First version of the SDE solver with the suggested parameters

These parameters are left constant thoughout the whole assignment.

```
S0 = 14;
K = 15;
r = 0.1;
sigma = 0.25;
T = 0.5;
```

Calculate the estimated price of a call option, using the Euler method

```
N_samples = 100000;
gamma = 0.8;
n_timeponts = 100;
V_vec = STD_solverv1(N_samples, n_timeponts, T, S0, sigma, gamma, K, r);
V_est = exp(-r * T) * mean(V_vec);

disp("Estimated price of call option")
```

```
Estimated price of call option
```

```
disp("V = " + num2str(V_est))
```

```
V = 0.46218
```

Error analysis of first version

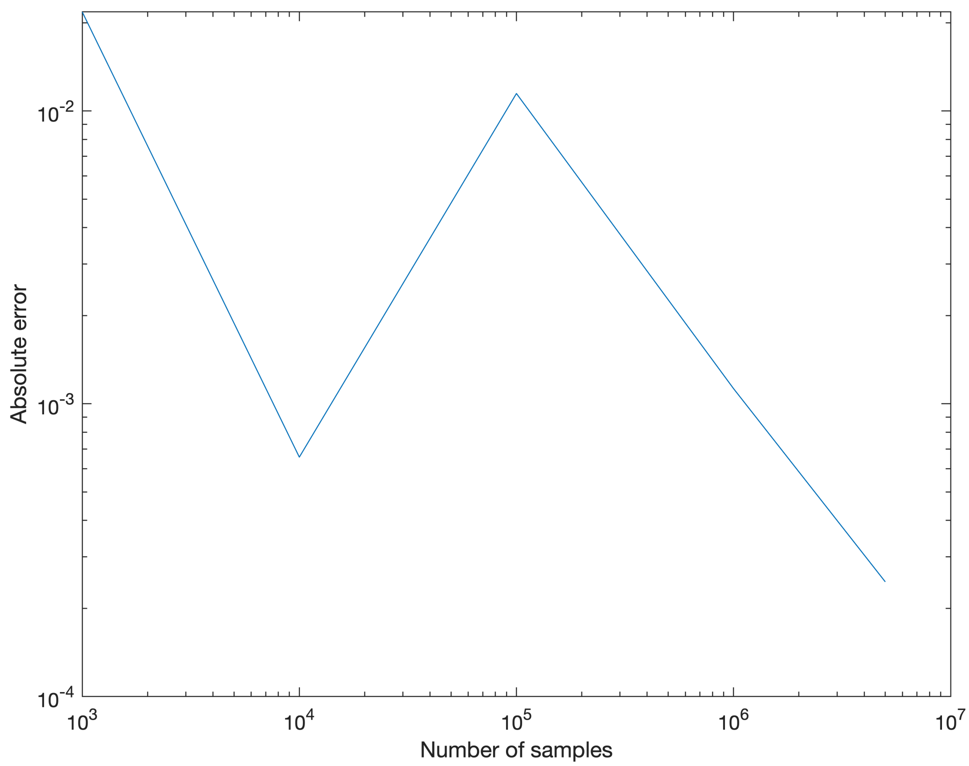
The sample error as a function of number of samples.

```

gamma = 1;
V_analytical = bsexact(sigma, r, K, T, S0) % Theoretical value for the given parameters
```

```
V_analytical = 0.8670
```

```

N_samples_vec = [1000, 10000, 100000, 1000000, 5000000];
V_est_vec1 = zeros(1,length(N_samples_vec));

n_timeponts = 256;

for k = 1:length(N_samples_vec)

    N_samples = N_samples_vec(k);
    V_vec1 = STD_solverv1(N_samples, n_timeponts,T, S0, sigma, gamma, K, r);
    V_est_vec1(k) = exp(-r * T) * mean(V_vec1);
end

err_sample1 = abs(V_analytical -  V_est_vec1);
figure()
loglog(N_samples_vec, err_sample1)
xlabel("Number of samples")
```

```
ylabel("Absolute error")
```



To obtain the sample error as a function of the number of sample paths we estimate the option price using different amount of sample paths. Then we can calculate the error using the theoretical value obtained by the Black Scholes equation. The sample error should be going down with an increase in the number of samples, which our results confirm. Since we keep the timestep low, we can assume the discretisation error to be negligable.

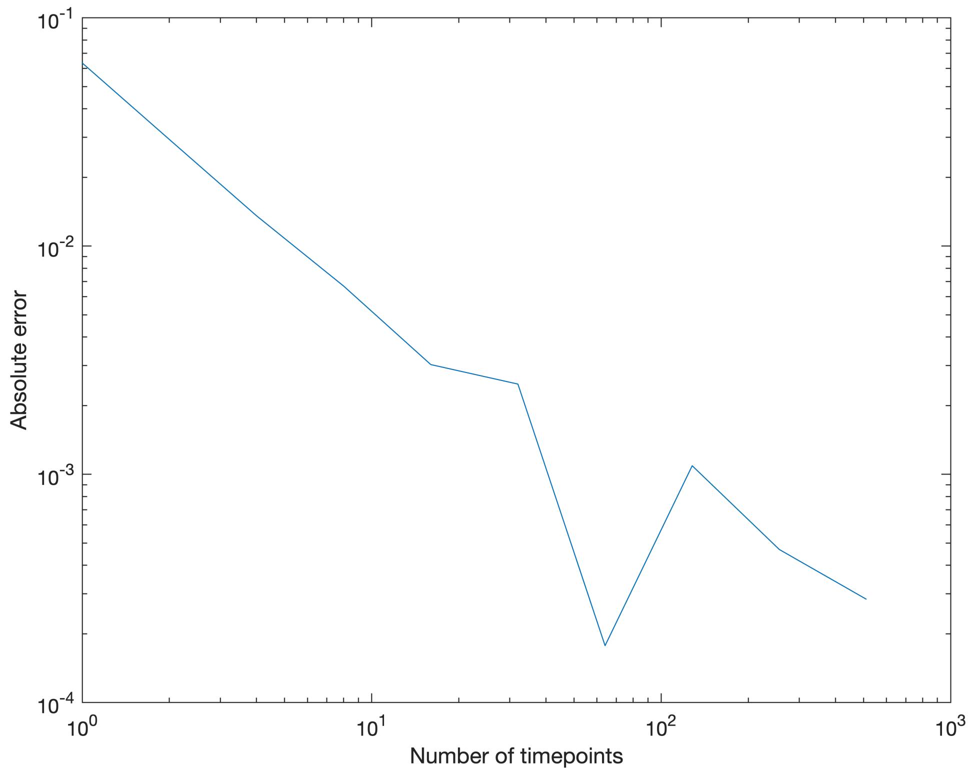
```
p = polyfit(log(N_samples_vec), log(err_sample1),1);
disp("Estimated rate of convergence: " + num2str(-p(1)))
```

```
Estimated rate of convergence: 0.38068
```

The discretization error

```
N_samples = 5000000;
n_points_vec = 2 .^ [0, 1, 2, 3, 4, 5, 6, 7, 8,9];
V_est_vec1 = zeros(1,length(n_points_vec));

for k = 1:length(n_points_vec)

    n_timeponts = n_points_vec(k);
    V_vec1 = STD_solverv1(N_samples, n_timeponts, T, S0, sigma, gamma, K, r);
    V_est_vec1(k) = exp(-r * T) * mean(V_vec1);
end
```

```
err_time1 = abs(V_analytical -  V_est_vec1);

figure()
loglog(n_points_vec, err_time1)
xlabel("Number of timepoints")
ylabel("Absolute error")
```



Similarly we can calculate the discretization error as a function of the time step. We calculate different estimates of the option price and compare with the result from the Black Scholes equation, and calculate the error. The discretization error should be going down when the number of time points increases, which our results confirms. When calculating the discretization error we keep the number of samples high so we can neglect the sample error.

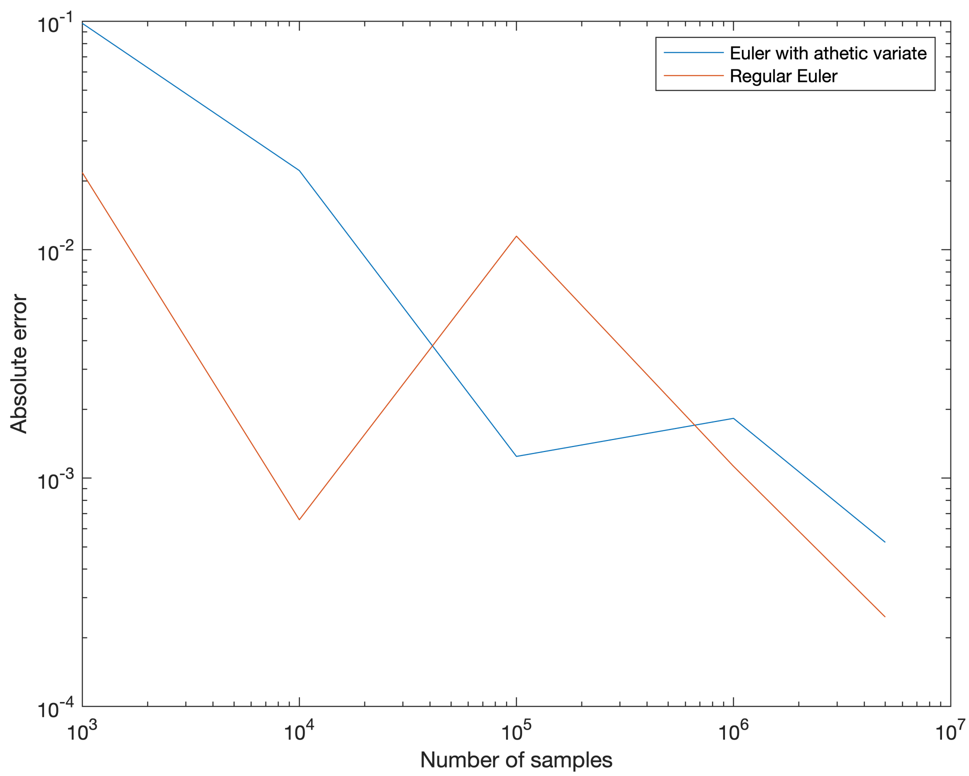
```
p = polyfit(log(n_points_vec), log(err_time1),1);
disp("Estimated rate of convergence:  " + num2str(-p(1)))
```

```
Estimated rate of convergence: 0.88609
```

SDE solver with anthetic variates

```
V_est_vec2 = zeros(1,length(N_samples_vec));
n_timeponts = 256;

for k = 1:length(N_samples_vec)
    N_samples = N_samples_vec(k)/2; % We divide by 2 since the anthetic variate methods
```

```
                                        % for every iteration
    V_vec2 = STD_solverv1_anth(N_samples, n_timeponts,T, S0, sigma, gamma, K, r);
    V_est_vec2(k) = exp(-r * T) * mean(V_vec2);
end

err_sample1_anth = abs(V_analytical -  V_est_vec2);
figure()
loglog(N_samples_vec, err_sample1_anth)
hold on;
loglog(N_samples_vec, err_sample1)
hold off;
xlabel("Number of samples")
ylabel("Absolute error")
legend("Euler with athetic variate","Regular Euler")
```



Antithetic variates did not seem to give a significant improvement in this case.

```
p = polyfit(log(N_samples_vec), log(err_sample1_anth),1);
disp("Estimated rate of convergence: " + num2str(-p(1)))
```

```
Estimated rate of convergence: 0.60217
```

SDE solver with anthetic variates and Runge-Kutta scheme

```
V_vec = STD_solverv2(N_samples, n_timeponts, T, S0, sigma, gamma, K, r);
```

```
V_est = exp(-r * T) * mean(V_vec);
%V_std = std(V_vec);

disp("Estimated price of call option")
```

```
Estimated price of call option
```

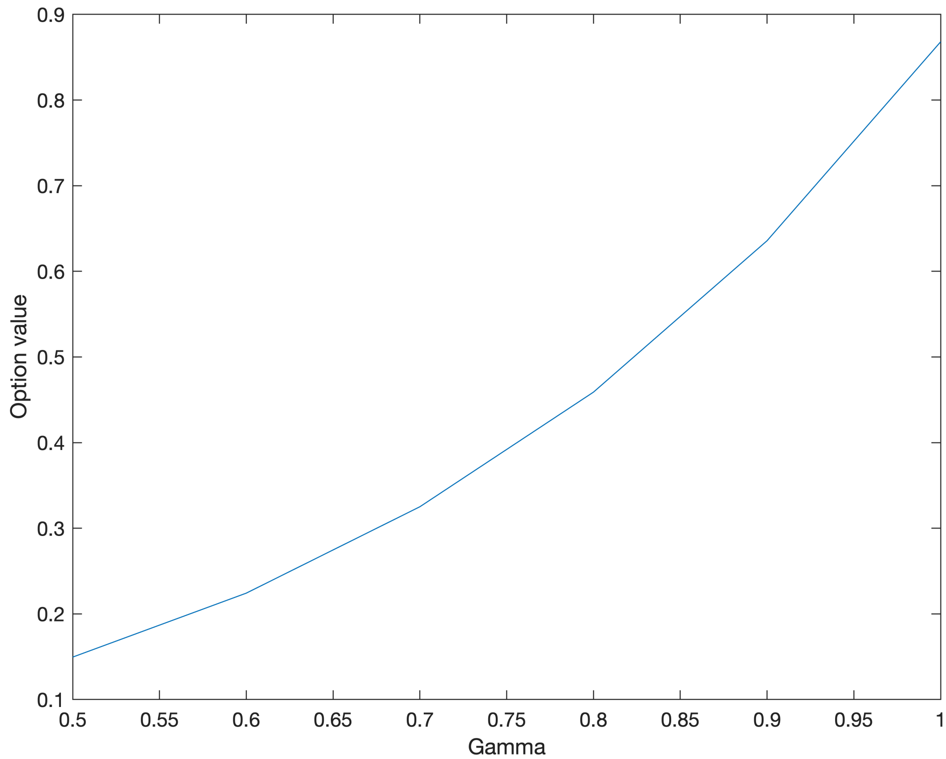
```
disp("V = " + num2str(V_est))
```

```
V = 0.86711
```

Gamma versus the price

In this section we examine the relationship between gamma and the value of the option.

```
gammavec = 0.5:0.1:1;
V_est_vec = zeros(1, length(gammavec));
N_samples = 1E6;
n_timeponts = 256;
ind = 1;
for gamma = gammavec

    V_vec = STD_solverv2(N_samples, n_timeponts, T, S0, sigma, gamma, K, r);
    V_est_vec(ind) = exp(- r * T) * mean(V_vec);
    ind = ind + 1;
end

figure()
plot(gammavec, V_est_vec)
xlabel("Gamma")
ylabel("Option value")
```

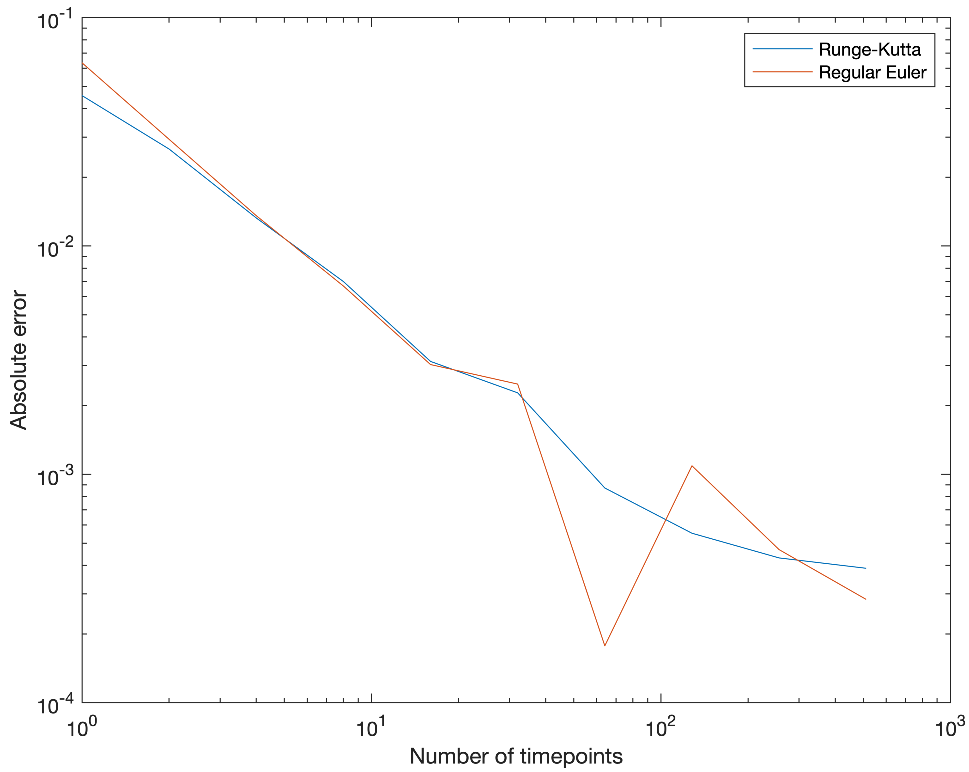


As we can see from our results, the price of a CEV (Constant elasticity of variance model) modeled option increases when gamma increases. An increase in gamma leads to an increased variance which leads to an increase in the price for an option like this.

The discretization error analysis of the second version, Euler versus Runge-Kutta

```
N_samples = 5000000 / 2;
V_est_vec = zeros(1,length(n_points_vec));

for k = 1:length(n_points_vec)

    n_timeponts = n_points_vec(k);
    V_vec = STD_solverv2(N_samples, n_timeponts, T, S0, sigma, gamma, K, r);
    V_est_vec(k)  = exp(-r * T)  * mean(V_vec);
end

err_time3 = abs(V_analytical -  V_est_vec);

figure()
loglog(n_points_vec, err_time3)

hold on;
loglog(n_points_vec, err_time1)
hold off;
xlabel("Number of timepoints")
```

```
ylabel("Absolute error")
legend("Runge-Kutta","Regular Euler")
```




```
p = polyfit(log(n_points_vec), log(err_time3),1);
disp("Estimated rate of convergence: " + num2str(-p(1)))
```

```
Estimated rate of convergence: 0.82372
```

As we can see, the convergence rate for the Euler method and the Runge-Kutta method is very similar. For the diffusion term Euler has a convergence order of the square root of the time step, and Runge Kutta has a convergence order of the time step. Both methods convergence order for the drift term is the time step. Depending on how big a impact the diffusion term has, the more different the convergence rate is when comparing the methods. Therefor it seems that in this case the diffusion term don't have a big impact, which leads to the convergence order of both methods being very similar.

Helper functions

```
function V_vec = STD_solverv1(N_samples, n_timepoints, T, S0, sigma, gamma, K, r)
    V_vec = zeros(1,N_samples);
    dt = T / n_timepoints;

    for i = 1:N_samples
        S_prev = S0;
        for j = 1:n_timepoints
            Z = randn;
            S = S_prev + r * S_prev * dt + sigma * S_prev ^ gamma * Z * sqrt(dt); % Eul
            S_prev = S;
```

```
        end
        V_vec(i) = max([S - K, 0]);

    end
end

function V_vec = STD_solverv1_anth(N_samples, n_timepoints, T, S0, sigma, gamma, K, r)
    V_vec = zeros(1,N_samples);
    dt = T / n_timepoints;
    for i = 1:N_samples
        S_prev1 = S0;
        S_prev2 = S0;
        for j = 1:n_timepoints
            Z1 = randn;
            Z2 = -Z1; % Here the anthetic variate is applied.
            S1 = S_prev1 + r * S_prev1 * dt + sigma * S_prev1 ^ gamma * Z1 * sqrt(dt);
            S2 = S_prev2 + r * S_prev2 * dt + sigma * S_prev2 ^ gamma * Z2 * sqrt(dt);
            S_prev1 = S1;
            S_prev2 = S2;
        end
        V_vec(i) = (max([S1 - K, 0]) + max([S2 - K, 0])) / 2;

    end
end

function V_vec = STD_solverv2(N_samples, n_timepoints, T, S0, sigma, gamma, K, r)
    V_vec = zeros(1,N_samples);
    dt = T / n_timepoints;
    for i = 1:N_samples
        S_prev1 = S0;
        S_prev2 = S0;
        for j = 1:n_timepoints
            Z1 = randn;
            Z2 = -Z1; % Here the anthetic variate method is applied. This makes that Z2
               % See section 4.3 in "Extra_reference_for_Anthethic.
            S1_h = S_prev1 + r * S_prev1 * dt + sigma * S_prev1 ^ gamma * sqrt(dt);
            S1 = S_prev1 + r * S_prev1 * dt + sigma * S_prev1 ^ gamma * Z1 * sqrt(dt) +
                (1 /(2 * sqrt(dt))) * (sigma * S1_h ^ gamma - ...
                sigma * S_prev1 ^ gamma) * ((sqrt(dt) * Z1)^ 2 - dt); % Runge kutta sch
            S2_h = S_prev2 + r * S_prev2 * dt + sigma * S_prev2 ^ gamma * sqrt(dt);
            S2 = S_prev2 + r * S_prev2 * dt + sigma * S_prev2 ^ gamma * Z2 * sqrt(dt) +
                (1 / (2 * sqrt(dt))) * (sigma * S2_h ^ gamma  - ...
                sigma * S_prev2 ^ gamma) * ((sqrt(dt) * Z2)^ 2 - dt);
            S_prev1 = S1;
            S_prev2 = S2;
        end
        V_vec(i) = (max([S1 - K, 0]) + max([S2 - K, 0])) / 2; % The result is the mean

    end
end
```