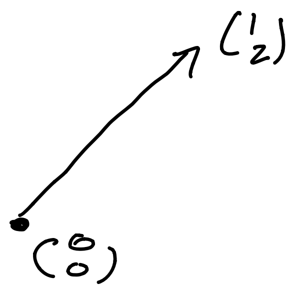
Vector – mathematical tool

- has a length and a direction
- represented in $\mathbb{R}^n$, $\begin{pmatrix} a \\ b \\ \vdots \\ n \end{pmatrix} = \vec{v}$
- thought of as a pointer $0 \rightarrow \mathbb{R}^n$



- can be scaled and added

$$c\vec{v}, \quad \vec{u} + \vec{v}$$

- linear combination of vectors can be interpreted as a matrix

$$c_1\vec{v}_1 + c_2\vec{v}_2 + \dots \implies \begin{pmatrix} a_1 & \dots \\ b_1 & \\ c_1 & \end{pmatrix}$$

Span - the set of all linear combinations of a set of vectors

$$\text{Span}\{v_1, v_2, \dots, v_n\} = \{c_1 v_1 + c_2 v_2 + \dots + c_n v_n \mid c \in \mathbb{R}^n\}$$

Matrix Equation

$$Ax = b$$

$$\left.\begin{array}{l} A = (v_1, v_2, \dots, v_n) \\ x = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} \end{array}\right\} Ax = [c_1 v_1, c_2 v_2, \dots]$$

b → value of linear combination, vector too!

→ by definition within the span!

→ all b within span of Ax exist

M is $m \times n$ matrix $m \rightarrow$ rows
$n \rightarrow$ columns

Homogeneous - $Ax = 0$

- $x = 0$ is the <u>trivial</u> solution
- $x \neq 0$ is a <u>nontrivial</u> solution

Inhomogeneous - $Ax = b$, $b \neq 0$

Linear Dependence

- describes a set of vectors, V
- V can have a vector $\vec{v}_1$ removed w/out effecting the size of the span
- one vector $\vec{v}_1$ is in the span of other vectors

- columns w/out pivots → dependence
  (parametric form)

Linear Independence - $Ax = 0$ has only the trivial solution

- $m \times n$ matrix w/ take up all $\mathbb{R}^n$ w/ its span

Subset (of $\mathbb{R}^n$)

- a set of points within $\mathbb{R}^n$

Subspace (of $\mathbb{R}^n$)

- a subset V of $\mathbb{R}^n$ where
  - V contains the zero vector
  - $u, v \in V \rightarrow u + v \in V$

- $v \in V \rightarrow cv \in V$

- a subspace contains the span of any vectors in it!
- every span is a subspace and every subspace is a span

Column Space - subspace generated by span of matrix columns

$Col(A)$

Null Space - defined by

$$Nul(A) = \{ x \text{ in } \mathbb{R}^n \mid Ax = 0 \}$$

- proven to be a subspace of $\mathbb{R}^n$

Basis - for a subspace V

- set of vectors $\{v_1, v_2, \ldots, v_n\}$ where $\text{span}(\{v_1, v_2, \ldots, v_n\}) = V$ and the set is linearly independent
- essentially forms a coordinate system

Dimension - for a subspace V

- # of vectors in the basis

Standard Basis

$$e_1 = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \quad e_2 = \begin{pmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{pmatrix}, \quad e_n = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix}$$

- unit vectors in each direction

* Pivot columns of a matrix form the basis

B-cordinate vector - describes a vector x w/ respect to a basis

$\beta = \{v_1, v_2, \ldots, v_3\}$, basis for subspace V

$$[x]_\beta = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ . \end{bmatrix}, \quad x = \beta[x_\beta]$$

Rank - dimension of col(A)

Nullity - dimension of Nul(A)

* Rank + Nullity = Dim of A

* Dim of Nullity is equal to dim of solutions for $Ax = b$ as long as it is consistent