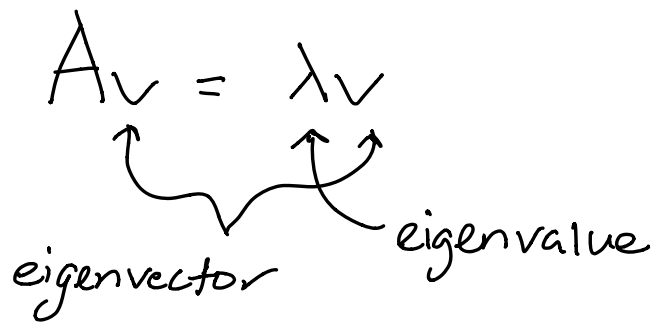


$$Av = \lambda v$$



- only for square matrices, v must remain the same dimension
- A scales the vector
- At most n eigenvalues
- eigenvectors are linearly independent

Eigenspaces

$$\begin{array}{l}
 Av = \lambda v \\
 Av - \lambda Iv = 0
 \end{array}
 \left. \vphantom{\begin{array}{l} Av = \lambda v \\ Av - \lambda Iv = 0 \end{array}} \right\} \text{Solving for } \lambda\text{'s}$$

$$(A - \lambda I)v = 0$$

$$\lambda\text{-eigenspace} = \text{Nul}(A - \lambda I)$$

- each eigenvalue has infinite eigenvectors

$$\text{characteristic polynomial } f(\lambda) = \det(A - \lambda I)$$

- λ is an eigenvalue if $f(\lambda) = 0$

$$f(\lambda) = (-1)^n \lambda^n + (-1)^{n-1} \text{Tr}(A) \lambda^{n-1} + \dots + \det(A).$$

- find roots using a computer to find λ 's

Similar Matrices

$$A, B \quad \exists C; \quad A = CBC^{-1}$$

- similar matrices have the same eigenvalues

Diagonalizable Matrix - A

$$A = CDC^{-1}, \quad D \text{ is a diagonal matrix}$$

- A has n linearly independent eigenvalues, (A, n x n)

Complex Eigenvalues

- Every matrix has a (possibly real) complex eigenvalue

Difference Equation

$$V_{t+1} = A V_t$$

- matrix transforms over time

Stochastic Matrix - A

$\forall i, j, a_{ij} > 0$ - all positive values

$\forall j, \sum_i a_{ij} = 1$ - columns sum to 1

• $\exists \lambda, \lambda = 1$

• $\forall \lambda, |\lambda| \leq 1$