JWST Project

Meeting Notes #5 (due 04/05/23)

Instructor: McCleary *Student: Eddie Berman*

Agenda

1. Attempts to plot residuals
2. Project Updates
3. Reading
4. Finalize funding now that I've been given award

Residuals Plot

Still this issue :

```
TypeError: '>' not supported between instances of 'list' and 'float'
```

Project Updates

Started working with julia,

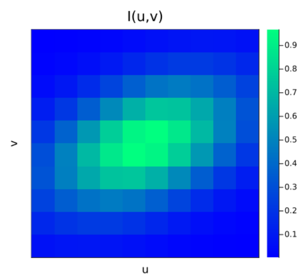

FIGURE 1: heatmap from shopt.jl

Now that I have input data, I need (1) a loss function. Will probably just use least squares. I also need (2) a metric, (3) an optimization scheme, and (4) a method to turn my constrained optimization problem into an unconstrained optimization problem.

Recall,

$$\begin{bmatrix} u' \\ v' \end{bmatrix} = \frac{s}{\sqrt{1-(g_1)^2-(g_2)^2}} \begin{bmatrix} 1+g_1 & g_2 \\ g_2 & 1-g_1 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}.$$

We've talked before about how Elliptical Gaussians are positive definite, it turns out the eigenvalues of this matrix are $\lambda = 1 \pm \sqrt{(g_1)^2 + (g_2)^2}$, which restricts $(g_1, g_2) \in B_2(r)$. *(For me, this makes the prefactor seem much more motivated).*

THIS SHOWS A GLARING PROBLEM IN PIFF. PIFF states that it uses conjugate (projective) gradient descent and makes no reference to constraining the parameters. Therefore it can make steps *outside* $B_2(r)$. We can fix this with a retraction.

Given a diffeomorphism $f : D \to \mathbb{R}^2$ via $f(z) = \frac{z}{1-\|z\|^2}$, we can construct an inverse via $\|z\| = 1+\frac{1}{2}\left(\frac{1}{\|x\|^2} - \sqrt{\frac{4}{\|x\|^2} - \frac{1}{\|x\|^4}}\right)$. You can verify that under appropriate leads this function is bounded in $B_2(r)$. In fact, this leads directly to the definition of $|\, e \,|$ show below.

According to our diagnostic plots our g_1, g_2 seemed to bounded between $(-1, 1)$, I wonder if they brute forced this at the end by normalizing. Are they not plotting outliers ? Something is weird to me or I am not understanding. Perhaps this is why we sometimes see outliers at the ends :

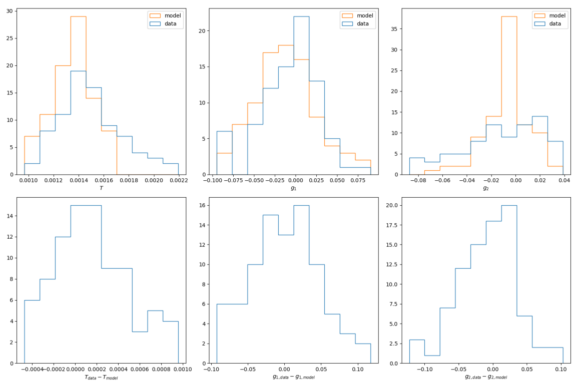

FIGURE 2: F115w 30mas Real Data

```
###This is what they do in PIFF
# Convert from sigma to T
        # Note: the hsm sigma is det(M)^1/4, not sqrt(T/2), so need to account for the effect
        #       of the ellipticity.
        #       If M = [ sigma^2 (1+e1)   sigma^2 e2   ]
        #              [   sigma^2 e2   sigma^2 (1-e1) ]
        #       Then:
        #         det(M) = sigma^4 (1-|e|^2) = sigma_hsm^4
        #         T = tr(M) = 2 * sigma^2
        #       So:
        #         T = 2 * sigma_hsm^2 / sqrt(1-|e|^2)
        # Using |e| = 2 |g| / (1+|g|^2), we obtain:
        #         T = 2 * sigma_hsm^2 * (1+|g|^2)/(1-|g|^2)
```

They just rescale it ? Does rejecting the HSM failures keep us bounded ?

More Thoughts

I've been thinking a lot about the idea of novelty lately. I want to be sure my contributions are unique to the literature. So, in doing this project, I've noted 3 things that I believe differentiate my approach from the literature :

1. The retraction that keeps $(g_1, g_2) \in B_2(r)$
2. The use of the Fisher-Rao metric. **NB : Need a noise model.**
3. The use of a Gauss-Newton Method *(or other quasi- method)* to do the optimization.

Reading

No sources in particular, but generally trying to learn about projective gradient descent and types of quasi-newton methods. Specifically interested in the relationship between accuracy and run time.

Co-op info

1. Start Date (with july 4th and the 1st being a Saturday)
2. End dates
3. Job description
4. Connect your NUworks account
5. Wage absent of award (details)