Fundamental problems concerning VHF and UHF propagation: **reflective coefficient of the earth** and **reflection**

An important stage in designing of mobile radio communications systems (VHF: 30MHz-300MHz; UHF: 300MHz-3GHz) consists in studying the mobile radio channel behaviour. The technical characteristics of the transmitter/receiver and antennae are adjusted/ chosen depending on the radio communication channel. The radio channel modifies randomly in time and frequency. Its behaviour may be predicted, in certain limits, using complex statistical propagation models.

The reflective coefficient of the earth

In case of propagation above the reflective surfaces, the received signal is a combination of the direct and reflected waves. In order to determine the resultant, the reflective coefficient of the earth which will affect the reflected wave must be taken into consideration.

Theoretical abstract

The amplitude and the phase of the reflected wave depend on the reflective coefficient of the earth in the reflection point. Their values for the horizontally polarized wave and for the vertically polarized wave are different. In practice the earth is neither a perfect conductor, nor a perfect dielectric, therefore the reflective coefficient depends on the dielectric constant ε and on the conductivity σ. For the horizontally polarized wave, incident on the surface of the Earth (considered perfectly smooth), the reflective coefficient is given by:

$$\rho_h = \frac{\sin\psi - \sqrt{(\varepsilon_r - j\chi) - (\cos\psi)^2}}{\sin\psi + \sqrt{(\varepsilon_r - j\chi) - (\cos\psi)^2}}$$

where ε_r is the relative dielectric constant of the Earth and

$$\chi = \frac{\sigma}{\omega\varepsilon_0}$$

where ε_0 is the dielectric constant of free space and ω is the angular frequency.

For the vertically polarized wave, the corresponding expression is:

$$\rho_v = \frac{(\varepsilon_r - j\chi)\sin\psi - \sqrt{(\varepsilon_r - j\chi) - (\cos\psi)^2}}{(\varepsilon_r - j\chi)\sin\psi + \sqrt{(\varepsilon_r - j\chi) - (\cos\psi)^2}}$$

It can be remarked that ρ_h and ρ_v are complex quantities and the reflected wave will differ both in amplitude and in phase as compared to the incident wave.

For the vertical polarization, the amplitude and the relative phase of the reflected wave decrease rapidly with the increase of ψ and, at an angle known as the ***pseudo-Brewster angle***, the amplitude reaches a minimum, and the phase shift has the value of – 90°. For values of ψ higher than the pseudo-Brewster angle, ρ_v increases again and the phase converges to zero. By definition, ***the Brewster angle*** (the polarization angle) represents that incidence angle for which the polarized wave is perfectly transmitted through the incident surface, without any reflections.

Remark:

- The polarization of an electromagnetic wave is given by the orientation of the electric field vector. The electric field vector is perpendicular both to the propagation direction of the wave, and on the magnetic field vector. Polarization can be thus defined as the geometric locus traced by the electric field vector on a (stationary) plane perpendicular on the propagation direction, while the wave propagates and goes beyond the respective plane. When this geometric locus is a line, the polarization is linear, and further on it can be classified in vertical and horizontal polarization.

Contents of the work

Run ***demo2rpr.m*** which creates the plots of the variations of the modulus and phases of the reflective coefficients in case of the vertically polarized planar wave and of the horizontal one, respectively, taking as a parameter the frequency *f*. Input data:

- the coefficients $\varepsilon_r = 15$, $\sigma = 12 \cdot 10^{-3}$;
- the values of the frequencies *f* = 1 MHz, 4 MHz, 12 MHz, 100 MHz and 1 GHz.
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Questions

1. For horizontal polarization describe the phase variation of the reflective coefficient as a function of the frequency. Give the approx. value of the relative phase of the incident and reflected waves.
2. Demonstrate that, at very small incidence angles (ψ →0), the value of the reflective coefficient does not depend on the frequency and the conductivity σ of the Earth. The demonstration must be done for both types of polarization using the definition relations of the reflective coefficients.
3. Estimate the approximate value of the reflective coefficients for both types of polarization of the wave, in the case of a small incidence angle (ψ →0). Read from the graphics the values for the modulus and for the phase, corresponding to each type of polarization.

4. Compute the value of the pseudo-Brewster angle in the domain of mobile communications (frequency higher than 800 MHz), according to the graphic which gives the modulus of the reflective coefficient ρ_v as a function of the incidence angle.

5. Determine from the graphic the value of the pseudo-Brewster angle at 100 MHz.

Propagation above planar reflective surfaces

In case of propagation above planar reflective surfaces, the received signal is a combination of the direct and reflected waves (Figure 1). The model of the propagation above the planar reflective surfaces is the fundament on which lay the majority of the empirical and semi-empirical models that will be studied later.

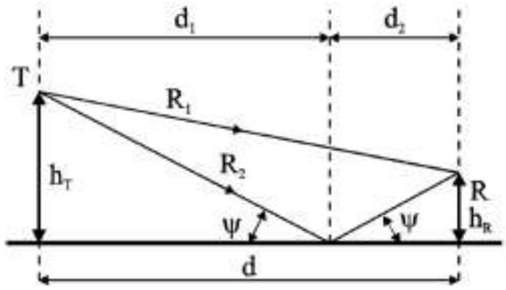

Figure 1. Propagation above a planar surface

Theoretical abstract

The propagation losses depending on the complex reflective coefficient are:

$$L = \frac{P_R}{P_T} = \frac{G_R G_T}{4d^2}\left(\frac{c}{2\pi f}\right)^2 [1 + |\rho| exp(-j\Delta\varphi - \theta)]^2$$

in which the equation of the electric field at the receiver was taken into account

$$E = E_d\left(1 + \rho e^{-j\Delta\varphi}\right)$$

For distances smaller than a few tens of km the curvature of the Earth can be neglected and it can be supposed that the surface is smooth. Moreover, the assumption that we are in the case of the incident wave at a very small angle ψ and therefore ρ = −1 can be done and thus the equation that highlights the intensity of the field at the receiver is:

$$E = E_d\left(1 - e^{-j\Delta\varphi}\right) = E_d(1 - \cos\Delta\varphi + j\sin\Delta\varphi)$$

Therefore:

$$|E| = |E_d|\sqrt{1 + cos^2\Delta\varphi - 2cos\Delta\varphi + sin^2\Delta\varphi} = 2|E_d|\sin\frac{\Delta\varphi}{2}$$

and, moreover, by using the geometry of the reflection from the figure above:

$$|E| = 2|E_d| \sin \frac{2\pi h_T h_R}{\lambda d}$$

Because the received power is proportional with the square of the field intensity according to:

$$P_R = \left(\frac{E\lambda}{2\pi}\right)^2 \frac{\pi G_R}{Z_0} = \left(\frac{E\lambda}{2\pi}\right)^2 \frac{G_R}{120}$$

and, because

$$\frac{E_d^2}{Z_0} = \frac{P_T G_T}{4\pi d^2}$$

therefore

$$E_d^2 = \frac{Z_0 P_T G_T}{4\pi d^2} = \frac{30 P_T G_T}{d^2}$$

it results

$$P_R = \frac{P_T G_T}{d^2}\left(\frac{\lambda}{2\pi}\right)^2 \frac{G_R}{4} \cdot 4sin^2\left(\frac{2\pi h_T h_R}{\lambda d}\right) = 4 P_T G_T G_R \left(\frac{\lambda}{4\pi d}\right)^2 sin^2\left(\frac{2\pi h_T h_R}{\lambda d}\right)$$
$$= 4 P_T G_T G_R \left(\frac{c}{4\pi f d}\right)^2 sin^2\left(\frac{2\pi h_T h_R f}{cd}\right) \qquad (4)$$

If $d \gg h_T$ and $d \gg h_R$, the equation becomes:

$$\frac{P_R}{P_T} = G_T G_R \left(\frac{h_T h_R}{d^2}\right)^2 \qquad (5)$$

Equation (5) is known as the **equation of propagation above smooth surfaces**.

Contents of the work

Run ***demo3rpr.m*** which creates the plots of the propagation losses in the case of the reflection on planar surfaces, as a function of the distance, taking as a parameter the frequency *f*. Input data:

- the distance between the transmitter and the receiver *d* =10 m...10 km;
- the values of the frequencies *f* = 1 MHz, 4 MHz, 12 MHz, 100 MHz and 1 GHz;
- the height of the antennae $h_T = 10m$, $h_R = 1m$;
- the gain of the antennae $G_T = G_R = 1$.

Remark:

- Equation (5) can be applied only when the conditions $d \gg h_T$ and $d \gg h_R$ are fulfilled. In the proximity of the transmitter the exact formula must be used which allows the evaluation of the maxima and minima of the power of the signal.

Questions

1. What are the essential differences between the propagation equation in free space and the propagation equation above planar reflective surfaces?

2. Determine the slope of the propagation losses above planar reflective surfaces, as a function of the distance.

3. What can be stated about the frequency dependence of the propagation losses at a high distance from the transmitter?

4. How can be explained the presence of the local minima in the proximity of the transmitter?

5. Determine the propagation losses in the situation in which the distance between the transmitter and the receiver is d=1km, the frequency of the signal is f=900MHz, the heights of the antennae$h_T = 10m$, $h_R = 1m$ and the gains of the antennae$G_T = G_R = 1$. Indication: use the Matlab function: *ReflPlan*.

6. Determine the propagation losses with the relation obtained with the approximation of a planar surface in the situation in which the distance between the transmitter and the receiver is d=1km, the frequency of the signal is f=1GHz, the heights of the antennae are $h_T = 10m$, $h_R = 1m$ and the gains of the antennae are $G_T = G_R = 1$. The Matlab function: use true*ReflPlan*.