$$Av = \lambda v$$

eigenvalue

eigenvector

- only for square matrices, v must remain the same dimension

- A scales the vector

- At most n eigenvalues

- eigenvectors are linearly independant

Eigenspaces

$$Av = \lambda v$$

$$Av - \lambda I v = 0$$

Solving for λ's

$(A - \lambda I)v = 0$)

$$\boxed{\lambda\text{-eigenspace} = \mathrm{Nul}(A - \lambda I)}$$

- each eigenvalue has infinite eigenvectors

$$\boxed{\text{characteristic polynomial} \quad f(\lambda) = \det(A - \lambda I)}$$

- λ is an eigenvalue if $f(\lambda) = 0$

$f(\lambda) = (-1)^n \lambda^n + (-1)^{n-1}\,\mathrm{Tr}(A)\lambda^{n-1} + \cdots + \det(A).$

- find roots using a computer to find λ's

Similar Matrices

$A, B \quad \exists C ; \; A = CBC^{-1}$

- similar matrices have the same eigenvalues

Diagonizable Matrix - A

$A = CDC^{-1}$, D is a diagonal matrix

- A has n linearly independent eigenvalues, $(A, n \times n)$

Complex Eigenvalues

- Every matrix has a (possibly real) complex eigenvalue

Difference Equation

$$v_{t+1} = A v_t$$

- matrix transforms over time

Stochastic Matrix - A

$\forall i,j,\ a_{ij} > 0$ - all positive values

$\forall j,\ \sum_i a_{ij} = 1$ - columns sum to 1

- $\exists \lambda,\ \lambda = 1$
- $\forall \lambda,\ |\lambda| \leq 1$