Dot Product

$$x \cdot y = x_1 y_1 + x_2 y_2 + \ldots = \|x\| \, \|y\| \cos\theta$$

θ - angle between x and y

Length $\quad \|x\| = \sqrt{x \cdot x} = \sqrt{x_1^2 + x_2^2 + \ldots}$

Distance $\quad \text{dist}(x,y) = \|y - x\|$

Unit Vector $\quad \|x\| = 1$

$$x_{unit} = x / \|x\|$$

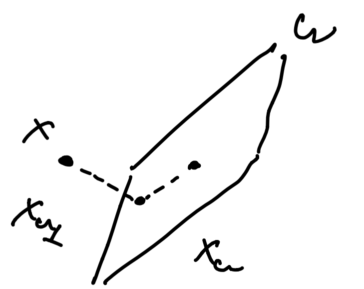
- scale by length of vector

Orthoginal - x, y are orthogonal if $x \cdot y = 0$

Orthoginal Compliment - to W subset of $\mathbb{R}^n$

$$W_\perp = \{ v \in \mathbb{R}^n \mid \forall w \in W, \; v \cdot w = 0 \}$$

Orthoginal Decomposition



$$x = x_w + x_{w_\perp}$$

* compressing $x \to x_w$ loses the least amount of info while getting x in W

Orthoginal Projection x, W

$x \to x_w$

* Can use a transformation matrix $T: x \to x_w$

Orthoginal Set

$S = \{u_1, u_2, \ldots, u_n\}$; $u_i \cdot u_j = 0 \quad \forall i,j \quad i \neq j$

Orthonormal Set

- orthoginal set where $\forall i \quad u_i \cdot u_i = 1$