$\det : \{n \times n \text{ matrix}\} \longrightarrow \mathbb{R}$

- function satisfying the following properties
  - row replacement doesn't change $\det(A)$
  - $\det(cA) = c[\det(A)]$
  - swapping two rows $\rightarrow$ $-\det(A)$
  - $\det(I) = 1$

Invertibility

- A square matrix is only invertible if and only if $\det(A) \neq 0$

Multipilicativity

$\det(AB) = \det(A)\det(B)$

Tranpose

$$\det(A) = \det(A^{-1})$$

Multilinearity

$$T(x_i) = \det(\ldots, x_i, \ldots)$$

- is a linear function

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minor i,j - A_{ij} the $n \times n$ matrix with row i and column j removed

cofactor i,j - $C_{ij} = (-1)^{i+j} \det(A_{ij})$

$$\det(A) = \sum_{i=1}^{n} a_{ij} C_{ij} \quad \text{or} \quad \sum_{j=1}^{n} a_{ij} C_{ij}$$

- satisfies principles defined earlier

- can be defined recursively from $1 \to N$

Kramer's Rule

$$A^{-1} = \frac{1}{\det(A)} \begin{pmatrix} C_{11} & C_{21} & \cdots \\ C_{12} & C_{22} & \\ \vdots & & \ddots \end{pmatrix}$$

$$\boxed{x_i = \frac{\det(A_i)}{\det(A)}}$$

$$P = \begin{pmatrix} | & | & \\ v_1 & v_2 & \cdots \\ | & | & \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ \vdots \end{pmatrix} \quad ; \quad \forall i, \; a_i \leq 1$$

$$\det(A) = \mathrm{vol}(P(A))$$

- The determinant is the volume of the parallelpiped of the matrix

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$T(S) = \{ T(x) \mid x \in S \}$ - image of S

$$\mathrm{vol}(T(S)) = |\det(A)| \cdot \mathrm{vol}(S)$$

- shows the shrinking or expansion of the volume of S
