Assignment 2 – ”Charged particles in magnetic fields”

Learning Outcomes Assessed

1. Write functions describing the motion of a particle.
2. Numerically solve differential equations.
3. Plot results.

Introduction

The general motion of a charged particle in electric ($\vec{E}$) and magnetic ($\vec{B}$) fields is governed by

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

where q, m and $\vec{v}$ are the charge, mass, and velocity of the particle, and $\vec{F}$ is the force acting on it. The cross product operator is available in numpy as numpy.cross.

In this assignment, we will analyse the motion of charged particle by numerically solving the corresponding differential equation.

Be careful and consistent when choosing your units. SI units as in scipy.constants might be a good choice here.

Make sanity checks of your results. Orbits bigger than the Earth is bad, as is particles going faster than the speed of light.

Submission

You need to submit your solution as a Jupyter Notebook (file ending: .ipynb). Other formats will not be accepted.

The notebook should contain your code, the figures requested in the various questions, and short explanations of the procedures you used to get them.

Grading

Grade	Criteria
3 (Pass)	Substantially correct attempt at mandatory parts. Code quality satisfactory. Report including the required figures.
4	As 3, but good attempt, including some extended parts. Code quality is good, code commented. Report including the required figures and comments about how you created them.
5	As 4, but excellent attempt, including several extended parts. Code quality is excellent, code well commented. Report including the required figures and comments about how you created them and what they show.

We will make a judgement on the code quality. Good quality code (for this assignment:

- Uses correct naming conventions for variables, functions, constants, etc.
- Sensible names for variables, functions, constants, etc.
- Terse, well-written comments, where appropriate; and to indicate code relating to different sections. Correct spelling.
- Readable; avoids substantial, unnecessary complexity and repetition.
- No redundant or unused code.
- Comments for both individual lines and sections of code.

Mandatory Questions

Question 1

Consider a proton moving in x-direction in a homogeneous magnetic field with a field axis in z-direction. The proton has an energy of 1 MeV and the field strength is 3 T. Using an ordinary differential equation solver, follow the motion of this particle for 1 microsecond. Plot and describe the result.

Question 2

Launch the same proton (energy 1 MeV) in the same magnetic field (3 T, parallel to the z-axis), but this time the initial proton velocity has both and x- and a z-component. Follow the motion of this particle for 1 microsecond. Plot and describe the result.

Question 3

Repeat question 2, but make a 3D plot of your results. Depending on how you do that, you will have to include the line

```
from mpl_toolkits.mplot3d import Axes3D
```

in your import-section.

Extended questions

Question 4

Change the magnetic field so that it is cylindrically symmetric around the z-axis. If *R* is the distance from the z-axis, in m, the magnitude of the magnetic field is 9T/*R*. The direction of the B-field at any point should be along a ring around z in the clockwise direction.

Launch a 1 MeV proton at R = 3 meters straight up (in the z-direction) and plot the first microsecond. Describe the result. It can be noted that the field at R = 3m is very similar to the case in question 1 with a gentle gradient added. The solution should be very similar to that of question 1.

Note: Cartesian is not the only possible coordinate system. The cross product works in cylindrical coordinates, too. This might simplify things a bit. However, the acceleration and velocity terms are more complicated. See https://www.continuummechanics.org/cylindricalcoords.html

Question 5

Repeat question 4, but make a 3D plot of your results.

Question 6

Evaluate the drift velocity in question 4.