Numerical Optimization Algorithms

Group 1

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The Bisection Method

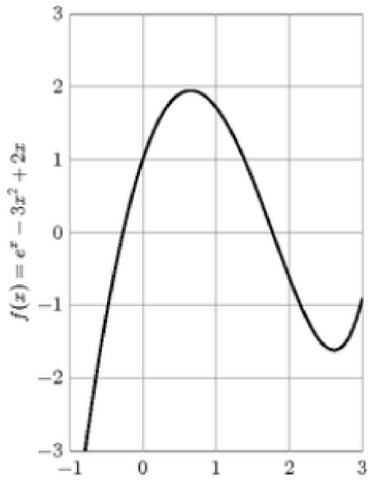


Figure 1: A sample objective function: $f(x) = e^x - 3x^2 + 2x$.

The algorithm:

1. Choose points a and b (where a<b) that bound the maximum. We will search in [a,b].
2. Take c to be the midpoint of a and b.
3. Take l to be the midpoint of the left interval, a and c and r to be the midpoint of the right interval, c and b.
4. Compare f(l), f(c) and f(r) and
   - i If f(c) is largest set a=l, and b=r.
   - ii If f(l) ir largest keep a, and set b=c.
   - iii If f(r) is largest keep b, and set a=c.
5. Repeat the algorithm until b-a is les than some prescribed tolerance.

The following code implements the algorithm in a rather basic way:

```
double f(double x)
{
  return( exp(x) - 3*x*x + 2*x );
}

double bisection(double a, double b)
{
    double c = (a+b)/2.0;
    while ( (b-a) > 1e-6)
    {
        c = (a+b)/2.0;
        double l = (a+c)/2.0, r=(c+b)/2.0;

        if ( (f(c) > f(l)) && (f(c) > f(r)) )
        {
            a=l;
            b=r;
        }
        else if ( f(l) > f(r) )
            b=c;
        else

            a=c;
    }
    return(c);
}
```

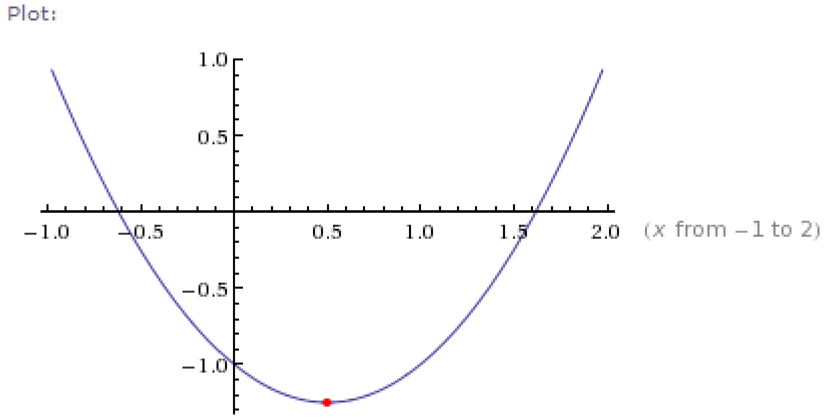
Newton Method

Algorithm

1. Get the function f(x) as input
2. Find the "jacobian" or first-order derivative f'(x)
3. Find the "hessian" or second order derivative f''(x)
4. Assume a starting point x_0
5. Solve iteratively for next value, x_{n+1} untilyou reach$x_n = x_{n-1}$

NOTE: To find more local optimums, try with different values of $x_0$and repeat steps 3-5

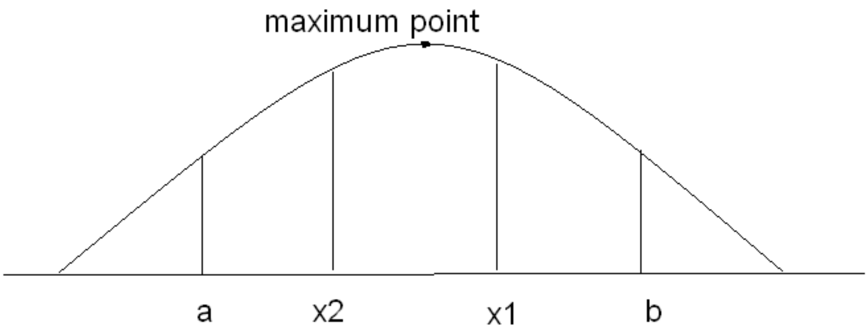
For the example below:



The relevant data value will solve as the steps below :

- f(x) =x^2 -x -1
- f'(x) = 2x -1
- f''(x) = 2
- X0 = 1
- X1 = ½
- X2 = ½ = X1

Golden Section Method



Algorithm

1. The relevent data values are read- the function, lower value a, upper value b, maximum tolerance and maximum iterations:
   READ*Function, a, b, maxIter, tolerance*
2. The golden ratio is calculated using the formula:
   $$R = \frac{\sqrt{5} - 1}{2}$$
3. F(a) and F(b) are calculated by substituting a and b into the function
   CALCULATE*f(a), f(b)*
4. The golden section method it then applied for a maximum amount of iterations
   LOOP*k*FROM*1*TO*maxIter*
5. Apply golden section method
   x1=a+R(b-a); f1=func(x1)
   x2=b-R(b-a); f2=func(x2)
6. If f1 is bigger than f2, we use the top three points
   IF *(f1>f2)*THEN
7. Assign new values
   a=x2; x2=x1; f2=f1
8. Calculate next value of x1
   x1=a+R(b-a); f1=func(x1)
9. If f2 is bigger than f1, we use the bottom three points and assign the new values
   ELSE
   b=x1; x1=x2; f1=f2
   x2=b-R(b-a); f2=func(x2)
   ENDIF
10. Output the new values
    WRITE*k,x1,x2*
11. Check the tolerence
    IF *(abs(x1-x2)<tolerance)*
    STOP

    END LOOP