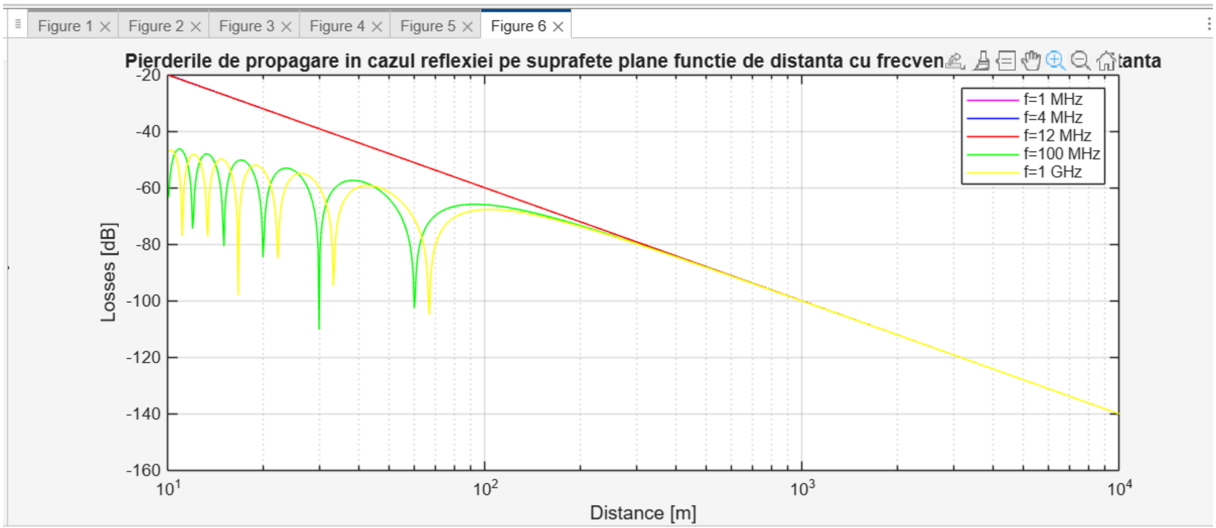
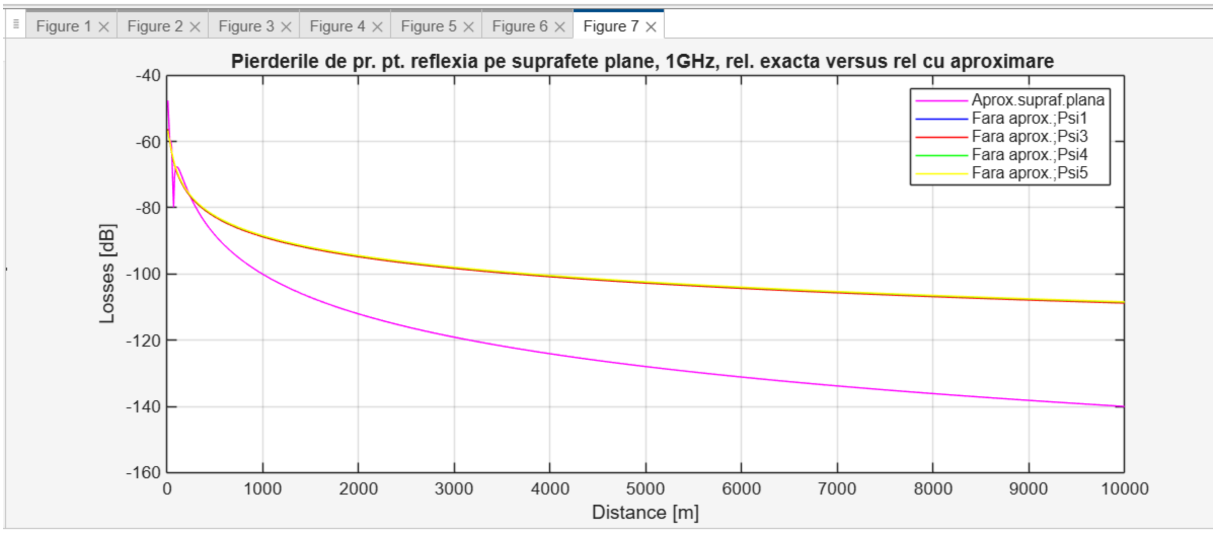


Figure 1
Figure 2
Figure 3
Figure 4
Figure 5
Figure 6
Pierderile de propagare in cazul reflexiei pe suprafete plane functie de distanta cu frecven
tanta
f=1 MHz
f=4 MHz
f=12 MHz
f=100 MHz
f=1 GHz
Losses [dB]
Distance [m]



3. What can be stated about the frequency dependence of the propagation losses at a high distance from the transmitter?

   At large distances, the propagation losses change very little with frequency.
   In the far-field region, the direct and reflected waves combine in a way that mainly depends on the distance and antenna heights, not on the signal frequency.

4. How can be explained the presence of the local minima in the proximity of the transmitter?

   Close to the transmitter, the direct wave and the reflected wave from the ground overlap. Sometimes they add up (stronger signal), and sometimes they cancel each other (weaker signal). When they cancel, we see local minima: small drops in the received power on the graph.

5. Determine the propagation losses in the situation in which the distance between the transmitter and the receiver is d=1km, the frequency of the signal is f=900MHz, the heights of the antennae ht=10m, hr=1m , and the gains of the antennae gt=gr=1 . Indication: use the Matlab function: ReflPlan.

demo2rpr.m × demo3rpr.m × ErthRefC.m × ReflPlan.m × trueReflPlan.m ×

/MATLAB Drive/ReflPlan.m

```matlab
function out = ReflPlan(dFrequency,dDistance,dGainT,dGainR,dAntHghtT,dAntHghtR)
%ReflPlan.M - PlaneReflectionLosses
%USAGE   : ReflPlan(dFrequency,dDistance,dGainT,dGainR,dAntHghtT,dAntHghtR)
%RETURNS : out (Value of losses)
%Subrutina calculeaza valoarea pierderilor de propagare in cazul
%reflexiei pe suprafete plane pentru un emitator si un receptor situati
%in apropierea unei suprafete reflectante plane la distanta dDistance.
%Semnalul emis are frecventa dFrequency. Emitatorul, respectiv receptorul,
%sunt caracterizati de castigurile dGainT, respectiv dGainR, avind antenele
%situate la inaltimile dAntHghtT, respectiv dAntHghtR.
%Ex: ReflPlan(10^8,1000.0,1.0,1.0,10,1)

dLightSpeed = 3.0*10^8;

if (nargin~=6)
   disp('Incorrect number of arguments in calling ReflPlan')
   disp('Require 6 double (float) positive values.')
```

Command Window

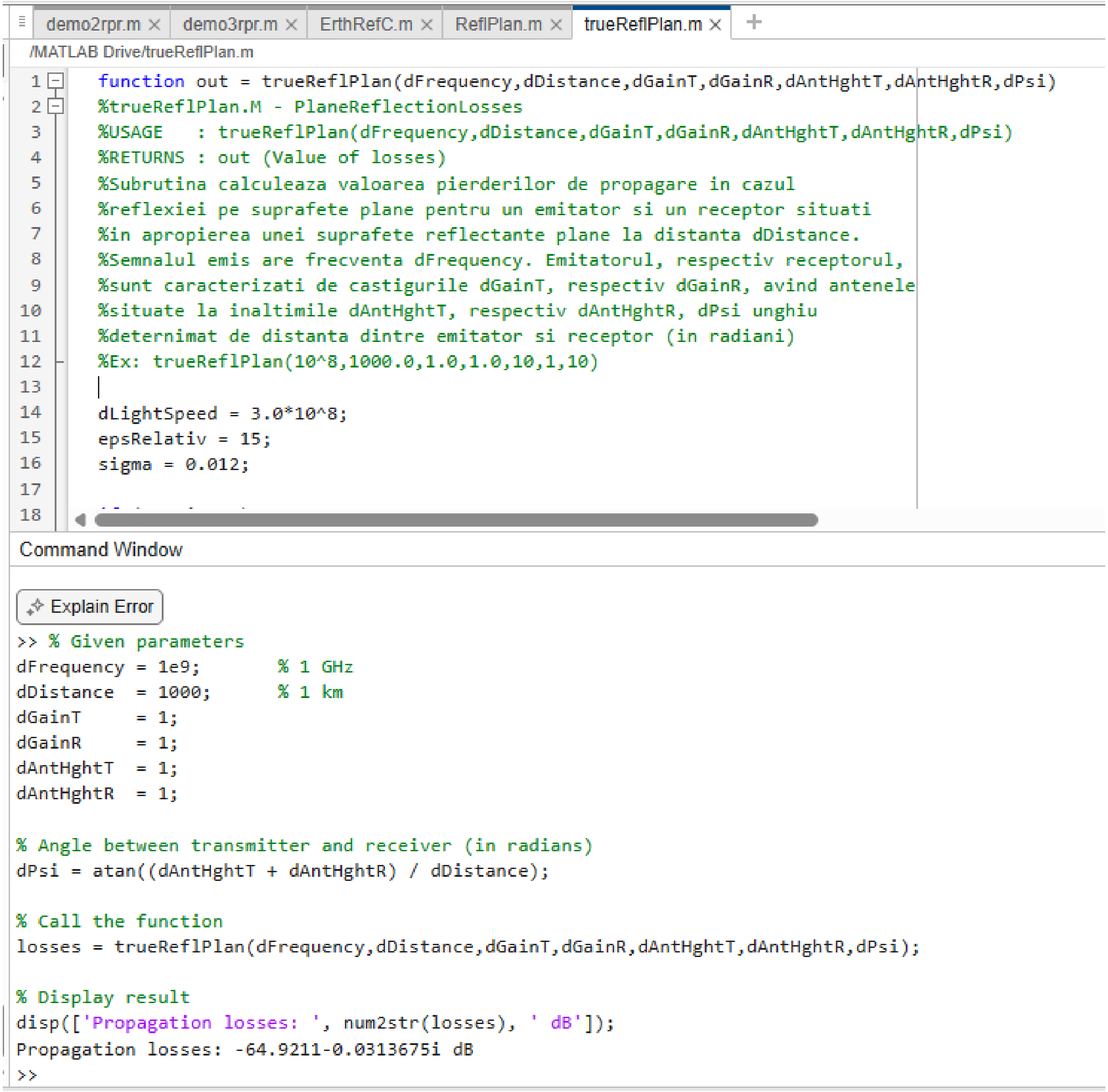
```matlab
>> ReflPlan
Incorrect number of arguments in calling ReflPlan
Require 6 double (float) positive values.
>>
>> % Given parameters
dFrequency = 900e6;    % Frequency: 900 MHz
dDistance = 1000;      % Distance: 1 km
dGainT = 1;            % Transmitter antenna gain
dGainR = 1;            % Receiver antenna gain
dAntHghtT = 10;        % Transmitter antenna height in meters
dAntHghtR = 1;         % Receiver antenna height in meters

% Call the ReflPlan function
losses = ReflPlan(dFrequency, dDistance, dGainT, dGainR, dAntHghtT, dAntHghtR);

% Display the result
disp(['Propagation losses: ', num2str(losses), ' dB']);
Propagation losses: -100.0515 dB
>>
```

- High loss (~-100 dB) is mainly due to long distance (1 km) vs. short wavelength (0.33 m) resulting in strong free-space attenuation.
- Low antenna heights (10 m & 1 m) make the reflection term very small resulting in weaker received signal.
- Antenna gains of 1 don't help reduce loss.
- **Ways to reduce loss:** increase antenna heights, use higher-gain antennas, or reduce distance / frequency.

6. Determine the propagation losses with the relation obtained with the approximation of a planar surface in the situation in which the distance between the transmitter and the receiver is d=1km, the frequency of the signal is f=1GHz, the heights of the antennae are ht=1m, hr=1m , and the gains of the antennae are gt=gr=1 . The Matlab function: use trueReflPlan.



The result is –64.92 – 0.03i dB. The real part (–64.9 dB) shows the actual propagation losses, while the imaginary part (–0.03i) appears because of the phase shift between the direct and reflected signals. Since the imaginary part is very small, we can ignore it and say the losses are about –65 dB. This shows that the phase difference slightly affects the signal but has negligible effect on power.