

# ShOpt.jl | A Julia Library for Empirical Point Spread Function Characterization of JWST NIRCам Data

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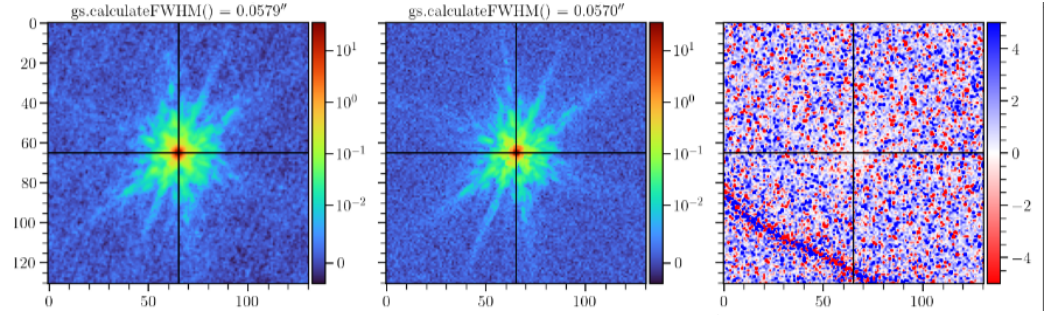
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## Summary

## Introduction

When astronomers take pictures of space, optical properties of the camera and atmospheric effects distort the incoming light. Stars are examples of what astronomers call point sources. Before any distortion is applied to an image, point sources of light can be thought of as delta functions. The aptly named point spread function (PSF) is a mathematical model that quantifies exactly how the light is being distorted. The point spread function takes as input a delta function and a position and outputs a distorted image. It can be thought of as the impulse response of an optical system to incoming light. The goal of point spread function characterization is to be able to point to any position on your camera and predict what the distortion looks like. Once we have a model that can do this well, we can deconvolve our images with the point spread function to obtain what the image would look like in the absence of distortion. The empirical way to do this is to take our images of distorted stars and separate them into training and validation sets. Our point spread function will be found by interpolating the training stars across the field of view of the camera and validated by comparing the reserved stars to the point spread function model's prediction.

Shear Optimization with ShOpt.jl introduces modern techniques for empirical point spread function characterization across the field of view tailored to the data captured by the James Webb Space Telescope (JWST). We can approximate our stars with analytic profiles. This gives us a rough idea of the size and shape of the point spread function and doubles as a mechanism to clean data. We adopt a multivariate gaussian profile because it is computationally cheap to fit to an image. That is, it is easy to differentiate and doesn't involve any numeric integration or other costly steps to calculate. Other common fits such as Kolmogorov fits involve numeric integration and thus take much longer to fit. The JWST point spread functions are very "spikey" as seen below, and as a result, analytic profiles are limited in their ability to model the point spread function. Thus, the usual advantages of a more expensive analytic profile are muted.



**Figure 1:** The plot on the left represents the average cutout of all of the stars in a supplied catalog. Likewise, plot in the middle represents the average point spread function model. The plot on the right represents the normalized error between the observed star cutouts and the point spread function model.

Our multivariate gaussian is parameterized by three variables,  $[s, g_1, g_2]$ , where  $s$  corresponds to size and  $g_1, g_2$  correspond to shear. A shear matrix has the form

$$\begin{pmatrix} 1 + g_1 & g_2 \\ g_2 & 1 - g_1 \end{pmatrix}$$

. Given a point  $[u, v]$ , we obtain coordinates  $[u', v']$  by applying a shear and then a scaling by  $\frac{s}{\sqrt{1 - g_1^2 - g_2^2}}$ . Then, we choose  $f(r) := Ae^{-r^2}$  to complete our fit, where  $A$  makes the fit sum to unity over the cutout of our star. After we fit this function to our stars with `Optim.jl` (Mogensen & Riseth, 2018) and `ForwardDiff.jl` (Revels et al., 2016), we interpolate the parameters across the field of view according to position. Essentially, each star is a datapoint, and the three variables are treated as polynomials in focal plane coordinates of degree  $n$ , where  $n$  is supplied by the user. The focal plain refers to the set of points where an image appears to be in perfect focus. This is instead of pixel coordinates, where one just uses  $(x, y)$  as measured on an image. For a more precise model, we also give each pixel in our star stamp a polynomial and interpolate it across the field of view. That is, each pixel in position  $(i, j)$  of a star cutout gets its own polynomial, interpolated over  $k$  different star cutouts at different locations in the focal plane. This is referred to in the literature as a pixel basis (Jarvis et al., 2020).

## Notation

1. For the set  $B_2(r)$ , we have:

$$B_2(r) \equiv \{[x, y] : x^2 + y^2 < 1\} \subset \mathbb{R}^2$$

2. For the set  $\mathbb{R}_+$ , we have:

$$\mathbb{R}_+ \equiv \{x : x > 0\} \subset \mathbb{R}$$

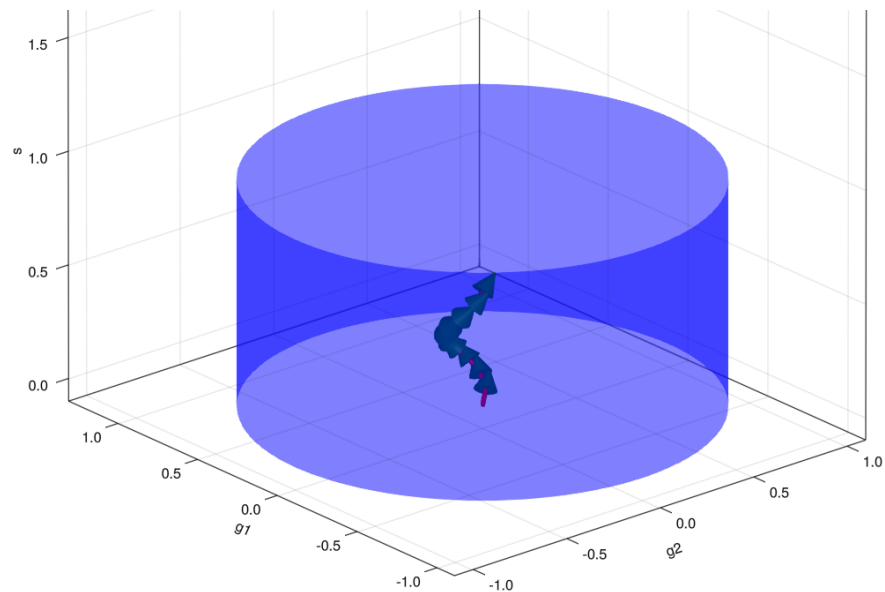
3. For the Cartesian product of sets  $A$  and  $B$ , we have:

$$A \times B \equiv \{(a, b) : a \in A, b \in B\}$$

## Methods

`ShOpt.jl` takes inspiration from a number of algorithms outside of astronomy. Mainly, `SE-Sync` (Rosen et al., 2019), an algorithm that provides a certifiably correct solution to a robotic mapping problem by considering the manifold properities of the data. `SE-Sync` proves that

54 with sufficiently clean data, their algorithm will descend to a global minimum constrained to  
 55 the manifold  $SE(d)^n/SE(d)$ . Likewise, we are able to put a constraint on the solutions we  
 56 obtain to  $[s, g_1, g_2]$  to a manifold. (Bernstein & Jarvis, 2002) outlined that the solution to  
 57  $[s, g_1, g_2]$  is constrained to the manifold  $B_2(r) \times \mathbb{R}_+$ . While it was known that this constrain  
 58 existed in the literature, the parameter estimation task is generally framed as an unconstrained  
 59 problem (Jarvis et al., 2020). For a more rigorous treatment of optimization on manifolds see  
 60 (Absil et al., 2008) and (Boumal, 2023). Julia has lots of support for working with manifolds  
 61 with Manopt, which we may leverage in future releases (Bergmann, 2022).



**Figure 2:** LFBGS algorithm used to find parameters subject to the cylindrical constraint.  $s$  is arbitrarily capped at 1 as a data cleaning method.

62 ShOpt.jl provides two modes for pixel grid fits, PCA mode and Autoencoder mode. PCA mode,  
 63 outlined below, reconstructs its images using the first  $n$  principal components. Autoencoder  
 64 mode uses a neural network to reconstruct the image from a lower dimensional latent space.  
 65 The network code written with Flux.jl is also outlined below (Innes, 2018). Both modes  
 66 provide the end user with tunable parameters that allow for both perfect reconstruction of  
 67 star cutouts and low dimensional representations. The advantage of these modes is that they  
 68 provide good reconstructions of the distorted images that can learn the key features of the  
 69 point spread function without overfitting the background noise. In this way it generates a  
 70 datapoint for our algorithm to train on and denoises the image in one step.

71 PCA mode

```
function pca_image(image, ncomponents)
    #Load img Matrix
    img_matrix = image

    # Perform PCA
    M = fit(PCA, img_matrix; maxoutdim=ncomponents)

    # Transform the image into the PCA space
    transformed = MultivariateStats.transform(M, img_matrix)

    # Reconstruct the image
    reconstructed = reconstruct(M, transformed)
```

```

# Reshape the image back to its original shape
reconstructed_image = reshape(reconstructed, size(img_matrix)...)
end

72 Autoencoder mode

# Encoder
encoder = Chain(
    Dense(r*c, 128, leakyrelu),
    Dense(128, 64, leakyrelu),
    Dense(64, 32, leakyrelu),
)

#Decoder
decoder = Chain(
    Dense(32, 64, leakyrelu),
    Dense(64, 128, leakyrelu),
    Dense(128, r*c, tanh),
)

#Full autoencoder
autoencoder = Chain(encoder, decoder)

#x_hat = autoencoder(x)
loss(x) = mse(autoencoder(x), x)

# Define the optimizer
optimizer = ADAM()

```

## 73 Statement of need

74 We can trace the first empirical PSF fitters back to DAOPHOT (Stetson, 1987). PSFex made  
75 major advancements in precise PSF modeling. With PSFex, you could interpolate several  
76 different bases, including a basis of pixels, instead of relying on simple parametric functions.  
77 PSFex was built as a general purpose tool and to this day is widely used. Newer empirical  
78 PSF fitters are geared toward large scale surveys and the difficulties that arise specific to  
79 those datasets. As an example, The Dark Energy Survey and DESCam (Flaugher et al., 2015;  
80 Jarvis et al., 2020) sparked the creation of PIFF. The recent data from the James Webb Space  
81 Telescope poses new challenges.

- 82 (1) The JWST PSFs are not well approximated by analytic profiles as seen in Figure 1. This  
83 calls for well thought out parametric free models that can capture the full dynamic range  
84 of the Point Spread Function without fixating on the noise in the background. Previously,  
85 Rowe statistics and other parametric equations were used to diagnose PSF accuracy  
86 (Rowe, 2010). ShOpt provides a suite of parametric free summary statistics out of the  
87 box.
- 88 (2) The NIRCcam detectors are 0.03"/pix or 0.06" /pix (Gardner et al., 2006). To capture  
89 an accurate description of the point spread function at this scale we need images that  
90 are 131 by 131 to 261 by 261 pixels across. These vignette sizes are much larger in  
91 comparison to the sizes needed for previous large surveys such as DES (Jarvis et al.,  
92 2020) and SuperBIT (McCleary et al., 2023) and forces us to evaluate how well existing  
93 PSF fitters scale to this size. The DES and SuperBIT surveys needed PSF sizes of 17 by  
94 17 and 48 by 48, an order of magnitude less than the JWST PSF sizes.

## 95 State of the Field

96 There are several existing empirical PSF fitters in addition to a forward model of the JWST  
97 PSFs developed by STScI (Jarvis et al., 2020 ; Bertin, 2011; Perrin et al., 2014 ; Perrin et  
98 al., 2012). We describe them here and draw attention to their strengths and weaknesses to  
99 motivate the development of ShOpt.jl. As described in the statement of need, PSFex was one  
100 of the first precise and general purpose tools used for empirical PSF fitting. However, PSFex  
101 produced a systematic size bias of the point spread function with how it calculated spatial  
102 variation across the field of view (Jarvis et al., 2020). It was discovered via the analytic profile  
103 fits that the size of the point spread function, governed by the variable  $[s]$ , was underestimated.

104 PIFF (Point Spread Functions in the Full Field of View) followed PSFex in the effort to correct  
105 this issue. The DES camera was 2.2 degrees across, which was large enough for the size bias  
106 to become noticable for their efforts. PIFF works in focal plane coordinates as opposed to sky  
107 coordinates which fixes the systematic size bias. Jarvis and DES also used the Python libraries  
108 of astropy (Astropy Collaboration et al., 2022) and Galsim (Rowe et al., 2015) to make the  
109 software more accessible than PSFex to programmers in the astrophysics community. PSFex  
110 was written in C and has been active for more than 20 years. Due to being so old and written  
111 in a low level language it is much less approachable for a community of open source developers.  
112 One of the motivations of ShOpt was to write astrophysics specific software in Julia, because  
113 Julia provides a nice balance of readability and speed with it's high level functional paradigm  
114 and just in time compiler.

115 While we do have forwards models of the JWST PSF, these models are for single exposure  
116 images. The JWST images are either single exposure or mosaics (Perrin et al., 2014, 2012).  
117 Mosaiced images are essentially single exposure detector images averaged together. To account  
118 for the rotation of the camera between the capture of images and the wide field of view,  
119 there are a number of steps that make applying these forward models to mosaics a non trivial  
120 procedure.

121 The COMOS-Web survey is the largest JWST extragalactic survey according to area and  
122 prime time allocation (Casey et al., 2023), and takes up  $0.54 \text{ deg}^2$  (Beichman et al., 2012;  
123 Rieke et al., 2023). This is a large enough portion of the sky that we should prepare to see a  
124 lot of variation across the field of view. This gives ShOpt the opportunity to validate PIFF's  
125 correction for handling PSF variations and test how impactful (or not impactful) PSFex's size  
126 bias is.

## 127 Future Work

128 We speculate that petal diagrams may be able to approximate the spikey natures of JWST  
129 PSFs. Consider  $r = A \cos(k\theta + \gamma)$ , shown below in figure 3 for different  $[A, k]$  values where  
130  $\gamma = 0$ . In practice,  $[A, k, \gamma]$  could be learnable parameters. We could then choose some  
131  $f(r) \propto \frac{1}{r}$  such that the gray fades from black to white. We would define  $f(r)$  piece wise such  
132 that it is 0 outside of the petal and decreases radially with  $r$  inside the petal.

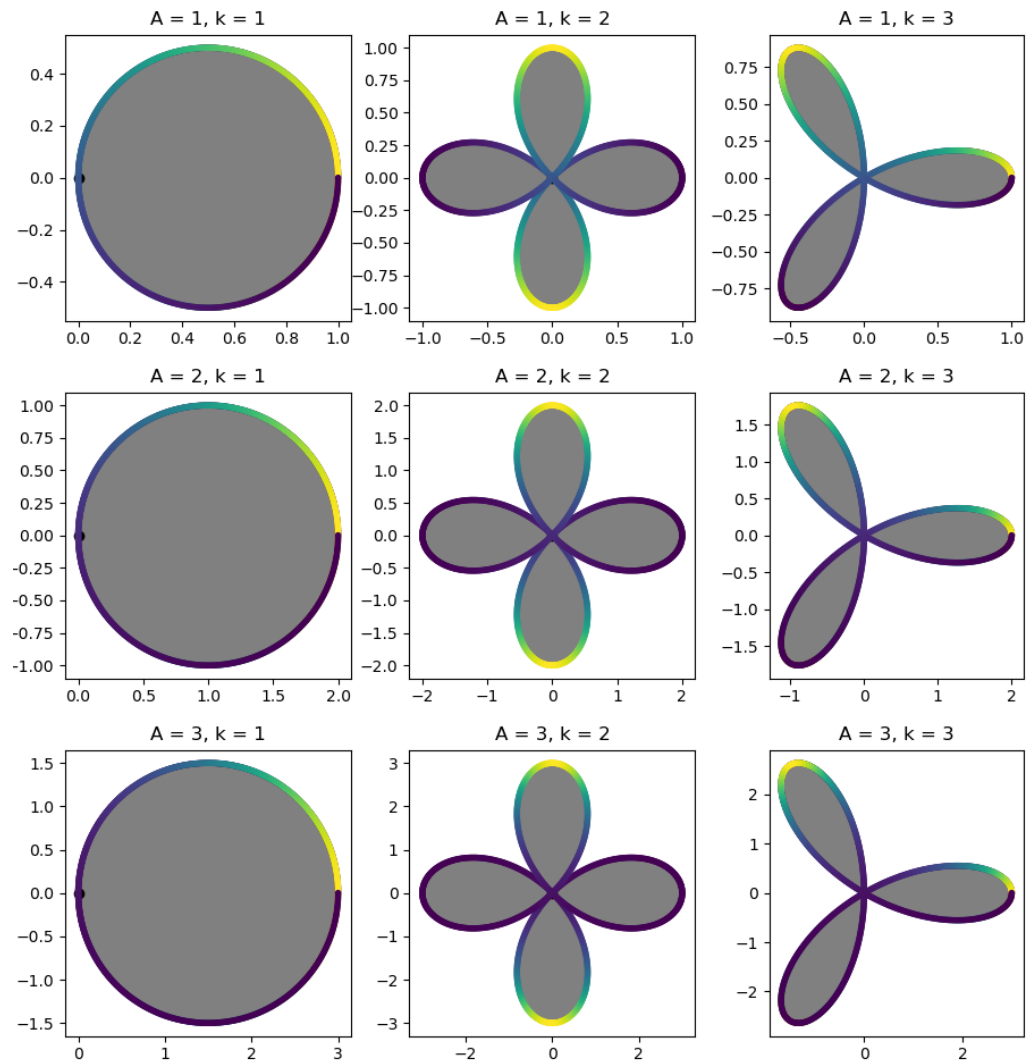


Figure 3: Petal Diagram

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## References

- Absil, P.-A., Mahony, R., & Sepulchre, R. (2008). *Optimization algorithms on matrix manifolds* (p. xvi+224). Princeton University Press. ISBN: 978-0-691-13298-3
- Astropy Collaboration, Price-Whelan, A. M., Lim, P. L., Earl, N., Starkman, N., Bradley, L., Shupe, D. L., Patil, A. A., Corrales, L., Brasseur, C. E., Nöthe, M., Donath, A., Tollerud, E., Morris, B. M., Ginsburg, A., Vaher, E., Weaver, B. A., Tocknell, J., Jamieson, W., ... Astropy Project Contributors. (2022). The Astropy Project: Sustaining and Growing a Community-oriented Open-source Project and the Latest Major Release (v5.0) of the Core Package. *935*(2), 167. <https://doi.org/10.3847/1538-4357/ac7c74>
- Beichman, C. A., Rieke, M., Eisenstein, D., Greene, T. P., Krist, J., McCarthy, D., Meyer, M., & Stansberry, J. (2012). Science opportunities with the near-IR camera (NIRCam) on the James Webb Space Telescope (JWST). In M. C. Clampin, G. G. Fazio, H. A. MacEwen, & J. M. O. Jr. (Eds.), *Space telescopes and instrumentation 2012: Optical, infrared, and millimeter wave* (Vol. 8442, p. 84422N). International Society for Optics; Photonics; SPIE. <https://doi.org/10.1117/12.925447>
- Bergmann, R. (2022). Manopt.jl: Optimization on manifolds in Julia. *Journal of Open Source Software*, *7*(70), 3866. <https://doi.org/10.21105/joss.03866>
- Bernstein, G. M., & Jarvis, M. (2002). Shapes and shears, stars and smears: Optimal measurements for weak lensing. *The Astronomical Journal*, *123*(2), 583. <https://doi.org/10.1086/338085>
- Bertin, E. (2011). Automated Morphometry with SExtractor and PSFEx. In I. N. Evans, A. Accomazzi, D. J. Mink, & A. H. Rots (Eds.), *Astronomical data analysis software and systems XX* (Vol. 442, p. 435).
- Boumal, N. (2023). *An introduction to optimization on smooth manifolds*. Cambridge University Press. <https://doi.org/10.1017/9781009166164>
- Casey, C. M., Kartaltepe, J. S., Drakos, N. E., Franco, M., Harish, S., Paquereau, L., Ilbert, O., Rose, C., Cox, I. G., Nightingale, J. W., Robertson, B. E., Silverman, J. D., Koekemoer, A. M., Massey, R., McCracken, H. J., Rhodes, J., Akins, H. B., Amvrosiadis, A., Arango-Toro, R. C., ... Zavala, J. A. (2023). *COSMOS-web: An overview of the JWST cosmic origins survey*. <https://arxiv.org/abs/2211.07865>
- Flaugher, B., Diehl, H. T., Honscheid, K., Abbott, T. M. C., & others. (2015). The dark energy camera. *AJ*, *150*, 150. <https://doi.org/10.1088/0004-6256/150/5/150>
- Gardner, J. P., Mather, J. C., Clampin, M., Doyon, R., Greenhouse, M. A., Hammel, H. B., Hutchings, J. B., Jakobsen, P., Lilly, S. J., Long, K. S., Lunine, J. I., Mccaughrean, M. J., Mountain, M., Nella, J., Rieke, G. H., Rieke, M. J., Rix, H.-W., Smith, E. P., Sonneborn, G., ... Wright, G. S. (2006). The james webb space telescope. *Space Science Reviews*, *123*(4), 485–606. <https://doi.org/10.1007/s11214-006-8315-7>
- Innes, M. (2018). Flux: Elegant machine learning with julia. *Journal of Open Source Software*. <https://doi.org/10.21105/joss.00602>
- Jarvis, M., Bernstein, G. M., Amon, A., Davis, C., Lé get, P. F., Bechtol, K., Harrison, I., Gatti, M., Roodman, A., Chang, C., Chen, R., Choi, A., Desai, S., Drlica-Wagner, A., Gruen, D., Gruendl, R. A., Hernandez, A., MacCrann, N., Meyers, J., ... and, R. D. W. (2020). Dark energy survey year 3 results: Point spread function modelling. *Monthly Notices of the Royal Astronomical Society*, *501*(1), 1282–1299. <https://doi.org/10.1093/mnras/staa3679>
- McCleary, J. E., Everett, S. W., Shaaban, M. M., Gill, A. S., Vassilakis, G. N., Huff, E. M., Massey, R. J., Benton, S. J., Brown, A. M., Clark, P., & others. (2023). Lensing in the

- 190 blue II: Estimating the sensitivity of stratospheric balloons to weak gravitational lensing.  
191 *arXiv Preprint arXiv:2307.03295*.
- 192 Mogensen, P. K., & Riseth, A. N. (2018). Optim: A mathematical optimization package for  
193 julia. *Journal of Open Source Software*, 3(24), 615. <https://doi.org/10.21105/joss.00615>
- 194 Perrin, M. D., Sivaramakrishnan, A., Lajoie, C.-P., Elliott, E., Pueyo, L., Ravindranath, S., &  
195 Albert, Loic. (2014). Updated point spread function simulations for JWST with WebbPSF.  
196 In Jr. Oschmann Jacobus M., M. Clampin, G. G. Fazio, & H. A. MacEwen (Eds.), *Space*  
197 *telescopes and instrumentation 2014: Optical, infrared, and millimeter wave* (Vol. 9143, p.  
198 91433X). <https://doi.org/10.1117/12.2056689>
- 199 Perrin, M. D., Soummer, R., Elliott, E. M., Lallo, M. D., & Sivaramakrishnan, A. (2012).  
200 Simulating point spread functions for the James Webb Space Telescope with WebbPSF. In  
201 M. C. Clampin, G. G. Fazio, H. A. MacEwen, & Jr. Oschmann Jacobus M. (Eds.), *Space*  
202 *telescopes and instrumentation 2012: Optical, infrared, and millimeter wave* (Vol. 8442, p.  
203 84423D). <https://doi.org/10.1117/12.925230>
- 204 Revels, J., Lubin, M., & Papamarkou, T. (2016). Forward-mode automatic differentiation in  
205 Julia. *arXiv:1607.07892 [Cs.MS]*. <https://arxiv.org/abs/1607.07892>
- 206 Rieke, M. J., Kelly, D. M., Misselt, K., Stansberry, J., Boyer, M., Beatty, T., Egami, E., Florian,  
207 M., Greene, T. P., Hainline, K., Leisenring, J., Roellig, T., Schlawin, E., Sun, F., Tinnin,  
208 L., Williams, C. C., Willmer, C. N. A., Wilson, D., Clark, C. R., ... Young, E. T. (2023).  
209 Performance of NIRCcam on JWST in flight. *Publications of the Astronomical Society of*  
210 *the Pacific*, 135(1044), 028001. <https://doi.org/10.1088/1538-3873/acac53>
- 211 Rosen, D. M., Carlone, L., Bandeira, A. S., & Leonard, J. J. (2019). SE-sync: A certifiably cor-  
212 rect algorithm for synchronization over the special euclidean group. *The International Jour-*  
213 *nal of Robotics Research*, 38(2-3), 95–125. <https://doi.org/10.1177/0278364918784361>
- 214 Rowe, B. (2010). Improving PSF modelling for weak gravitational lensing using new methods  
215 in model selection. *404*(1), 350–366. <https://doi.org/10.1111/j.1365-2966.2010.16277.x>
- 216 Rowe, B., Jarvis, M., Mandelbaum, R., Bernstein, G. M., Bosch, J., Simet, M., Meyers, J.  
217 E., Kacprzak, T., Nakajima, R., Zuntz, J., Miyatake, H., Dietrich, J. P., Armstrong, R.,  
218 Melchior, P., & Gill, M. S. S. (2015). *GalSim: The modular galaxy image simulation*  
219 *toolkit*. <https://arxiv.org/abs/1407.7676>
- 220 Stetson, P. B. (1987). DAOPHOT: A COMPUTER PROGRAM FOR CROWDED-FIELD  
221 STELLAR PHOTOMETRY. *Publications of the Astronomical Society of the Pacific*,  
222 99(613), 191. <https://doi.org/10.1086/131977>