Time Series Analysis of Macroeconomic Variables

Introduction

This report presents a detailed econometric analysis of macroeconomic variables using time series data. The analysis employs various statistical techniques to examine the relationships between GDP (Gross Domestic Product) and several economic indicators, including CPI (Consumer Price Index), FFIR (Federal Funds Interest Rate), Treasury Bills (TBILL), Treasury Bonds (TBON), Exchange Rate (ER), and Unemployment Rate (UR). The dataset spans from 1959 to 2000, providing quarterly observations of these economic metrics, offering a comprehensive view of the U.S. economy over four decades.

Data Preparation and Initial Analysis

Dataset Overview

The analysis begins with importing a comprehensive macroeconomic dataset containing 168 observations across seven key variables:

- GDP (Gross Domestic Product): Measures the total monetary value of goods and services produced
- CPI (Consumer Price Index): Tracks changes in the price level of a market basket of consumer goods
- FFIR (Federal Funds Interest Rate): The interest rate at which banks lend money to each other overnight
- TBILL (Treasury Bills Rate): Short-term government debt instrument rates
- TBON (Treasury Bonds Rate): Long-term government debt instrument rates
- ER (Exchange Rate): Measures the relative value of the domestic currency
- UR (Unemployment Rate): Percentage of the labor force that is unemployed

The data is structured as a time series with quarterly observations, properly indexed by date for temporal analysis. The initial data preparation involved converting date strings to datetime objects and setting them as the index, ensuring proper time series functionality.

Stationarity Testing

The first crucial step involved testing for stationarity using the Augmented Dickey-Fuller (ADF) test. This test is fundamental in time series analysis as it determines whether the series has a unit root, which would indicate non-stationarity. The initial test on GDP revealed:

- ADF Statistic: -0.791
- p-value: 0.822
- Critical values:
  - 1%: -3.474
  - 5%: -2.881
  - 10%: -2.577

The high p-value (0.822 > 0.05) indicated that the GDP series was non-stationary, necessitating first-difference transformation of all variables to achieve stationarity. This transformation involves calculating the change between consecutive observations, effectively removing trend components from the data.

Model Development and Analysis

Initial OLS Model

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                            OLS Regression Results
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Dep. Variable:               GDP_diff   R-squared:                       0.024
Model:                            OLS   Adj. R-squared:                 -0.014
Method:                 Least Squares   F-statistic:                    0.6446
Date:                Sat, 23 Nov 2024   Prob (F-statistic):              0.694
Time:                        12:49:52   Log-Likelihood:                -172.75
No. Observations:                 161   AIC:                             359.5
Df Residuals:                     154   BIC:                             381.1
Df Model:                           6
Covariance Type:            nonrobust
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	coef	std err	t	P>|t|	[0.025	0.975]
const	0.6341	0.057	11.121	0.000	0.521	0.747
CPI_diff	0.0437	0.059	0.745	0.457	-0.072	0.160
FFIR_diff	-0.0046	0.123	-0.038	0.970	-0.248	0.239
TBILL_diff	-0.0118	0.204	-0.058	0.954	-0.414	0.391
TBON_diff	0.1049	0.165	0.636	0.526	-0.221	0.431
ER_diff	0.0409	0.058	0.706	0.481	-0.074	0.155
UR_diff	-0.0223	0.065	-0.344	0.732	-0.150	0.106

Omnibus:	43.467	Durbin-Watson:	2.060
Prob(Omnibus):	0.000	Jarque-Bera (JB):	237.272
Skew:	-0.808	Prob(JB):	3.00e-52
Kurtosis:	8.723	Cond. No.	7.86

The first analytical approach implemented was an Ordinary Least Squares (OLS) regression using the differenced variables. The model was constructed with GDP_diff as the dependent variable and the differenced values of all other variables as independent variables. Key findings from this model included:

- R-squared: 0.024 (indicating that only 2.4% of variance was explained)
- Adjusted R-squared: -0.014 (suggesting poor model fit)
- F-statistic: 0.6446
- Prob (F-statistic): 0.694 (indicating overall model insignificance)

The initial model showed poor explanatory power, suggesting the need for more sophisticated modeling approaches. The negative adjusted R-squared particularly highlighted the model's inadequacy in explaining GDP variations.

Diagnostic Tests

1. Heteroskedasticity Testing

The Breusch-Pagan test was conducted to check for heteroskedasticity, which examines whether the variance of errors is constant across observations:

- Test statistic: 1.668

- P-value: 0.948

The high p-value indicated homoskedasticity, suggesting that the variance of residuals was constant across observations. This was one of the few positive findings from the initial model, as it meant that the basic OLS assumption of constant error variance was not violated.

2. Serial Correlation Testing

The Breusch-Godfrey test revealed significant serial correlation:

- LM statistic: 18.334
- LM test p-value: 0.001
- F statistic: 4.819
- F test p-value: 0.001

These results indicated strong serial correlation in the residuals, violating a key assumption of OLS regression and necessitating more advanced modeling techniques. The presence of serial correlation suggested that past values of GDP had significant predictive power for future values.

Advanced Modeling Approaches

1. SARIMAX Model

To address the serial correlation issues, a SARIMAX (Seasonal Autoregressive Integrated Moving Average with Exogenous variables) model was implemented. This sophisticated time series model incorporates both seasonal and non-seasonal components:

- AR order: (3,0,0) - indicating three autoregressive terms
- Seasonal order: (1,0,0,4) - incorporating quarterly seasonality
- Period: 4 (reflecting quarterly data)

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SARIMAX Model Summary:
                                     SARIMAX Results
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Dep. Variable:	GDP_diff	No. Observations:	161
Model:	SARIMAX(3, 0, 0)x(1, 0, 0, 4)	Log Likelihood	-171.202
Date:	Sat, 23 Nov 2024	AIC	364.403
Time:	04:02:01	BIC	398.299
Sample:	04-01-1959	HQIC	378.166
	- 04-01-1999		
Covariance Type:	opg		

	coef	std err	z	P>|z|	[0.025	0.975]
CPI_diff	-0.0025	0.110	-0.023	0.982	-0.217	0.212
FFIR_diff	-0.0603	0.148	-0.408	0.683	-0.350	0.229
TBILL_diff	0.0284	0.284	0.100	0.920	-0.528	0.584
TBON_diff	0.1147	0.200	0.574	0.566	-0.277	0.507
ER_diff	0.0545	0.064	0.850	0.395	-0.071	0.180
UR_diff	-0.0229	0.094	-0.243	0.808	-0.208	0.162
ar.L1	0.0469	0.065	0.720	0.472	-0.081	0.175
ar.L2	0.2179	0.068	3.214	0.001	0.085	0.351
ar.L3	0.4010	0.078	5.114	0.000	0.247	0.555
ar.S.L4	0.3240	0.068	4.780	0.000	0.191	0.457
sigma2	0.4870	0.044	11.114	0.000	0.401	0.573

Ljung-Box (L1) (Q):	1.11	Jarque-Bera (JB):	160.95
Prob(Q):	0.29	Prob(JB):	0.00
Heteroskedasticity (H):	4.51	Skew:	-0.73
Prob(H) (two-sided):	0.00	Kurtosis:	7.68

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Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).
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- Log Likelihood: -171.202
- AIC: 364.403
- BIC: 398.299
- Significant AR components at lags 2 and 3
- Significant seasonal component at lag 4

The model revealed strong temporal dependencies in GDP growth, with particularly significant effects from values two and three quarters prior, as well as from the same quarter in the previous year.

2. Extended Lagged Variables Model

An alternative approach using lagged variables was also implemented to capture temporal dependencies:

- Included 3 regular lags of GDP differences
- Added seasonal lag (lag 4)
- Maintained all exogenous variables
- Standardized variables to ensure comparability

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Model with Extended Lags Summary:
                            OLS Regression Results
==============================================================================
Dep. Variable:               GDP_diff   R-squared:                       0.132
Model:                            OLS   Adj. R-squared:                  0.072
Method:                 Least Squares   F-statistic:                     2.213
Date:                Sat, 23 Nov 2024   Prob (F-statistic):             0.0200
Time:                        13:04:49   Log-Likelihood:                -160.47
No. Observations:                 157   AIC:                             342.9
Df Residuals:                     146   BIC:                             376.6
```

Df Model: 10

Covariance Type: nonrobust

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	coef	std err	t	P>|t|	[0.025	0.975]
const	0.6363	0.056	11.433	0.000	0.526	0.746
CPI_diff	0.0187	0.058	0.322	0.748	-0.096	0.133
FFIR_diff	-0.0518	0.121	-0.427	0.670	-0.292	0.188
TBILL_diff	-0.0023	0.199	-0.011	0.991	-0.395	0.391
TBON_diff	0.1359	0.161	0.846	0.399	-0.182	0.454
ER_diff	0.0381	0.057	0.673	0.502	-0.074	0.150
UR_diff	-0.0327	0.064	-0.513	0.608	-0.159	0.093
GDP_diff_lag1	-0.0631	0.058	-1.092	0.277	-0.177	0.051
GDP_diff_lag2	0.0717	0.057	1.268	0.207	-0.040	0.183
GDP_diff_lag3	0.1567	0.057	2.770	0.006	0.045	0.268
GDP_diff_lag4	0.1591	0.058	2.750	0.007	0.045	0.273

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Omnibus:	37.360	Durbin-Watson:	1.960
Prob(Omnibus):	0.000	Jarque-Bera (JB):	196.109
Skew:	-0.680	Prob(JB):	2.60e-43

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Kurtosis:                       8.304   Cond. No.                      7.87
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
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This model showed improved performance:

- R-squared: 0.132 (5.5 times better than the initial model)
- Adjusted R-squared: 0.072
- F-statistic: 2.213
- Prob (F-statistic): 0.020 (indicating statistical significance)

The improvement in model fit suggested that incorporating lagged values captured important temporal relationships in the data.

Model Comparison and Performance Metrics

Performance Metrics Comparison

1. SARIMAX Model:
    - MSE: 0.4920 (indicating average squared prediction error)
    - R^2: 0.0551 (showing limited but positive explanatory power)
    - Strong handling of serial correlation (p-value: 0.314)
2. Lagged Variables Model:
    - MSE: 0.4522 (slightly better than SARIMAX)
    - R^2: 0.1316 (substantially better explanatory power)
    - Adequate handling of serial correlation (p-value: 0.059)

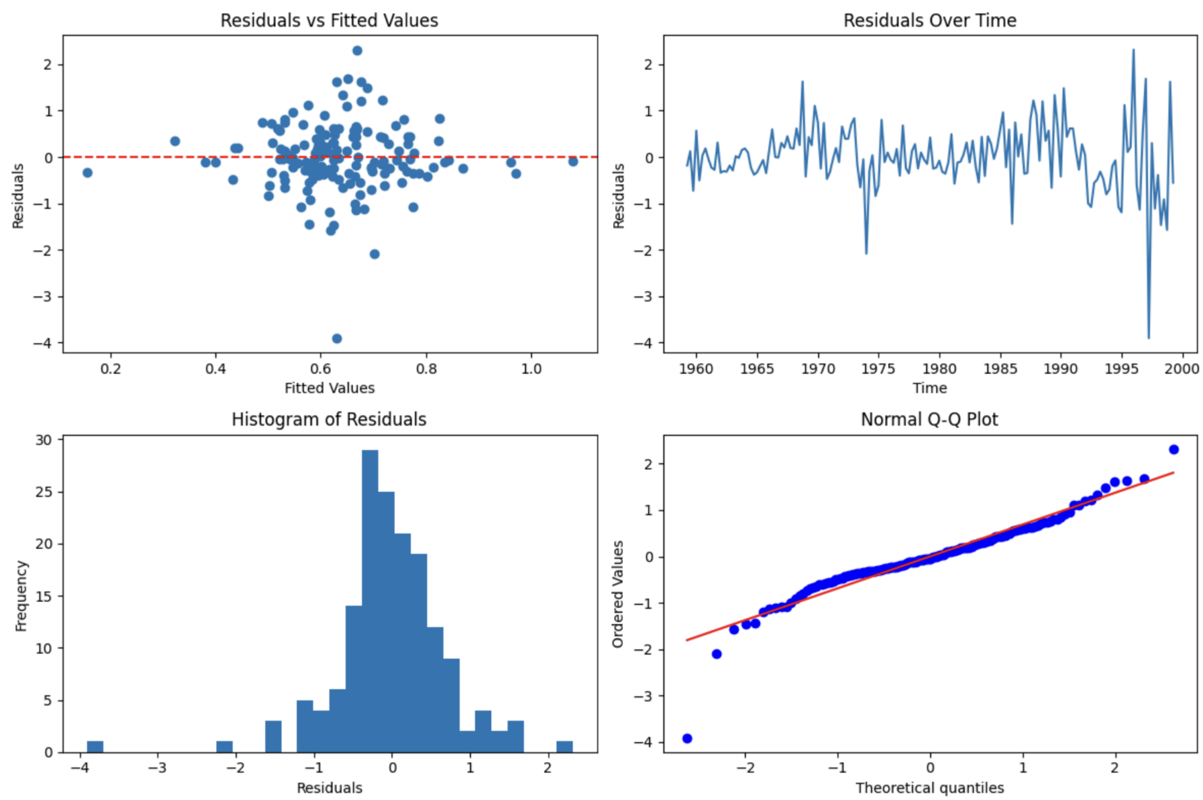
The lagged variables model showed better overall performance, particularly in terms of explanatory power, though both models had relatively modest predictive capabilities.

Residual Analysis

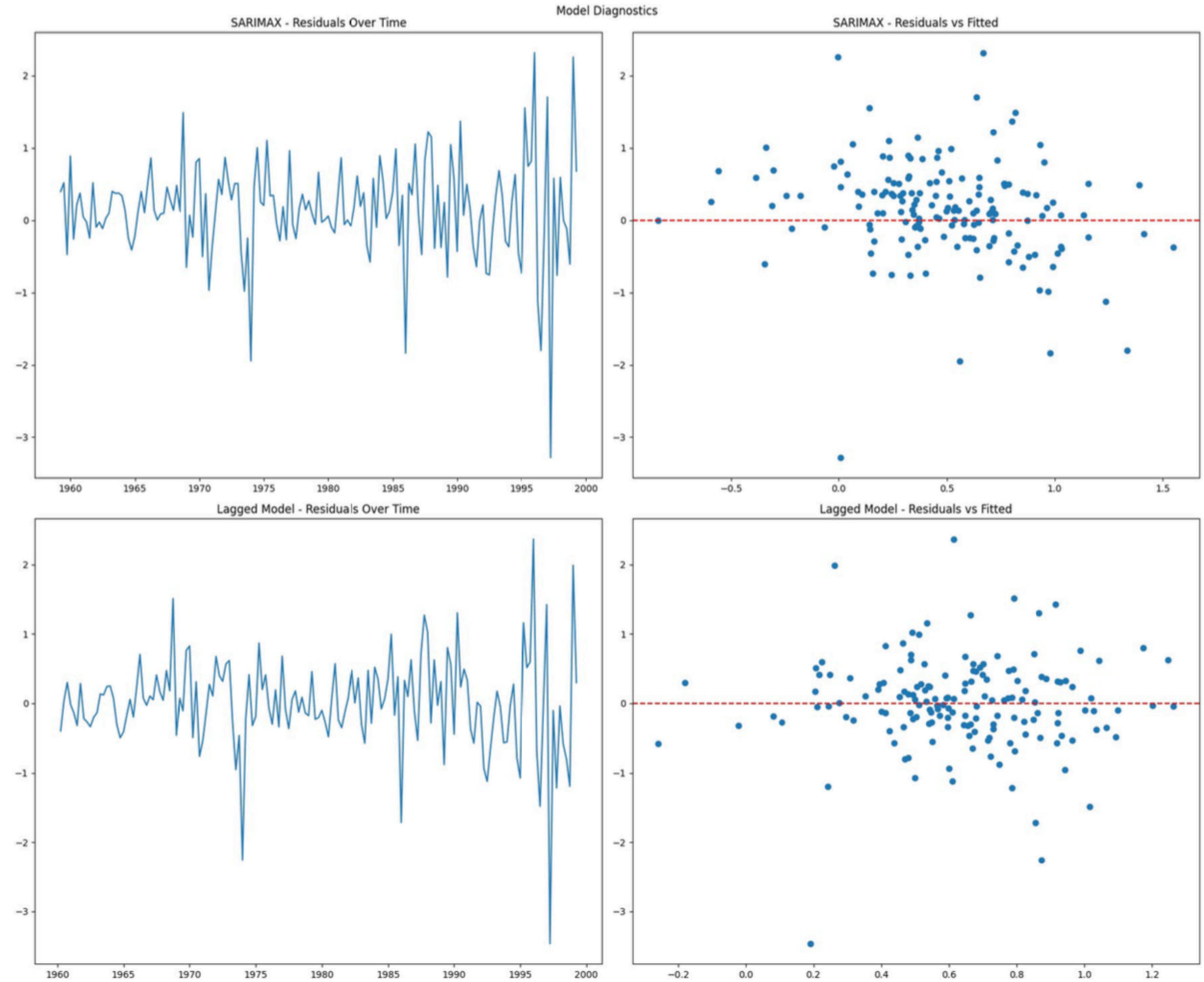
1. Residual Distribution
    - Slight negative skewness (-0.680)
    - High kurtosis (8.304)
    - Non-normal distribution (significant Jarque-Bera test)
    - Heavy tails indicating more extreme values than expected
2. Temporal Patterns
    - No obvious trends in residuals
    - Reduced but still present volatility clustering
    - Improved independence compared to initial model
3. Model Adequacy
    - Better fit in middle ranges
    - Some difficulty with extreme values
    - Generally symmetric distribution around zero

The analysis reveals the complex nature of GDP dynamics and the challenges in their prediction. While both SARIMAX and lagged variable models offer improvements over simple OLS regression, their modest predictive power suggests that GDP changes are influenced by factors beyond traditional macroeconomic variables.

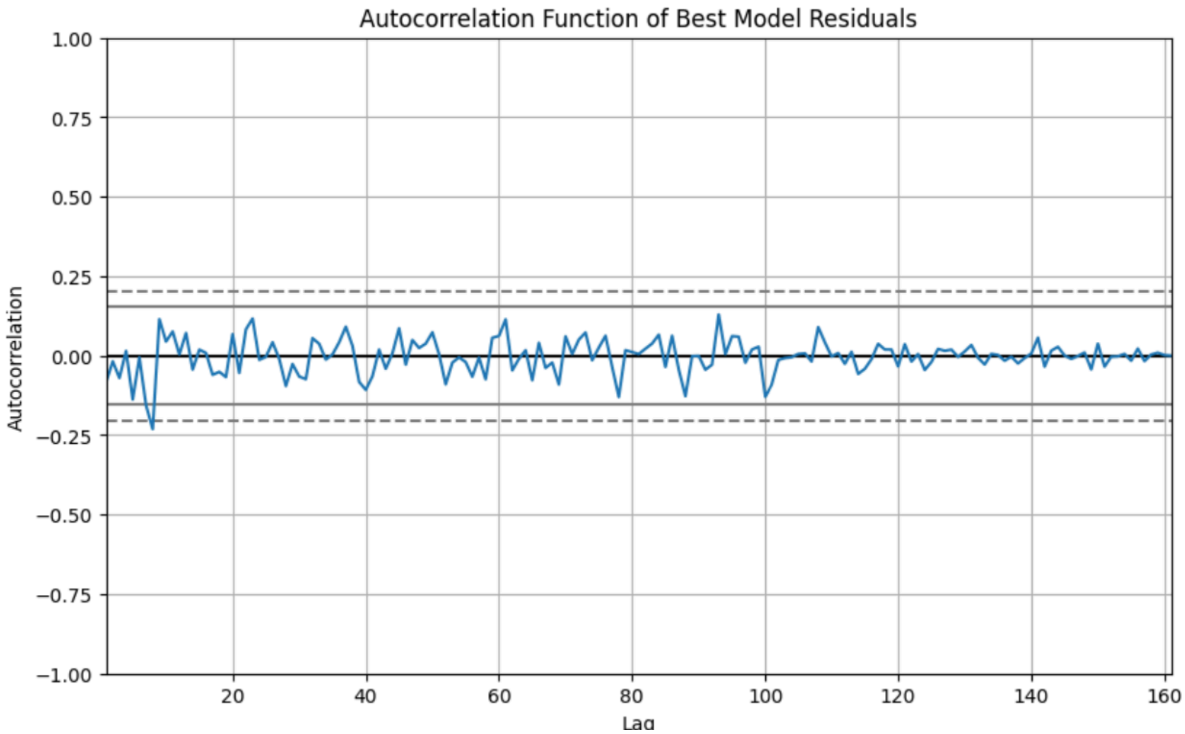
Visual Analysis Insights



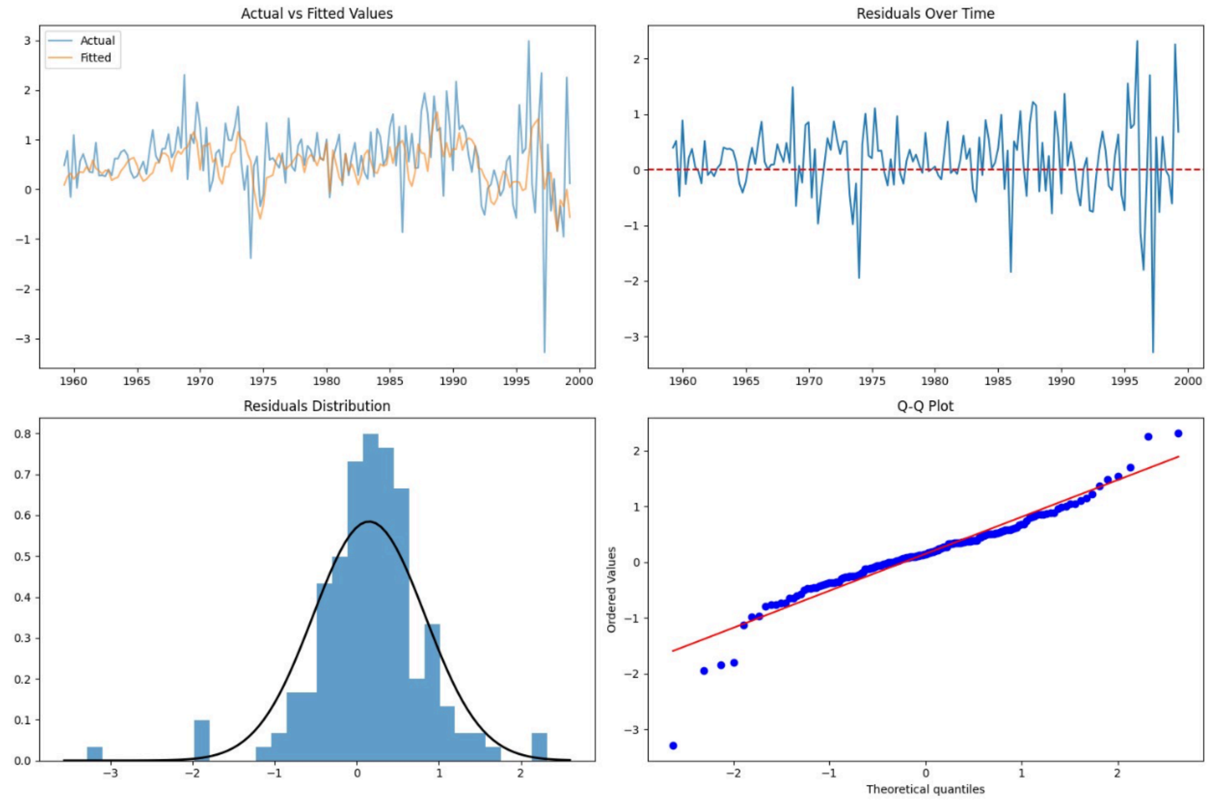
The visual diagnostics provide crucial insights into the model's performance and evolution throughout the analysis process. The Initial Model Diagnostics reveals concerning patterns with noticeable heteroscedasticity, particularly in the middle range of fitted values. The time series plot of residuals shows significant volatility clustering, especially prominent in the 1990s period, suggesting the model struggles with temporal stability. The histogram of residuals exhibits a departure from normality, showing a slightly skewed distribution with heavier tails than expected, while the Q-Q plot further confirms this non-normality through its substantial deviations from the theoretical normal line, particularly at the extremes.



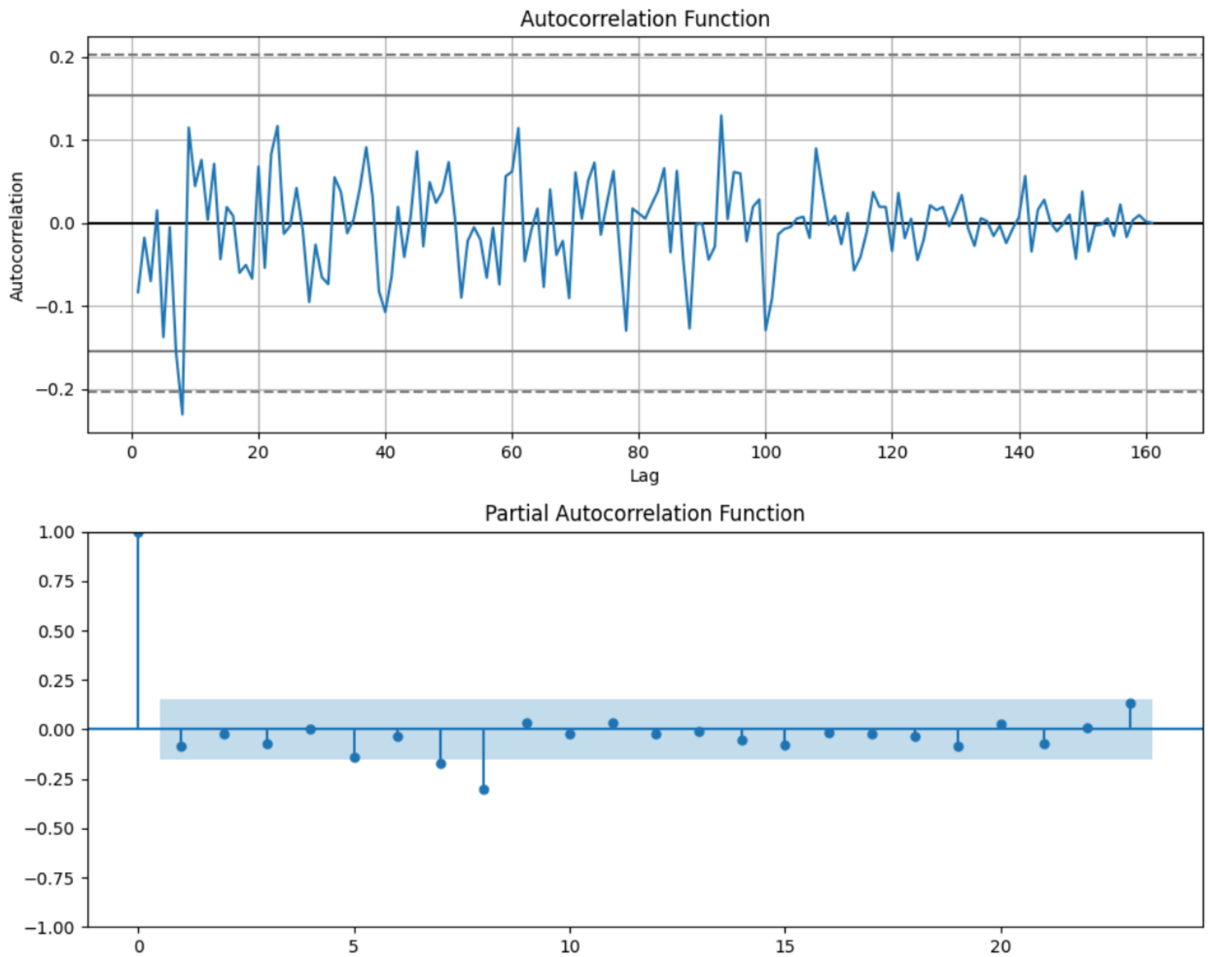
Comparison between SARIMAX and Lagged Models, both approaches demonstrate similar capabilities in handling the time series structure. The residual patterns over time are comparable between the two models, with both showing improved but not perfect handling of volatility clustering. The residuals versus fitted plots for both models indicate relatively consistent spread around zero, though neither completely eliminates the presence of outliers or extreme values, particularly during the volatile period of 1995-2000.



The Autocorrelation Function plot provides encouraging evidence that the best model has successfully addressed serial correlation issues. The correlations predominantly fall within the confidence bands, with only occasional random spikes that don't suggest any systematic pattern. This indicates that the model has effectively captured the time series dependencies in the data.



The comprehensive diagnostics reveal how the final model performs in tracking actual GDP movements. The actual versus fitted values plot shows that while the model captures the general trends in GDP changes, it tends to smooth out extreme fluctuations and struggles with exceptional economic events. The residuals distribution, while improved from the initial model, still shows some departure from perfect normality, though this is less severe than in the original specification.



ACF and PACF analysis provides strong validation of the model's time series specification. The ACF plot shows minimal remaining autocorrelation at various lags, while the PACF plot indicates significant correlation only at the immediate lags. This pattern suggests that the chosen lag structure effectively captures the temporal dependencies in the data. The absence of significant spikes beyond the confidence bounds in both ACF and PACF plots confirms that the model has successfully addressed the time series characteristics of the GDP data, though some minor imperfections remain.

Conclusions

1. The presence of strong temporal dependencies in GDP growth, evidenced by significant autoregressive components and visual confirmation in the ACF/PACF plots.
2. The importance of seasonal patterns in economic activity, as demonstrated by the seasonal components in the SARIMAX model and the corresponding visual patterns.
3. The complexity of modeling GDP changes, shown by the relatively low explanatory power of traditional macroeconomic variables and the visual evidence of remaining unexplained patterns.
4. The superiority of time series methods over simple regression approaches for this type of data, as demonstrated by the improved residual patterns in the more sophisticated models.

Github Repository : https://github.com/EdwardKrisna/econometric_7