

### **1.3: Forms of fractions, decimals and percentages**

#### **1.3.1 Fractions**

A fraction is a number which is represented by one integer – the *numerator* – divided by another integer – the *denominator* (or the *divisor*). For example,  $\frac{3}{5}$  is a fraction with numerator 3 and denominator 5. Because fractions are written as one integer divided by another – a *ratio* – they are called *rational* numbers. Fractions are either *proper*, *improper* or *mixed*:

- in a proper fraction the numerator is less than the denominator, for example,  $\frac{4}{7}$
- in an improper fraction the numerator is greater than the denominator, for example  $\frac{12}{5}$
- a mixed fraction is in the form of an integer and a fraction, for example  $6\frac{2}{3}$

So that  $-\frac{8}{11}$  is a ..... fraction?

## Multiplying fractions

Two fractions are multiplied by multiplying their respective numerators and denominators independently. For example:

$$\frac{2}{3} \times \frac{5}{7} = \frac{2 \times 5}{3 \times 7} = \frac{10}{21}$$

Try this one for yourself.

$$\frac{5}{9} \times \frac{2}{7} = \dots\dots\dots$$

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$\frac{10}{63}$
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Because

$$\frac{5}{9} \times \frac{2}{7} = \frac{5 \times 2}{9 \times 7} = \frac{10}{63}$$

*Correct? Then on to the next frame*

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## Of

The word 'of' when interposed between two fractions means multiply. For example:

Half of half a cake is one-quarter of a cake. That is

$$\frac{1}{2} \text{ of } \frac{1}{2} = \frac{1}{2} \times \frac{1}{2} = \frac{1 \times 1}{2 \times 2} = \frac{1}{4}$$

So that, for example:

$$\frac{1}{3} \text{ of } \frac{2}{5} = \frac{1}{3} \times \frac{2}{5} = \frac{1 \times 2}{3 \times 5} = \frac{2}{15}$$

So that  $\frac{3}{8} \text{ of } \frac{5}{7} = \dots\dots\dots$

## Equivalent fractions

Multiplying the numerator and denominator by the same number is equivalent to multiplying the fraction by unity, that is by 1:

$$\frac{4 \times 3}{5 \times 3} = \frac{4}{5} \times \frac{3}{3} = \frac{4}{5} \times 1 = \frac{4}{5}$$

Now,  $\frac{4 \times 3}{5 \times 3} = \frac{12}{15}$  so that the fraction  $\frac{4}{5}$  and the fraction  $\frac{12}{15}$  both represent the same number and for this reason we call  $\frac{4}{5}$  and  $\frac{12}{15}$  *equivalent fractions*.

A second fraction, equivalent to a first fraction, can be found by multiplying the numerator and the denominator of the first fraction by the same number.

So that if we multiply the numerator and denominator of the fraction  $\frac{7}{5}$  by 4 we obtain the equivalent fraction .....

Check your result in Frame 35

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$\frac{28}{20}$
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Because

$$\frac{7 \times 4}{5 \times 4} = \frac{28}{20}$$

We can reverse this process and find the equivalent fraction that has the smallest numerator by *cancelling out* common factors. This is known as reducing the fraction to its *lowest terms*. For example:

$\frac{16}{96}$  can be reduced to its lowest terms as follows:

$$\frac{16}{96} = \frac{4 \times 4}{24 \times 4} = \frac{4 \times \cancel{4}}{24 \times \cancel{4}} = \frac{4}{24}$$

by cancelling out the 4 in the numerator and the denominator

## Dividing fractions

The expression  $6 \div 3$  means the number of 3's in 6, which is 2. Similarly, the expression  $1 \div \frac{1}{4}$  means the number of  $\frac{1}{4}$ 's in 1, which is, of course, 4. That is:

$$1 \div \frac{1}{4} = 4 = 1 \times \frac{4}{1} \quad \text{Notice how the numerator and the denominator of the divisor are switched and the division replaced by multiplication.}$$

Two fractions are divided by switching the numerator and the denominator of the divisor and multiplying the result. For example:

$$\frac{2}{3} \div \frac{5}{7} = \frac{2}{3} \times \frac{7}{5} = \frac{14}{15}$$

So that  $\frac{7}{13} \div \frac{3}{4} = \dots\dots\dots$

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$\frac{28}{39}$
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Because

$$\frac{7}{13} \div \frac{3}{4} = \frac{7}{13} \times \frac{4}{3} = \frac{28}{39}$$

In particular:

$$1 \div \frac{3}{5} = 1 \times \frac{5}{3} = \frac{5}{3}$$

The fraction  $\frac{5}{3}$  is called the *reciprocal* of  $\frac{3}{5}$

So that the reciprocal of  $\frac{17}{4}$  is  $\dots\dots\dots$

## Adding and subtracting fractions

Two fractions can only be added or subtracted immediately if they both possess the same denominator, in which case we add or subtract the numerators and divide by the common denominator. For example:

$$\frac{2}{7} + \frac{3}{7} = \frac{2+3}{7} = \frac{5}{7}$$

If they do not have the same denominator they must be rewritten in equivalent form so that they do have the same denominator - called the *common denominator*. For example:

$$\frac{2}{3} + \frac{1}{5} = \frac{10}{15} + \frac{3}{15} = \frac{10+3}{15} = \frac{13}{15}$$

The common denominator of the equivalent fractions is the LCM of the two original denominators. That is:

$$\frac{2}{3} + \frac{1}{5} = \frac{2 \times 5}{3 \times 5} + \frac{1 \times 3}{5 \times 3} = \frac{10}{15} + \frac{3}{15} \text{ where 15 is the LCM of 3 and 5}$$

So that  $\frac{5}{9} + \frac{1}{6} = \dots\dots\dots$

*The result is in Frame 42*

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$\frac{13}{18}$
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Because

The LCM of 9 and 6 is 18 so that

$$\frac{5}{9} + \frac{1}{6} = \frac{5 \times 2}{9 \times 2} + \frac{1 \times 3}{6 \times 3} = \frac{10}{18} + \frac{3}{18} = \frac{10+3}{18} = \frac{13}{18}$$

## 49 Percentages

A percentage is a fraction whose denominator is equal to 100. For example, if 5 out of 100 people are left-handed then the fraction of left-handers is  $\frac{5}{100}$  which is written as 5%, that is 5 *per cent* (%).

So if 13 out of 100 cars on an assembly line are red, the percentage of red cars on the line is .....

Arithmetic

25

13%

50

Because

The fraction of cars that are red is  $\frac{13}{100}$  which is written as 13%.

Try this. What is the percentage of defective resistors in a batch of 25 if 12 of them are defective?

48%

51

Because

The fraction of defective resistors is  $\frac{12}{25} = \frac{12 \times 4}{25 \times 4} = \frac{48}{100}$  which is written as 48%.

Notice that this is the same as:

$$\left(\frac{12}{25} \times 100\right)\% = \left(\frac{12}{25} \times 25 \times 4\right)\% = (12 \times 4)\% = 48\%$$

*A fraction can be converted to a percentage by multiplying the fraction by 100.*

To find the percentage part of a quantity we multiply the quantity by the percentage written as a fraction. For example, 24% of 75 is:

$$\begin{aligned} 24\% \text{ of } 75 &= \frac{24}{100} \text{ of } 75 = \frac{24}{100} \times 75 = \frac{6 \times 4}{25 \times 4} \times 25 \times 3 = \frac{6 \times \cancel{4}}{\cancel{25} \times \cancel{4}} \times \cancel{25} \times 3 \\ &= 6 \times 3 = 18 \end{aligned}$$

So that 8% of 25 is .....

*Work it through and check your results with the next frame*

2

52

Because

$$\frac{8}{100} \times 25 = \frac{2 \times 4}{25 \times 4} \times 25 = \frac{2 \times \cancel{4}}{\cancel{25} \times \cancel{4}} \times \cancel{25} = 2$$

