2018 Spring Semester. Optional Research Plan Using Functions of a Complex Variable.

1. The IEA (Incremental Effect Additivity) Equation when using AIVP-defined IDERs

An IDER (individual dose effect relation) giving effect E as a function of dose d is defined by solving the following AIVP (autonomous initial value problem) for an (in general non-linear) ODE:

(1.1) (A) $\mathrm{d}E/\mathrm{d}d = F(E)$; (B) $E = 0$ when $d = 0$,

with the slope function $F(E)$ sufficiently well behaved that there is one and only one solution for all sufficiently small non-negative doses.

The main references for this research plan are RR18.docx and RR18A.docx. These are basically a 2018 paper our last year's URAP class just published. Among other things they explain acronyms like IDERs and AIVPs in the title above; for example AIVP is defined in RR18A5.2 (section 5.2 of RR18A.docx) and also in a glossary RR18A1.2 (table 2 in section 1 of RR18A). I use RR18.docx because it has section numbering that the Journal removed (I think some copy editor started to read the paper, decided carelessly he or she didn't like numbering, didn't realize till half way through that the results would make the paper much less clear, lost information while removing the numbering, and then wasn't able to restore the numbering without a huge effort so just continued to the bitter end).

Suppose we have a mixture of K components, each with IDER $E_k(d_k)$ defined by choosing an F_k in Eq, (1.1) by using biophysical ideas and/or by looking at data in a graph and taking a guess; suppose the proportions r_1, r_2, ..., r_K are constants that obey Eq. (1) of RR18, namely

(1.2) $d_k = r_k d$; $r_k > 0$; $\sum_{k=1}^{K} r_k = 1$,

where $d = \sum_{k=1}^{K} d_k$ is total mixture dose.

Then the equation of incremental effect additivity becomes

(1.3) $dI/dd = \sum_{k=1}^{K} r_k F_k(I)$; $d = 0 \Leftrightarrow I = 0$,

which, as discussed in RRA5, is much more useful than the original IEA equation {Siranart, 2016 #265}, basically because it does not require the existence of IDER inverse functions.

The purpose of this optional research plan is to explore whether choosing the F_k functions in Eq. (1.3) with the help of the theory of functions of a complex variable can further increase the usefulness of Eq. (1.3). This requires investigating the results of RRA5 on the case where F_k is a polynomial more systematically and generalizing to F_k that are not polynomials. Hopefully by looking at many specific examples we may be able to spot trends and with luck find some actual theorems. That may not succeed and even if successful will almost surely take a long time; that's how research works.