MURANG'A UNIVERSITY OF TECHNOLOGY
DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE
Programme: Diploma in Supply Chain Management/
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Unit name: Numeracy Skills

Direct and inverse proportions determination

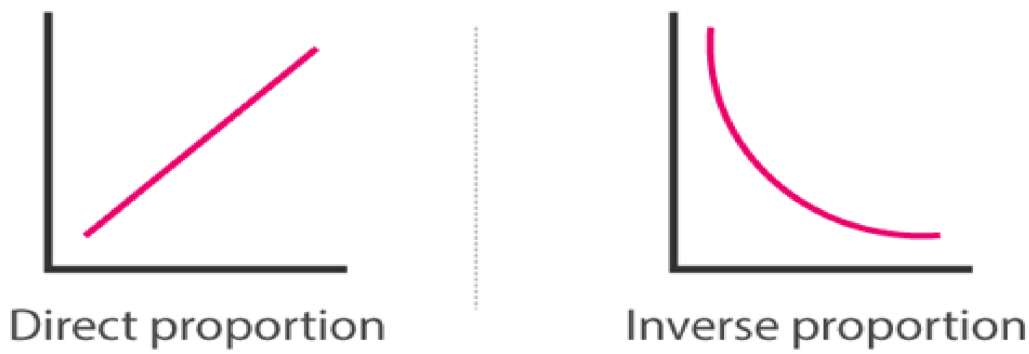
A **direct and inverse proportion** are used to show how the quantities and amount are related to each other. They are also mentioned as directly proportional or inversely proportional. The symbol used to denote the proportionality is'∝'. For example, if we say, *a* is proportional to *b*, then it is represented as '$a \propto b$' and if we say, *a* is inversely proportional to *b*, then it is denoted as 'a∝1/b'. These relations are governed by some proportionality rules. Now in both cases, the value of 'a' changes in terms of 'b' or when the value of 'b' changes, the value of 'a' also change. The change in both values is equated with a **constant of proportionality**. Basically, a proportion states that two ratios like a/b and c/d are equal to each other, in such a way, a/b = c/d.

Direct and Inverse Proportion Definitions

The proportion is said to be a **direct proportion** between two values when one is a multiple of the other.

For example, 1 cm is equal to 10 mm.

Here, in order to convert cm to mm, the multiplier should be 10.



Direct Proportion

Two quantities a and b are said to be in direct proportion if they increase or decrease together. In other words, the ratio of their corresponding values remains constant. This means that,

a/ b = k

where k is a positive number, then the quantities a and b are said to vary directly.

In such a case if the values b_1, b_2 of *b* corresponding to the values a_1, a_2 of *a* respectively then it becomes;

$a_1/b_1 = a_2/b_2$

The direct proportion is also known as **direct variation**.

Direct proportion or direct variation is the relation between two quantities where the ratio of the two is equal to a constant value. It is represented by the **proportional symbol, ∝**. In fact, the same symbol is used to represent inversely proportional, the matter of the fact that the other quantity is inverted here. Two quantities existing in direct proportion can be expressed as;

$x \propto y$

$x/y = k$

$x = ky$

k is a non-zero constant of proportionality.

Where x and y are the value of two quantities and k are a constant known as the **constant of proportionality**. If x_1, $y_1$is the initial values and x_2, y_2 are the final values of quantities existing in direct proportion. They can be expressed as,

$$\frac{x_1}{y_1} = \frac{x_2}{y_2}$$

For example, x and y are two quantities or variables which are linked with each other directly, then we can say x ∝ y. When we remove the proportionality symbol, the ratio of x and y becomes equal to a constant, such as $\frac{x}{y}$= C, where C is a constant. But in the case of inverse proportion, x and y are denoted as x ∝ 1/y or xy = C.

Consider the statement,

a is directly proportional to *b*

This can be written using the symbol as:

$a \propto b$

Consider the other statement, $a = 2b$

In this case, it shows that a is proportional to b, and the value of one variable can be found if the value of another variable is given.

For example:

Let $b=7$

Therefore, $a = 2 \times 7 = 14$

Similarly, if you take the value of "a" as 14, you will find the value of b

Such that

$14 = 2 \times b$

$14/ 2 = b$

Therefore, $b=7$

Direct proportion Examples in Real Life

In our day-to-day life, we observe that the variations in the values of various quantities depending upon the variation in values of some other quantities.

For example: if the number of individuals visiting a restaurant increases, earning of the restaurant also increases and vice versa. If a greater number of people are employed for the same job, the time taken to accomplish the job decreases.

Sometimes, we observe that the variation in the value of one quantity is similar to the variation in the value of another quantity that is when the value of one quantity increases then the value of other quantity also increases in the same proportion and vice versa. In such situations, two quantities are termed to exist in direct proportion.

Some more examples are:

Speed is directly proportional to distance.

The cost of the fruits or vegetable increases as the weight for the same increases.

Example 1:

A machine manufactures 20units per hour. How many units can it manufacture in 4 hours?

The units that machine manufactures is **directly proportional** to hours it has worked.

More works the machine does, more are the units manufactured; in direct proportion.

This could be written as:

Units ∝ Hours Worked

If the machine works for 2 hours, we get 40 Units

If the machine works for 4 hours, we get 80 Units

Example 2:

An electric pole, 7 meters high, casts a shadow of 5 meters. Find the height of a tree that casts a shadow of 10 meters under similar conditions.

Solution:

Let the height of the tree be x meters. We know that if the height of the pole increases the length of shadow will also increase in same proportion. Hence, we observe that the height of the tree and the length of its shadow exist in direct proportion. In other words height of pole is directly proportional to the length of its shadow. Thus,

$\frac{x_1}{y_1} = \frac{x_2}{y_2}$ $\frac{7}{5} = \frac{X}{10}$ x = 14 meters

Example 3:

A train travels 200 km in 5 hours. How much time it will take to cover 600 km?

Solution:

Let the time taken be T hours. We know that time taken is directly proportional to distance covered. Hence,

$\frac{x_1}{y_1} = \frac{x_2}{y_2}$ $\frac{200}{5} = \frac{600}{T}$ T = 15 hours

Example 4:

The scale of a map is given as **1:20000000**. Two cities are **4cm** apart on the map. Find the actual distance between them.

Solution: Map distance is 4 cm.

Let the actual distance be x cm, then 1:20000000 = 4:x.

$\frac{1}{20000000} = \frac{4}{X}$ ⇒ x = 80000000 cm = 800 km

Proportion Formula

An equation is said to be in proportion when the elements in it, say, a, b, c and d are in proportion. **a** and **d** are called extremes, whereas **b** and **c** are called mean terms. The product of means in the ratio is equal to the product of extremes. Two ratios are said to be equal if their cross products are equal.

The Proportion Formula is given as,

a:b::c:d ⇒ $\frac{a}{b} = \frac{c}{d}$

Example: What is the value of x in 6 : x :: 2 : 5?

Solution:

6 : x :: 2 : 5
6 : x = 2 : 5

$\frac{6}{x} = \frac{2}{5}$

$6 \times \frac{5}{2} = x$

$x = \frac{30}{2}$

x = 15

Example

You know that to make 20 pancakes you have to use 2 eggs. How many eggs are needed to make 100 pancakes?

	Eggs	pancakes
Small amount	2	20
Large amount	x	100

$$\frac{eggs}{pancakes} = \frac{eggs}{pancakes} \; or \; \frac{pancakes}{eggs} = \frac{pancakes}{eggs}$$

If we write the unknown number in the nominator then we can solve this as any other equation

$$\frac{x}{100} = \frac{2}{20}$$

Multiply both sides with 100

$$100 \cdot \frac{x}{100} = 100 \cdot \frac{2}{20}$$

$$x = \frac{200}{20}$$

$$x = 10$$

Example 1

John lives with three dogs. His daughter asks him to look after her dog for a week while she goes away. John normally buys two tins of dog food a day for the three dogs. How many tins should he buy for the four dogs for a week?

Solution

There are several ways to do this. Here is one.

3 dogs eat 2 tins a day

1 dog consumes $\frac{2}{3}$ tin a day

4 dogs consume $4 \times \frac{2}{3} = \frac{8}{3}$ tins a day

4 dogs consume $7 \times \frac{8}{3} = \frac{56}{3} = 18\frac{2}{3}$ tins a week

Therefore, John should buy 19 tins for the week.

NB: It helped to simplify the problems by considering 1 dog, rather than going straight to 4 dogs.

Example 2

Debbie is checking her phone bill. Her mobile phone calls have all been charged at the same rate, 30 pence per minute. (Call charges are rounded to the nearest penny and charged to the nearest second.)

(a) She wants to check the cost of a call to her friend. The call lasted 7 minutes and 34 seconds. How much should she have been charged?

(b) How long, at this rate, can she speak to her friend if the call charge is to cost no more than £2.50?

Solution

(a) 1minute cost 30 p. (i.e. 60 seconds for 30 p).

- 1 second for $\frac{30}{60}$ p = 0.5 p.
- 7 minutes 34 seconds is 454 seconds, costing 454 × 0.5 p = 227 p.
- The call should have been charged at £2.27.

(b) 0.5 p will allow her to talk for 1 second.

- 1 p will allow her to talk for 2 seconds.
- £2.50 (250 p) will allow her to talk for 250 × 2 seconds, = 500 seconds, i.e. 8 minutes 20 seconds.

Inverse Proportion

The value is said to inversely proportional when one value increases, and the other decreases. The proportionality symbol is used in a different way. Consider an example; we know that the more workers on a job would reduce the time to complete the task. It is represented as

Number of workers ∝ (1/ Time taken to complete the job)

Inverse Proportion Definition

Two quantities *a* and *b* are said to be in inverse proportion if an increase in the quantity a, there will be a decrease in the quantity b, and vice-versa. In other words, the product of their corresponding values should remain constant. Sometimes, it is also known as inverse variation

That is, if ab = k, then ***a*** and ***b*** are said to vary inversely. In this case, if b_1, b_2 are the values of b corresponding to the values a_1, a_2 of a respectively then a_1 b_1 = a_2 b_2 or $a_1/a_2 = b_2/b_1$

The statement 'a is inversely proportional to b is written as

a ∝ 1/b

Here, an equation is given that involves the inverse proportions that can be used to calculate the other values.

Let,

a = 25/b

Here a is inversely proportional to b

If one value is given, the other value can be easily found.

Say b=10

a= 25/10 = 2.5

Similarly, if a = 2.5, the value of b can be obtained.

2.5 = 25/b

b= 25/2.5 = 10

Direct and Inverse Proportion Examples

Example 1:

A train is moving at a uniform speed of 75 kilometres/hour.

(i)How many kilometres are covered by train in 20 minutes?

(ii) Find the time required to cover a distance of 250 kilometres.

Solution:

Let the distance travelled (in km) in 20 minutes be a and time taken (in minutes) to cover 250 km be b.

Distance travelled (in km)	75	a	250
Time taken (in minutes)	60	20	b

We know that 1 hour = 60 minutes

Since the speed of the train is uniform, therefore, the distance covered would be directly proportional to time.

(i) We have 75 /60 = a /20

or (75 /60) 20 = a

or a = 25

So, the train will cover a distance of 25 kilometres in 20 minutes.

(ii) Also, 75/60=250/ b or

b= (250 x 60)/ 75

b = 200 minutes or 3 hours 20 minutes.

Therefore, 3 hours 20 minutes is required to cover a distance of 250 kilometres.

Alternatively, when a is known, then one can determine b, using the relation

a/20 =250/ b

Example 2: The value f is directly proportional to g. When f = 20, g = 10. Find an equation relating f and g.

Solution:

Given, $f \propto g$

or we can write,

f = kg, where k is the constant proportionality.

20 = k x 10

k = 2

Therefore, the required equation is;

f = 2g

Example 3:

If 15 workers can finish a task in 42 hours, calculate the number of workers required to complete the same task in 30 hours.

solution

In this situation, the number of workers varies indirectly with the time required to finish a task.

Thus, they are inversely proportional. Now, assume that the number of workers required to complete the task in 30 hours be "x". Here, the number of workers $\propto$ 1/hours

Or,

Number of workers = C/hours (here "C" is the constant of proportionality)

Now, consider the first case: "15 workers can finish a task in 42 hours"

Here, 15 = C/42

=> C = 15 × 42 = 630.

Now, consider the second case: "x workers can finish a task in 30 hours"

Here, x = C/30

=> x = 630/30

Or, x = 21

So, the number of 21 workers are required to complete the task in 30 hours.

Exercise

1. **A car travels 14 km in 25 minutes. Find out how far the car can travel in 5 hours if the speed remains the same?**

Systems of Measurement

When you want to tell someone how big or how far away something is, you need a ‘common system’ for communicating this information. There are two most common systems of measurement i.e., the metric system, used widely in Europe and most of the rest of the world, and the Imperial or British system, a form of which is now chiefly used in the USA.

British Imperial vs Metric Systems

The Imperial and U.S. Customary systems of measurement both originate from an amalgamation of early British systems of measurement. The imperial system was originally formalised by the British Weights and Measures Act of 1824 in order to provide a rapidly-developing industrial society with much-needed consistency.

However, this was half a century after American independence, and the system used in the US is based on earlier 18th Century British systems. The two are predominantly the same, but there are some differences, such as the measurement of volumes.

The British Imperial System uses units such as pounds and ounces for mass, miles, yards, feet and inches for distance, and pints and gallons for volume. It’s not a simple or intuitive system and its origins have baffled many scientists over the decades.

For example, there are 12 inches in a foot, 3 feet in a yard, and 16 ounces in a pound. What’s more, because the units are not in nice easy numbers, it can make converting quantities from one unit to another a bit of a challenge, so it really helps if you have a good understanding of **fractions**.

The metric system was officially adopted as a standardised system of measurement by the French in the late 18th century, although it was ‘invented’ over a century earlier. Believe it or not, the length of a ‘metre’ was derived from measurements of the earth’s circumference, which at the time aroused much curiosity and suspicion! However, it is the simplicity of the system that led to its rapid adoption throughout much of the industrialised world.

All the units in the metric system are in multiples of 10: there are 10mm in 1cm, 100cm in a metre, 1000m in a kilometre, and so on. It means that calculations can be done as **decimals**, so multiples of units can be calculated by dividing and multiplying by 10 and its powers. This is much easier to work out in your head and is easily adaptable in all sorts of applications, particularly in science and engineering.

The British Imperial System of Measurement

There are 20 or more ‘base units’ in the imperial system, unlike the metric system, which has fewer than 10. This is a lot to remember. Some of them are no longer in common usage, such as ‘fathom’, which is a unit of length used for measuring the depth of water. Others are used in very specific applications, such as ‘furlong’, which is still the recognised unit of distance in horse racing, and ‘chain’, which is a unit of distance used in the railway industry.

1). Length or distance

Lengths and distances are measured in inches, feet, yards and miles:

12 inches (in) = 1 foot (ft)
3 feet = 1 yard
1760 yards = 1 mile

2). Fluid volume

Fluids are measured in fluid ounces, cups, pints, quarts and gallons.

In the American system:

8 fluid ounces (fl oz) = 1 cup
2 cups = 1 pint
2 pints = 1 quart
4 quarts = 1 gallon

In the British imperial system, 20 fluid ounces = 1 pint, and ‘cups’ are not used at all.

This is only likely to be a problem in recipes. However, it’s usually clear whether you have an English or American recipe by the use of cups as a standard measure, and you can therefore amend your other quantities accordingly.

3). Mass

When we 'weigh' something to find its weight, what we are actually measuring is its **mass**.

The *weight* of an object is the combination of its *mass* and the effect of *gravity* acting on it. Weight can change, depending on the influence of gravity, but mass always stays the same.

So, the *weight* of an apple down here on the surface of the earth is over 6 times greater than its weight on the moon, because there is very little gravity on the moon. However, the mass of the apple on the earth and on the moon is the same. The physical make-up of its skin, flesh and core do not change on its journey from Earth to Moon, it is only the effect of gravity that changes. In the imperial system, mass is measured in ounces, pounds and tons:

16 ounces (oz) = 1 pound (lb)
2,000 pounds = 1 ton

The Metric (SI) System

The metric system is much simpler than the imperial system. There are a series of *base units*, one for each of *distance*, *mass*, and *volume*, and a series of prefixes to tell you what multiple of the base unit is being used.

The International System of Units (SI) is the standard metric system that is currently used, and consists of seven SI base units of *length, mass, time, temperature, electric current, luminous intensity, and amount of substance*. Although SI is used almost universally in science (including in the US), some countries such as the United States still use their own system of units. This is partly due to the substantial financial and cultural costs involved in changing a measurement system compared to the potential benefit of using a standardized system. Since US customary units (USC) are so entrenched in the United States, and SI is already used in most applications where standardization is important, everyday use of USC is still prevalent in the United States, and is unlikely to change. As such, many unit converters including this Conversion Calculator exist, and will continue to do so to ensure that people globally are able to communicate different measurements effectively.

These are the most common:

Basic Unit	Symbol	Measuring
Metre/meter	**m**	**Distance**
Gram	**g**	**Mass**
Second	**s**	**Time**
Litre/liter	**l**	**Volume**
Newton	**N**	**Weight/Force**

Other standard units in the metric system include the Kelvin (K) to measure temperature, ampere (A) to measure electric current, the candela (cd) to measure light intensity, and the mole (mol) to measure the amount of a substance in a scientific (molecular) context. Some of these are only used in scientific applications, so you are unlikely to come across them in day to day usage.

1). Measuring Volume

Volume is usually quoted either in litres (L), or in cubic metres, m^3.

$1cm^3$ is equal to 1 millilitre (ml). One litre is equivalent to 1000ml, so it follows that one litre is also the same as $1000cm^3$.

Volume is a cubic measurement. We can **calculate the volume** of a box (cuboid shape) by multiplying length by width by depth.

$1m^3$ is NOT 1 litre!

$1m^3$ is 1m x 1m x 1m. A litre is $1000cm^3$, which is the same as 10cm x 10cm x 10cm, which is a lot smaller. There are 1000 litres in one cubic metre ($1m^3$) and is another useful relationship is that 1 litre of water weighs exactly 1kg.

The mass of $1cm^3$ or 1ml of water is equal to 1g. For everyday purposes, this can be regarded as true all of the time. Mass, volume and density are related, so 1 litre of sea water has a slightly bigger mass than 1 litre of pure water.

2). Measuring Temperature

There are three scales commonly used for measuring temperature: Fahrenheit, Celsius or Centigrade, and Kelvin.

Fahrenheit is the oldest scale and was formerly used across Europe but has now been replaced by the Centigrade scale. It is, however, still widely used in the USA. This scale was originally defined by 18th Century German physicist Fahrenheit as 180 equal intervals between the temperature at which water freezes and the temperature at which it boils. The exact measurement of these temperatures has undergone some refinement since then; freezing point is now 32°F and boiling point is 212°F. This is why it is not the most intuitive temperature scale.

Celsius / Centigrade is used across most of the rest of the world apart from the USA and its associated territories. It was developed to provide a simpler and more scientifically exact scale than the original Fahrenheit system. The freezing temperature of water is 0°C, and the boiling point is 100°C. 'Centigrade' broadly translates as '100 steps' in Latin. The Celsius scale was named after the Swedish Astronomer Anders Celsius, who created a virtually identical scale with 100 intervals between the two reference temperatures. 'Celsius' is the more commonly-used unit, but is interchangeable with Centigrade.

The weather is the most common reason for needing to understand the alternative scale. Anything below 10°C or 50°F is cool to cold, 20°C or 68°F is warm, and anything above 30°C, 86°F, is hot.

Kelvin is the scientific measurement scale, and the SI unit for temperature. It has exactly the same increments as the Celsius / Centigrade scale. The zero point, or 0K, is -273°C, which is absolute zero. Nothing can be colder than absolute zero, because this is the temperature at which all thermal motion of particles ceases and no thermal energy is left in a substance. Conversion to Celsius is therefore very easy: you simply add 273 to the Kelvin temperature.

Converting Between Metric and British Imperial Systems

In order to convert from metric to imperial systems, you just multiply by the desired 'conversion factor'. It is often useful to be able to convert *approximately*, for example, to estimate driving distance or maximum speed limit when travelling in another country.

There are a series of useful approximations which you can use. For example:

- 1 yard is approximately 1 metre
- 1 mile is about 1.5 kilometres (km), and a km is about two thirds of a mile.
- 1 litre is about 1 American quart
- 1 (UK) pint is about 500ml (half a litre)
- 1 kilogram (kg) is about 2 pounds (lb)

To measure quantities like this, units of measurement are required. Calculations in metric system is easy, because they are based on powers of 10 just like in our decimal system. The following units are most commonly used and it is important to remember their symbols (or shortened form).

Quantity	Name of unit	Symbol	Value
length	millimetre centimetre **metre** kilometre	mm cm m km	10mm = 1cm 100cm = 1m 1000m = 1km
mass	milligram **gram** kilogram tonne	mg g kg t	1000mg = 1g 1000g = 1kg 1000kg = 1t
time	second minute **hour** day	s min h day	60s = 1min 60min = 1h 24h = 1day
temperature	degrees Celcius	°C	
area	square millimetre square centimetre **square metre** hectare	mm^2 cm^2 m^2 ha	$1cm^2 = 100\ mm^2$ $1\ m^2 = 10,000\ cm^2$ $1\ ha = 10,000\ m^2$
volume	cubic millimetres cubic centimetres **cubic metres**	mm^3 cm^3 m^3	$1cm^3 = 1000\ mm^3$ $1\ m^3 = 1,000,000\ cm^3$
capacity (volume of fluids)	millilitre **litre** kilolitre	mL L kL	1000ml = 1 L 1000L = 1kL
speed	metres per second **kilometres per hour**	ms^{-1} kmh^{-1}	

- **Units in blue are standard units.**

Perimeters of Regular and Irregular Figures

Perimeter is the distance around a two-dimensional shape. The perimeter of a polygon is defined as the total distance around the outside of a polygon and is measured in meters, kilometres etc.

The perimeter of a polygon is calculated by taking the sum of all side lengths of a particular polygon. You can also find the perimeter of all polygons, whether they are regular or irregular polygons.

For a regular polygon, the perimeter is equal to the product of one side length and the number of sides of the polygon.

Perimeter of a regular polygon = (length of one side) × number of sides

For example, the perimeter of a regular pentagon whose side length 8 cm, is given by;

Perimeter of a regular pentagon = 8 x 5 = 40 cm.

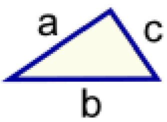
For an irregular polygon, the perimeter is calculating by adding together the individual side lengths.

For example, the perimeter of an irregular pentagon whose side lengths are; 5 cm, 4 cm, 6 cm, 10 cm and 9 cm.

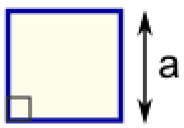
Perimeter = (5 + 4 + 6 + 10 + 9) cm

= 34 cm.

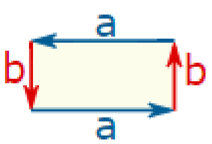
Perimeter Formulas



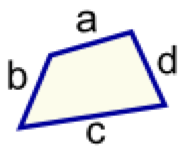
Triangle
Perimeter = a + b + c



Square
Perimeter = 4 × a
a = length of side

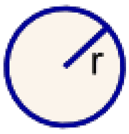


Rectangle
Perimeter = 2 × (a + b)



Quadrilateral

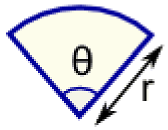
Perimeter = a + b + c + d



Circle

Circumference = $2\pi r$

r = radius



Sector

Perimeter = $r(\theta+2)$

r = radius

θ = angle in radians

Example 1

Find the perimeter of a triangle whose sides are 20 cm, 15 cm and 18 cm.

Solution

Perimeter of a triangle = a + b + c

= (20 + 15 + 18) cm

= 53 cm

Example 2

Calculate the perimeter of an equilateral triangle whose side length is 12 cm.

Solution

Perimeter of an equilateral triangle = 3a

= (3 x 12) cm

= 36 cm

Example 3

Determine the value of x for a triangle whose side lengths are, (x + 20) cm, (4x – 5) cm,

(2x + 15) cm and the perimeter is 100 cm.

Solution

Perimeter = a + b + c

(x + 20) + (4x – 5) + (2x + 15) = 100 cm

Simplify.

x + 20 + 4x – 5 + 2x + 15 = 100

Collect the like terms.

7x + 30 = 100

Subtract 30 on both sides.

7x = 70

Divide both sides by 7 to get,

x = 10.

Therefore, the value of x = 10 cm.

So, the three side lengths are;

⇒ (x + 20) = (10 + 20) = 30 cm

⇒ (4x – 5) = 4(10) – 5 = 35 cm

⇒ (2x + 15) = 2(10) + 15 = 35 cm.

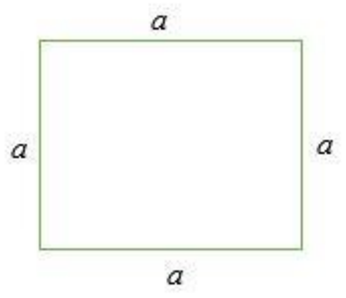
Perimeter of a square and a rhombus

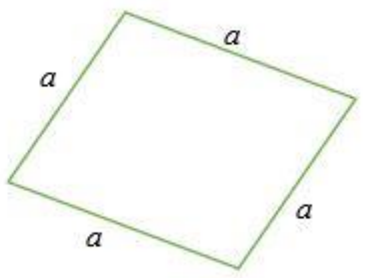
The perimeter of a square is given by,

P = a + a + a + a

P= 4a

Where, a = the length of the side length of a square.





Since a square and a rhombus have both 4 sides that are equal, then the perimeter of a rhombus is equal to the perimeter of a square.

Example 4

Calculate the perimeter of a square which has the length of 10 ft.

Solution

P= 4a

= (4 x 10) ft

= 40 ft.

Example 5

Find the perimeter of a rhombus with a side length of 4 inches.

Solution

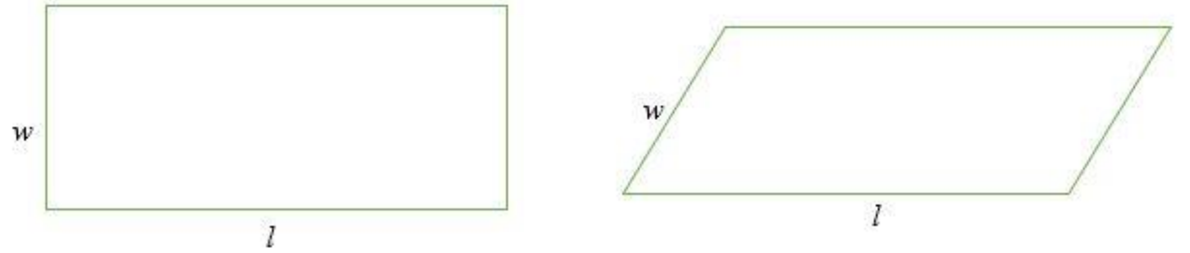
Perimeter of a rhombus = 4a

= (4 x 4) inches.

Perimeter of a rectangle and parallelogram

The perimeter of a rectangle and parallelogram are equal. The formula for calculating the perimeter of a rectangle is given by,

Perimeter of a rectangle = Length + Length + width + width



Perimeter of a rectangle = 2 (L + W)

where,

L = length of a rectangle or parallelogram and

W = width of the rectangle or parallelogram.

Example 6

What is the perimeter of a rectangle with the length as 100 mm and width as 80 mm?

<u>Solution</u>

Perimeter of a rectangle = 2 (L + W).

= 2 (100 + 80) mm

= 2 x 180 mm

P = 360 mm

Example 7

Find the perimeter of a parallelogram whose length is 12 yards and width, is 5 yards.

<u>Solution</u>

Perimeter of a parallelogram = 2(L + W).

= 2 (12 + 5) yards.

= 2 x 17 yards

P = 34 yards

Example 8

The width of a rectangle 5 m less than the length. Find the length and width of the rectangle if its perimeter is 34 m.

<u>Solution</u>

Width is 5 m less than the length.

Let, the length = x.

Width = x – 5

But, the perimeter = 2 (L + W)

34 = 2 (x – 5 + x)

34 = 2 (2x – 5)

34 = 4x -10

Add 10 on both sides.

44 = 4x

Divide both sides by 4.

x = 11

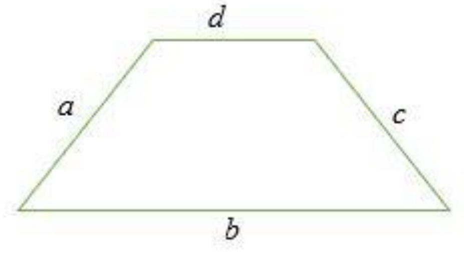
Therefore, the length of the rectangle is 11m and width is 6 m.

Perimeter of a trapezoid

The perimeter of a trapezoid is given by,

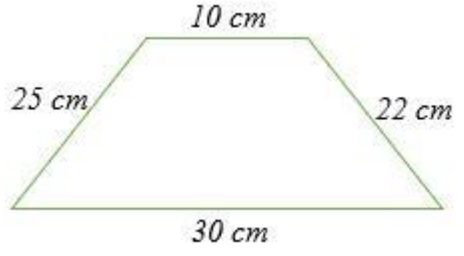
P = a + b + c + d

where a, b, c and d are the lengths of each side.



Example 9

Calculate the perimeter of the trapezoid shown below.



<u>Solution</u>

Perimeter of a trapezoid = a + b + c + d

= (25 + 30 + 22 + 10)

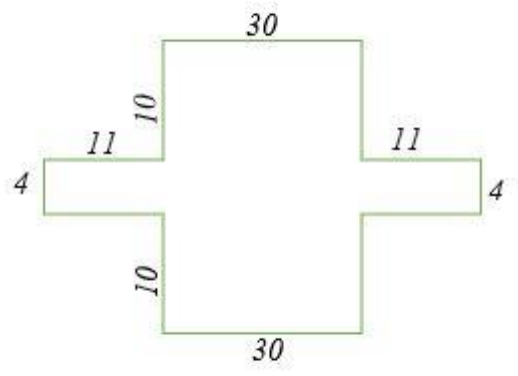
= 87 cm.

Perimeter of irregular polygons

As stated earlier, the perimeter of an irregular polygon is equal to the sum of all side lengths.

Example 10

Calculate the perimeter of the diagram shown below if the dimensions are in mm.



<u>Solution</u>

Perimeter = (4 + 11 + 10 + 30 + 10 + 11 + 4 + 11 + 10 + 30+ 10 + 11) mm

P = 152 mm

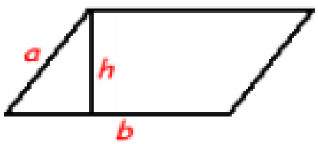
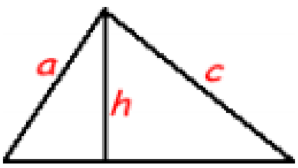
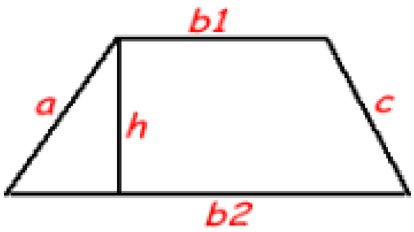
Exercise

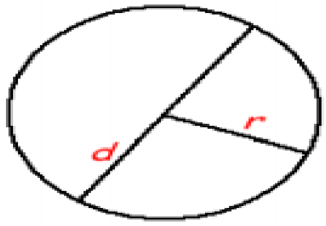
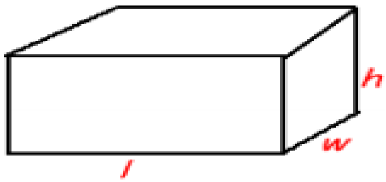
1. What is the perimeter of a rectangular garden 15 meters long and 12 meters wide?
2. Mike jogs onto the track of a park for 14 minutes and covered a distance of 3500 meters. If it takes 3 minutes to complete one round of the track, what is the perimeter of the park? Assume the track runs with the boundary of the park.

Area of Regular Polygons

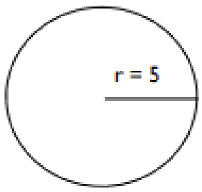
A regular polygon is a polygon with congruent sides and angles. Recall that the perimeter of a square is 4 times the length of a side because each side is congruent. We can extend this concept to any regular polygon.

FORMULAS FOR PERIMETER, AREA, SURFACE, VOLUME

Shapes	**Formulas**
	**Rectangle** **Area** = Length X Width $A = lw$ **Perimeter** = 2 X Lengths + 2 X Widths $P = 2l + 2w$
	**Parallelogram** **Area** = Base X Height $A = bh$ **Perimeter** = add the length of all sides $P = 2a + 2b$
	**Triangle** **Area** = 1/2 of the base X the height $A = \frac{1}{2}bh$ **Perimeter** = $a + b + c$ (add the length of the three sides)
	**Trapezoid** **Area** = 1/2 of the base X the height $A = (\frac{b1+b2}{2})h$ **Perimeter** = add lengths of all sides $P = a + b1 + b2 + c$

	**Circle** **Radius** = the distance from the center to a point on the circle (r). **Diameter** = the distance between two points on the circle through the center (d = 2r). **Circumference** = the distance around the circle ($C = \pi d = 2\pi r$). (Assume $\pi \approx 3.14$) **Area** = πr^2
	**Rectangular Solid** **Volume** = Length X Width X Height $V = lwh$ **Surface** = $2lw + 2lh + 2wh$

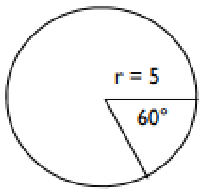
For Circles area = πr^2



Area = πr^2

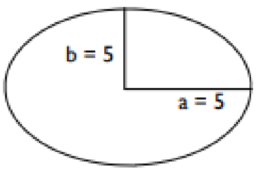
$= 3.142 \times 5^2 = 78.54\text{cm}^2$

For a sector of a Circle area = area of circle $\times \frac{\text{sector angle}}{360}$



Area of sector = $\pi r^2 \times \frac{60}{360} = 13.1\text{cm}^2$

For Ellipse area = πab



Area = $3.142 \times 10 \times 5 = 157\text{cm}^2$

Area of Irregular Shapes

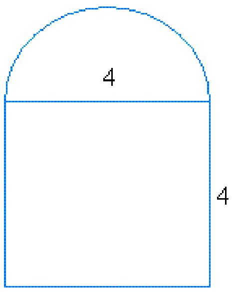
Irregular shapes are the polygons with five or more sides of varying lengths. These shapes or figures can be decomposed further into triangles, squares, and quadrilaterals to evaluate the area. To find the area of irregular shapes, the first thing to do is to divide the irregular shape into regular shapes that you can recognize such as triangles, rectangles, circles, squares and so forth. Then, find the area of these individual shapes and add them up.

Calculating the area of an irregular shape with curved edges,

- **Dividing the irregular shape in two or more regular shapes**.

Use this method for irregular shapes, which are a combination of triangles and polygons. Use predefined formulas to calculate the area of such shapes and add them together to obtain the total area. For example, an irregular shape we divide multiple edges into a triangle and three polygons.

Example 1: Find the area of the figure below:



Square

Area $_{\text{square}}$ = s^2

Area $_{\text{square}}$ = 4^2

Area $_{\text{square}}$ = 16

Circle

Area $_{\text{circle}}$ = pi × r^2

Notice that the radius of the circle is 4/2 = 2

Area $_{\text{circle}}$ = 3.14 × 2^2 Area $_{\text{circle}}$ = 3.14 × 4

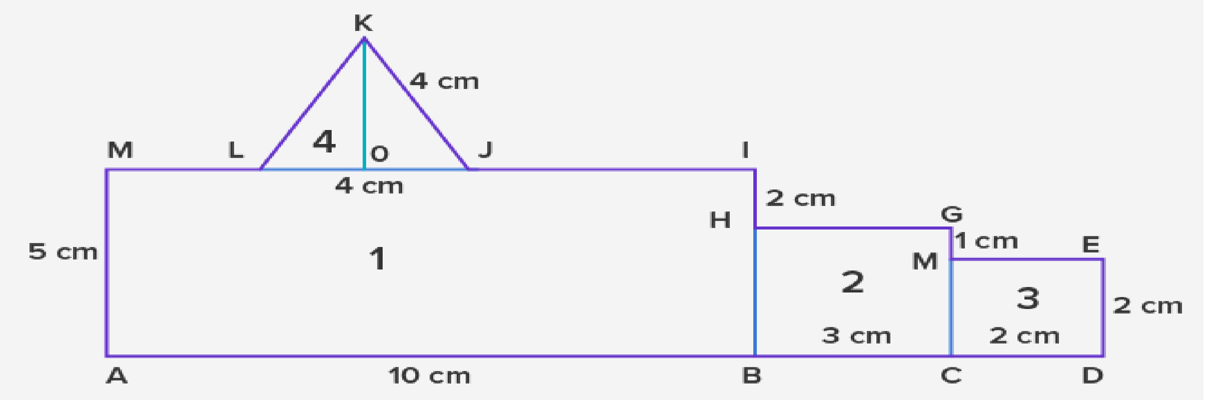
Area $_{\text{circle}}$ = 12.56

Since you only have half a circle, you have to multiply the result by 1/2

1/2 × 12.56 = 6.28

Area of this shape = 16 + 6.28 = 22.28

Example 2



The total area of the figure is given as:

⇒ Area = Area (ABIM) + Area (BCGH) + Area (CDEF) + Area (JKL)

⇒ Area = (AB × BI) + (BC × CG) + (CD × DE) + ($\frac{1}{2}$× LJ × KO)

⇒ Area = (10 × 5) + (3 × 3) + (2 × 2) + ($\frac{1}{2}$× 4 × 4)

⇒ Area = 50 + 9 + 4 + 8

⇒ Area = 71 cm2

- **Dividing the irregular shape with curves in two or more regular shapes**

In this method, decompose an irregular shape into multiple squares, triangles, or other quadrilaterals. Depending on the shape and curves, a part of the figure can be a circle, semicircle or quadrant as well.

The following figure is an irregular shape with 8 sides, including one curve. Determine the unknown quantities by the given dimensions for the sides. Decompose the figure into two rectangles and a semicircle.

The area of the shape ABCDEF is:

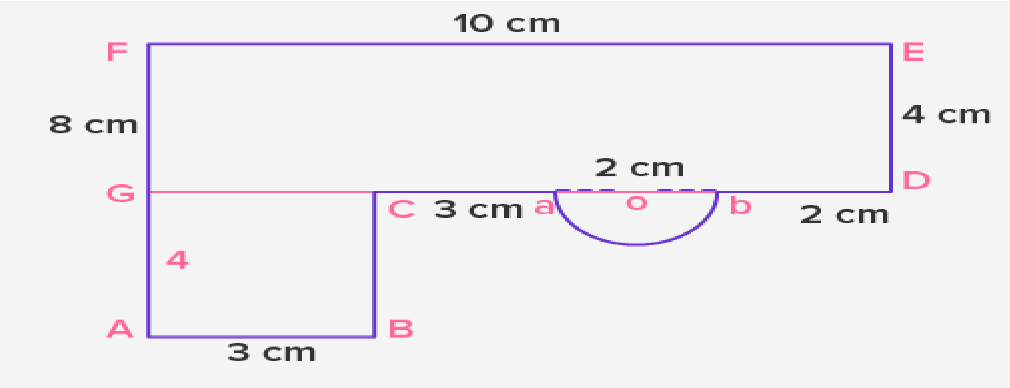
Area (ABCDEF) = Area (ABCG) + Area (GDEF) + Area (aob)

Area = (AB × AG) + (GD × DE) + ($\frac{1}{2}$ × π × ob^2)

Area = (3 × 4) + (10 × 4) + ($\frac{1}{2}$ × 3.14 × 1^2)

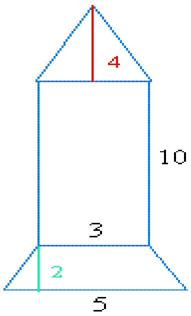
Area = 12 + 40 + 1.57

Area = 53.57 cm^2



Example 3.

Find the area of the figure below:



The figure above has three regular shapes. Starting from top to bottom, it has a triangle, a rectangle, and a trapezoid

Find the area for each of those three shapes and add the results

Triangle

Area $_{\text{triangle}}$ = (base × height)/2

Area $_{\text{triangle}}$ = (3 × 4)/2

Area $_{\text{triangle}}$ = 12/2

Area $_{\text{triangle}}$ = 6

Rectangle

Area $_{\text{rectangle}}$ = length × width

Area $_{\text{rectangle}}$ = 3 × 10

Area $_{rectangle}$ = 30

Trapezoid

Area $_{trapezoid}$ = $((b_1 + b_2) \times h)/2$

Area $_{trapezoid}$ = $((3 + 5) \times 2)/2$

Area $_{trapezoid}$ = $(8) \times 2/2$

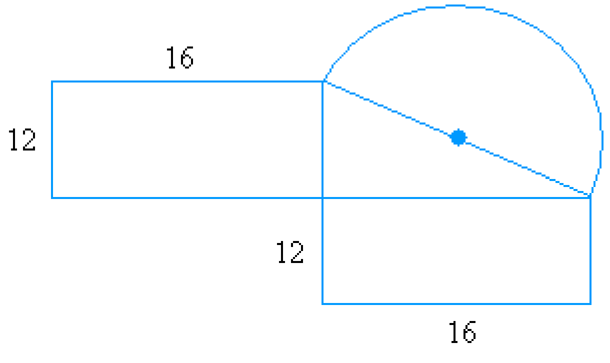
Area $_{trapezoid}$ = 16/2

Area $_{trapezoid}$ = 8

Area of this shape = 6 + 30 + 8 = 44

Example 4

Find the total area of the figure below:



The figure above has 4 regular shapes. It has a triangle, two rectangles, and half a circle

Find the area for each of those 4 shapes and add the results

Rectangle

Area $_{rectangle}$ = length × width

Area $_{rectangle}$ = (12 × 16)

Area $_{rectangle}$ = 192

Since we have two of the same rectangle, the area is 192 + 192 = 384

Triangle

Notice that the longest side of the rectangle is the base of the triangle and the short side of the rectangle is the height of the triangle

So,

$Area_{triangle} = (base \times height)/2$

$Area_{triangle} = (16 \times 12)/2$

$Area_{triangle} = (192)/2$

$Area_{triangle} = 96$

Circle

To get the area of the half circle, we need to know the diameter

Notice that the diameter is the hypotenuse of a right triangle, so use the Pythagorean Theorem to find the length of the diameter

$c^2 = a^2 + b^2$

$c^2 = 12^2 + 16^2$

$c^2 = 144 + 256$

$c^2 = 400$

$c = \sqrt{400}$

$c = 20$

Therefore, the diameter is 20. Since the diameter is 20, the radius is 10

$Area_{circle} = pi \times r^2$

$Area_{circle} = 3.14 \times 10^2$

$Area_{circle} = 3.14 \times 100$

$Area_{circle} = 314$

Since you only have half a circle, you have to multiply the result by 1/2

$1/2 \times 314 = 157$

Area of this shape = 384 + 96 + 157 = 637

Application

The estimation of area for irregular figures is an essential method for drawing maps, building architecture, and marking agricultural fields. We apply the concept in the cutting of fabrics as per the given design. In higher grades, the technique lays a basis for advanced topics such as calculating volume, drawing conic sections and figures with elliptical shapes.

Volume of Regular shapes

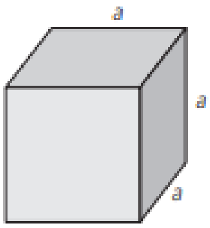
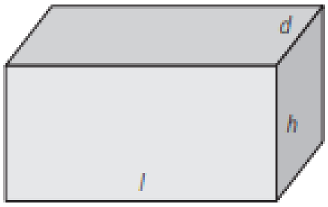
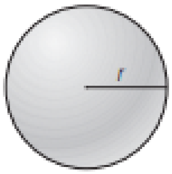
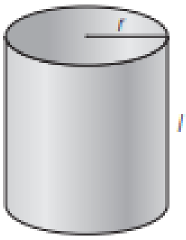
The total space occupied by the body is called volume. In SI system, the unit of volume is a cubic meter (m^3). Other similar units are mm^3, cm^3, ml, l, etc. The volume of solid is measured in mm^3, cm^3, m^3, etc. Measuring cylinders are used for the measurement of the volume of liquids. The volume of liquids is measured in ml, l, etc,

1 ml = $1cm^3$ or 1cc (cubic centimetre)
1000 ml = 1l (litre)
1000 cm^3 = 1l

Measurement of Volume of Regular Solids

Volumes of Solid Figures

The methods of calculating the volume of regular shaped objects are given in the table below:

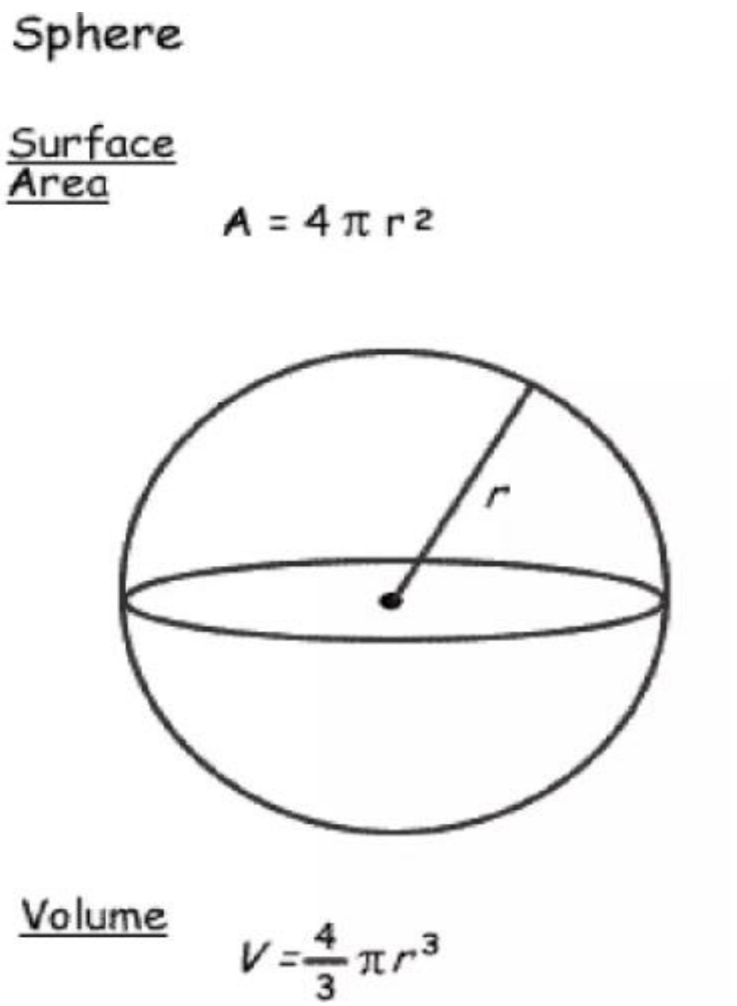
Object	Dimensions	Volume/m^3
Cube		$a \times a \times a$
Rectangular block		$l \times h \times d$
Sphere		$\frac{4}{3}\pi r^3$
Cylindrical block		$\pi r^2 l$

Surface Area and Volume of a Sphere

A three-dimensional circle is known as a sphere. In order to calculate either the surface area or the volume of a sphere, you need to know the radius (**r**). The radius is the distance from the centre of the sphere to the edge and it is always the same, no matter which points on the sphere's edge you measure from.

Once you have the radius, the formulas are rather simple to remember. Just as with the circumference of the circle, you will need to use pi (**π**). Generally, you can round this infinite number to 3.14 or 3.14159.

- **Surface Area = $4\pi r^2$**
- **Volume = $4/3\ \pi r^3$**



Surface Area and Volume of a Cone

A cone is a pyramid with a circular base that has sloping sides which meet at a central point. In order to calculate its surface area or volume, you must know the radius of the base and the length of the side.

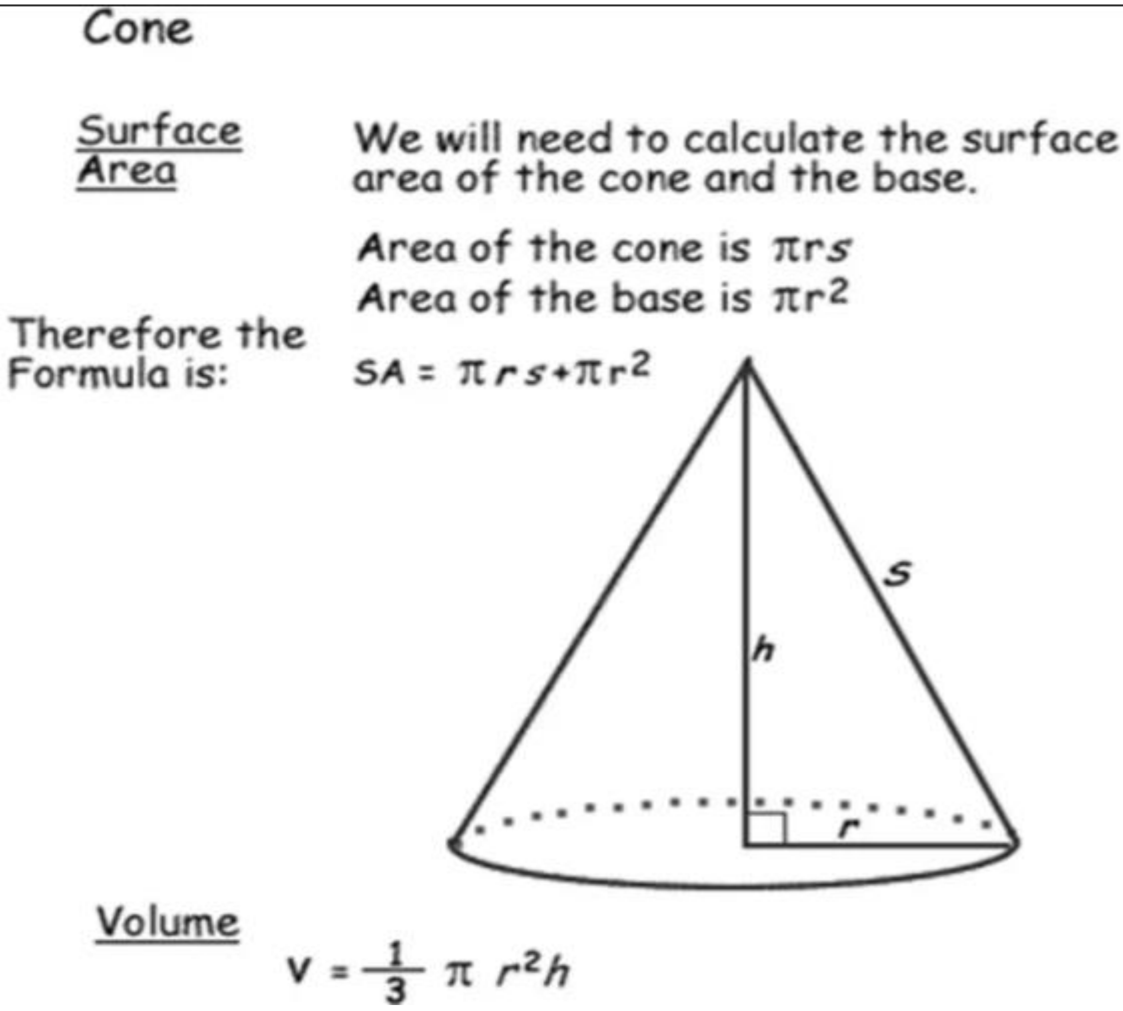
If you do not know it, you can find the side length (**s**) using the radius (**r**) and the cone's height (**h**).

$$\mathbf{s = \sqrt{(r2 + h2)}}$$

With that, you can then find the total surface area, which is the sum of the area of the base and area of the side.

- **Area of Base: πr^2**
- **Area of Side: πrs**
- **Total Surface Area = $\pi r^2 + \pi rs$**

To find the volume of a sphere, you only need the radius and the height.



Surface Area and Volume of a Cylinder

This shape has a circular base and straight, parallel sides. This means that in order to find its surface area or volume, you only need the radius (**r**) and height (**h**).

However, you must also factor in that there is both a top and a bottom, which is why the radius must be multiplied by two for the surface area.

- **Surface Area = $2\pi r^2 + 2\pi rh$**
- **Volume = $\pi r^2 h$**

Cylinder

Surface Area

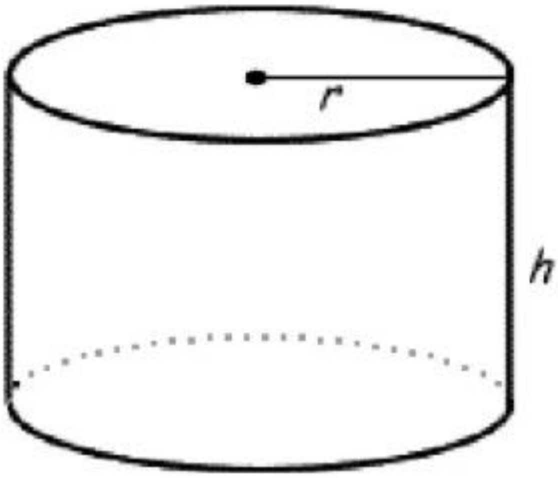
We will need to calculate the surface area of the top, base and sides.

Area of the top is πr^2

Area of the bottom is πr^2

Area of the side is $2\pi rh$

Therefore the Formula is: $A = 2\pi r^2 + 2\pi rh$



Volume $V = \pi r^2 h$

Surface Area and Volume of a Rectangular Prism

A rectangular in three dimensions becomes a rectangular prism (or a box). When all sides are of equal dimensions, it becomes a cube. Either way, finding the surface area and the volume require the same formulas.

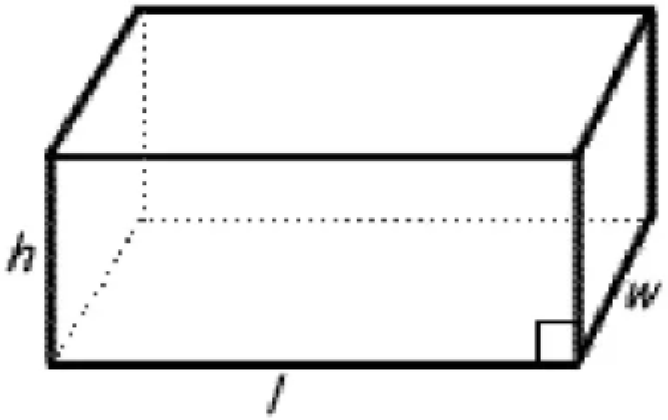
For these, you will need to know the length (**l**), the height (**h**), and the width (**w**). With a cube, all three will be the same.

- **Surface Area = 2(lh) + 2(lw) + 2(wh)**
- **Volume = lhw**

Rectangular Prism

Surface Area

$$A = 2(wh + lw + lh)$$



Volume

$$V = lwh$$

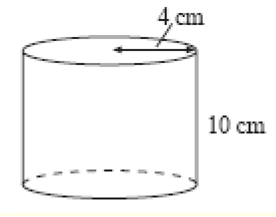
Exercises

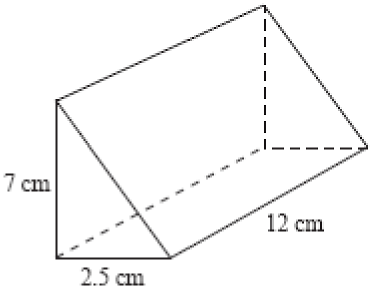
- Find the surface area of a non-cubic prism with the following measurements:

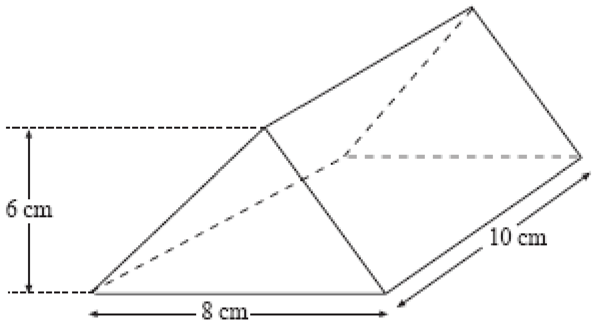
  $l=7;\ w=6;\ h=2$

- A small rectangular jewellery box has two square ends with areas of 36 square centimetres, and a width of 10 centimetres. What is the surface area of the outside of the jewellery box?
- The length of a garden is thrice its breadth. A playground measuring 180 sq. m occupies 1/15th of the total area of the garden. Find the length of the garden.

Find the surface area and volumes of the following shapes.







The diagram shows a wooden block that has had a hole drilled in it. The diameter of the hole is 2 cm.
Calculate the volume of this solid, giving your answer correct to 2 decimal places.