Matrix as a function

$$Ax = b$$

function from $\mathbb{R}^n$ to $\mathbb{R}^m$, $A \mapsto m \times n$

- $\mathbb{R}^n$ is the <u>domain</u>
- $\mathbb{R}^m$ is the <u>codomain</u>

- b is the image of x under A

<u>Range</u> - set of possible b vectors
  - column space of A

<u>One-to-one</u> - function A if no more than 1 vector x such that $Ax = b$
  - linearly independent
  - cannot be wide

Onto - function A if $\forall b, \exists x : Ax = b$

- col(A) spans $\mathbb{R}^m$
- cannot be tall

Linear Transformation

- $T(u + v) = T(u) + T(v)$
- $T(cu) = cT(u)$
- $(T + U)x = T(x) + U(x)$
- $(cT)x = cT(x)$

Standard Coordinate Vectors - make the default coordinate system

$$e_n = \underbrace{\begin{pmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{pmatrix}}_{n \text{ digits}}$$

$$A e_i = v_i$$

$$A = \begin{pmatrix} T(e_1) & T(e_2) & \dots & T(e_n) \end{pmatrix}, \quad T(x) = Ax$$

- Matrices are a linear transformation

<u>Composition</u> $(T \circ U)(x) = T(U(x)) = TUx$

<u>Matrix Algebra</u>

$A + B = C$	$cA = B$
$a_{ij} + b_{ij} = c_{ij}$	$ca_{ij} = b_{ij}$

$$B = \begin{pmatrix} v_1 & v_2 & \dots & v_n \end{pmatrix}$$

$$AB = \begin{pmatrix} Av_1 & Av_2 & \dots & Av_n \end{pmatrix}$$

$A \to m \times n$

$B \to n \times p$

$AB \to mp$

$c_{ij} = a_i b_j$

<u>Invertible Matrix</u> - A is invertible if

$$\exists B : AB = I_n = BA$$

$$B = A^{-1}$$

- A must be square

$$Ax = b, \quad A^{-1}Ax = A^{-1}b = x$$

- use to solve for x