

# **MINISTRY OF EDUCATION CERTIFICATE IN INFORMATION COMMUNICATION TECHNOLOGY**

**KENYA INSTITUTE OF CURRICULUM DEVELOPMENT  
STUDY NOTES**

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## **BASIC ELECTRONICS**

**MODULE I: SUBJECT NO 10**

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# TOPIC 1: INTRODUCTION TO ELECTRICAL CIRCUITS

## Electrical Quantities & Units

### POTENTIAL

Potential refers to the possibility of doing work

Any charge has the potential to do the work of moving another charge, by either attraction or repulsion. When we consider two unlike charge, they have a difference of, potential. The charge has a certain amount of potential, corresponding to the amount of work this charge can do.

**electromotive force**, abbr. emf, difference in electric potential, or voltage, between the terminals of a source of electricity, e.g., a battery from which no current is being drawn. When current is drawn, the potential difference drops below the emf value. Electromotive force is usually measured in volts

### UNIT

The unit of potential difference [**electromotive force, abbr. emf**] is volt named after Alessandro Volta. The volt is the measure of work needed to move an electric charge.

### VOLT

One volt is the potential difference (voltage) between two points when one joule of energy is used to move one coulomb of charge from one point to the other

OR

Potential difference is one volt if 0.7376 (foot-pound) of work is required to move  $6.25 \times 10^{18}$  electrons.

Since

$$6.25 \times 10^{18} \text{ electrons} = 1 \text{ coulomb}$$

$$0.7376 \text{ foot-pound} = 1 \text{ joule}$$

Hence definition will be:

**One volt is equal to one joule of work per coulomb of charge ( $V = W/Q$ )**

Its symbol is V.

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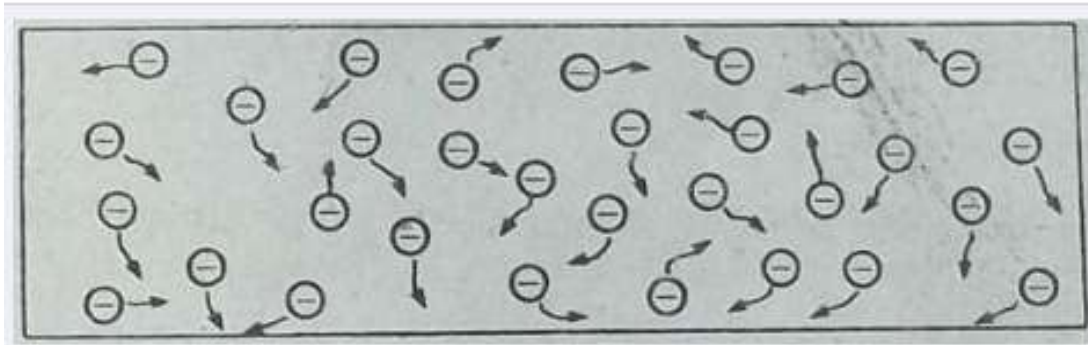
### CURRENT

The continuous motion of free electrons by applying potential difference is called current.

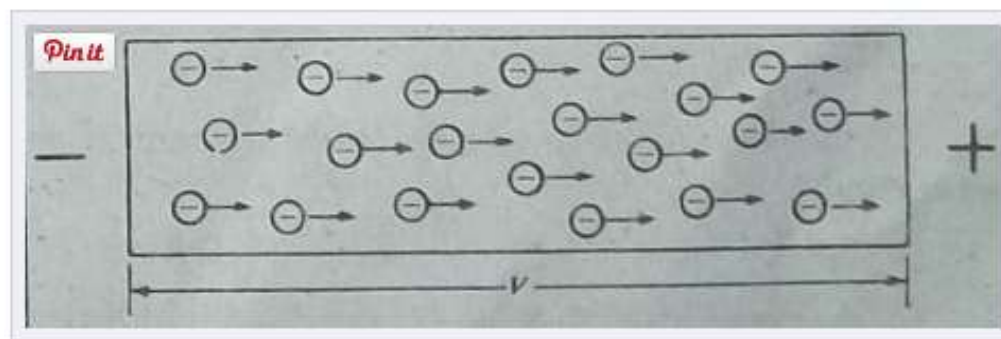
OR

Electrical current is defined as the rate of flow of electrons in a conductive or semi-conductive Material.

$$I = Q/t$$



Free electrons are available in all conductive and semi-conductive materials. These electrons drift randomly in all directions, from atom to atom, within the structure of the material. Now if a voltage is placed across. The conductive or semi-conductive material, one end becomes positive and the other end negative as indicated in figure. The repulsive force between the negative voltages at the left end causes the free electrons (negative charges) to move toward the right. The attractive force between the positive voltage at the right end pulls the free electrons from the negative end of the material to the positive end, as shown in figure.



The movement of the free electrons from the negative end of the material to the positive end is the electrical current, symbolized by **I**

#### UNIT

The unit of current is Ampere symbolized by **A** it is named after Andre Ampere

#### AMPERE

The amount of current is one ampere if  $6.25 \times 10^{18}$  electrons move per second.

OR

The amount of current is one Ampere if one coulomb (1c) of charge flows through a conductor for one second.

---

## **RESISTANCE**

Resistance is the blocking of free electrons while moving through a conductor.

OR

The opposition that limits the amount of current produced by applied voltage is called resistance.

When current flows in a material and occasionally collide with atoms. These collisions cause the electrons to lose some of their energy and thus their movement is restricted. The more collisions, more the flow of electrons is restricted. This restriction varies and is determined by the type of material. The property of a material that restricts the flow of electrons is called resistance designated R.

## **UNIT**

Resistance R, is expressed in the unit of Ohm, named after George Simon Ohm and symbolized by the Greek letter Omega ( $\Omega$ ).

## **OHM**

Ohm is defined as:

There is one Ohm ( $1\Omega$ ) of resistance when one Ampere (1A) of current flows in a material with one volt (1V) applied.

OR

The resistance in which steady current of one Ampere generates heat of 1 joule per second.

OR

A resistance that develops 0.24 calorie of heat by currents for one second has one Ohm resistance

---

## **Difference between conventional current and electron current**

### **CONVENTIONAL CURRENT**

The motion of positive charges, in the opposite direction of the flow of electrons is called conventional current.

This direction is generally used for analyzed circuit in electrical engineering. The reason is based on some traditional definitions in the science of physics. By the definitions of force and work with positive values, positive potential is considered above the negative potential. Conventional current corresponds to a motion of positive charges (Falling down hill) from positive to a negative potential.

An example of positive charges in motion for conventional current is the current of whole charges in P-type semi-conductor. Also a current of positive ions in liquids and gasses moves in opposite direction of electron flow.

### **ELECTRON CURRENT**

The current which is due to the motion of electrons only in the opposite direction of conventional current is called electron current.

The direction of electron drift for current I is out from negative side of the voltage source, current I passes through external circuit with R and returns to positive side of voltage source.

Inside the battery, the electron moves towards the negative terminal to produce potential difference. The battery is doing work of separation of negative and positive charges, accumulated electrons at negative terminal and protons at positive terminal. The potential difference allows electrons to move from negative terminal passes through external circuit, returns to positive terminal.

However the direction of electron or electronics current flow is from negative potential to positive potential

By formal definition, any form of power (e.g. electrical, mechanical, thermal, etc) is the rate at which energy or work is performed. The standard unit of power is the watt (or joules per second).

Electrical power is the rate at which electrical energy is delivered to a load (via an electrical circuit) and converted into another form of energy (e.g. heat, light, sound, chemical, kinetic, etc). In terms of electrical quantities current and voltage, power can be calculated by the following standard formula:

$$P = VI$$

Where P is power in watts, V is potential difference in volts and I is current in amperes.

**NOTE:** Electrical power and electrical energy are quantities equivalent to power and energy known from other technical and scientific fields, e.g., mechanics, physics, chemistry, etc. The only formal difference is that electrical power and energy are related to electric circuits and other electrical quantities. For example, the power Pin a circuit with a steady voltage V and current I and with application of Ohm's law can be simply calculated as:

$$P = V \cdot I = I^2 \cdot R = \frac{V^2}{R}$$

Electrical power can be also expressed as a change of electrical energy E in time t:

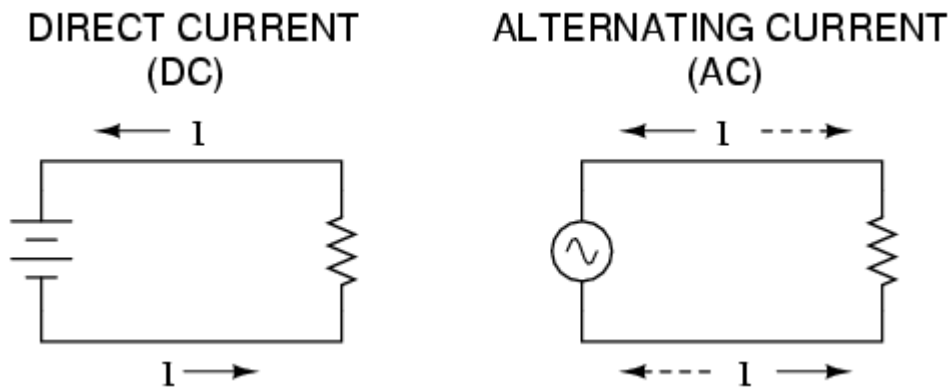


$$p(t) = \frac{\mathrm{d}E}{\mathrm{d}t}$$

## TOPIC 2&3: BASIC DC/AC CIRCUITS

Most students of electricity begin their study with what is known as *direct current* (DC), which is electricity flowing in a constant direction, and/or possessing a voltage with constant polarity. DC is the kind of electricity made by a battery (with definite positive and negative terminals), or the kind of charge generated by rubbing certain types of materials against each other.

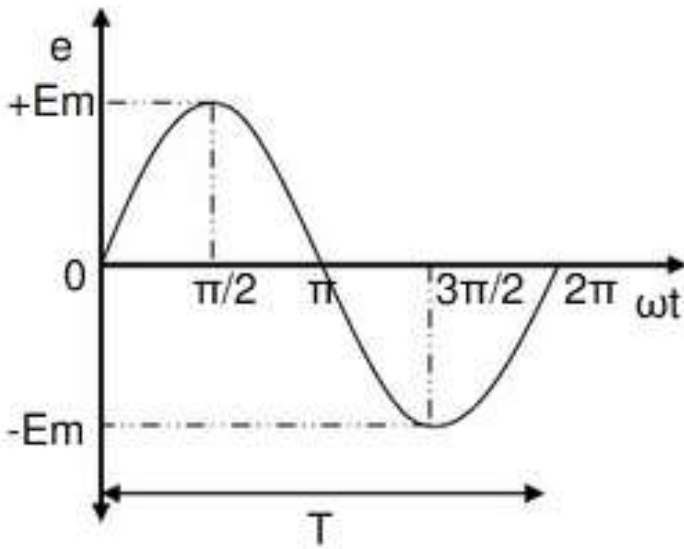
As useful and as easy to understand as DC is, it is not the only “kind” of electricity in use. Certain sources of electricity (most notably, rotary electro-mechanical generators) naturally produce voltages alternating in polarity, reversing positive and negative over time. Either as a voltage switching polarity or as a current switching direction back and forth, this “kind” of electricity is known as Alternating Current (AC): Figure [below](#)



### SIMPLE AC CIRCUITS

#### Single Phase AC Circuit

Definition of Alternating Quantity



An alternating quantity changes continuously in magnitude and alternates in direction at regular intervals of time. Important terms associated with an alternating quantity are defined below.

**1. Amplitude**

It is the maximum value attained by an alternating quantity. Also called as maximum or peak value

**2. Time Period (T)**

It is the Time Taken in seconds to complete one cycle of an alternating quantity

**3. Instantaneous Value**

It is the value of the quantity at any instant

**4. Frequency (f)**

It is the number of cycles that occur in one second. The unit for frequency is Hz or cycles/sec. The relationship between frequency and time period can be derived as follows.

- Time taken to complete  $f$  cycles = 1 second
- Time taken to complete 1 cycle =  $1/f$  second

$$T = 1/f$$

Cycle - A complete repeat of a sinusoidal wave form

## Passive Components in AC Circuits

**Active:** Those devices or components which produce energy in the form of Voltage or Current are called as Active Components

**Passive:** Those devices or components which store or maintain Energy in the form of Voltage or Current are known as Passive Components

## COMPLEXER DIAGRAM/PHASOR DIAGRAM

## Introduction

In AC electrical theory every power source supplies a voltage that is either a sine wave of one particular frequency or can be considered as a sum of sine waves of differing frequencies. The neat thing about a sine wave such as  $V(t) = A\sin(\omega t + \delta)$  is that it can be considered to be directly related to a vector of length  $A$  revolving in a circle with angular velocity  $\omega$  - in fact just the y component of the vector. The phase **constant**  $\delta$  is the starting angle at  $t = 0$ . In Figure 1, shows this



Figure 1

A 2D drawing of a rotating vector shows the vector inscribed in the centre of a circle as indicated in Figure 2 below. The angular frequency  $\omega$  may or may not be indicated.

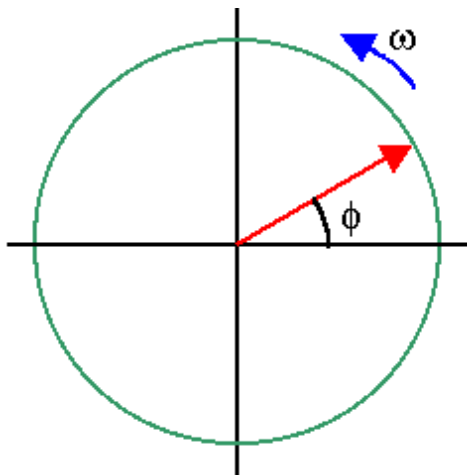


Figure 2

When two sine waves are produced on the same display, one wave is often said to be *leading* or *lagging* the other. This terminology makes sense in the revolving vector picture as shown in Figure 3. The blue vector is said to be leading the red vector or conversely the red vector is

lagging the blue vector.

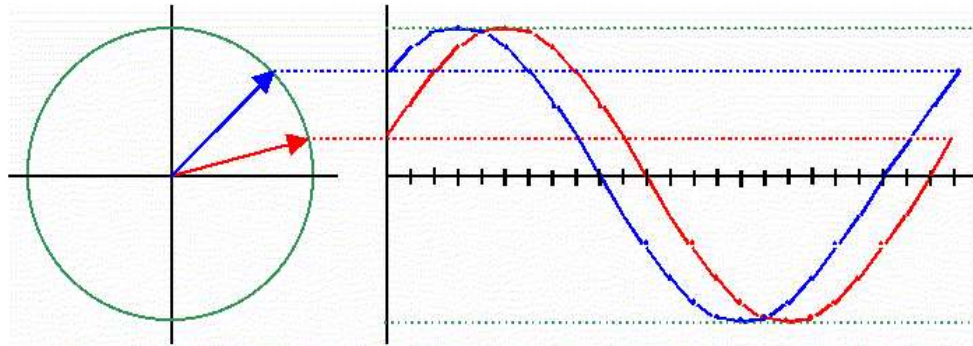


Figure 3

## Phasor Diagrams

A diagram giving relationship between sinusoidal alternating currents and volts in simple ac circuits

### Operator J

X-axis → reference direction

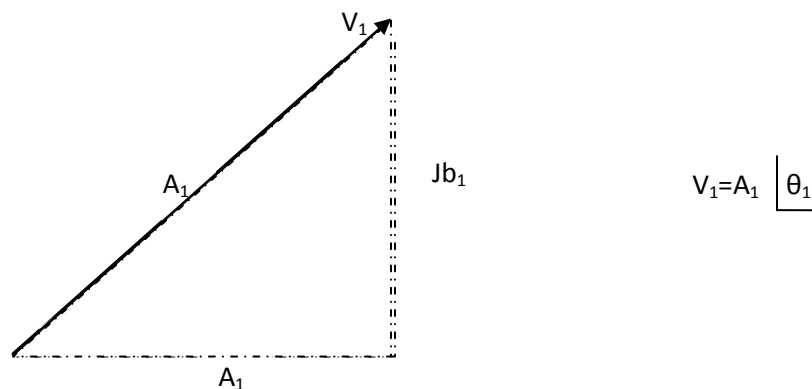
Y-axis s → quadrate direction

Complexer → subject of measurement

Phase angle → Complexer angle represented as the angle turned through (anti clockwise) from the positive reference direction to the direction of the Complexer

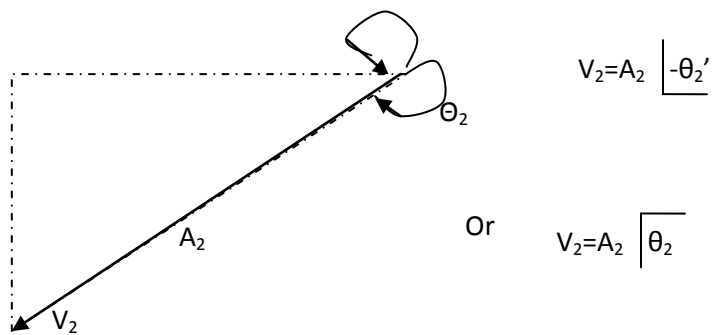
A Complexer may be completely described by :

- i. Statement, or its magnitude, with respect to a given scale unit.
- ii. A statement or its phase with respect to a reference direction.



IF PHASE ANGLE IS  $>180^\circ$ , negative phase angle (phase angle measured in clockwise direction) is stated

$\theta_2$



Or  $V_2 = A_2 \angle \theta_2$

Where

$$\theta_2' = 360 - \theta_2$$

This method described a Complexer is called polar notation.

**A**=magnitude operator or modulus.

**$\theta$** =phase operator or anticlockwise direction into which a Complexer in the reference direction must be turned to take the direction of the given Complexer

i.e.  $-1$  and  $180^\circ$  or  $\angle -180^\circ$  are identical

**j** represents the operation of turning a complexer trough  $90^\circ$  in an anti-clockwise direction hence

$$jb = b \angle 90^\circ$$

**jb** is a Complexer of length **b** in the quadrate direction

$$-jb = -(jb) = -b \angle 90^\circ$$

Above is a Complexer of length **b** with negative quadrate direction

Note

It's convenient to represent a Complexer by the sum of 2 components, one of which is in either the positive or negative reference direction while the other is either the positive or negative quadrant direction thus

$$V_1 = A_1 \angle \theta_1 = a_1 + jb_1$$

Where

$$a_1 = A_1 \cos \theta_1 \text{ and } b_1 = A_1 \sin \theta_1$$

And

$$V_2 = A_2 \angle \theta_2 = -a_2 - jb_2$$

Where

$$a_2 = -A_2 \cos \theta_2 = A_2 \cos (\pi - \theta_2) \quad \text{and} \quad b_2 = -A_2 \sin \theta_2 = -A_2 \sin (\pi - \theta_2)$$

The 'a + jb' method of describing Complexes is termed rectangular notation

$$V_1 = A_1 (\cos \theta + j \sin \theta) \text{ -trigonometry notation}$$

Since **j** is an operation that turns a Complex number  $+90^\circ$  without changing its size,  $j^2$  operations by **j** will turn a Complex number a total of  $180^\circ$  from the original reference direction.

$$j(j a) = j^2 a = -1 \times a$$

Algebraically  $j^2 = -1$

$$j = 90^\circ$$

$$j^2 = 180^\circ$$

$$j^3 = 270^\circ$$

## SIMPLE AC IMPEDANCE

Impedance –limitations of the subject quantity or measurement. Impedance is quantity that can be represented by complex operation.

### Pure resistance

Suppose a sinusoidal current by Complex **I** is passed through a pure resistance **R**. The pd across **R** will be a sinusoidal voltage represented by Complex **V**, where **V** and **I** are in phase with one another and where  $|V|$  and  $|I|$  is equal to **R**.

If **I** is chosen as positive reference Complex, then:

$$I = |I| \angle 0^\circ = |I| \angle 0^\circ$$

HENCE

$$V = |V| \angle 0^\circ = |V| \angle 0^\circ$$

And the impedance is given by

$$Z = \frac{V}{I} = \frac{|V| \angle 0^\circ}{|I| \angle 0^\circ} = R$$

### Pure inductance

Suppose that a semisolid current represented by the Complexer I is passed through a pure inductance L, the pd across L will be a semisolid voltage represented by Complexer V, where V leads I by  $90^\circ$ .

IF  $|V|/|I|$  is equal to  $\omega L$

$$\omega = 2\pi f$$

If  $|I|$  is chosen as the ref Complexer, then

$$I = |I| \angle 0^\circ \quad \text{AND} \quad V = |V| \angle 90^\circ = jV$$

Therefore the impedance

$$Z = \frac{V}{I} = \frac{|V|}{|I|} \angle 90^\circ = j\omega L = j \times L$$

### Pure capacitance

Suppose that a semisolid current by the Complexer is passed through a pure capacitance C, the PD across C will be a semisolid conductor voltage represented by the Complexer V, where V lags I by  $90^\circ$  and  $V/I$  is equal to  $1/\omega C$

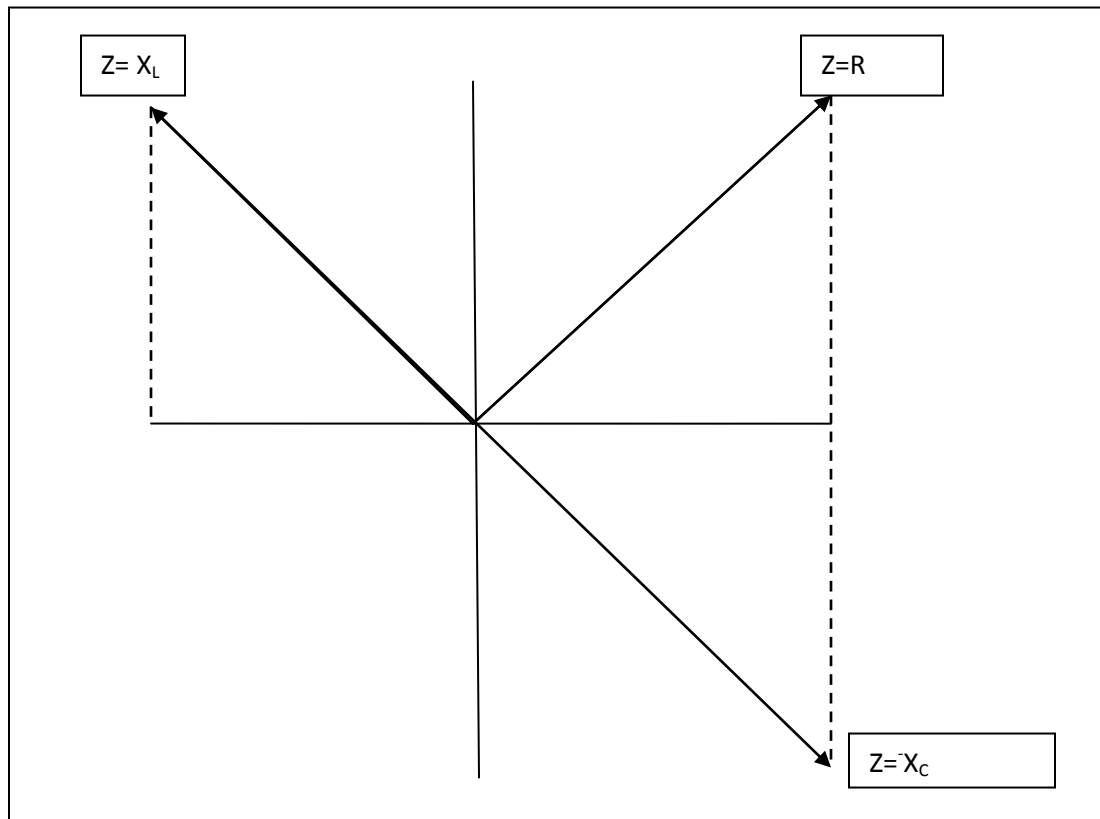
If I is chosen as the reference Complexer then

$$I = |I| \angle 0^\circ \quad \text{AND} \quad V = |V| \angle -90^\circ = -jV$$

$$Z = \frac{V}{I} = \frac{|V|}{|I|} \angle -90^\circ = \frac{-j}{\omega C} = \frac{-j}{j\omega C} = \frac{1}{j\omega C}$$

$$\text{i.e. } Z = \frac{1}{j\omega C} = -j \times C$$





If the resistance and reactance of circuit one expressed as reference and quadrature operations than the total impedance of the circuit may be determined

$Z = R + j\omega L$  - representing impedance of a circuit in which a resistance  $R$  is connected in series with an inductive reactance  $\omega L$  across a supply of frequency

$Z = R - j/\omega C$  - for a resistance and capacitance in series

For a circuit with  $R$ ,  $L$  and  $C$  in series, the impedance in complex form is

$$Z = R + j\omega L - \frac{j}{\omega C}$$

$$= R + j \left( \omega L - \frac{1}{\omega C} \right)$$

$$= R + j(X_L - X_C)$$

$$= Z_1 + Z_2 + Z_3$$

## Direct Current Circuits

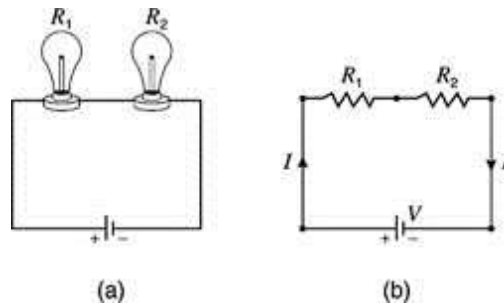
In a simple circuit that is used to light a bulb with a battery, the battery provides **direct current**—a current flowing in only one direction. This section is concerned with the analysis of

simple direct current circuits of two types: (1) those with combinations of resistor elements and (2) those with batteries in different branches of a multiple-loop circuit.

### Series and parallel resistors

Resistance, at least to some degree, exists in all electrical elements. The resistors might be light bulbs, heating elements, or components specifically manufactured for their resistance. It is assumed that the resistance in the connecting wires is negligible.

The series connection of two resistors ( $R_1$  and  $R_2$ ) is shown in Figure 1. What is the equivalent resistor for this combination?



**Figure 1**

Two resistors connected in series. The drawing (a) is equivalent to the schematic (b).

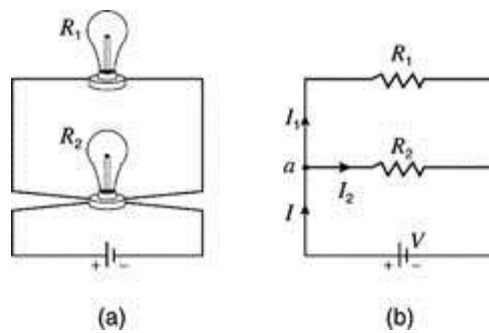
Because there is only one pathway for the charges, the current is the same at any point in the circuit, that is,  $I = I_1 = I_2$ . The potential difference supplied by the battery equals the potential drop over  $R_1$  and the potential drop over  $R_2$ . Thus,

$$V = V_1 + V_2$$

from Ohm's law,  $V = IR_1 + IR_2$   
 and  $V = I(R_1 + R_2)$   
 therefore,  $R_{eq} = R_1 + R_2$

When resistors are in series, the equivalent resistance is the sum of the individual resistances. Compare this result with adding capacitors in series. For series resistors, the current is the same; while for series capacitors, the charge is the same. (Note that the equivalent resistance is a simple sum, but the equivalent capacitance is given by a reciprocal expression.)

The parallel connection for two resistors ( $R_1$  and  $R_2$ ) is shown in Figure 2. What is the equivalent resistance for this combination?



**A circuit illustrating the application of Kirchhoff's rules, and the resulting equations.**

**Figure 2**

Two resistors connected in parallel. The drawing (a) is equivalent to the schematic (b).

At point *a* for the circuit diagram—see Figure (b)—the current branches so that part of the total current in the circuit goes through the upper branch and part through the lower branch. The potential drop of the current is the same regardless of which path is taken; therefore, the voltage difference is the same over either resistor ( $V_{batt} = V_1 = V_2$ ). The currents sum to the total

current:  $I = I_1 + I_2$  from Ohm's law,  $I = \frac{V}{R_1} + \frac{V}{R_2}$ , and  $I = V \left( \frac{1}{R_1} + \frac{1}{R_2} \right)$  therefore,  

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

Thus, the reciprocal of the equivalent resistance is equal to the sum of the reciprocals of the individual resistors in the parallel combination. Compare this result with adding capacitors in parallel. For parallel resistors, the voltages across the resistors are equal, and the same is true for parallel capacitors. (Note that the equivalent resistance is a reciprocal expression, but the equivalent capacitance for parallel combination is a simple sum.)

### Kirchhoff's rules

If a circuit has several batteries in the branches of multiloop circuits, the analysis is greatly simplified by using **Kirchhoff's rules**, which are forms of conservation laws:

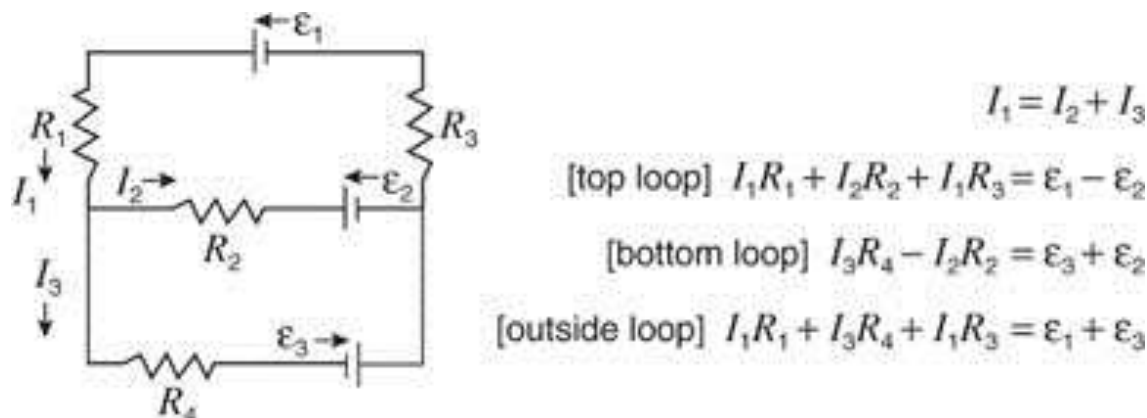
- The sum of the currents entering a junction must equal the sum of the currents leaving the junction. This rule, sometimes called the **junction rule**, is a statement of conservation of charge. Because charge neither builds up at any place in the circuit nor leaves the circuit, the charge entering a point must also leave that point.
- The algebraic sum of the drops in potential across each element around any loop must equal the algebraic sum of the emfs around any loop. This rule expresses conservation of energy. In other words, the charge moving around any loop must gain as much energy from batteries as it loses when going through resistors.

When applying Kirchhoff's rules, use consistent sign conventions. Refer to the directions selected for the currents in Figure. Fewer mistakes will be made if one direction is consistently used—for example, clockwise in all loops. If an incorrect direction for one current is selected

initially, the solution for that current will be negative. Use the following sign conventions when applying the loop rule:

- If the resistor is traveled in the direction of the current, the change in potential is negative, and if traveled opposite to the selected direction of the current, it is positive.
- If a source of emf is traveled in the direction of the emf (from – to + between the terminals), then the change in potential is positive, and if traveled opposite to the direction of the emf, it is negative.

Check the equations for Figure 3.



**Figure 3**

A circuit illustrating the application of Kirchhoff's rules, and the resulting equations.

Imagine that the values of the resistances and voltage were given for this problem. Then, it would be possible to write four different equations: the junction equation, the top loop, the bottom loop, and the outside loop. Only three currents exist, however, so only three equations are necessary. In this case, solve the set of equations that are the easiest to manipulate.

## Resistivity of conductors

### Overview

- Resistivity is a measure of the resistance to electrical conduction for a given size of material. Its opposite is electrical conductivity ( $=1/\text{resistivity}$ ).
- Metals are good electrical conductors (high conductivity and low resistivity), while non-metals are mostly **poor** conductors (low conductivity and high resistivity).
- The **more** familiar term electrical resistance measures how difficult it is for a piece of material to conduct electricity - this depends on the size of the piece: the resistance is higher for a longer or narrower section of material.

- To remove the effect of size from resistance, resistivity is used - this is a material property which does not depend on size.
- Resistivity is affected by temperature - for most materials the resistivity increases with temperature. An exception is semiconductors (e.g. silicon) in which the resistivity decreases with temperature.
- The ease with which a material conducts heat is measured by thermal conductivity. As a first estimate, good electrical conductors are also good thermal conductors.

### Design issues

- Resistivity is important in any product which conducts electricity. Components which must conduct easily (called "conductors") must have low resistivity, while those which must not conduct (called "insulators") must have high resistivity.
- Many products will contain both conductors and insulators, e.g. a 13A plug - the conductors take the electricity where it is wanted (the machine or appliance) and the insulators prevent it from getting where it isn't wanted (i.e. the user!)
- The resistivity of insulators and conductors differ by a huge factor - typically one million, million, million! Within a given class of materials (e.g. metals), the resistivity can still vary by a factor of 1000 or more. The differences look small compared to the difference between metals and insulators like polymers, but can be very significant in choosing the metal for a conductor - 1000 times more current for the same voltage.
- Electrical and thermal conductivity are closely related. Examples of products in which good thermal conduction is required are radiators and saucepans; **thermal insulation** is required for pan handles and cookers.

### Measurement

Observations of how the resistance varies with size of a sample suggest that resistivity obeys the relationship:

Electrical resistivity = resistance \* cross-sectional area / length

$$R = \frac{\rho l}{A}$$

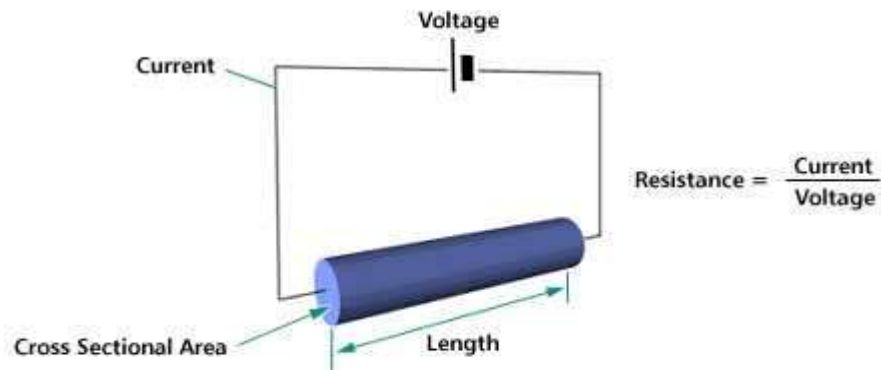
**R** is the resistance of the conductor in Ohms

**A** is the cross sectional area in m<sup>2</sup>

**l** is the length of the wire in meters

**ρ** is the resistivity of the material in Ohm(meters)

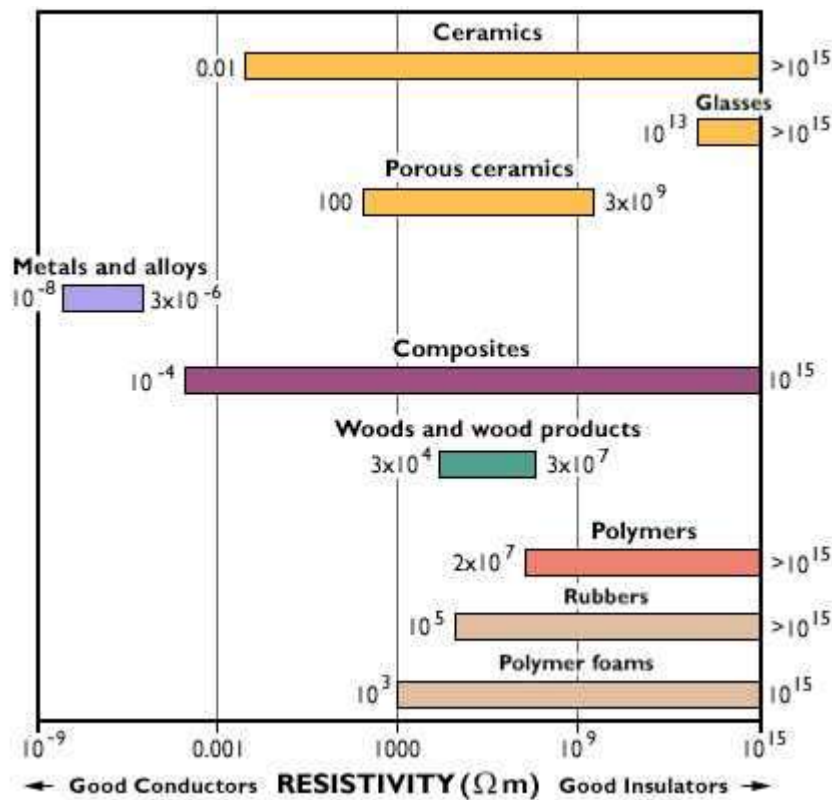
The resistivity can therefore be calculated quite easily by measuring the resistance of a piece of wire of **constant** cross-section and known length.



In practice resistivity is difficult to measure accurately because of the effect of temperature. More sophisticated techniques need to be used if accurate measurements are required, e.g. a Wheatstone bridge.

### Units & Values

From the definition of resistivity it can be seen that it has units of ohm-metres. Giving exact values can be difficult because of the strong temperature dependence. Data are usually given for room temperature.



# TOPIC 4: ELECTRONIC COMPONENTS

## Introduction & Characteristics of Electronic Components

An **electronic component** is any basic **discrete device** or physical entity in an electronic system used to affect [electrons](#) or their associated [fields](#). Electronic components are mostly [industrial products](#), available in a singular form and are not to be confused with [electrical elements](#), which are conceptual abstractions representing idealized electronic components.

Electronic components have a number of [electrical terminals](#) or [leads](#). These leads connect to create an [electronic circuit](#) with a particular function (for example an [amplifier](#), [radio receiver](#), or [oscillator](#)). Basic electronic components may be packaged discretely, as arrays or networks of like components, or integrated inside of packages such as [semiconductor integrated circuits](#), [hybrid integrated circuits](#), or [thick film](#) devices. The following list of electronic components focuses on the discrete version of these components, treating such packages as components in their own right.

## Classification & Application of Electronic Components

Components can be classified as [passive](#), [active](#), or [electromechanic](#). The strict physics definition treats passive components as ones that cannot supply energy themselves, whereas a [battery](#) would be seen as an active component since it truly acts as a source of energy.

However, [electronic engineers](#) who perform [circuit analysis](#) use a more restrictive definition of [passivity](#). When only concerned with the energy of [signals](#), it is convenient to ignore the so-called [DC](#) circuit and pretend that the power supplying components such as [transistors](#) or [integrated circuits](#) is absent (as if each such component had its own battery built in), though it may in reality be supplied by the DC circuit. Then, the analysis only concerns the AC circuit, an abstraction that ignores DC voltages and currents (and the power associated with them) present in the real-life circuit. This fiction, for instance, lets us view an oscillator as "producing energy" even though in reality the oscillator consumes even more energy from a DC power supply, which we have chosen to ignore. Under that restriction, we define the terms as used in [circuit analysis](#) as:

- **Active components** rely on a source of energy (usually from the DC circuit, which we have chosen to ignore) and usually can inject power into a circuit, though this is not part of the definition.<sup>[1]</sup> Active components include amplifying components such as [transistors](#), triode [vacuum tubes](#) (valves), and [tunnel diodes](#).
- **Passive components** can't introduce net energy into the circuit. They also can't rely on a source of power, except for what is available from the (AC) circuit they are connected to. As a consequence they can't amplify (increase the power of a signal), although they may increase a voltage or current (such as is done by a transformer or resonant circuit). Passive components include two-terminal components such as resistors, capacitors, inductors, and transformers.
- **Electromechanical components** can carry out electrical operations by using moving parts or by using electrical connections

Most passive components with more than two terminals can be described in terms of [two-port parameters](#) that satisfy the principle of [reciprocity](#)—though there are rare exceptions.<sup>[2]</sup> In contrast, active components (with more than two terminals) generally lack that property.

## Examples of Electrical Components

### 1. Resistors

Resistors- a device that limits the VMS applied voltage per unit VMS current flowing

-a device that measures the applied voltage per unit current, flowing in a connection terminal.

We have 2types of resistors

1. fixed resistor
2. variance resistors

**Fixed resistors** –include wire wound carbon composition, cracked carbon and tin-oxide resistors  
.have a fixed resistor

**Variable resistors**- a device with 3 terminals and resistance maybe measured at the two outer terminals and its value maybe valid.

Include thermostat, meter bridges etc.

### Color coding

The value of fixed resistors is usually indicated by color code.

The first color of this indicates the first digit of value of the resistor.

The second color the second digit and the third color indicates the number of zeros that follow the digit. A fourth color is often used to indicate tolerance limit of the resistor (if no four color, tolerance is taken as+- 20%)

Color	figure	Tolerance	
Black	0	Golden	±3%
Brown	1	Silver	±10%
Red	2	none	±20%
Orange	3		
Yellow	4		
Green	5		
Blue	6		
Violet	7		
Grey	8		



White	9	
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## Resistance

### Series

$$R_T = R_1 + R_2 + R_3$$

### Parallel

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2}$$

## 2. Capacitors

**Passive devices** – a device that is dependant of external power supply. They do not produce their own energy. I.e. resistors, capacitors etc

A capacitor is a passive electric component consisting of a pair of conductors separated by a dielectric (insulator) phase. If a voltage (potential difference) is applied across the conductors, a static electric field develops in the dielectric that stores energy and produces a mechanical force between the conductors. An ideal capacitor is characterized by a single constant value, capacitance, measured in farads. This is the ratio of the electric charge on each conductor to the PD between them.

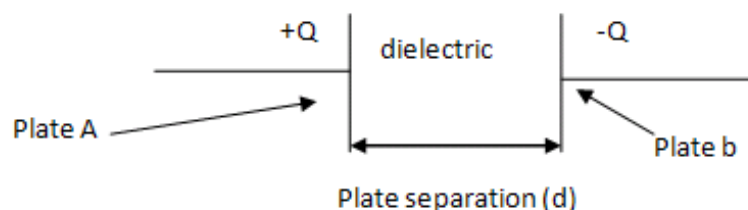
### Application

Electric circuits for blocking direct current, while allowing alternating current to pass.

Filter networks for smoothing the o/p of power supply in the resonant circuits that tune radios to a particular frequency.

### Operation

A capacitor consists of two conductors separated by a non-conductor region called the dielectric medium through it may be a vacuum or a semiconductor depletion region chemically identical to the conductors.



A capacitor is assumed to be self contained and isolated, with no net electric charge and no influence from any external electric field.

The conductors' turns held equal and opposite charges on their facing surfaces

And the dielectric develops an electric field in SI units a capacitor of one farad means that one coulomb of charges each conductor lunches a voltage of one volt across the device.

An ideal capacitor is wholly characterized by a constant capacitance C, defined the ratio of charge +- Q on each conductor to the voltage V between them

$$C = \frac{Q}{V}$$

Sometime the charge built -up affects the capacitors mechanically, consisting its capacitance to vary in each case, capacitance is defined in terms of increment charged

$$C = \frac{dq}{dv}$$

### Current voltage relationship

The current  $i(t)$  through any component I electric CCT is defined as the rate of flow of charge  $Q(t)$  passing through it but actual charges electrons can not pass through it but actual charged, electrons cumulates on the negative plate for each one that leaves the positive plate ,resulting in an electron depletion and consequent positive charges on one electrode that is equal and positive to the accumulated negative charges on the other.

Thus the charge on the electrodes is equal to the integral of the current as well as propositional to the voltage.

$$V(t) = \frac{q(t)}{C} = \frac{1}{C} \int_0^t i(T) dT + V(t_0) \quad \text{Initial voltage}$$

Taking derivatives of this and multiplying by C yields

$$i(t) = \frac{dq(t)}{dt} = C \frac{dv(t)}{dt}$$

**Note**

The dual of the capacitor is the inductor which store energy in the magnetic field other than electric field its current voltage relationship is obtained from exchanging current and voltage in the capacitor equation and replacing C with L

Capacitors connections

### Series

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_n}$$

### Parallel

$$C = C_1 + C_2 + C_n$$

### Capacitor type

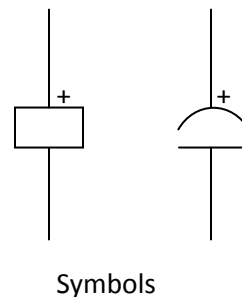
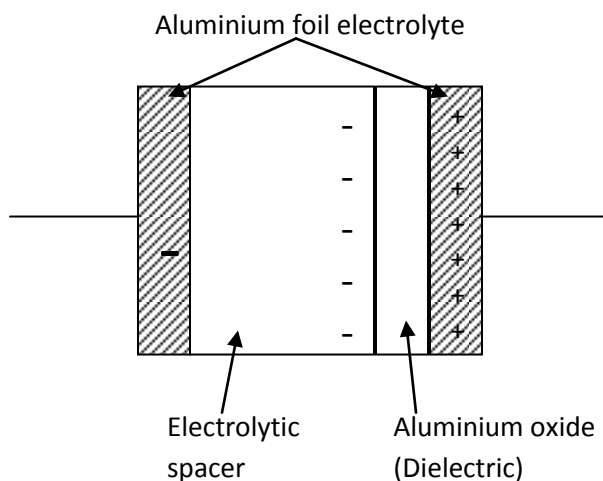
**Dielectric:** Usually the variable type, such as used for tuning transmitters, receivers and transistors radios.

**Film capacitors:** Most common consist of a relatively large family of capacitors including polyester, poly carbonate, metalized paper, teflon etc.

**Ceramic capacitors:** Made by coating two sides of a small porcelain or ceramic disc with silver then are stacked together to make a capacitor.

**Electrolytic capacitors:** For larger capacitance volume requirement, instead of thin mechanical film layer for one of the electrode, a semi liquid electrolyte solution in the form of jelly/paste is used to serve as the second electrode usually cathode.

The dielectric is a thin layer of oxide which is grown electro-chemically in production with the thickness of the film being less than 10microns



### 3. Inductor

An inductor/reactor is a passive electrical component that can store energy in a magnetic field created by the electric current passing through it. An inductor's ability to store magnetic energy is measured by its inductance, in units of henries.

An inductor is a conducting wire shaped as a coil, the loops helping to create a strong magnetic field inside the coil that induce a voltage that apposes the change in current. The current and voltage in an inductor change in time.

#### Operation:

Inductance (L) is an effect resulting from the magnetic field that forms around a current-carrying conductor which tends to resist change in the current. Electric current through the conductor creates a magnetic flux proportional to the current and a change in this current creates a corresponding change in magnetic flux which in turn generates an electromotive force (EMF) that oppose this change in current.

Inductance is a measure of the amount of EMF generated per unit Change in current.

i.e. – an inductor with inductance of 1henry produce an EMF of 1V when the current through the inductor change at the rate of 1amp/sec

$$V \text{ across } L \Rightarrow V = \frac{\Delta\phi}{\Delta T}$$

$$V \Rightarrow V = \frac{\Delta flux}{\Delta time}$$

But since  $\Delta flux$  is given by the inductance L and the change in current across the coil  $\Delta I$ , the voltage V is

$$V = L \times \frac{\Delta I}{\Delta T} \text{ - Electrical definition}$$

$$L = \mu N^2 \times A / l \text{ – physics definition}$$

Where

$\mu$  - Relative ease of current flow in the inductor (permeability)

N – Number of turns in the coil

A – Cross sectional area of the coil

l – Length of the coil

### Application

- Analog circuits and signal processing inductors + capacitors + other components form tuned circuits to emphasize or filter out specific signal frequencies.
- Two or more inductors which have coupled magnetic flux form a transformer
- Energy storage devices in some switched-mode power supplies
- Electrical transmission systems, where they are used to depress voltage from lightning strikes and to limit switching currents and fault current (reactors)

### Inductor connections (networks)

#### Parallel

$$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \dots + \frac{1}{L_n}$$

#### Series

$$L = L_1 + L_2 + \dots + L_n$$

### Types of inductors

- 1) Air core coil – the term refers to a coil wound on plastic, ceramic or other non magnetic forms as well as those that actually have air inside the winding.
- 2) Radio frequency inductor – RF, mostly are air core types but with specialized construction to minimize power loss
- 3) Ferro magnetic core - iron core, inductors using a magnetic core of ferromagnetic material such as iron or ferrite to increase inductance.
- 4) Laminated core inductor – the core is made of stacks of thin steel sheets or laminations oriented parallel to the field, with an insulating coating on the surface to avoid power loss.
- 5) Ferrite core inductor – ferrite is a ceramic ferromagnetic material that is non conductive so eddy current cannot flow within it.
- 6) Steroidal coils – for higher magnetic fields and inductance, we carry out windings of a coil on a steroidal or doughnut shaped ferrite core. The magnetic field lines form closed loops within the doughnut without leaving the core material. Since little of their magnetic flux is outside the core, they radiate less electromagnetic interference than straight coils.
- 7) Variable inductor - a variable inductor can be constructed by making one of the terminals of the device a sliding spring contact that can move along the surface of the coil, increasing/decreasing the number of turns of the coil in the circuit. An alternative construction method is to use a movable magnetic core which can slide in or out of the coil; moving the core further into the coil increase permeability thus inductance

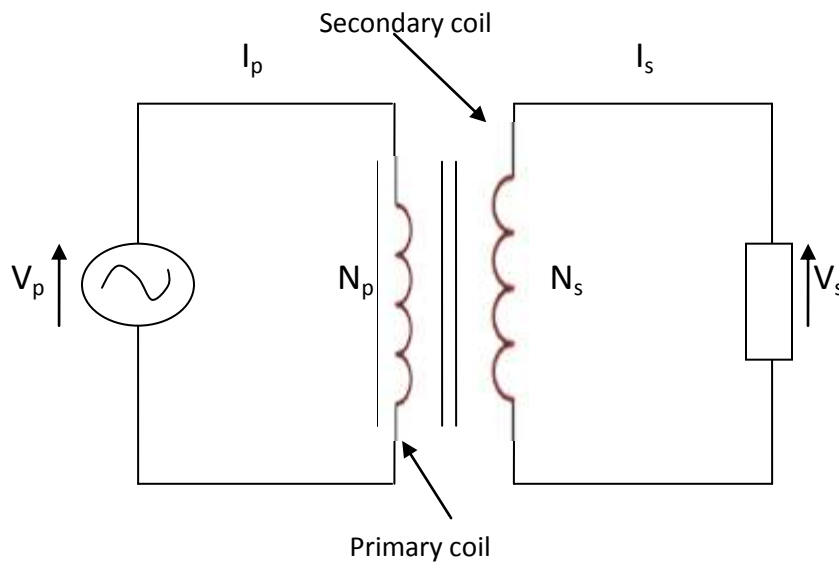
## **4. Transformer**

A device that transfers electrical energy from one circuit to another through inductively coupled conductors – the transformer coils.

A varying current in the first(primary winding) creates a varying magnetic flux in the transformer core, and thus a varying magnetic field through the secondary winding. This varying magnetic field induces a varying electrical force (EMF) or voltage in the secondary winding. This effect is called induction.

**Note** If in the secondary coil, there develops a magnetic field that will induce a voltage to appose the change in current, then the effect is called self induction.

### Basic operation



If the secondary coil is attached to a load, that allows current to flow , electrical power is transmitted from the primary circuit to the secondary circuit.

Ideally, the transformer is perfectly efficient; all the incoming energy is transformed from the primary circuit to the magnetic field and into the secondary circuit. If this condition is met, the incoming electric power must equal to outgoing power.

$$P_{\text{incoming}} = I_p V_p = P_{\text{out going}} = I_s V_s$$

$$\text{Therefore } \frac{V_p}{V_s} = \frac{I_s}{I_p}$$

By induction code

$$V_s = N_s \frac{d\phi}{dt} \quad \text{and} \quad V_p = N_p \frac{d\phi}{dt}$$

Therefore  $\frac{v_p}{v_s} = \frac{N_p}{N_s}$

The impedance in one circuit is transformed by a square of the turn's ratio

1. E.

- If impedance  $Z_2$  is attached across the terminals of the secondary coil, it appears too on the primary circuit as

$$Z_s \left( \frac{N_p}{N_s} \right)^2$$

- This means relation is reciprocal, so that the impedance  $Z_p$  of the primary circuit appears to the secondary as

$$Z_p \left( \frac{N_s}{N_p} \right)^2$$

### Types of transformers

- 1) Auto transformer - for stepping up/down between voltages in the 110-117-120 range and 220-230-240 range
- 2) Poly phase transformer – for three phase supplies
- 3) Leakage transformer
  - Also called stray-field transformer has a significantly higher leakage inductance than other transformers.
  - used for arc welding and high voltage discharge lamps (neon lamps and low cathode fluorescent lamps)
- 4) Resonant transformer
  - uses leakage inductance of its secondary winding in combination with external capacitors, to create one or more resonant circuits
  - Generates very high voltage and provides much higher current
  - Applied in CCFL inverter, couple between stages of a super heterodyne receiver
- 5) Audio transformer – designed for use in audio circuits

#### Application

- Block radio frequency interference or the DC component of an audio signal
- Split or combine audio signals
- Provide impedance matching between high and low impedance circuits i.e.
  - o between high impedance(valve) amplifier output and a low impedance loud speaker
  - o between a high impedance instrument output and the low impedance input of a mixing console

- 6) Instrument transformer - used for measuring voltage and current in electrical power systems and for power system protection and control. Where a voltage or current is too large to be conveniently used by an instrument, it can be scaled down to a standardized, low value

## 5. Transducers

Device that converts energy from one form to another. Most transducers convert electrical energy to mechanical or to a physical quantity as temperature, sound, light, vibrations, pressure etc or vice versa

### #Functions of a transducer

- 1) Sense the presence of a magnitude, charge or frequency of a measurand
- 2) Provides an electrical output which when appropriately processed and applied to a read out device gives an accurate quantitative data about a measurand.

Measurand – quantity, property or condition which transducer converts to electrical energy

### Classifications of transducers

This is according to following

- 1) Electrical principle involved in the operation
- 2) Application of the transducer which is based on the physical property or condition being measured
- 3) The excitation principle  
e.g.
  - passive transducers require an external power and its output is a measure of some variation as resistance, capacitance e.t.c.
  - self generating transducers (active) requires external power thus produce an analog voltage or current when stimulated by some physical form of energy.

There are two main categories of transducers

- Passive
- Active

**Note:** passive elements – is dependent of an external supply where as active elements generates part of or all its power supply

### Passive transducers

A) Variable resistance transducer

Class and example	Nature of device	Quantity measured or application
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1. Resistance strain gauge	Resistance wire foil semiconductor stress wire	Strain, force, torque, pressure
2. Resistance thermometer	Resistance wire thermistor with a large temp coefficient of resistivity	Temp: temp effect with radiant heat
3. Thermistor radio meter	Radiation transducer on the thermistor	Missile and satellite tracking

#### B) Variable inductance

Class and example	Nature of device	Quantity measured or application
1. Magnetostriction gauge	Magnetic property varied by pressure or stress	Sound, pressure, force
2. Hall-effect pick-up	Magnetic field interact with current through a semiconductor to give voltage	Field strength, current

#### C) Variable capacitance

Class and example	Nature of device	Quantity measured or application
1. Condenser microphone	Capacitance between a diaphragm and a fixed electrical varied by sound or pressure	Speech, music, noise vibrations
2. Dielectric gauge	Capacitance varied by change in dielectric	Liquid level thickness

#### Active transducers

Class and example	Nature of device	Quantity measured or application
1. Moving coil generator	Relative movement or the coil varies output voltage	Vibrations, velocity speed of displacement
2. Thermocouple or thermopile	Pair of dissimilar metal semi conductor at different temp	Temp differences, heat radiations, heat flow
3. Photovoltaic cell	semi conductor cell or transistor generates voltage from light	Exposure meter, light meter charging solar batteries
4. Piezo-electric pick-up	Quartz or other crystal mounted in compression or bending or twisting mode	Vibration, deceleration heat flow

## **Introduction & Characteristics of Integrated Circuit**

An **integrated circuit** or **monolithic integrated circuit** (also referred to as an **IC**, a **chip**, or a **microchip**) is a set of electronic circuits on one small flat piece (or "chip") of semiconductor material, normally silicon. The integration of large numbers of tiny transistors into a small chip results in circuits that are orders of magnitude smaller, cheaper, and faster than those constructed of discrete electronic components. The IC's mass production capability, reliability and building-block approach to circuit design has ensured the rapid adoption of standardized ICs in place of designs using discrete transistors. ICs are now used in virtually all electronic equipment and have revolutionized the world of electronics. Computers, mobile phones, and other digital home appliances are now inextricable parts of the structure of modern societies, made possible by the small size and low cost of ICs.

### **Merits and Demerits of ICs**

#### Advantages of Integrated Circuits

1. Miniature in size. As fabrication process is used for the integration of active and passive components on to a silicon chip, the IC becomes a lot smaller. When compared to a discrete circuit, it may be at least a thousand times smaller.
2. Due to small size, the weight of the IC also reduces, when compared to the discrete circuit.
3. To produce hundreds of discrete circuits on a PCB for the same logic takes more time and increase the cost factor. But for the production of hundreds of IC's the cost of production will be very low and less time consuming.
4. The PCB consisting soldered joints will be less reliable. This problem is omitted in IC's because of no soldered joints, with fewer interconnections, and thus highly reliable.
5. The small size of IC's causes lesser power consumption and lesser power loss.
6. In a discrete circuitry, if a single transistor becomes faulty, the whole circuit may fail to work. This transistor has to be desoldered and replaced. It is difficult to find out which component has failed. This problem can be omitted in an IC by replacing an entire IC as it is low in cost.
7. Increased operating speed because of absence of parasitic capacitance effect.
8. As the IC's are produced in bulk the temperature coefficients and other parameters will be closely matching.
9. Improved functional performance as more complex circuits can be fabricated for achieving better characteristics.
10. All IC's are tested for operating ranges in very low and very high temperatures.
11. As all the components are fabricated very close to each other in an IC, they are highly suitable for small signal operation, as there won't be any stray electrical pickup.

12. As all the components are fabricated inside the chip, there will not be any external projections.

#### Disadvantages of Integrated Circuits

1. Some complex IC's maybe costly. If such integrated circuits are used roughly and become faulty, they have to be replaced by a new one. They cannot be repaired as the individual components inside the IC are too small.
2. The power rating for most of the IC's does not exceed more than 10 watts. Thus it is not possible to manufacture high power IC's.
3. Some components like transformers and inductors cannot be integrated into an IC. They have to be connected externally to the semiconductor pins.
4. High grade P-N-P assembly is not possible.
5. The IC will not work properly if wrongly handled or exposed to excessive heat.
6. It is difficult to achieve low temperature coefficient.
7. It is difficult to fabricate an IC with low noise.
8. It is not possible to fabricate capacitors that exceed a value of 30pF. Thus, high value capacitors are to be connected externally to the IC.
9. There is a large value of saturation resistance of transistors.

#### **Characteristics of digital ICs**

1. Fan out
2. Power dissipation
3. Propagation Delay
4. Noise Margin
5. Fan In
6. Operating temperature
7. Power supply requirements

**1. Fan-out:** Fan out specifies the number of standard loads that the output of the gate can drive without impairment of its normal operation

**2. Power dissipation:** Power dissipation is measure of power consumed by the gate when fully driven by all its inputs.

**3. Propagation delay:** Propagation delay is the average transition delay time for the signal to propagate from input to output when the signals change in value. It is expressed in ns.

**4. Noise margin:** It is the maximum noise voltage added to an input signal of a digital circuit that does not cause an undesirable change in the circuit output. It is expressed in volts.

**5. Fan in:** Fan in is the number of inputs connected to the gate without any degradation in the voltage level.

**6. Operating temperature:** All the gates or semiconductor devices are temperature sensitive in nature. The temperature in which the performance of the IC is effective is called as operating temperature. Operating temperature of the IC vary from 00 C to 700 c.

# TOPIC 5: SEMICONDUCTOR

## 1. Atomic Structure & Semiconductor Theory

### Introduction

There are three basic **types of materials** that we are concerned with in electronics. These are conductors, semiconductors and insulators.

Materials that have very low electrical resistivity (in the order of  $1 \times 10^{-6}$  ohm-metres) are called conductors.

Materials that have very high electrical resistivity (in the order of  $1 \times 10^{13}$  ohm-metres) are called insulators.

Semiconductors are materials that have resistivity values in between those of conductors and insulators, they are neither good conductors nor good insulators.

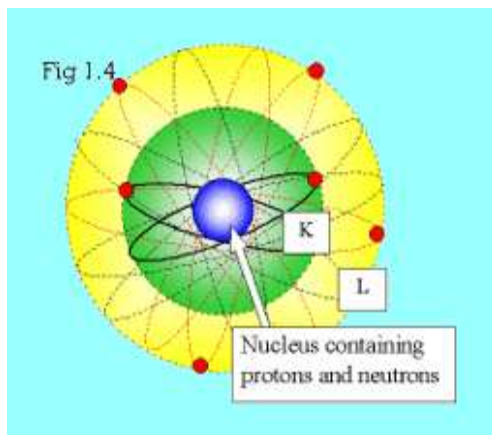
Examples of conductors:	Examples of insulators:	Examples of Semiconductors:
<ul style="list-style-type: none"><li>• Copper</li><li>• Aluminium</li><li>• Silver</li><li>• Gold</li></ul>	<ul style="list-style-type: none"><li>• Rubber</li><li>• PVC</li><li>• Paper</li><li>• Mica</li></ul>	<ul style="list-style-type: none"><li>• Silicon</li><li>• Germanium</li></ul>

Semiconductor materials are used to make a range of devices that are used in modern electronic circuits. In order to understand how these devices work we must first gain an understanding of the electrical properties of naturally occurring (**intrinsic**) semiconductors. We then need to learn about the electrical properties of **extrinsic** semiconductors. Extrinsic semiconductor material is just a naturally occurring pure semiconductor material that has been modified by a manufacturing process.

First we will look at the atomic structure of intrinsic semiconductors to understand their electrical properties. Then we will look at how this structure is modified to produce extrinsic semiconductor material and how this changes the materials electrical properties. Finally we will look at the construction and operation of a semiconductor diode which is the most basic semiconductor device used in electronic circuits. During this section we will consider "true" electron flow rather than conventional current flow which is used in **electrical circuit** analysis.

## 2. Electrons in Conductors & Semi-conductors (Outline of atomic theory)

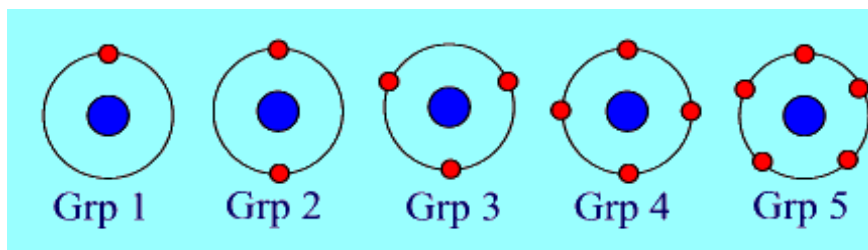
An atom can be thought of as a central positively charged nucleus orbited by negatively charged electrons. The positive charge of the nucleus is due to the positively charged protons it contains. For an atom in its natural state the total negative charge of the electrons is equal in magnitude to the positive charge of the nucleus. Therefore the atom is electrically neutral. The **orbits** of the electrons are arranged in shells. The first shell is closest to the nucleus and contains a maximum of two electrons. The next outer shell contains a maximum of eight electrons. The next shell also contains a maximum of eight electrons.



## Element groups

One way of categorising the atoms of different elements is by the number of electrons in their outer shell.

- An atom with one electron in its outer shell is called a group one element
- An atom with two electrons in its outer shell is called a group two element
- An atom with three electrons in its outer shell is called a group three element
- An atom with four electrons in its outer shell is called a group four element
- An atom with five electrons in its outer shell is called a group five element etc.



Semiconductors are group four elements i.e. they have four electrons in their outermost shell. (The electrons in the outer shell of an atom are called **valence electrons**). Note it is important to point out that the element groups refer to the number of electrons in the outermost shell of the atom, the total number of electrons in the atom can be greater i.e. for atoms that have inner shells that also contain electrons. Also many atoms have less electrons in their outer shell than the maximum number that the shell can hold. e.g. for group 4 materials silicon and germanium the outer shell can hold a maximum of 8 electrons so the outer shell is only half full. The outer shell of the atom is **more** stable when it is completely full.

## Periodic Table of Groups of Elements

1																	18
1 H	2											13	14	15	16	17	2 He
3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
11 Na	12 Mg	3	4	5	6	7	8	9	10	11	12	13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
55 Cs	56 Ba	*	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
87 Fr	88 Ra	**	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Uut	114 Uuq	115 Uup	116 Uuh	117 Uus	118 Uuo
* Lanthanides			57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu
** Actinides			89 Ac	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr

#### Color Key

Alkali Metal	Alkaline Earth	Inner- Transition	Transition Metal	Basic Metal	Semi Metal	Nonmetal	Halogen	Noble Gas
		Lanthanide						
		Actinide	Transactinide					

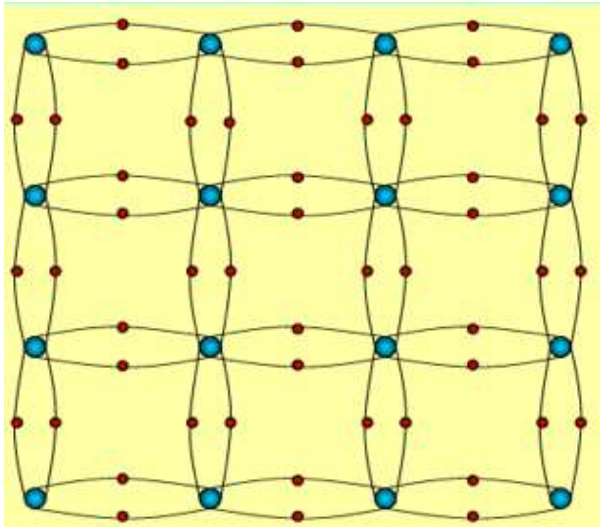
## 3. Semiconductor Materials

### Intrinsic semiconductor materials

The naturally occurring semiconductor materials that are used to manufacture electronic devices are Silicon and Germanium (Germanium is an older choice of material which is less used today). The following text refers to silicon but is equally applicable to germanium.

#### Pure Silicon

First a very pure crystal of silicon must be produced. The atomic structure of the silicon can be represented by the diagram below. Silicon like all semiconductors is a group 4 element and its atoms have only four electrons in the outer shell ( 4 valence electrons). It takes eight electrons to fill the outer shell and make it stable. The atoms share their valence electrons with neighbouring atoms so that each atom effectively contains eight electrons in the outer shell. This sharing of valence electrons with neighbouring atoms forms covalent bonds. It is these covalent bonds that bind the atoms together.



- The silicon atoms form a square lattice
- Each silicon nucleus has four electrons in its outer shell
- These electrons are paired with the corresponding electrons in adjacent atoms.
- These are called covalent bonds. Covalent bonds are what binds the material together
- The net result is that each nuclei (along with the electrons in the inner shells) are surrounded by eight outer electrons tightly bound in the atomic structure.

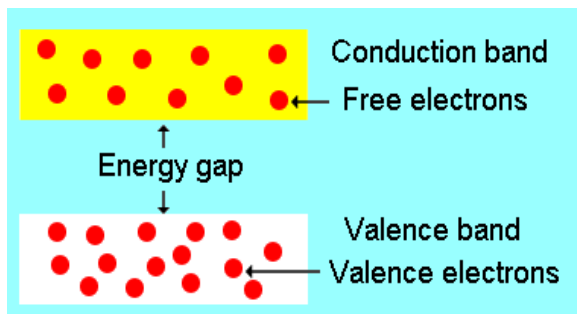
*Note this is a simplified diagram showing a 2 dimensional representation of the structure of silicon. Obviously silicon has a 3 dimensional structure and the covalent bonds do not really lie in a single plane as shown in the diagram. The actual arrangement of covalent bonds forms a shape called a tetrahedron.*

This diagram does give a good representation of how the electrons are bound to the atoms. This reflects the fact that there are no free electrons to produce an electrical current if a voltage is applied to the material. However an energy level diagram is better for explaining more about the electrical properties of silicon.

## Energy band diagrams

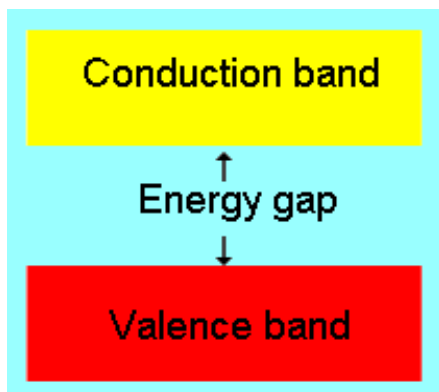
Energy band diagrams show the energy levels of the electrons in the material. We are only interested in two of the bands, the conduction band and the valence band. The valence band is occupied by the electrons with the highest energy level of those which are still attached to their parent atoms, these are the outer most (or valence) electrons. The conduction band is occupied by electrons which are free from their parent atoms. These electrons are free to move through the material. (When a voltage is applied these electrons will drift to produce an electrical current.) In semiconductors there is an gap between the valence and conduction bands. This energy gap reflects the amount of energy that would be needed to remove an electron from it's parent atom (ie to transfer it from the valence to the conduction band).





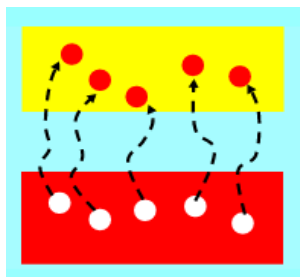
### Energy band diagram for silicon

The number of valence electrons in the pure silicon is enough to completely fill the valence band (so no movement of electrons can occur in the valence band). There are no free electrons, therefore the conduction band is completely empty.



### Electron-Hole pair generation

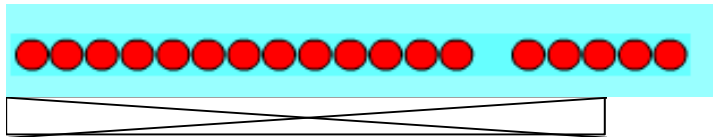
As previously stated there is an energy gap between the **conduction** and valence band in semiconductors. However the energy required to jump this gap can be supplied to the electrons from heat energy. (This means that if the material is heated some electrons will acquire enough energy to **break free** of their parent atoms to become free electrons.) At **room temperature** some electrons will have acquired the energy to jump into the conduction band. If the temperature is increased so will the number of electrons in the conduction band. This process is called **electron-hole pair generation**. This is because by supplying energy we transfer an electron from the valence band to the conduction band. This produces a free electron in the conduction band and leaves a hole (vacant electron position) in the valence band.



The free electrons are now available to contribute to an electrical current if a voltage is applied to the material. Also the holes (the vacant electron positions) in the valence band will now allow movement of electrons in the valence band, this can also contribute to an electrical current. The net effect is that heat increases the conduction properties of a pure semiconductor.

## Holes (virtual particles!)

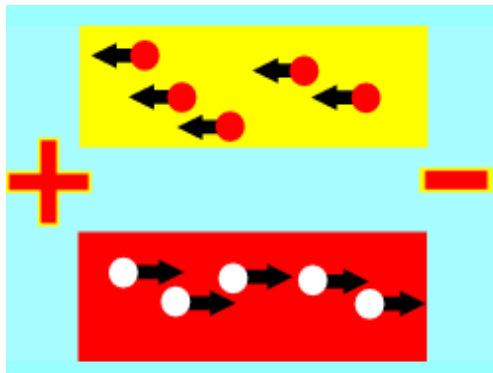
The movement of electrons in the valence band is complicated and as they move to fill the vacant electron positions the position of the hole appears to move in the opposite direction to the electrons. In fact it is easier to consider the movement of the hole and we can imagine it to be a positively charged particle because of the direction it moves in (opposite to that of negatively charged electrons).



At first this may seem a strange idea but later you will see that by considering the movement of these **virtual particles** it is much easier to understand how semiconductor devices work.

*However it is important to remember that although we regard holes as positively charged particles they are not real particles and all the effects we see are actually caused by the movement of electrons in the valence band.*

When an external voltage is applied the negatively charged electrons in the conduction band will move towards the positive terminal and the positively charge holes in the valence band will move towards the negative terminal.



## Leakage current

It is important to realise that although electron-hole pair generation in semiconductors means that there will be some current flow when a voltage is applied, this current is very small (typically millionths of an Amp) compared to the current which would flow through a conductor with the same voltage applied. This current is called leakage current.

### Comparison of Silicon and Germanium

All of the above applies equally to both Silicon and Germanium except for the actual value of leakage current. Germanium has a smaller energy gap between the valence and conduction band. Therefore **more** electron hole pairs are produced in Germanium resulting in a higher leakage current at any given temperature.

### Comparison of Conductor, semiconductor and insulator energy band diagrams

In conductors the valence band and conduction band overlap therefore electrons can move freely into the conduction band. The vast number of electrons in the conduction band drift to produce large currents when a voltage is applied. In insulators the energy gap between the valence and conduction band is very large. Therefore very few electrons manage to jump into the conduction band and leakage currents are extremely small.

### Electrical breakdown

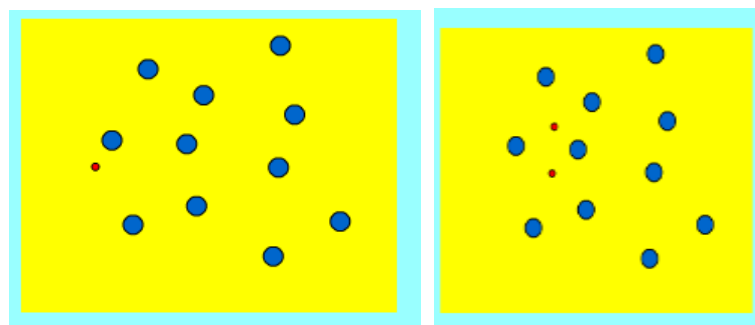
It is important to point out that when we are describing the conduction properties of materials we are considering fairly normal operating conditions and we are not talking about situations involving extreme voltages. Air for instance is an excellent insulator, however in thunderstorms voltages in the order of a hundred million volts can force a current through the air in the form of a lightning bolt. It would not take such an extreme voltage to break down a small piece of silicon and force it to conduct electricity. There are two stages that occur as a material begins to breakdown due a large applied voltage. These are **zener breakdown** and **avalanche breakdown**.

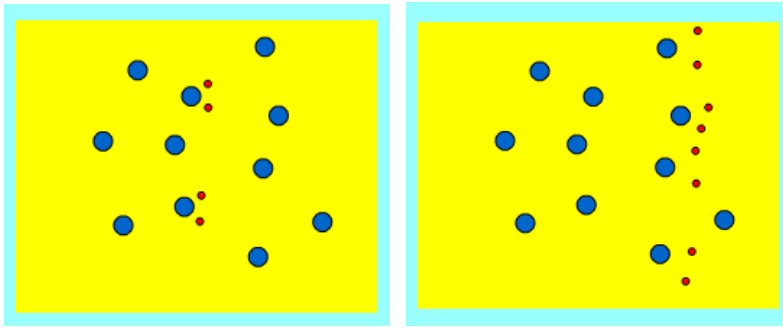
#### Zener breakdown

In Zener breakdown the **electrostatic** attraction between the negative electrons and a large positive voltage is so great that it pulls electrons out of their covalent **bonds** and away from their parent atoms. ie Electrons are transferred from the valence to the conduction band. In this situation the current can still be limited by the limited number of free electrons produced by the applied voltage so it is possible to cause Zener breakdown without damaging the semiconductor.

#### Avalanche breakdown

Avalanche breakdown occurs when the applied voltage is so large that electrons that are pulled from their covalent bonds are accelerated to great velocities. These electrons collide with the silicon atoms and knock off **more** electrons. These electrons are then also accelerated and subsequently collide with other atoms. Each collision produces more electrons which leads to more collisions etc. The current in the semiconductor rapidly increases and the material can quickly be destroyed.



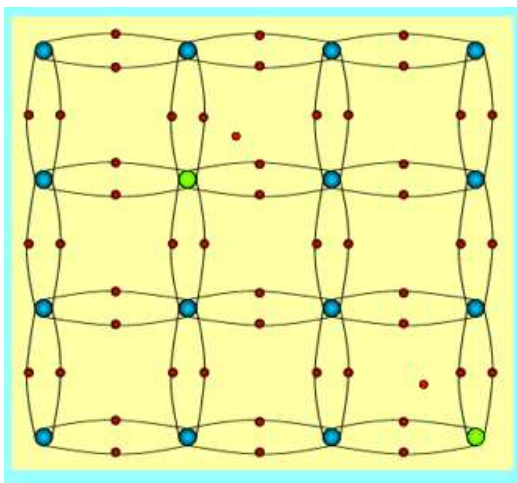


#### 4. Formation of P-type & N-type materials (Extrinsic semiconductor)

To produce extrinsic semiconductor material specific amounts of impurity are added to the pure intrinsic semiconductor. This process is called doping and the impurity atoms are called donor atoms. There are two types of extrinsic semiconductor which are manufactured, P type semiconductor and N type semiconductor. The production of extrinsic semiconductor will be described for the **more** common silicon semiconductor material but the process is identical for germanium.

##### N type semiconductor

The pure silicon is doped with a group 5 element such as phosphorus, antimony or arsenic. These materials have atoms with five valence electrons (pentavalent atoms). Four of these electrons will form covalent **bonds** with neighbouring silicon atoms. As there are only four covalent bonds binding the donor atom to the neighbouring silicon atoms the fifth electron is not part of a covalent bond, and is therefore a free electron. Every impurity atom will produce a free electron in the conduction band. These electrons will drift to produce an electrical current if a voltage is applied to the material and the N type semiconductor is a much better conductor than the intrinsic pure silicon material.



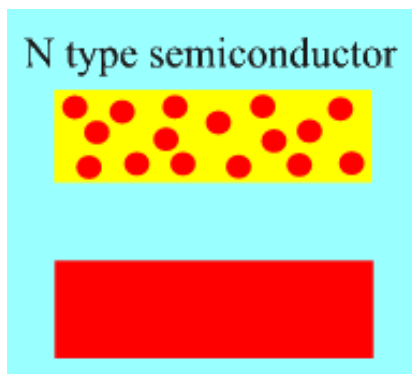
- The silicon atoms form a square lattice
- The green atoms represent the donor atoms
- Four of the five valence electrons form covalent bonds with neighbouring silicon atoms
- The fifth electron has no neighbouring electron to pair with and is a free electron

- Each donor atom produces a free electron

Note it is important to point out that the material is called N type semiconductor because the majority of charge carriers which will contribute to an electrical current through the material are negatively charged free electrons produced by the doping process. *There will be some contribution to the current flow from positively charge holes due to electron hole pair generation but these holes are the minority charge carriers in this material.* **The N type material itself is not negatively charged. The negative charge of the electrons of the donor atoms is balanced by the positive charge in the nucleus.**

### Energy band diagram

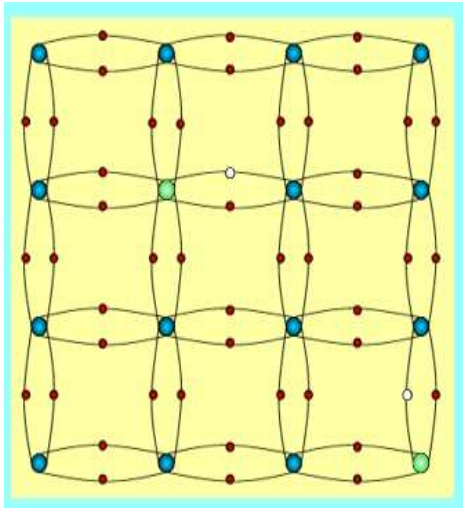
The diagram below shows an energy band diagram for N type semiconductor. The valence band is completely full as all of the covalent bonds are complete. The conduction band contains free electrons from the fifth valence electrons in the donor atoms.



Note this diagram does not show the **electron hole pairs** that would be present due to thermal energy. The electron hole pairs are minority charge carriers in N type semiconductors, the majority being the free electrons produced by the doping process.

### P type semiconductor

The pure silicon is doped with a group 3 element such as boron, aluminium or indium. These materials have atoms with three valence electrons (trivalent atoms). The three electrons will form covalent bonds with neighbouring silicon atoms. However there are not enough electrons to form the fourth covalent bond. This leaves a hole in the covalent bond structure and therefore a hole in the valence band of the energy level diagram. Every impurity atom will produce a hole in the valence band. These holes will drift to produce an electrical current if a voltage is applied to the material and the P type semiconductor is a much better conductor than the intrinsic pure silicon material.

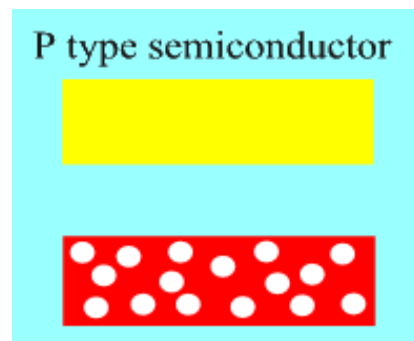


- The silicon atoms form a square lattice
- The green atoms represent the donor atoms
- three of the four covalent bonds are formed with neighbouring silicon atoms
- The fourth bond cannot be formed as there are not enough electrons, this leaves a hole in the valence band
- Each donor atom produces a hole in the valence band

Note it is important to point out that the material is called P type semiconductor because the majority of charge carriers which contribute to an electrical current are positively charged holes produced by the doping process. *There will be some contribution to the current flow from negatively charged electrons due to electron hole pair generation but these electrons are the minority charge carriers in this material.* **The P type material itself is not positively charged because the negative charge of the electrons of the donor atoms are balance by the positive charge in the nucleus.**

### Energy band diagram

The diagram below shows an energy band diagram for P type semiconductor. The valence band contains holes due to the incomplete covalent bond around each donor atom. The conduction band is empty as there are no free electrons.

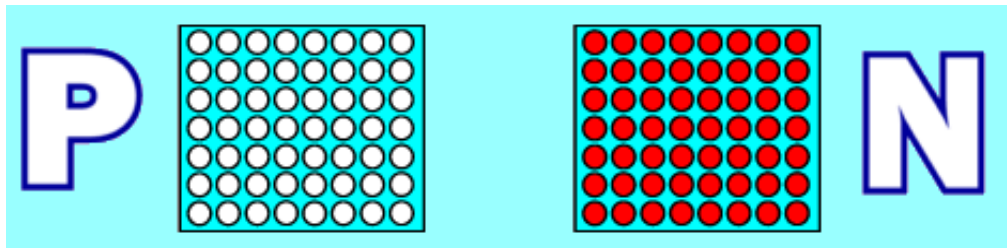


Note this diagram does not show the electron hole pairs that would be present due to thermal energy. The electron hole pairs are minority charge carriers in P type semiconductors, the majority being the holes produced by the doping process.

## 5. The PN junction diode & Operation

Understanding the operation of the semiconductor diode is the basis for an understanding of all semiconductor devices. The diode is actually manufactured as a single piece of material but it is much easier to explain the operation if we imagine producing two separate pieces of N type and P type material and then "sticking" them together.

Consider a piece of N type material. It contains mobile charge carriers in the form of free electrons. These electrons will be in motion due to thermal energy. *(It is important to realise that this motion does not result in an electrical current because the motion is random and there is not net movement of charge from one area of the material to another. This is similar to the way that even in a perfectly still glass of water the individual molecules will be moving randomly on a microscopic scale.)* The net result is that the random motion of the electrons results in them being evenly distributed throughout the N type material. In the P type material it is the positively charged holes that are mobile and for identical reasons to those previously described the holes are evenly distributed throughout the P type material.

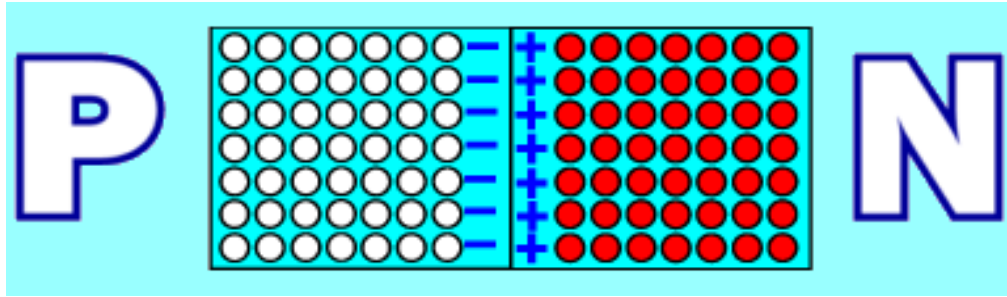


Now consider what will happen if these two separate pieces of P and N type material are joined together. The random motion of the mobile electrons in the N type material and the holes in the P type material would tend to cause an even distribution of electrons and holes throughout the semiconductor. And in fact this is what begins to happen.

Consider the electrons in the N type material. The electrons start to migrate across the junction of the two materials. When they cross into the P type material they recombine with the holes (ie they fill in the holes in the valence band by filling in the vacant electron positions around the trivalent donor atoms). This means that the number of holes near to the junction becomes depleted. Also as the electrons leave the previously neutral N type material a positive charge builds up at the junction. (This is because the positive charge from the nucleus of atoms near to the junction is now greater than the negative charge of the electrons in that region. This is due to the reduction in the number of electrons due to those which have moved across the junction.)

Similarly as holes migrate from the P to N type material they recombine with electrons (the free electron from the pentavalent atoms completes the fourth covalent bond around the trivalent atom). This leaves a depletion of free electrons near the junction in the N type material. Also a

negative charge builds up near the junction in the P type material due to the loss of positively charged holes.



The net result is that the migration of electrons from N to P type material and the migration of holes from P to N has two effects. It results in a depletion of mobile charge carriers near the junction ( a depletion of electrons in the N type material and a depletion of holes in the P type material). This depletion layer is typically about 1 micrometre wide ( 1 millionth of a meter!). Also a voltage is produced across the junction which is called a barrier voltage. The N type material develops a positive charge close to the junction and the P type develops a negative charge. This prevents any further migration of mobile charge carriers.

### **The effect of the barrier voltage**

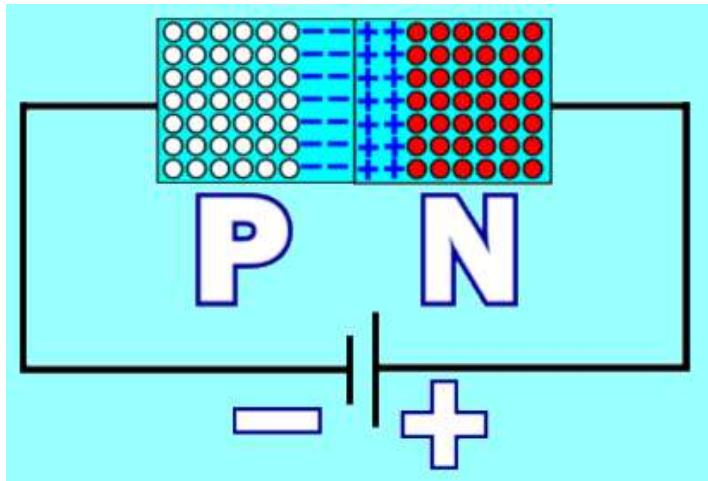
The positive charge at the N side of the junction repels any positively charged holes that would tend to migrate across the junction from the P type material. It also attracts free electrons and therefore prevents them moving out of the N type material. Similarly the negative charge in the P type material close to the junction repels electrons which would tend to migrate from the N type material and it attracts the holes and prevents them moving out of the P type material. The migration of mobile charge carriers across the junction would stop when the barrier voltage had built up to a sufficient level to prevent any further migration. For Silicon this is about 0.6 to 0.7 volts for Germanium it is about 0.2 to 0.3 volts.

### **Reverse Bias**

Consider applying an external voltage to the diode as shown below with the positive terminal connected to the N type material and the negative terminal connected to the P type material. The external voltage would tend to cause the movement of electrons from the negative terminal of the supply through the diode and back to the positive terminal (electron flow).

To do this the negative terminal would tend to inject electrons into the P type material causing a further depletion of holes. This would produce a widening of the depletion layer and an increase in the negative charge at the junction until it was equal in magnitude to the applied voltage. The negative charge at the junction would oppose the negative terminal of the external voltage and this would prevent any further injection of electrons into the P type material.





Similarly, the positive terminal would tend to pull electrons from the N type material. This would further deplete the N type material of electrons, widening the depletion layer and increasing the positive charge at the junction until it was equal to the magnitude of the applied voltage. This would then prevent any further loss of electrons.

The net effect is that when an external voltage is connected this way the effect of the barrier voltage opposes the external voltage. Any initial movement of charge due to the external voltage will just increase the barrier voltage until it is equal to the applied voltage and therefore no current will flow through the diode. When an external voltage is connected to a diode with this polarity we say that it is reverse biased.

Note as holes are the majority current carriers in P type material it is more common to consider the movement of holes rather than electrons in the P type material. Therefore we can say that the negative terminal tends to remove holes rather than injecting electrons in the same way that we considered the positive terminal removing electrons from the N type material. The effect is the same, the removal of holes from the P type material would increase the depletion layer and increase the barrier voltage.

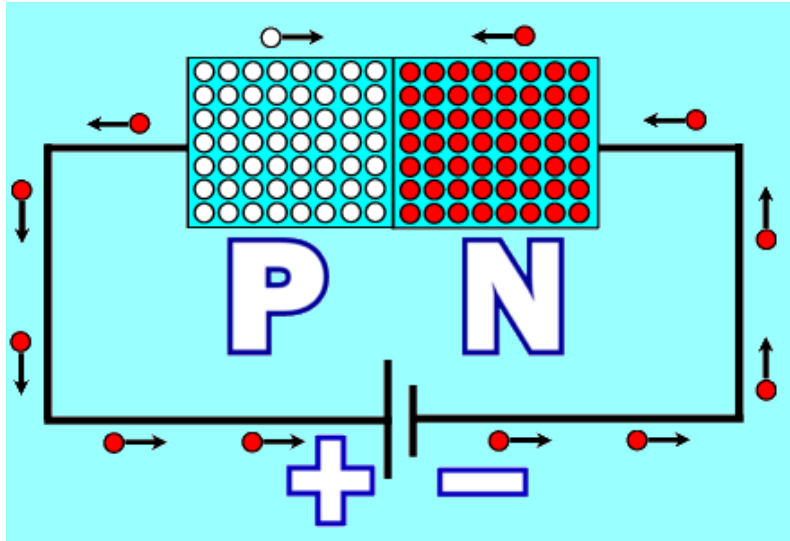
### **Forward bias**

Consider applying an external voltage to the diode as shown below with the positive terminal connected to the P type material and the negative terminal connected to the N type material. The external voltage would tend to cause the movement of electrons from the negative terminal of the supply through the diode and back to the positive terminal (electron flow).

The negative terminal would tend to inject electrons into the N type material. This would increase the number of electrons and therefore reduce depletion layer. This would reduce the positive charge at the junction. Similarly the positive terminal would tend to pull electrons from the P type material. This would increase the number of holes, reducing the depletion layer and reducing the negative charge at the junction.

The net effect is that when the external voltage is connected this way it reduces the barrier voltage and if the applied voltage is greater than the barrier voltage it will overcome it and

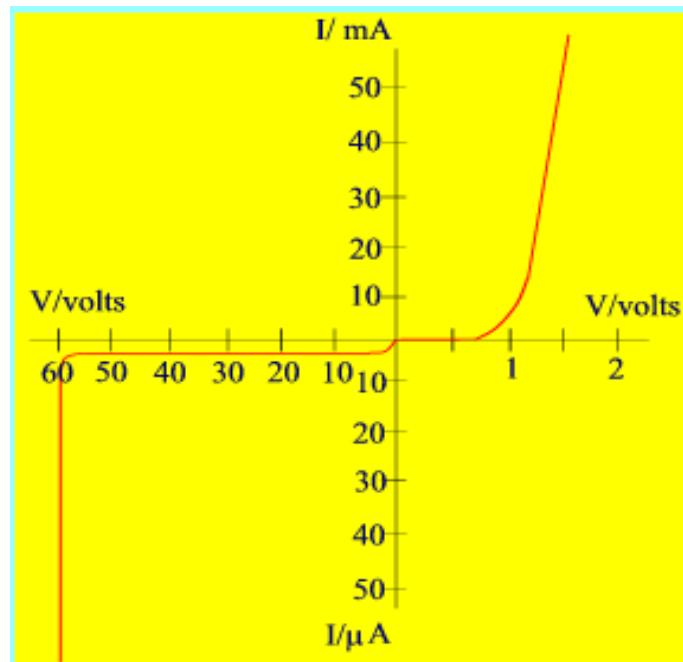
produce a current flow through the diode. When an external voltage is connected to a diode with this polarity we say that it is forward biased.



Note as holes are the majority current carriers in P type material it is more common to consider the movement of holes rather than electrons in the P type material. Therefore we can say that the positive terminal injects holes rather than removes electrons in the same way that we considered the negative terminal injecting electrons into the N type material. The effect is the same the injection of holes would reduce the depletion layer and reduce the barrier voltage.

### Diode Characteristic

A diode characteristic is simply a graph of the voltage applied to a diode and the current it produces. The negative part of the voltage axis corresponds to when the diode is reverse biased and the positive part is when the diode is forward biased. The negative part of the current axis shows current flowing in the reverse direction through the diode.



- **The main features of the characteristic are:**
- No current flows when the diode is forward biased until the barrier voltage is overcome (0.6V - 0.7V for silicon 0.2V - 0.3V for germanium)
- The forward characteristic is non linear (not a straight line). This shows that the resistance is not constant.
- the gradient of the forward characteristic quickly becomes very steep. This shows that the forward resistance is very low
- The negative current axis is on a different scale (showing millionths of an amp rather than thousandths) this is so we can indicate the very small leakage current which flows due to electron hole pair generation (ie due to the natural conduction properties of the pure silicon). The leakage current flows in both directions but is too small to indicate on the current scale used on the forward part of the characteristic
- If a large enough reversed bias voltage is applied the diode will eventually conduct due to zener then avalanche breakdown (ie due to the natural conduction properties of the pure silicon). The actual voltage that breakdown occurs varies for individual diodes and can be determined by the manufacturing process

# TOPIC 6: MEMORIES

## Definition & Characteristics of Memory

**Memory** is the electronic holding place for instructions and data that your computer's microprocessor can reach quickly. When your computer is in normal operation, its memory usually contains the main parts of the operating system and some or all of the application programs and related data that are being used.

Memory is sometimes distinguished from *storage*, or the physical medium that holds the much larger amounts of data that won't fit into RAM and may not be immediately needed there.

### Characteristics of Computer Memory

**1. Electrical Characteristics** - The voltage and current requirements depend on the manufacturing technology of the device. The voltage level is not of major concern because most of the semiconductor memory devices operate at TTL voltage levels.

**2. Speed** - There is a finite time delay between the application of address and the availability of stable and accurate data on the data lines. This memory delay depends on the manufacturing technology and other factors such as size.

**3. Capacity** representing the global volume of information (in bits) that the memory can store. Memory is small in size and hence its storage is relatively low

## Types of Memories

Memory is primarily of three types –

- Cache Memory
- Primary Memory/Main Memory
- Secondary Memory

### Cache Memory

Cache memory is a very high speed semiconductor memory which can speed up the CPU. It acts as a buffer between the CPU and the main memory. It is used to hold those parts of data and program which are most frequently used by the CPU. The parts of data and programs are transferred from the disk to cache memory by the operating system, from where the CPU can access them.



### Advantages

The advantages of cache memory are as follows –

- Cache memory is faster than main memory.
- It consumes less access time as compared to main memory.

- It stores the program that can be executed within a short period of time.
- It stores data for temporary use.

### Disadvantages

The disadvantages of cache memory are as follows –

- Cache memory has limited capacity.
- It is very expensive.

### **Primary Memory (Main Memory)**

Primary memory holds only those data and instructions on which the computer is currently working. It has a limited capacity and data is lost when power is switched off. It is generally made up of semiconductor device. These memories are not as fast as registers. The data and instruction required to be processed resides in the main memory. It is divided into two subcategories RAM and ROM.



### Characteristics of Main Memory

- These are semiconductor memories.
- It is known as the main memory.
- Usually volatile memory.
- Data is lost in case power is switched off.
- It is the working memory of the computer.
- Faster than secondary memories.
- A computer cannot run without the primary memory.

### **Secondary Memory**

This type of memory is also known as external memory or non-volatile. It is slower than the main memory. These are used for storing data/information permanently. CPU directly does not access these memories, instead they are accessed via input-output routines. The contents of secondary memories are first transferred to the main memory, and then the CPU can access it. For example, disk, CD-ROM, DVD, etc.



### Characteristics of Secondary Memory

- These are magnetic and optical memories.
- It is known as the backup memory.
- It is a non-volatile memory.
- Data is permanently stored even if power is switched off.
- It is used for storage of data in a computer.
- Computer may run without the secondary memory.
- Slower than primary memories.

## Examples of Secondary Memory

**Magnetic Disks:-** Magnetic storage or magnetic recording is the storage of data on a magnetized medium. **Magnetic** storage uses different patterns of magnetization in a magnetisable material to store data and is a form of non-volatile **memory**. The information is accessed using one or more read/write heads.

### Examples

1. Hard Disk Drive
2. Floppy Disk
3. Memory Stick

**Optical Disks:-** *Optical* storage is the storage of data on an optically readable medium. Data is recorded by making marks in a pattern that can be read back with the aid of light, usually a beam of laser light precisely focused on a spinning *optical disc*.

### Examples

1. CD
2. DVD
3. Blue-ray Disk

**Solid state disk:** SSD incorporates the storage technique implemented in microchip-based flash memory, where data is electronically stored on flash memory chips. An SSD is an entirely electronic storage device, and its physical assembly contains no mechanical objects.

### Examples

1. Pen Drive
2. Flash Drive
3. Thumb Drive

## Units of memory

1 Bit = Binary Digit

8 Bits = 1 Byte = 2 Nibble

1024 Bytes = 1 KB (Kilobyte)

1024 KB = 1 MB (Megabyte)

1024 MB = 1 GB (Giga Byte)

1024 GB = 1 TB (Terabyte)

1024 TB = 1 PB (Petabyte)

1024 PB = 1EB (Exabyte)

1024 EB = 1 ZB (Zettabyte)

1024 ZB = 1 YB (Yottabyte)

1024 YB = 1 (Brontobyte)

1024 Brontobytes = 1 (Geopbyte)

The above information helps the aspiring candidates in their preparation for the bank exams as the computer awareness section is an important section. The candidates need to learn the basic computer information in order to clear the sectional wise cutoff which is important to crack the

# TOPIC 7: NUMBER SYSTEM

## Introduction to Number System

When we type some letters or words, the computer translates them in numbers as computers can understand only numbers. A computer can understand the positional number system where there are only a few symbols called digits and these symbols represent different values depending on the position they occupy in the number.

A **numeral system** (or **system of numeration**) is a writing **system** for expressing **numbers**; that is, a mathematical notation for representing **numbers** of a given set, using digits or other symbols in a consistent manner.

The value of each digit in a number can be determined using –

- The digit
- The position of the digit in the number
- The base of the number system (where the base is defined as the total number of digits available in the number system)

## Decimal Number System

The number system that we use in our day-to-day life is the decimal number system. Decimal number system has base 10 as it uses 10 digits from 0 to 9. In decimal number system, the successive positions to the left of the decimal point represent units, tens, hundreds, thousands, and so on.

Each position represents a specific power of the base (10). For example, the decimal number 1234 consists of the digit 4 in the units position, 3 in the tens position, 2 in the hundreds position, and 1 in the thousands position. Its value can be written as

$$\begin{aligned} & (1 \times 1000) + (2 \times 100) + (3 \times 10) + (4 \times 1) \\ & (1 \times 10^3) + (2 \times 10^2) + (3 \times 10^1) + (4 \times 10^0) \\ & 1000 + 200 + 30 + 4 \\ & 1234 \end{aligned}$$

As a computer programmer or an IT professional, you should understand the following number systems which are frequently used in computers.

S.No.	Number System and Description
1	<b>Binary Number System</b> Base 2. Digits used : 0, 1
2	<b>Octal Number System</b> Base 8. Digits used : 0 to 7
3	<b>Hexa Decimal Number System</b> Base 16. Digits used: 0 to 9, Letters used : A- F



## Binary Number System

Characteristics of the binary number system are as follows –

- Uses two digits, 0 and 1
- Also called as base 2 number system
- Each position in a binary number represents a **0** power of the base (2). Example  $2^0$
- Last position in a binary number represents a **x** power of the base (2). Example  $2^x$  where **x** represents the last position - 1.

### Example

Binary Number:  $10101_2$

Calculating Decimal Equivalent –

Step	Binary Number	Decimal Number
Step 1	$10101_2$	$((1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0))_{10}$
Step 2	$10101_2$	$(16 + 0 + 4 + 0 + 1)_{10}$
Step 3	$10101_2$	$21_{10}$

**Note** –  $10101_2$  is normally written as 10101.

## Octal Number System

Characteristics of the octal number system are as follows –

- Uses eight digits, 0,1,2,3,4,5,6,7
- Also called as base 8 number system
- Each position in an octal number represents a **0** power of the base (8). Example  $8^0$
- Last position in an octal number represents a **x** power of the base (8). Example  $8^x$  where **x** represents the last position - 1

### Example

Octal Number:  $12570_8$

Calculating Decimal Equivalent –

Step	Octal Number	Decimal Number
Step 1	$12570_8$	$((1 \times 8^4) + (2 \times 8^3) + (5 \times 8^2) + (7 \times 8^1) + (0 \times 8^0))_{10}$
Step 2	$12570_8$	$(4096 + 1024 + 320 + 56 + 0)_{10}$
Step 3	$12570_8$	$5496_{10}$

**Note** –  $12570_8$  is normally written as 12570.

## Hexadecimal Number System

Characteristics of hexadecimal number system are as follows –

- Uses 10 digits and 6 letters, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F
- Letters represent the numbers starting from 10. A = 10, B = 11, C = 12, D = 13, E = 14, F = 15
- Also called as base 16 number system
- Each position in a hexadecimal number represents a **0** power of the base (16). Example,  $16^0$
- Last position in a hexadecimal number represents a **x** power of the base (16). Example  $16^x$  where **x** represents the last position - 1

### Example

Hexadecimal Number:  $19FDE_{16}$

Calculating Decimal Equivalent –

Step	Binary Number	Decimal Number
Step 1	$19FDE_{16}$	$((1 \times 16^4) + (9 \times 16^3) + (F \times 16^2) + (D \times 16^1) + (E \times 16^0))_{10}$
Step 2	$19FDE_{16}$	$((1 \times 16^4) + (9 \times 16^3) + (15 \times 16^2) + (13 \times 16^1) + (14 \times 16^0))_{10}$
Step 3	$19FDE_{16}$	$(65536 + 36864 + 3840 + 208 + 14)_{10}$
Step 4	$19FDE_{16}$	$106462_{10}$

**Note** –  $19FDE_{16}$  is normally written as 19FDE.

## Computer - Number Conversion

There are many methods or techniques which can be used to convert numbers from one base to another. In this chapter, we'll demonstrate the following –

- Decimal to Other Base System
- Other Base System to Decimal
- Other Base System to Non-Decimal
- Shortcut method - Binary to Octal
- Shortcut method - Octal to Binary
- Shortcut method - Binary to Hexadecimal
- Shortcut method - Hexadecimal to Binary

### Decimal to Other Base System

**Step 1** – Divide the decimal number to be converted by the value of the new base.

**Step 2** – Get the remainder from Step 1 as the rightmost digit (least significant digit) of the new base number.

**Step 3** – Divide the quotient of the previous divide by the new base.

**Step 4** – Record the remainder from Step 3 as the next digit (to the left) of the new base number.

Repeat Steps 3 and 4, getting remainders from right to left, until the quotient becomes zero in Step 3.

The last remainder thus obtained will be the Most Significant Digit (MSD) of the new base number.

#### Example

Decimal Number:  $29_{10}$

Calculating Binary Equivalent –

Step	Operation	Result	Remainder
Step 1	$29 / 2$	14	1
Step 2	$14 / 2$	7	0
Step 3	$7 / 2$	3	1
Step 4	$3 / 2$	1	1
Step 5	$1 / 2$	0	1

As mentioned in Steps 2 and 4, the remainders have to be arranged in the reverse order so that the first remainder becomes the Least Significant Digit (LSD) and the last remainder becomes the Most Significant Digit (MSD).

Decimal Number :  $29_{10}$  = Binary Number :  $11101_2$ .

### **Other Base System to Decimal System**

**Step 1** – Determine the column (positional) value of each digit (this depends on the position of the digit and the base of the number system).

**Step 2** – Multiply the obtained column values (in Step 1) by the digits in the corresponding columns.

**Step 3** – Sum the products calculated in Step 2. The total is the equivalent value in decimal.

#### Example

Binary Number:  $11101_2$

Calculating Decimal Equivalent –

Step	Binary Number	Decimal Number
Step 1	$11101_2$	$((1 \times 2^4) + (1 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0))_{10}$
Step 2	$11101_2$	$(16 + 8 + 4 + 0 + 1)_{10}$
Step 3	$11101_2$	$29_{10}$

Binary Number :  $11101_2$  = Decimal Number :  $29_{10}$

## Other Base System to Non-Decimal System

**Step 1** – Convert the original number to a decimal number (base 10).

**Step 2** – Convert the decimal number so obtained to the new base number.

### Example

Octal Number :  $25_8$

Calculating Binary Equivalent –

### **Step 1 - Convert to Decimal**

Step	Octal Number	Decimal Number
Step 1	$25_8$	$((2 \times 8^1) + (5 \times 8^0))_{10}$
Step 2	$25_8$	$(16 + 5)_{10}$
Step 3	$25_8$	$21_{10}$

Octal Number :  $25_8$  = Decimal Number :  $21_{10}$

### **Step 2 - Convert Decimal to Binary**

Step	Operation	Result	Remainder
Step 1	$21 / 2$	10	1
Step 2	$10 / 2$	5	0
Step 3	$5 / 2$	2	1
Step 4	$2 / 2$	1	0
Step 5	$1 / 2$	0	1

Decimal Number :  $21_{10}$  = Binary Number :  $10101_2$

Octal Number :  $25_8$  = Binary Number :  $10101_2$

### **Shortcut Method – Binary to Octal**

**Step 1** – Divide the binary digits into groups of three (starting from the right).

**Step 2** – Convert each group of three binary digits to one octal digit.

### Example

Binary Number :  $10101_2$

Calculating Octal Equivalent –

Step	Binary Number	Octal Number
Step 1	$10101_2$	010 101
Step 2	$10101_2$	$2_8 5_8$
Step 3	$10101_2$	$25_8$

Binary Number :  $10101_2 = \text{Octal Number} : 25_8$

### Shortcut Method – Octal to Binary

**Step 1** – Convert each octal digit to a 3-digit binary number (the octal digits may be treated as decimal for this conversion).

**Step 2** – Combine all the resulting binary groups (of 3 digits each) into a single binary number.

#### Example

Octal Number :  $25_8$

Calculating Binary Equivalent –

Step	Octal Number	Binary Number
Step 1	$25_8$	$2_{10} 5_{10}$
Step 2	$25_8$	$010_2 101_2$
Step 3	$25_8$	$010101_2$

Octal Number :  $25_8 = \text{Binary Number} : 10101_2$

### Shortcut Method – Binary to Hexadecimal

**Step 1** – Divide the binary digits into groups of four (starting from the right).

**Step 2** – Convert each group of four binary digits to one hexadecimal symbol.

#### Example

Binary Number :  $10101_2$

Calculating hexadecimal Equivalent –

Step	Binary Number	Hexadecimal Number
Step 1	$10101_2$	$0001 0101$
Step 2	$10101_2$	$1_{10} 5_{10}$
Step 3	$10101_2$	$15_{16}$

Binary Number :  $10101_2 = \text{Hexadecimal Number} : 15_{16}$

### Shortcut Method - Hexadecimal to Binary

**Step 1** – Convert each hexadecimal digit to a 4-digit binary number (the hexadecimal digits may be treated as decimal for this conversion).

**Step 2** – Combine all the resulting binary groups (of 4 digits each) into a single binary number.

#### Example

Hexadecimal Number :  $15_{16}$

Calculating Binary Equivalent –

Step	Hexadecimal Number	Binary Number
Step 1	15 <sub>16</sub>	1 <sub>10</sub> 5 <sub>10</sub>
Step 2	15 <sub>16</sub>	0001 <sub>2</sub> 0101 <sub>2</sub>
Step 3	15 <sub>16</sub>	00010101 <sub>2</sub>

Hexadecimal Number : 15<sub>16</sub> = Binary Number : 10101<sub>2</sub>

## Binary Arithmetic

Binary arithmetic is essential part of all the digital computers and many other digital system. Arithmetic circuits form point of the CPU. Mathematical operations include

Subtraction, multiplication, division and addition

### Binary Addition

It is a key for binary subtraction, multiplication, division. There are four rules of binary addition.

Case	A	+	B	Sum	Carry
1	0	+	0	0	0
2	0	+	1	1	0
3	1	+	0	1	0
4	1	+	1	0	1

In fourth case, a binary addition is creating a sum of (1 + 1 = 10) i.e. 0 is written in the given column and a carry of 1 over to the next column.

#### Example – Addition

$$\begin{array}{r}
 0011010 + 001100 = 00100110 \\
 \begin{array}{r}
 11 \quad \text{carry} \\
 0011010 = 26_{10} \\
 + 0001100 = 12_{10} \\
 \hline
 0100110 = 38_{10}
 \end{array}
 \end{array}$$

### Binary Subtraction

**Subtraction and Borrow**, these two words will be used very frequently for the binary subtraction. There are four rules of binary subtraction.

Case	A	-	B	Subtract	Borrow
1	0	-	0	0	0
2	1	-	0	1	0
3	1	-	1	0	0
4	0	-	1	0	1

### Example – Subtraction

$$\begin{array}{r}
 0011010 - 001100 = 00001110 \\
 \begin{array}{r}
 \phantom{00}11 \text{ borrow} \\
 00\cancel{1}1010 = 26_{10} \\
 -0001100 = 12_{10} \\
 \hline
 0001110 = 14_{10}
 \end{array}
 \end{array}$$

## Binary Multiplication

Binary multiplication is similar to decimal multiplication. It is simpler than decimal multiplication because only 0s and 1s are involved. There are four rules of binary multiplication.

Case	A	x	B	Multiplication
1	0	x	0	0
2	0	x	1	0
3	1	x	0	0
4	1	x	1	1

### Example – Multiplication

Example:

$$\begin{array}{r}
 0011010 \times 001100 = 100111000 \\
 \begin{array}{r}
 0011010 = 26_{10} \\
 \times 0001100 = 12_{10} \\
 \hline
 0000000 \\
 0000000 \\
 0011010 \\
 0011010 \\
 \hline
 0100111000 = 312_{10}
 \end{array}
 \end{array}$$

## Binary Division

Binary division is similar to decimal division. It is called as the long division procedure.

### Example – Division

$$101010 / 000110 = 000111$$

$$\begin{array}{r} \phantom{000}111 \phantom{00} = 7_{10} \\ 000110 \overline{) 101010} \phantom{00} = 42_{10} \\ \underline{-110} \phantom{00} = 6_{10} \\ \phantom{00}1001 \\ \underline{-110} \\ \phantom{000}110 \\ \underline{-110} \\ \phantom{00000}0 \end{array}$$

## Binary Codes

In the coding, when numbers, letters or words are represented by a specific group of symbols, it is said that the number, letter or word is being encoded. The group of symbols is called as a code. The digital data is represented, stored and transmitted as group of binary bits. This group is also called as **binary code**. The binary code is represented by the number as well as alphanumeric letter.

### Advantages of Binary Code

Following is the list of advantages that binary code offers.

- Binary codes are suitable for the computer applications.
- Binary codes are suitable for the digital communications.
- Binary codes make the analysis and designing of digital circuits if we use the binary codes.
- Since only 0 & 1 are being used, implementation becomes easy.

### Classification of binary codes

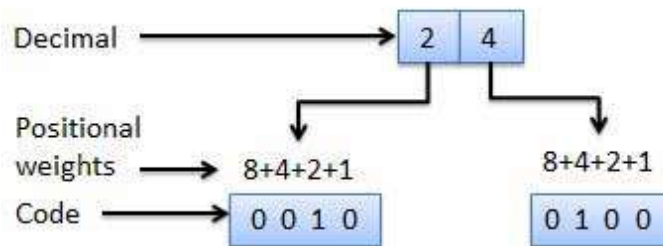
The codes are broadly categorized into following four categories.

- Weighted Codes
- Non-Weighted Codes
- Binary Coded Decimal Code
- Alphanumeric Codes
- Error Detecting Codes
- Error Correcting Codes



## Weighted Codes

Weighted binary codes are those binary codes which obey the positional weight principle. Each position of the number represents a specific weight. Several systems of the codes are used to express the decimal digits 0 through 9. In these codes each decimal digit is represented by a group of four bits.

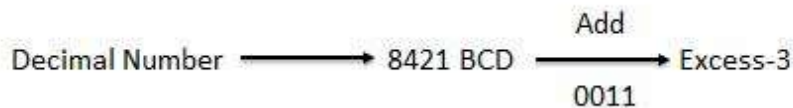


## Non-Weighted Codes

In this type of binary codes, the positional weights are not assigned. The examples of non-weighted codes are Excess-3 code and Gray code.

### Excess-3 code

The Excess-3 code is also called as XS-3 code. It is non-weighted code used to express decimal numbers. The Excess-3 code words are derived from the 8421 BCD code words adding (0011)<sub>2</sub> or (3)<sub>10</sub> to each code word in 8421. The excess-3 codes are obtained as follows –



### Example

Decimal	BCD	Excess-3 BCD + 0011
	8 4 2 1	
0	0 0 0 0	0 0 1 1
1	0 0 0 1	0 1 0 0
2	0 0 1 0	0 1 0 1
3	0 0 1 1	0 1 1 0
4	0 1 0 0	0 1 1 1
5	0 1 0 1	1 0 0 0
6	0 1 1 0	1 0 0 1
7	0 1 1 1	1 0 1 0
8	1 0 0 0	1 0 1 1
9	1 0 0 1	1 1 0 0

## Gray Code

It is the non-weighted code and it is not arithmetic codes. That means there are no specific weights assigned to the bit position. It has a very special feature that, only one bit will change each time the decimal number is incremented as shown in fig. As only one bit changes at a time, the gray code is called as a unit distance code. The gray code is a cyclic code. Gray code cannot be used for arithmetic operation.

Decimal	BCD	Gray
0	0 0 0 0	0 0 0 0
1	0 0 0 1	0 0 0 1
2	0 0 1 0	0 0 1 1
3	0 0 1 1	0 0 1 0
4	0 1 0 0	0 1 1 0
5	0 1 0 1	0 1 1 1
6	0 1 1 0	0 1 0 1
7	0 1 1 1	0 1 0 0
8	1 0 0 0	1 1 0 0
9	1 0 0 1	1 1 0 1

#### Application of Gray code

- Gray code is popularly used in the shaft position encoders.
- A shaft position encoder produces a code word which represents the angular position of the shaft.

### **Binary Coded Decimal (BCD) code**

In this code each decimal digit is represented by a 4-bit binary number. BCD is a way to express each of the decimal digits with a binary code. In the BCD, with four bits we can represent sixteen numbers (0000 to 1111). But in BCD code only first ten of these are used (0000 to 1001). The remaining six code combinations i.e. 1010 to 1111 are invalid in BCD.

Decimal	0	1	2	3	4	5	6	7	8	9
BCD	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001

#### Advantages of BCD Codes

- It is very similar to decimal system.
- We need to remember binary equivalent of decimal numbers 0 to 9 only.

#### Disadvantages of BCD Codes

- The addition and subtraction of BCD have different rules.

- The BCD arithmetic is little more complicated.
- BCD needs more number of bits than binary to represent the decimal number. So BCD is less efficient than binary.

## Alphanumeric codes

A binary digit or bit can represent only two symbols as it has only two states '0' or '1'. But this is not enough for communication between two computers because there we need many more symbols for communication. These symbols are required to represent 26 alphabets with capital and small letters, numbers from 0 to 9, punctuation marks and other symbols.

The alphanumeric codes are the codes that represent numbers and alphabetic characters. Mostly such codes also represent other characters such as symbol and various instructions necessary for conveying information. An alphanumeric code should at least represent 10 digits and 26 letters of alphabet i.e. total 36 items. The following three alphanumeric codes are very commonly used for the data representation.

- American Standard Code for Information Interchange (ASCII).
- Extended Binary Coded Decimal Interchange Code (EBCDIC).
- Five bit Baudot Code.

ASCII code is a 7-bit code whereas EBCDIC is an 8-bit code. ASCII code is more commonly used worldwide while EBCDIC is used primarily in large IBM computers.

## BCD Encoding & Arithmetic

- BCD is a binary code of the ten decimal digits. It is not a binary equivalent.
- To perform BCD addition
  1. Add the BCD digits as regular binary numbers.
  2. If the sum is 9 or less and no carry was generated, it is a valid BCD digit.
  3. If the sum produces a carry, the sum is invalid and the number 6 (0110) must be added to the digit.
  4. If the sum is greater than nine, the sum is invalid and the number 6 (0110) must be added to the digit.
  5. Repeat for each of the BCD digits.

### Note

**The binary-coded decimal (BCD)** is an encoding for decimal numbers in which each digit is represented by its own binary sequence.

A widely used variation of the two-digits-per-byte encoding is called **packed BCD (or simply packed decimal)**, where numbers are stored with two decimal digits "packed" into one byte each, and the last digit (or nibble) is used as a sign indicator. The preferred sign values are 1100 (hex C) for positive (+) and 1101 (hex D) for negative (−); other allowed signs are 1010 (A) and 1110 (E) for positive and 1011 (B) for negative. Some implementations also provide unsigned BCD values with a sign nibble of 1111 (hex F). In packed BCD, the number +127 is represented as the bytes 00010010 01111100 (hex 12 7C), and −127 as

00010010 01111101 (hex 12 7D).

## Encoding a BCD Number

**Definition:** BCD represents each of the digits of an unsigned decimal as the 4-bit binary equivalents.

**Unpacked BCD** - Unpacked BCD representation contains only one decimal digit per byte. The digit is stored in the least significant 4 bits; the most significant 4 bits are not relevant to the value of the represented number.

**Packed BCD** - Packed BCD representation packs two decimal digits into a single byte.

Decimal	Binary	BCD	
		Unpacked	Packed
0	0000 0000	0000 0000	0000 0000
1	0000 0001	0000 0001	0000 0001
2	0000 0010	0000 0010	0000 0010
3	0000 0011	0000 0011	0000 0011
4	0000 0100	0000 0100	0000 0100
5	0000 0101	0000 0101	0000 0101
6	0000 0110	0000 0110	0000 0110
7	0000 0111	0000 0111	0000 0111
8	0000 1000	0000 1000	0000 1000
9	0000 1001	0000 1001	0000 1001
10	0000 1010	0000 0001 0000 0000	0001 0000
11	0000 1011	0000 0001 0000 0001	0001 0001
12	0000 1100	0000 0001 0000 0010	0001 0010
13	0000 1101	0000 0001 0000 0011	0001 0011
14	0000 1110	0000 0001 0000 0100	0001 0100
15	0000 1111	0000 0001 0000 0101	0001 0101
16	0001 0000	0000 0001 0000 0110	0001 0110
17	0001 0001	0000 0001 0000 0111	0001 0111

18	0001 0010	0000 0001 0000 1000	0001 1000
19	0001 0011	0000 0001 0000 1001	0001 1001
20	0001 0100	0000 0010 0000 0000	0010 0000

**Invalid BCD Numbers,** These binary numbers are not allowed in the BCD code: 1010, 1011, 1100, 1101, 1110, 1111

## Packing a Two-Byte BCD

To pack a two-byte unpacked BCD number into a single byte creating a packed BCD number, shift the upper byte left four times, then **OR** the results with the lower byte.

*For example,*

0000 0111 0000 1001<sub>(unpacked BCD)</sub> = 0111 1001<sub>(packed BCD)</sub>

0000 0111 << 4 = 0111 0000 (SHIFT LEFT 4)  
0111 0000 + 0000 1001 = 0111 1001 (OR)

## Converting between Decimal and BCD

**From Decimal to Unpacked BCD:** To convert a decimal number into an unpacked BCD number, assign each decimal digit its 8-bit binary equivalent.

*For example,*

194<sub>(base 10)</sub> = 00000001 00001001  
00000100<sub>(unpacked BCD)</sub>

1	9	4 (base 10)
<hr/>		
00000001	00001001	00000100 (BCD)
MSB		LSB

**From Decimal to Packed BCD:** To convert a decimal number into a packed BCD number, assign each digit of the decimal to its 4-bit equivalent, padding the upper nybble with zeroes if necessary.

*For example,*

238<sub>(base 10)</sub> = 00000010 00111000<sub>(packed BCD)</sub>

2	3	8 (base 10)
<hr/>		
0010	0011	1000 (BCD)
MSB		LSB

**From BCD to Decimal:** To convert a BCD into a decimal number, just reverse the appropriate process from above; beginning with the LSB, group the binary digits by either 4 or 8 bits for packed and unpacked, respectively, then convert each set into its decimal equivalent.

## Converting between Binary and BCD

**From Binary to Unpacked BCD:** To convert a binary number into an unpacked BCD, divide the binary number by decimal 10 and place the quotient in the most significant byte and the remainder in the least significant byte.

*For example,*

$$00110101_{(\text{base } 2)} = 00000101 \ 00000011_{(\text{unpacked BCD})}$$

$$\begin{array}{rcl} 0011 \ 0101 & = & 53_{(\text{base } 2)} \\ \div \ 0000 \ 1010 & = & 10_{(\text{base } 10)} \\ \hline 0000 \ 0101 & = & 5 \\ \text{with a remainder of} & \Rightarrow & \begin{array}{c} \hline 0000 \ 0101 \ 0000 \ 0011 \\ \hline \end{array} \\ 0000 \ 0011 & = & 3 \qquad \qquad \text{MSB} \qquad \qquad \text{LSB} \end{array}$$

**From Two-Byte Unpacked BCD to Binary:** To convert from a two-byte unpacked BCD to a binary number, multiply the most significant byte of the BCD by decimal ten, then add the product to the least significant byte.

*For example,*

$$00001001 \ 00000010_{(\text{unpacked BCD})} = 01011100_{(\text{base } 2)}$$

$$\begin{array}{rcl} 0000 \ 1001 & = & 9 \\ \times \ 0000 \ 1010 & = & 10_{(\text{base } 10)} \\ \hline 0101 \ 1010 & = & 90_{(\text{base } 10)} \\ \\ 0101 \ 1010 & = & 90_{(\text{base } 10)} \\ + \ 0000 \ 0010 & = & 2 \\ \hline 0101 \ 1100 & = & 92_{(\text{base } 10)} \end{array}$$

## BCD Addition

Either packed or unpacked BCD numbers can be summed. BCD addition follows the same rules as **binary addition**. However, if the addition produces a carry and/or creates an invalid BCD number, an adjustment is required to correct the sum. The correction method is to add 6 to the sum in any digit position that has caused an error.

*For example,*

$$24 + 13 = 37$$

$$\begin{array}{r} 0010 = 24 \\ 0100 \\ + 0001 = 13 \\ 0011 \\ \hline 0011 = 37 \\ 0111 \end{array}$$

$$15 + 9 = 24$$

$$\begin{array}{r} 0001 = 15 \\ 0101 \\ + 0000 = 9 \\ 1001 \\ \hline 0001 = 1? \text{ (invalid)} \\ 1110 \end{array}$$

$$\begin{array}{r} 0001 = 1? \text{ (invalid)} \\ 1110 \\ + 0000 = 6 \text{ (adjustment)} \\ 0110 \\ \hline 0010 = 24 \\ 0100 \end{array}$$

$$19 + 28 = 47$$

$$\begin{array}{r} 0001 = 19 \\ 1001 \\ + 0010 = 28 \\ 1000 \\ \hline 0100 = 41 \text{ (error)} \\ 0001 \end{array}$$

$$\begin{array}{r} 0100 = 41 \text{ (error)} \\ 0001 \\ + 0000 = 6 \text{ (adjustment)} \\ 0110 \\ \hline 0100 = 47 \\ 0111 \end{array}$$

## BCD Subtraction

Either packed or unpacked BCD numbers can be subtracted. BCD subtraction follows the same rules as **binary subtraction**. However, if the subtraction causes a borrow and/or creates an invalid BCD number, an adjustment is required to correct the answer. The correction method is to subtract 6 from the difference in any digit position that has caused an error.

*For example,*

$$37 - 12 = 25$$

$$\begin{array}{r} 0011 = 37 \\ 0111 \\ - 0001 = 12 \\ 0010 \\ \hline 0010 = 25 \\ 0101 \end{array}$$

$$65 - 19 = 46$$

$$\begin{array}{r} 0110 = 65 \\ 0101 \\ - 0001 = 19 \\ 1001 \\ \hline 0100 = 4? \text{ (invalid)} \\ 1100 \end{array}$$

$$0100 = 4? \text{ (invalid)}$$

$$41 - 18 = 23$$

$$\begin{array}{r} 0100 = 41 \\ 0001 \\ - 0001 = 18 \\ 1000 \\ \hline 0010 = 29 \text{ (error)} \\ 1001 \end{array}$$

$$0010 = 29 \text{ (error)}$$

$$\begin{array}{r}
 1100 \\
 - 0000 \\
 \hline
 0110
 \end{array}
 = 6 \text{ (adjustment)}$$

$$\begin{array}{r}
 0100 \\
 0110
 \end{array}
 = 46$$

$$\begin{array}{r}
 1001 \\
 - 0000 \\
 \hline
 0010
 \end{array}
 = 6 \text{ (adjustment)}$$

$$\begin{array}{r}
 0010 \\
 0011
 \end{array}
 = 23$$

## Unpacked BCD Multiplication

Multiplication cannot be performed on packed BCD; the 4 most significant bits must be zeroed for the adjustment to work. Multiply the two unpacked BCD numbers using the rules for **binary multiplication**. To adjust the product divide it by decimal 10, then place the quotient in the most significant byte and the remainder in the least significant byte (**convert the binary answer to unpacked BCD**).

*For example,*

$$00001001 \times 00000100 = 00000011 \ 00000110$$

$$\begin{array}{r}
 0000 \ 1001 = 9 \\
 \times 0000 \ 0100 = 4 \\
 \hline
 0010 \ 0100 = 24 \quad \text{(error)}
 \end{array}$$

$$\begin{array}{r}
 0010 \ 0100 = 24 \quad \text{(error)} \\
 \div 0000 \ 1010 = 10_{(\text{base } 10)} \text{ (adjustment)}
 \end{array}$$

$$\begin{array}{r}
 0000 \ 0011 = 3 \\
 \text{with a remainder of} \Rightarrow \begin{array}{cc} 3 & 6 \\ \hline 0000 \ 0011 & 0000 \ 0110 \\ \text{MSB} & \text{LSB} \end{array} \\
 0000 \ 0110 = 6
 \end{array}$$

## Unpacked BCD Division

BCD division also cannot be performed on packed numbers. Before dividing an unpacked BCD number, the division adjustment is made by **converting the BCD numbers to binary**. Adjust the two-byte BCD number by multiplying the upper byte by decimal 10 and adding the product to the lower byte. After the adjustment, divide the two binary numbers using the rules of **binary arithmetic**. Finally, **convert the binary quotient into an unpacked BCD number** if necessary.

*For example,*

$$00000010 \ 00001000 \div 00000111 = 00000100$$



$$(28 \div 7 = 4)$$

$$\begin{array}{rcl} & 0000 & 0010 = 2 \\ \times & 0000 & 1010 = 10_{(\text{base } 10)} \text{ (adjustment)} \\ \hline \end{array}$$

$$0001 \ 0100 = 20_{(\text{base } 10)}$$

$$\begin{array}{rcl} & 0001 & 0100 = 20_{(\text{base } 10)} \\ + & 0000 & 1000 = 8 \\ \hline \end{array}$$

$$0001 \ 1100 = 28_{(\text{base } 10)}$$

$$\begin{array}{rcl} & 0001 & 1100 = 28_{(\text{base } 10)} \\ \div & 0000 & 0111 = 7 \\ \hline \end{array}$$

$$0000 \ 0100 = 4$$

$$00000101 \ 00000010 \div 00000100 = 00000001 \ 00000011$$

$$(52 \div 4 = 13)$$

$$\begin{array}{rcl} & 0000 & 0101 = 5 \\ \times & 0000 & 1010 = 10_{(\text{base } 10)} \text{ (adjustment)} \\ \hline \end{array}$$

$$0011 \ 0010 = 50_{(\text{base } 10)}$$

$$\begin{array}{rcl} & 0011 & 0010 = 50_{(\text{base } 10)} \\ + & 0000 & 0010 = 2 \\ \hline \end{array}$$

$$0110 \ 0100 = 52_{(\text{base } 10)}$$

$$\begin{array}{rcl} & 0110 & 0100 = 52_{(\text{base } 10)} \\ \div & 0000 & 0100 = 4 \\ \hline \end{array}$$

$$0000 \ 1101 = ? \quad (\text{invalid})$$

$$\begin{array}{rcl} & 0000 & 1101 = ? \quad (\text{invalid}) \\ \div & 0000 & 1010 = 10_{(\text{base } 10)} \text{ (adjustment)} \\ \hline \end{array}$$

$$0000 \ 0001 = 1 \quad \Rightarrow \quad 1 \quad 3$$

with a remainder of

$$0000\ 0011 = 3$$

<hr/>				<hr/>			
0000	0001	0000	0011				
		MSB				LSB	

# TOPIC 8: LOGIC GATES & BOOLEAN ALGEBRA

## Introduction to Logic Mathematics

**Mathematical logic** is a subfield of **mathematics** exploring the applications of formal **logic** to **mathematics**. It bears close connections to meta-mathematics, the foundations of **mathematics**, and theoretical computer science.

## Set theory

A *set* can be defined as a collection of *things* that are brought together because they obey a certain *rule*.

These 'things' may be anything you like: numbers, people, shapes, cities, bits of text ..., literally anything.

The key fact about the 'rule' they all obey is that it must be *well-defined*. In other words, it enables us to say for sure whether or not a given 'thing' belongs to the collection. If the 'things' we're talking about are English words, for example, a well-defined rule might be:

'... has 5 or more letters'

A rule which is not well-defined (and therefore couldn't be used to define a set) might be:

'... is hard to spell'

## Requirement of a set

1. A set must be well defined i.e. it must not leave any room for ambiguities e.g sets of all students- which? Where? When?

A set must be defined in terms of space and time

2. The objective (elements or members) from a given set must be distinct i.e each object must appear once and only once, Must appear but not more than once
3. The order of the presentation of elements of a given set is immaterial  
e.g  $1,2,3 = 1,3,2 = 3,2,1$

## Types of Sets

In set theory, there are different types of sets. All the operations in set theory could be based on sets. Set should be a group of individual terms in domain. The universal set has each and every element of domain. We are having different types of sets. We will see about the different types of sets.

## Different Types of Sets

There are different types of sets in set theory. They are listed below:

- Universal Set

- Empty set
  - Singleton set
  - Finite and Infinite set
  - Union of sets
  - Intersection of sets
  - Difference of sets
  - Subset of a set
  - Disjoint sets
  - Equality of two sets

### **Universal Set**

The set of all the 'things' currently under discussion is called the ***universal set*** (or sometimes, simply the ***universe***). It is denoted by **U**.

The universal set doesn't contain everything in the whole universe. On the contrary, it restricts us to just those things that are relevant at a particular time. For example, if in a given situation we're talking about numeric values – quantities, sizes, times, weights, or whatever – the universal set will be a suitable set of numbers (see below). In another context, the universal set may be {alphabetic characters} or {all living people}, etc.

### **Singleton Set:**

A set which contains only one element is called a singleton set.

For example:

- $A = \{x : x \text{ is neither prime nor composite}\}$   
It is a singleton set containing one element, i.e., 1.
- $B = \{x : x \text{ is a whole number, } x < 1\}$

This set contains only one element 0 and is a singleton set.

- Let  $A = \{x : x \in \mathbb{N} \text{ and } x^2 = 4\}$

Here A is a singleton set because there is only one element 2 whose square is 4.

- Let  $B = \{x : x \text{ is an even prime number}\}$

Here B is a singleton set because there is only one prime number which is even, i.e., 2.

### **Finite Set:**

A set which contains a definite number of elements is called a finite set. Empty set is also called a finite set.

For example:

- The set of all colors in the rainbow.
  - $N = \{x : x \in \mathbb{N}, x < 7\}$
  - $P = \{2, 3, 5, 7, 11, 13, 17, \dots, 97\}$

### **Infinite Set:**

The set whose elements cannot be listed, i.e., set containing never-ending elements is called an infinite set.

For example:

- Set of all points in a plane
  - $A = \{x : x \in \mathbb{N}, x > 1\}$

- Set of all prime numbers
- $B = \{x : x \in W, x = 2n\}$

**Note:**

All infinite sets cannot be expressed in roster form.

For example:

The set of real numbers since the elements of this set do not follow any particular pattern.

**Cardinal Number of a Set:**

The number of distinct elements in a given set A is called the cardinal number of A. It is denoted by  $n(A)$ .

For example:

- $A = \{x : x \in N, x < 5\}$   $A = \{1, 2, 3, 4\}$

Therefore,  $n(A) = 4$

- $B =$  set of letters in the word ALGEBRA

$B = \{A, L, G, E, B, R\}$

Therefore,  $n(B) = 6$

**Equivalent Sets:**

Two sets A and B are said to be equivalent if their cardinal number is same, i.e.,  $n(A) = n(B)$ .

The symbol for denoting an equivalent set is ' $\leftrightarrow$ '.

For example:

$A = \{1, 2, 3\}$  Here  $n(A) = 3$   $B = \{p, q, r\}$  Here  $n(B) = 3$  Therefore,  $A \leftrightarrow B$

**Equal sets:**

Two sets A and B are said to be equal if they contain the same elements. Every element of A is an element of B and every element of B is an element of A.

For example:

$A = \{p, q, r, s\}$   $B = \{p, s, r, q\}$

Therefore,  $A = B$

The various types of sets and their definitions are explained above with the help of examples.

**Empty Set**

In mathematics, empty set is a set theory related topic. A set without any elements is said to be an empty set. This article helps you understand empty set by giving a clear idea about empty set with some example problems.

**Empty Set Definition**

The other name of empty set is null set  $\phi$ . Consider two sets  $X = \{a, b, c, d\}$  and  $Y = \{1, 2, 3, 4, 5, 6\}$ . Consider another set  $Z$  which represents the intersection of  $X$  and  $Y$ . There is no common element for the set  $X$  and  $Y$ . So, intersection of  $X$  and  $Y$  is null.

$Z = \{ \}$       **The representation of empty set is  $\{ \}$ .**

### Empty Set or Null Set:

- A set which does not contain any element is called an empty set, or the null set or the void set and it is denoted by  $\emptyset$  and is read as phi. In roster form,  $\emptyset$  is denoted by  $\{ \}$ . An empty set is a finite set, since the number of elements in an empty set is finite, i.e., 0.
- **For example:** (a) The set of whole numbers less than 0.  
(b) Clearly there is no whole number less than 0.

Therefore, it is an empty set.

(c)  $N = \{x : x \in N, 3 < x < 4\}$

- Let  $A = \{x : 2 < x < 3, x \text{ is a natural number}\}$

Here  $A$  is an empty set because there is no natural number between 2 and 3.

- Let  $B = \{x : x \text{ is a composite number less than } 4\}$ .

Here  $B$  is an empty set because there is no composite number less than 4.

#### Note:

$\emptyset \neq \{0\} \therefore$  has no element.

$\{0\}$  is a set which has one element 0.

**The cardinal number of an empty set, i.e.,  $n(\emptyset) = 0$**

### Cardinality of Empty Set:

Since we know that the cardinal number represents the number of elements that are present in the set and by the definition of an empty set, we know that there are no element in the empty set. Hence, the cardinal number or cardinality of an empty is zero.

### Properties of Preparation for Empty Set:

1. Empty set is considered as subset of all sets.  $\phi \subset X$
2. Union of empty set  $\phi$  with a set  $X$  is  $X$ .  $A \cup \phi = A$

Intersection of an empty set with a set  $X$  is an empty set.

### Solved Examples

**Question 1:**  $A$  is a set of alphabets and  $B$  is a set of numbers. What is the intersection of  $A$  and  $B$ ?

**Solution:**  $A \cap B = \{ \}$

**Question 2:** Write the set A which is a set of goats with 10 legs.

**Solution:**  $A = \{ \}$

### Power Set of the Empty Set

A set is called the power set of any set, if it contains all subsets of that set. We can use the notation  $P(S)$  for representing any power set of the set. Now, from the definition of an empty set, it is clear that there is no element in it and hence, the power set of an empty set i.e.  $P(\phi)$  is the set which contain only one empty set, hence  $P(\phi) = \{\phi\}$

### Cartesian Product Empty Set

The Cartesian product of any two sets say A and B are denoted by  $A \times B$ . There are some conditions for Cartesian product of empty sets as follows:

If we have two sets A and B in such a way that both the sets are empty sets, then  $A \times B = \phi \times \phi = \phi$ . It is clear that, the cartesian product of two empty sets is again an empty set.

If A is an empty set and  $B = \{1, 2, 3\}$ , then the cartesian product of A and B is as follows:  $A \times B = \{\phi\} \times \{1, 2, 3\} = \{\phi \times 1, \phi \times 2, \phi \times 3\} = \{\phi, \phi, \phi\} = \{\phi\}$

So, we say that if one of the set is an empty set from the given two sets, then again the Cartesian product of these two sets is an empty set.

### Examples of Empty Sets

Given below are some of the examples of empty sets.

#### Solved Examples

**Question 1:** Which of the following represents the empty set?

1. A set of cats with 4 legs
2. A set of apples with red color
3. A set of positive numbers in which all are less than 1
4. A set of rectangles with 4 sides

**Solution:**

**Option 1:** A set of cats with 4 legs. This set is possible where cats are having 4 legs.

**Option 2:** A set of apples with red color. This set is possible where apple is in red color.

**Option 3:** A set of positive numbers in which all are less than 1.

This set is not possible because the positive numbers must be greater than 1. So, this set is considered as empty set.

**Answer: 3**

**Question 2:** A is a set of numbers from 1 to 10 B is a set of negative numbers. What is the intersection of A and B?

**Solution:**

Given:

A = set of number from 1 to 10. = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} B = set of negative numbers  
= {-1, -2, -3, -4, ...} Intersection of A and B =  $A \cap B = \{ \}$

**Answer:** The intersection of given sets is an empty set.

## **Subset**

Consider the sets, X = set of all students in your school and Y = set of all students in your class. It is obvious that set of all students in your class will be in your school. So, every element of Y is also an element of X. We say that Y is a subset of X. The fact that Y is a subset of X is expressed in symbol as  $Y \subset X$ . The symbol  $\subset$  stands for "is a subset of" or "is contained in". If Y is a subset of X, then X is known to be a superset of Y. The subset of a set will have elements equal to or less than the elements in the given set.

### **Subset Definition**

A set A is said to be a subset of a set B, if every element of A is also an element of B. In other words,  $A \subset B$  if whenever  $a \in A$ , then  $a \in B$ . It is often convenient to use the symbol  $\Rightarrow$  which means "implies". Suppose, for two sets A and B,  $A = \{1, 2, 3\}$  and  $B = \{1\}$  then B is the subset of A.

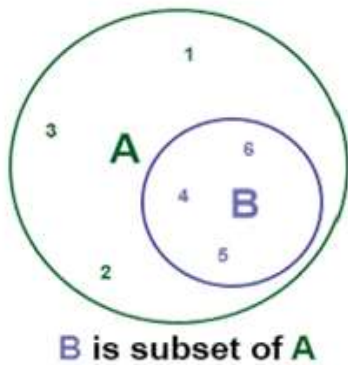
### **Subset Symbol:**

Using the symbol  $\Rightarrow$ , we can write the definition of subset as follows:

$A \subset B$  if  $a \in A \Rightarrow a \in B$

We read it as "A is a subset of B if a is an element of A, which implies that a is also an element of B". If A is not a subset of B, we write A is not a subset of B. If  $A = \{1, 2, 3, 4, 5, 6\}$  and  $B = \{4, 5, 6\}$ , then we can draw a Venn diagram for this as follows:



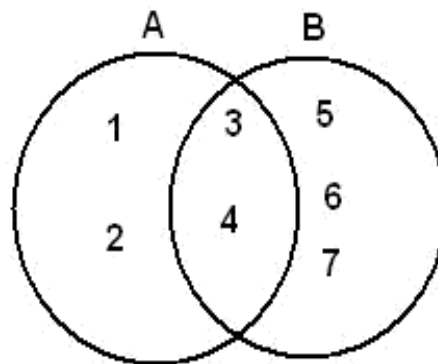


## Operation of a Set

### Union of Sets

Set is an important part of the mathematics. It is applied in almost many branch of mathematics. Set is the relation of some given data. There are many functions of set like union, intersection. Here, we will discuss about union of sets.

We denote the union of  $A$  and  $B$  by  $A \cup B$ . Thus,  $A \cup B = \{x | x \in A \text{ or } x \in B \text{ or } x \in A \text{ and } B\}$ . We write  $A \cup B = \{x | x \in A \text{ or } x \in B\}$  where, it is understood that the word 'or' is used in the inclusive sense. That is,  $x \in A$  or  $x \in B$  stands for  $x \in A$  or  $x \in B$  or  $x \in A \text{ and } B$ .

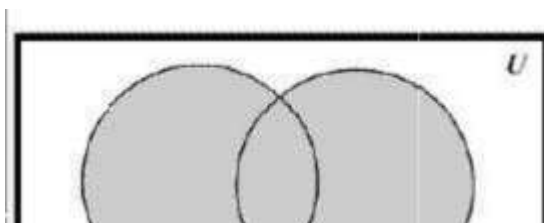


### **Union of Two Sets**

Let we have two sets  $A$  and  $B$ , then the union of these two sets is the set of all elements of each sets i.e. the set of those elements which are in either sets.

If  $A = \{1, 2, 3, 4\}$  and  $B = \{3, 4, 5, 6, 7\}$  then  $A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$ .

With the help of Venn diagram, we can prove it.



## Union of Countable Sets

A set of natural numbers which is a subset of a set with the same number of elements is called the countable set. The union of two countable sets is again a countable set. Let  $X$  and  $Y$  be two countable sets then  $X \cup Y$  is countable. Clearly, if  $X \cup Y$  is countable, then  $X$  and  $Y$  are each countable, as they are subsets of a countable set.

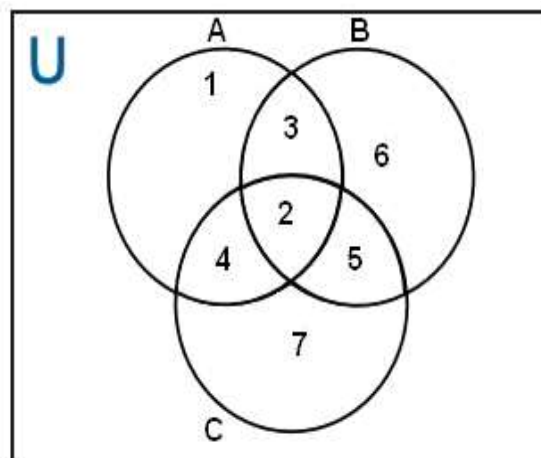
Conversely, let us suppose that we have two countable sets  $X$  and  $Y$ . And, we can define two surjection functions  $f: \mathbb{N} \rightarrow X$  and  $g: \mathbb{N} \rightarrow Y$ . Let  $Z = X \cup Y$ . Then, we can define  $h: \mathbb{N} \rightarrow Z$  in a way that  $h(2n + 1) = f(n)$  for  $n = 0, 1, \dots$  and  $h(2n) = g(n)$ ,  $n = 1, 2, \dots$ . Then,  $h$  is well defined function for every value of  $i \in \mathbb{N}$  is either odd or even, so  $h(i)$  is defined. Since  $h$  is onto function for any  $z \in Z$ , then  $z \in X$  or  $z \in Y$ . If  $z \in X$ , then  $h(2q + 1) = z$  for some value of  $q$  and if  $z \in Y$  then  $h(2p) = z$  for some value of  $p$ . Hence,  $Z$  is countable. So, we can say that the union of two countable sets is again a countable set.

## Union of Three Sets

If we have three sets say  $A$ ,  $B$  and  $C$ , then the union of these three sets is the set that contains all the elements or all contains that belongs to either  $A$  or  $B$  or  $C$  or to all three sets.

$A = \{1, 2, 3, 4\}$ ,  $B = \{2, 3, 5, 6\}$  and  $C = \{2, 4, 5, 7\}$ . Then,  $A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7\}$

We can show it in the Venn diagram as follows:



## Union of Sets Examples

Given below are some of the examples on union of sets.

## Solved Examples

**Question 1:** Find the union of each of the following two sets:

1.  $X = \{1, 3, 6\}$   $Y = \{1, 2, 6\}$
2.  $X = \{a, e, i, o, u\}$   $Y = \{a, e, c\}$
3.  $X = \{3, 4, 5\}$   $B = \varnothing$

**Solution:**

$$X \cup Y = \{1, 2, 3, 6\}$$

$$X \cup Y = \{a, c, e, i, o, u\}$$

$$X \cup Y = \{3, 4, 5\}$$

**Question 2:**

If  $X = \{1, 2, 5, 6\}$ ,  $Y = \{3, 4, 6, 9\}$ ,  $Z = \{3, 5, 6, 9\}$  and  $W = \{3, 6, 9, 11\}$ . Find

1.  $X \cup Y$
2.  $X \cup Z$
3.  $Y \cup Z$
4.  $Y \cup W$
5.  $X \cup Y \cup Z$
6.  $X \cup Y \cup W$
7.  $Y \cup Z \cup W$

**Solution:**

1.  $X \cup Y = \{1, 2, 3, 4, 5, 6, 9\}$
2.  $X \cup Z = \{1, 2, 3, 5, 6, 9\}$
3.  $Y \cup Z = \{3, 4, 5, 6, 9\}$
4.  $Y \cup W = \{3, 4, 5, 6, 9, 11\}$
5.  $X \cup Y \cup Z = \{1, 2, 3, 4, 5, 6, 9\}$
6.  $X \cup Y \cup W = \{1, 2, 3, 4, 5, 6, 9, 11\}$
7.  $Y \cup Z \cup W = \{3, 4, 5, 6, 9, 11\}$

**Find the Union of the Sets**

Here, we will learn how to find the union of the sets with the help of the following examples.

**Solved Examples**

**Question 1:**

Two sets are given.

$$A = \{5, 12, 13, 16, 19\}$$

$$B = \{5, 10, 13, 16, 19\}$$

Find  $A \cup B$

**Solution:**

Given sets are:

$$A = \{5, 12, 13, 16, 19\}$$

$$B = \{5, 10, 13, 16, 19\}$$

$$A \cup B = \{5, 10, 12, 13, 16, 19\}$$

Here, common elements in A, B are 5,13,16,19

So, it is taken only one times.

**Question 2:**

Find  $X \cup Y$  for the following set.

$$X = \{4, 6, 8, 9, 11\}$$

$$Y = \{3, 5, 6, 8, 11\}$$

**Solution:**

Given sets are

$$X = \{4, 6, 8, 9, 11\}$$

$$Y = \{3, 5, 6, 8, 11\}$$

$$\text{So, } X \cup Y = \{3, 4, 5, 6, 8, 9, 11\}$$

Here, common element is taken only one time.

**Intersection of Sets**

Intersection is an operation on sets. It is just opposite to union. It is a very useful and important concept in set theory. Before we learn about intersection, we need to understand some basic concept like what is set.

A set is a well-defined collection of data. It's data is known as it's members or elements. We represent the set by capital letters A, B, C, X, Y, Z, etc. We use the concept of set in daily life. For example, a team has five members. So, this is a set.

**Find the Intersection of the Sets**

For finding the intersection of two sets, we usually select those elements which are common in both the sets. If there are three sets, then we select those elements which are common in all three sets. Hence, if there are n number of sets, then we select only those elements which are common in all the n sets. In this way, we find the intersection of sets

**Intersecting Set:** Two sets A and B are said to be intersecting if  $A \cap B \neq \phi$

**Disjoint set:** Two sets A and B are said to be disjoint if  $A \cap B = \phi$

### Solved Examples

#### Question 1:

If  $A = \{1, 3, 4, 6, 9\}$  and  $B = \{2, 4, 6, 8\}$ , find  $A \cap B$ . What do you conclude?

#### Solution:

We have given that  $A = \{1, 3, 4, 6, 9\}$  and  $B = \{2, 4, 6, 8\}$

We have to find the intersection of A and B.

So,  $A \cap B = \{1, 3, 4, 6, 9\} \cap \{2, 4, 6, 8\}$

$A \cap B = \{4, 6\}$

#### Question 2:

If  $A = \{1, 3, 5, 7, 9\}$  and  $B = \{2, 4, 6, 8\}$ , find  $A \cap B$ . What do you conclude?

#### Solution:

We have  $A \cap B = \{1, 3, 5, 7, 9\} \cap$

$\{2, 4, 6, 8\} = \phi$

If no data match in both the sets, both the sets are known as **disjoint sets**. Thus, A and B are disjoint sets.

#### Question 3:

If  $A = \{1, 2, 3, 4, 5, 6, 7\}$ ,  $B = \{2, 4, 6, 8, 10\}$  and  $C = \{4, 6, 7, 8, 9, 10, 11\}$ , then find  $A \cap B$  and  $A \cap B \cap C$ .

#### Solution:

Given sets are

$A = \{1, 2, 3, 4, 5, 6, 7\}$

$B = \{2, 4, 6, 8, 10\}$

$C = \{4, 6, 7, 8, 9, 10, 11\}$

First, we have to find  $A \cap B$ . Then, we have to treat  $A \cap B$  as a single

set. For  $A \cap B$ , we select those elements which are common in sets A and

B. So,  $A \cap B = \{2, 4, 6\}$

For  $(A \cap B) \cap C$ , we select those elements which are common in sets  $A \cap B$  and  $C$ .

So,  $(A \cap B) \cap C = \{4, 6\}$

So,  $A \cap B \cap C = \{4, 6\}$

#### Question 4:

If  $A = \{1, 3, 5, 7, 9\}$ ,  $B = \{2, 4, 6, 8\}$  and  $C = \{2, 3, 5, 7, 11\}$ , find  $(A \cap B)$  and  $(A \cap C)$  What do you conclude?

#### Solution:

We have given that

$$A = \{1, 3, 5, 7, 9\}$$

$$B = \{2, 4, 6, 8\}$$

$$C = \{2, 3, 5, 7, 11\}$$

$$A \cap B = \{1, 3, 5, 7, 9\} \cap \{2, 4, 6, 8\} = \phi$$

Thus,  $A$  and  $B$  are disjoint sets

$$A \cap C = \{1, 3, 5, 7, 9\} \cap \{2, 3, 5, 7, 11\} = \{3, 5, 7\}$$

Thus,  $A$  and  $B$  are disjoint sets while  $A$  and  $C$  are intersecting sets.

#### Intersection of Two Sets

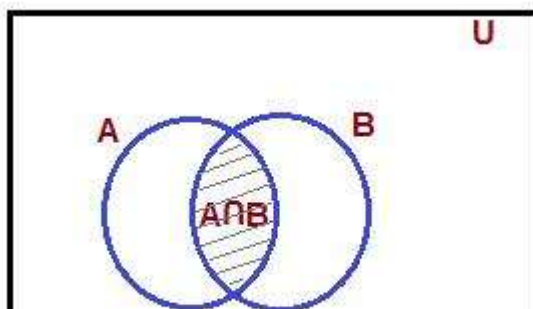
The intersection of two sets is the set of all the elements of two sets that are common in both of them. If we have two sets  $A$  and  $B$ , then the intersection of them is denoted by  $A \cap B$  and it is read as  $A$  intersection  $B$ .

Let  $X = \{2, 3, 8, 9\}$  and  $Y = \{5, 12, 9, 16\}$  are two sets.

Now, we are going to understand the concept of **Intersection of set**. It is represented by the symbol " $\cap$ ".

If we want to find the intersection of  $A$  and  $B$ , the common part of the sets  $A$  and  $B$  is the intersection of  $A$  and  $B$ . It is represented as  $A \cap B$ . That is, if an element is present in both  $A$  and  $B$ , then that will be there in the intersection of  $A$  and  $B$ . It will be more clear with the below figure.

Let  $A$  and  $B$  are two sets. Then, the intersection of  $A$  and  $B$  can be shown as below.



The intersection of A and B is denoted by  $A \cap B$ .

Thus,  $A \cap B = \{x : x \in A \text{ and } x \in B\}$ .

Clearly,  $x \in A \cap B$  i.e.,  $x \in A$  and  $x \in B$

In the above figure, the shaded area represents  $A \cap B$ .

In the same way, if  $A_1, A_2, \dots, A_n$  is a finite family of sets, then their intersection is represented by  $A_1 \cap A_2 \cap \dots \cap A_n$ .

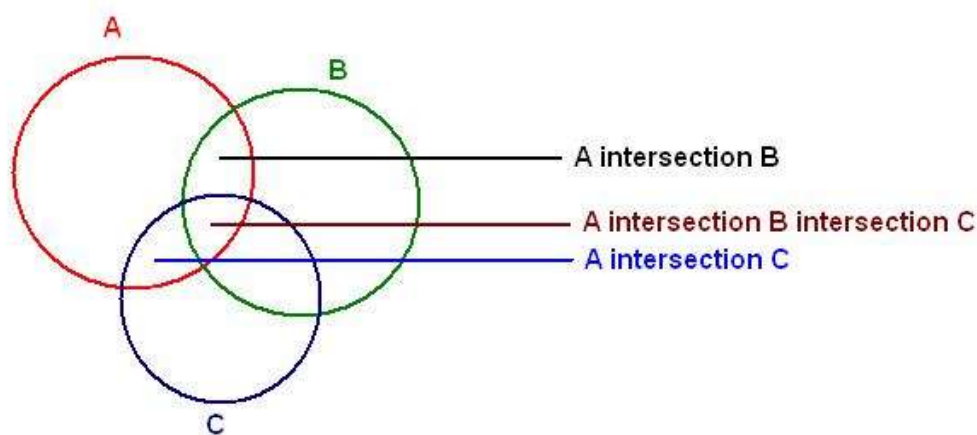
### Intersection of Convex Sets

In a Vector space, a set is called convex set if all the elements of the line joining two points of that set also lies on that set. In other words, we can say that the set S is convex set if for any points  $x, y \in S$ , there are no points on the straight line joining points x and y are not in the set S.

The intersection of two convex set is again a convex set. We can prove it with the help of contradiction method. So, let's suppose that A and B are the two convex sets. And, let we have two points x and y in such a way that  $x \in A \cap B$  and  $y \in A \cap B$ , then  $x \in A$ ,  $x \in B$ ,  $y \in A$  and  $y \in B$  and there exists a point z in such a way that z is not in A or B or both. This is the contradiction of our assumption that A and B are the convex sets. So there is no such point x, y and z can exist and  $A \cap B$  is a convex set.

### Intersection of Three Sets

If we have A, B and C, then the intersection of these three sets are the set of all elements A, B and C that are common in these three sets.



### Solved Example

**Question:**

If we have  $A = \{1, 3, 5, 7, 6, 8\}$ ,  $B = \{2, 4, 6, 8, 9\}$  and  $C = \{1, 3, 6, 8\}$ , then find the  $A \cap B \cap C$ .

**Solution:**

Given that  $A = \{1, 3, 5, 7, 6, 8\}$ ,  $B = \{2, 4, 6, 8, 9\}$  and  $C = \{1, 3, 6, 8\}$ .

Then, it is clear that the elements 6 and 8 are common in all the three given sets.

Hence, we get  $A \cap B \cap C = \{6, 8\}$ .

**Intersection of Open Sets**

Every intersection of open sets is again an open set. Let us have two open sets  $A_1$  and  $A_2$ . If the intersection of both of them is empty and empty set is again an open set. Hence, the intersection is an open set.

If  $A_1$  and  $A_2$  are open sets, then there exists some  $x \in A_1 \cap A_2$ . Since the given sets are open, we have some  $r_1$  and  $r_2$  in such a way that  $B_{r_1}(x) \subset A_1$  and  $B_{r_2}(x) \subset A_2$ . So, we can choose a number  $B_r(x) \subset A_1 \cap A_2$ .

So, we can say that if the intersection is not empty, then by the use of definition of intersection and non emptiness, there exists any  $x \in A_i$  for all  $A_i$ 's, where all  $A_i$ 's are open sets. Then, we have  $B_{r_i}(x) \subset A_i$  for some  $r_i > 0$ .

**Complement of a Set**

In set theory, complement set is one of the branch. Set of all elements in the universal set that are not in the initial set are said to be complement set. The complement of a set is represented by the symbol  $A'$ . The set is a collection of the object. Set is denoted by the symbols  $\{\}$ . In this article, we see in detail about the complement set.

**Complement of a Set Definition**

If we have a set  $A$ , then the set which is denoted by  $U - A$ , where  $U$  is the universal set is called the complement of  $A$ . Thus, it is the set of everything that does not belong to  $A$ . So, the complement of a set is the set of those elements which does not belong to the given set but belongs to the universal set  $U$ . Mathematically, we can show it as  $A^c = \{x \mid x \notin A \text{ but } x \in U\}$

Since we know that every set is the subset of the universal set  $U$ , then the complementary set is also the subset of  $U$ . The total number of elements in the complementary set is equal to the difference between the number of elements of the set  $U$  and the number of elements of the given set (say  $A$ ). If  $A$  is the given set, then the complement of  $A$  is denoted as  $A^c$  or  $A'$ .

For example,  $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$  and a set  $A = \{2, 3, 4, 5, 6\}$ . Then, the complement of  $A$  is denoted by  $A^c$  or  $A'$ .

$A^c = \{1, 7, 8, 9\}$ . We can show this with the help of Venn diagram





### **Complement of a Set Example**

Given below are some of the examples on complement of a set.

### **Solved Examples**

**Question 1:** Value of set  $U = \{2, 4, 6, 7, 8, 9, 10\}$  and  $A = \{7, 8, 9, 10\}$  and  $B = \{8, 9, 10\}$ . Find the complement of A, complement of B, complement of A union B.

**Solution:**

**Step 1:** Given

$$U = \{2, 4, 6, 7, 8, 9, 10\}$$

$$A = \{7, 8, 9, 10\}$$

$$B = \{8, 9, 10\}$$

**Step 2:** The element of set U is  $\{2, 4, 6, 7, 8, 9, 10\}$ . The element that does not belong to A is  $\{2, 4, 6\}$ . Complement of A is  $\{2, 4, 6\}$ .

**Step 3:** Complement of B is  $\{2, 4, 6, 7\}$

**Step 4:** Complement of AB is  $\{2, 4, 6, 8, 9, 10\}$ .

**Question 2:** Values of set  $U = \{3, 5, 7, 8, 9, 10, 12\}$  and  $A = \{8, 9, 10, 12\}$ . Find the complement of A.

**Solution:**

**Step 1:** Given

$$U = \{3, 5, 7, 8, 9, 10, 12\}$$

$$A = \{8, 9, 10, 12\}$$

**Step 2:** The element of set U is  $\{3, 5, 7, 8, 9, 10, 12\}$ . Elements  $\{3, 5, 7\}$  does not belong to the set A. So,  $A' = \{3, 5, 7\}$

**Step 3:** Complement of A is  $\{3, 5, 7\}$ .

**Question 3:** Values of set  $U = \{1, 4, 6, 7, 8, 10\}$  and  $A = \{6, 7, 8\}$ . Find the complement of A

**Solution:**

**Step 1:** Given

$$U = \{1, 4, 6, 7, 8, 10\}$$

$$A = \{6, 7, 8\}$$

**Step 2:** The element of set U is  $\{1, 4, 6, 7, 8, 10\}$ . Elements  $\{1, 4, 10\}$  does not belong to the set A.  $A'$  is  $\{1, 4, 10\}$ .

**Step 3:** Complement of A is  $\{1, 4, 10\}$ .

## **Set Difference**

Here, we are going to learn about an operation on set called difference of sets. In mathematics, a set can have a limited number of elements. Set is a collection of data. We can perform many operations on set. The difference operation is one of them. The subtract(difference) symbol in the function represents the removal of the values from the second set from the first set. The operation of subtraction is a removing or taking away objects from group of object.

### **Difference of Two Sets**

Difference of sets is defined as a method of rearranging sets by removing the elements which belong to another set. Difference of sets is denoted by either by the symbols - or  $\setminus$ . P minus Q can be written either  $P - Q$  or  $P \setminus Q$ .

The differences of two sets P and Q, is written as  $P - Q$ , **It contains elements of P which are not present in elements of Q**. Here, result  $P - Q$  is obtained. Take set P as usual and compare with set Q. Now, remove those element in set P which matches with set Q. If  $P = \{a, b, c, d\}$  and  $Q = \{d, e\}$ , then  $P - Q = \{a, b, c\}$ .

### **Definition for difference of sets**

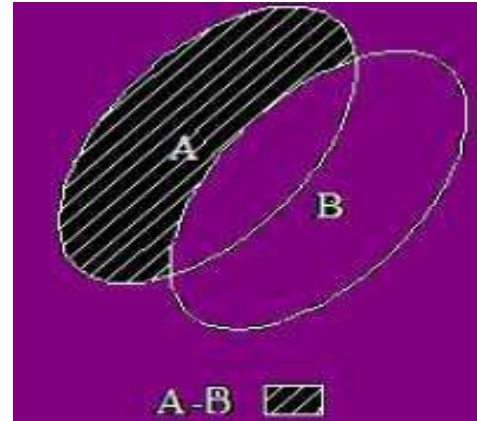
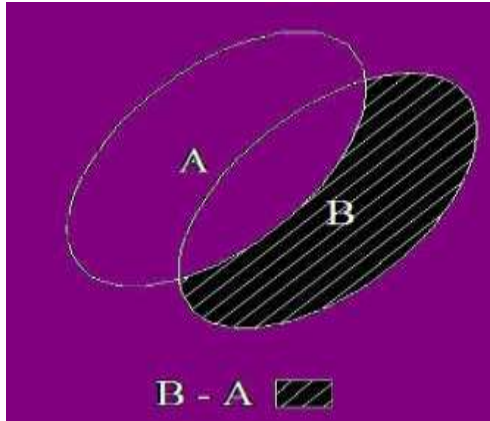
The difference between two sets A and B are represented in the order as the set of all those elements of A which are not in B. It is denoted by  $A - B$ .

In symbol, we write it as

$$A - B = \{x: x \in A \text{ and } x \notin B\}$$

$$\text{Similarly } B - A = \{x: x \in B \text{ and } x \notin A\}.$$

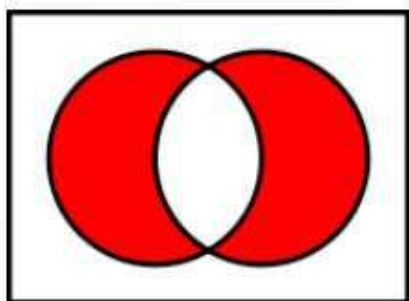
By representing it in the Venn diagram,



### Symmetric Difference of Sets

If we have two sets A and B, then the symmetric difference of these two sets A and B is the set of all elements those are either in A or in B not in both sets. So, we can say that the symmetric difference of two sets is the union without the intersection. We can use the symbol  $\triangle$  for this and denoted as follows:

$$A \triangle B = \{x \mid x \in A \setminus B \vee x \in B \setminus A\}$$



Symmetric Difference of Sets  
 $A \triangle B$

The symmetric difference of sets is associative. So, if we have three sets A, B and C, then  
 $(A \triangle B) \triangle C = A \triangle (B \triangle C)$

The symmetric difference of two sets is commutative i.e. for all sets A and B, we have  
 $A \triangle B = B \triangle A$

### Set Difference Examples

Given below are some of the problems based on difference of sets.

#### Solved Examples

**Question 1:** Consider the two sets  $A = \{11, 12, 13, 14, 15, 16\}$ ,  $B = \{12, 14, 16, 18\}$ . Find the difference between the two sets?

#### Solution:

Given  $A = \{11, 12, 13, 14, 15, 16\}$

$B = \{12, 14, 16, 18\}$

$A - B = \{11, 13, 15\}$

$B - A = \{18\}$

The set of all elements are present in A or in B. But, not in both is called the symmetric difference set.

**Question 2:**  $A = \{2, 3, 4, 1, 8, 9\}$  and  $B = \{2, 3, 4, 1, 8, 12\}$ . What is  $A - B$  and  $B - A$ ?

#### Solution:

Given  $A = \{2, 3, 4, 1, 8, 9\}$

$B = \{2, 3, 4, 1, 8, 12\}$

Here, all elements of A is available in B except 9.

So, the difference  $A - B = \{9\}$ .

Here, all elements of B are available in A except 12.

So, the difference  $B - A = \{12\}$ .

**Question 3:** Consider two sets  $A = \{a, b, f, g, h\}$ ,  $B = \{f, g, a, k\}$ . Find  $A - B$  and  $B - A$ ?

#### Solution:

Given  $A = \{a, b, f, g, h\}$

$B = \{f, g, a, k\}$  So,  $A - B = \{b, h\}$  and  $B - A = \{k\}$

**Question 4:** Consider given sets  $P = \{19, 38, 57, 76, 95\}$  and  $Q = \{7, 19, 57, 75, 94\}$ . Find  $P - Q$  and  $Q - P$ .

**Solution:**

Given  $P = \{19, 38, 57, 76, 95\}$

$Q = \{7, 19, 57, 75, 94\}$  So,  $P - Q = \{38, 76, 95\}$  and  $Q - P = \{7, 75, 94\}$

## Venn Diagrams

In mathematics, we can use the graphs and diagrams to solve some problems in geometry as well as in algebra. To follow this procedure, we can show some relations in set theory with the help of diagram, which is called as the **Venn diagram**. It is also known as **set diagram**. Venn diagrams are named so in the name of its founder John Venn in around 1880.

In set theory, Venn diagrams are studied. A set is defined as a collection of the same types of things. Venn diagram is an important and unique way of representing sets and various operations on them. It is a pictorial representation of sets. It is an easy way to understand about set theory. Venn diagrams are everywhere in set theory. With the help of Venn diagrams, we are able to show the operations of union, intersection, difference, complement etc. on the given sets.

In this page, we can discuss about these things with the help of a Venn diagram. In this process, the sets are represented by circles. Venn diagrams are generally used to represent operations on two or three sets. In order to learn about set theory in detail, one needs to command on Venn diagrams. In this article, students will learn about different types of Venn diagrams. So, go ahead with us and understand about Venn diagrams in detail.

### What is a Venn Diagram?

A Venn diagram is a pictorial representation of sets by set of points in the plane. The universal set  $U$  is represented pictorially by interior of a rectangle and the other sets are represented by closed figures viz circles or ellipses or small rectangles or some curved figures lying within the rectangle.

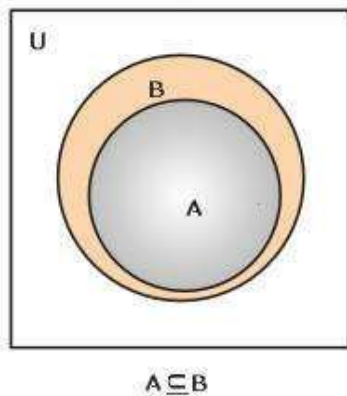
Venn diagram is a graphical tool in which we use overlapping circles to visually presentation among some given sets information. In Venn diagram, we can use two or more than two circles to show sets.

### Make a Venn Diagram

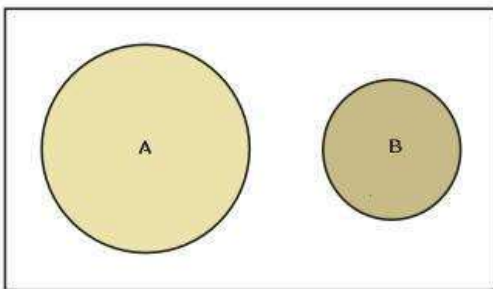
To make a Venn diagram, first we draw a rectangle to show the universal set  $U$  and mark  $U$  inside the rectangle. After that, we will make circles for given sets and name them as  $A$ ,  $B$ ,  $C$  etc. Then, according to the given relation of the sets, we can make a diagram for these sets in the

rectangle to show the relationship of the sets. Sometimes, we have some elements for the individual sets, then fill all the elements in their respective sets and as per the given relation of the sets.

For example, if A and B are any two arbitrary sets, elements such that, some elements are in A but not in B, some are in B but not in A, some are in both A and B, and some are in neither A nor B, we represent A and B in the pictorial form as in shown in the Venn diagram.

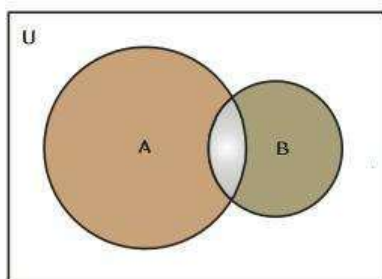


A is a subset of B and is represented as shown in the Venn diagram.



### Disjoint Sets

A and B are disjoint sets as shown in the Venn diagram.

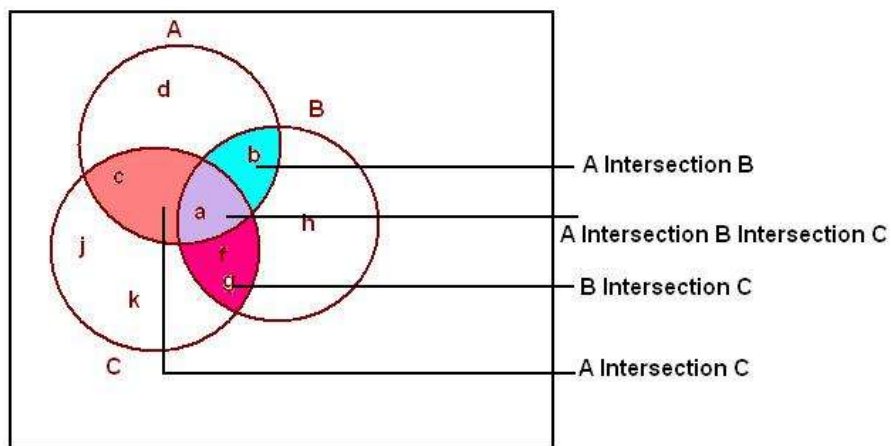


## Triple Venn Diagram

For the triple Venn diagram, we need three sets as A, B and C. In the triple Venn diagram, we have to show some relationship between these three sets.

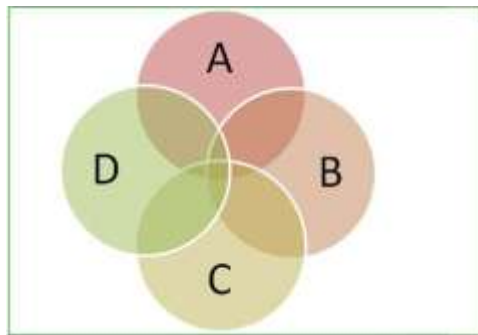
For example, let  $A = \{a, b, c, d, e\}$ ,  $B = \{a, b, f, g, h\}$  and  $C = \{a, c, e, f, g, j, k\}$ . Here, we can find  $A \cap B$ ,  $B \cap C$ ,  $A \cap C$  and  $A \cap B \cap C$  with the help of triple Venn diagram.

Given  $A = \{a, b, c, d, e\}$ ,  $B = \{a, b, f, g, h\}$  and  $C = \{a, c, f, g, j, k\}$ . Now,  $A \cap B = \{b\}$ ,  $B \cap C = \{f, g\}$ ,  $A \cap C = \{c\}$  and  $A \cap B \cap C = \{a\}$



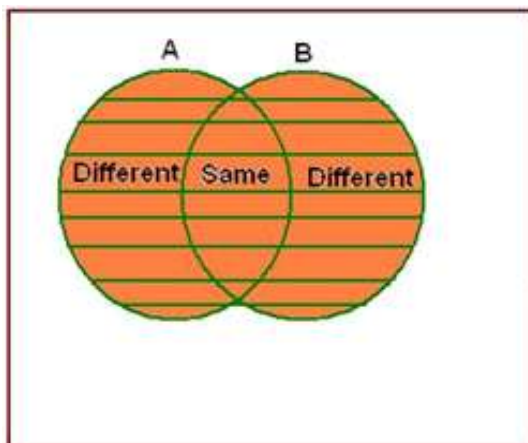
## 4 Circle Venn Diagram

Some times, we have four sets in a given problem and we want to show their relationship with the help of Venn diagram. For this, we can draw four circles in a rectangle box, each circle represents a unique set. Then, according to sets relation fill all the elements at their place.



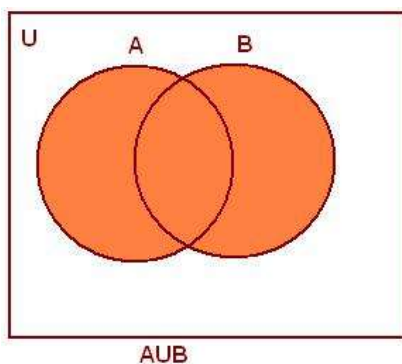
### Venn Diagram With Lines

In mathematics, sometimes we use the lines in the Venn diagram to show the union, intersection, difference etc. for the given sets. If we have sets A and B, then with the line Venn diagram we can show as:



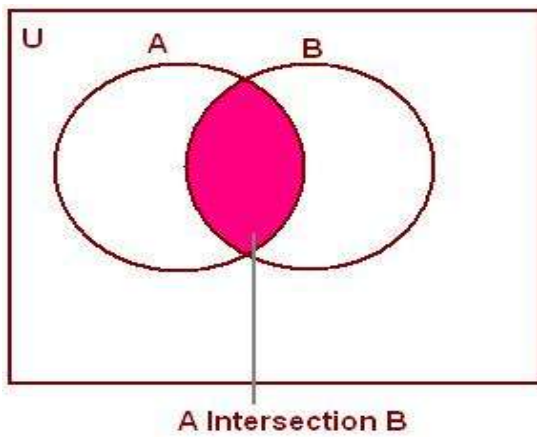
### Picture of a Venn Diagram

If we have two sets A and B, then  $A \cup B$  i.e. A union B:

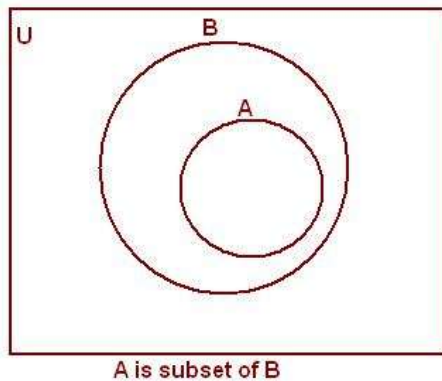


$A \cap B$  i.e. A intersection B:





**A and B are disjoint sets:**

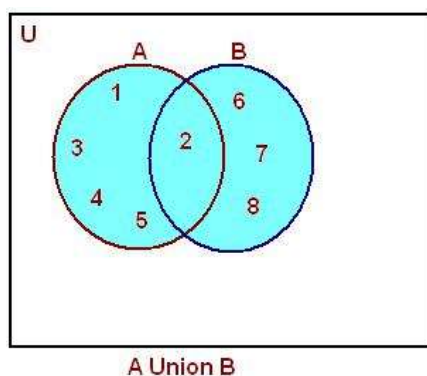


**A subset B:**

### Venn Diagram Union

If we have two sets A and B, then  $A \cup B$  is the set of all elements that are in set A and in the set B. If any element common in these two set, then we will take that one only one time. So, we can say that the union of the set A and B is everything which are either in set A or in the set B.

Let  $A = \{1,2,3,4,5\}$  and  $B = \{2,6,7,8\}$  then  $A \cup B = \{1,2,3,4,5,6,7,8\}$ . To show this union, we can use the Venn diagram also as



## Venn diagram Word Problems

Given below are some of the word problems on Venn diagram.

### Solved Example

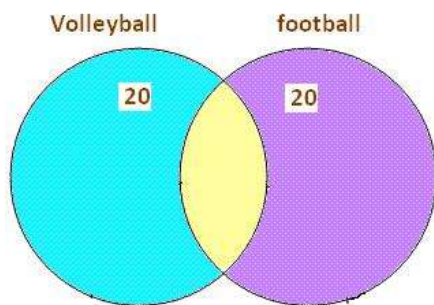
**Question:** There are 40 players participated in tournament match. In that, 20 players play in volley ball match and 20 players play in football match and 5 players play in both volley ball and football match. Solve this problem by using Venn diagram. How many of the players are either in match and how many are in neither match?

### Solution:

There are two categories, one is volleyball and other one is football.

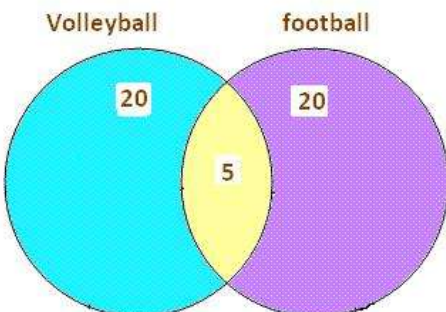
### Step 1:

Draw Venn diagram depending up on the classification given in the problem.



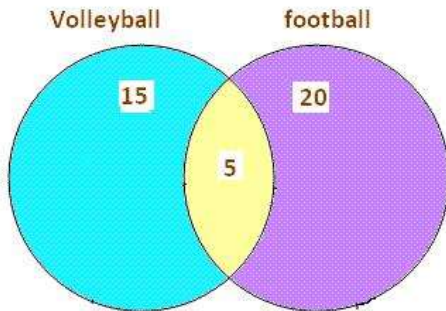
### Step 2:

Note that 5 players play both volleyball and football match



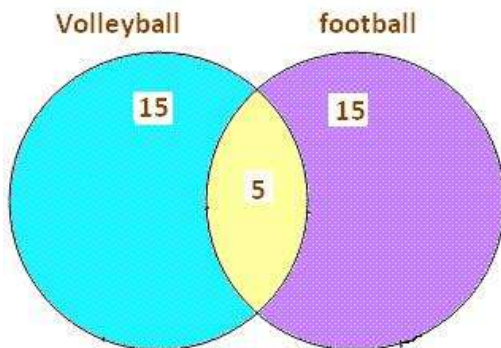
### Step 3:

Here, we accounted for 5 of the 20 players in volleyball match, leaving 15 players taking volleyball match but not football match. So, I will put "15" in the "volleyball only" part of the "volley ball" circle.



**Step 4:**

Here, we accounted for 5 of the 20 players in football match, leaving 15 players taking football match but not volleyball match. So, I will put "15" in the "football only" part of the "football" circle.



**Step 5:**

The total of  $5 + 15 + 15 = 35$  players are in either volley ball match or football match (or both). The total numbers of players are 40 and participating players are 35 only.

$$40 - 35 = 5 \text{ players}$$

## Boolean Algebra

In 1850, George Boole, an English mathematician developed rules and theorems that became Boolean algebra.

Boole's work was an outcrop of work in physiology called LOGIC.

Logic can be used to break down complex problems to simple and understandable problems.

The binary nature of logic problems was studied by Claude Shannon of MIT in 1938. Shannon applied Boolean algebra to relay logic switching circuits as means of realizing electric circuits.

Electric circuits used for digital computers are designed to generate only two voltage levels

Eg – high level ( $\approx 5V$ ) and low level ( $\approx 0V$ )

The binary number system requires two symbols hence its logical to identify a binary symbol with each voltage level. If we interpolate the high level as a binary 1 and low level as a binary 0, then we are using a positive logic system.

### Terminologies in Boolean Algebra

#### logic function and logic gates

**Logic circuit** - A computer switching/electronic circuit that consists of a number of logic gates and performs logical operations on data

**A logic gate** is an idealized or physical device implementing a Boolean function; that is, it performs a logical operation on one or more binary inputs, and produces a single binary output. A logic gate is a small transistor circuit, basically a type of amplifier, which is implemented in different forms within an integrated circuit. Each type of gate has one or more (most often two) inputs and one output.

**Boolean operation** is any logical operation in which each of the operands and the result take one of two values, as "true" and "false" or "circuit on" and "circuit off."

**A Boolean Function** is a description of operation (logic operation) on algebraic expression called **Boolean expression** which consists of binary variables, the constants 0 and 1, carried out in digital/electronic circuits and the logic outputting there off. The logic operation is well expressed in truth tables.

#### Truth tables

A truth table is a breakdown of a logic function by listing all possible values the function can attain. Such a table typically contains several rows and columns, with the top row representing the logical variables and combinations, in increasing complexity leading up to the final function.

### Logic Functions gates and circuitry

From Boolean algebra, we get three basic logic functions that form the basis of all digital computer functions. These basic functions are: AND, OR and NOT

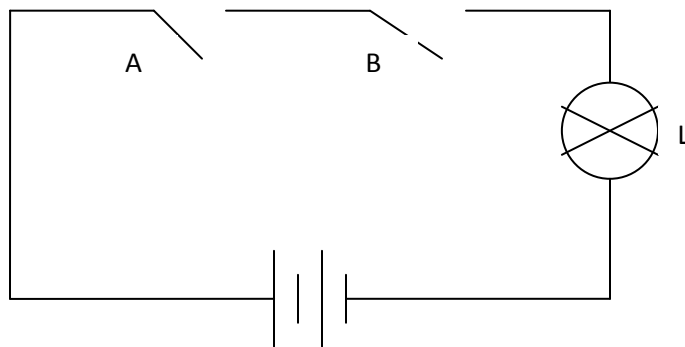
These functions can be expressed mathematically using Boolean algebra as given.

NOTE – The input and output variables are usually represented by letters as ABC or XYZ

- The logic state of these variables is represented by binary numbers 0 and 1

### **AND function**

The AND function can be thought of as a series circuit containing two or more switches



Circuit diagram

The logic indicator L will be ON only when logic switches A and B are both closed. Switches A and B have two possible logic states, open and closed. This can be represented in binary form as 0 – open and 1 – closed.

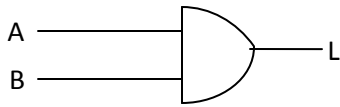
Logic indicator L also has two possible states 0 and 1

Truth table		
A	B	L(x.y)
0	0	0
0	1	0
1	0	0
1	1	1

The truth table is used to illustrate all the possible combinations of input and output conditions that can exist in a logic circuit. The Boolean expression used to represent an AND function is as follows

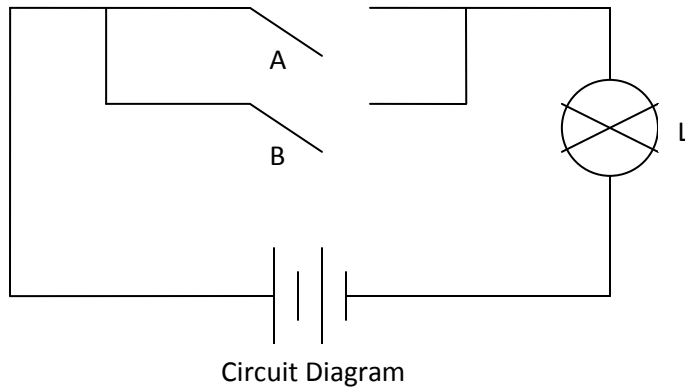
$$A.B=L$$

And is symbolized as



### **OR function**

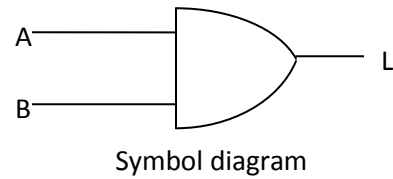
The function can be thought of as a parallel circuit containing two or more logic switches



Here, the logic indicator L will be ON whenever logic switch A and B are crossed. The truth table, Expression and Symbol of OR function is as follows

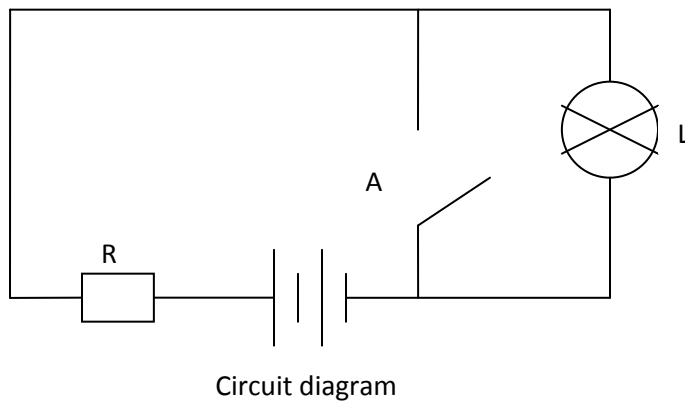
Truth table		
A	B	$L(x+y)$
0	0	0
0	1	1
1	0	1
1	1	1

$$A+B=L$$



### **NOT function**

It can be thought of as an inverter or negative circuit.

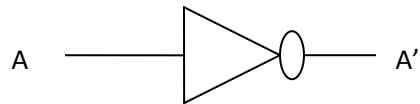


The logic indicator L will be ON whenever logic switch A is open.

The truth table, Expression and Symbol of NOT function is as follows

Truth table	
A	$L(x)'$
0	1
1	0

$$A = A'$$



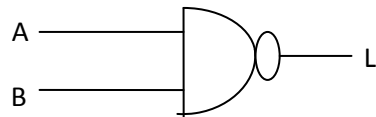
Symbol diagram

### **NAND**

If an AND gate is followed by an NOT gate then the combination is called an NAND gate and has following truth table and Boolean expression.

Truth table		
A	B	$L(x.y)'$
0	0	1
0	1	0
1	0	0
1	1	0

$$(A.B)' = L$$



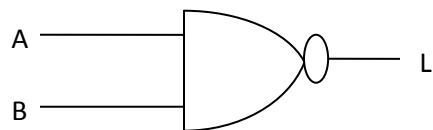
Symbol diagram

### **NOR**

If an OR gate is followed by an NOT gate then the combination is called an NOR gate and has following truth table and Boolean expression.

Truth table		
A	B	$L(x+y)'$
0	0	1
0	1	1
1	0	1
1	1	0

$$(A+B)' = L$$

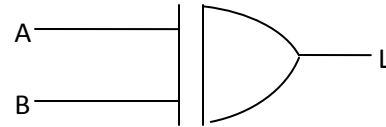
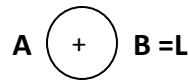


Symbol diagram

### **XOR**

This output strictly on condition that input is either high but not 2 highs

Truth table		
A	B	L (x±y)
0	0	0
0	1	1
1	0	1
1	1	0

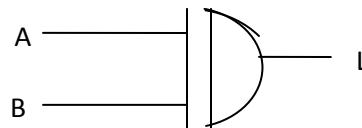


Symbol diagram

## **XNOR**

This output strictly on condition that input is either high but not 2 highs

Truth table		
A	B	L (x±y)'
0	0	1
0	1	0
1	0	0
1	1	1



## **Theorems of Boolean Algebra**

Boolean algebra deals with algebraic expressions between Boolean variables. Boolean algebra is a mathematical style dealing in logic. A fundamental rule relating to Boolean variables is called Boolean theorems.

### **Boolean theorems**

- Cumulative laws
  - $A+B=B+A$
  - $AB=BA$
- Associative laws
  - $(A+B)+C=A(B+C)=A+B+C$
  - $A(BC)=A(BC)=ABC$
- Distributive laws
  - $A(B+C)=AB+AC$
  - $A+BC=(A+B)(A+C)$

- This state that an expression can be expanded by multiplying term by term just like ordinary algebra. It indicates thus we can factor an expression

i.e –  $AB'C+A'B'C'=B'(AC+A'C')$  – Common factor is  $B'$

- Simplifying by distributive law

$$Y=AB'C+AB'D'=AB'(D+D')=AB' \text{ – since } D+D'=1+0=1 \text{ by distributive law}$$

- Identity law



i.  $A+A=A$       ii.  $AA=A$

5. Negative law

i.  $A'=A'$  ii.  $A''=A$

6. Redundancy laws

- i.  $A+AB=A(1+B)=A(1)=1$       N/b  $1+n=1$  where  $n=\text{any num/char}$
- ii.  $A(A+B)=AA+AB=A+AB=A$
- iii.  $0+A=A$
- iv.  $0A=0$
- v.  $1+A=1$
- vi.  $1A=A$
- vii.  $A'+A=1$
- viii.  $A'A=0$
- ix.  $A+AB'=A+B$
- x.  $A(A'+B)=AB$

EXAMPLE

$$Z=(A'+B)(A+B)=AA'+A'B+AB+BB=0+A'B+AB+B=B(A'+A+1)=B(1+1)=B$$

**Proves**

i.  $AC+ABC=AC$

Let  $y=AC+ABC$

$$=AC(1+B)=AC \text{ since } 1+n=1$$

ii.  $(A+B)(A+C)=A+BC$

Let  $y=(A+B)(A+C)$

$$=A(A+C)+B(A+C)=AA+AC+AB+CB=A+AC+AB+CB$$

$$=A(1+B)+AC+BC=A+AC+BC=A(1+C)+CB=A+BC$$

iii.  $A+A'B=A+B$

Let  $y=A+A'B$

$$=A.1+A'B=A(1+B)+A'B=A.1+AB+A'B=A+AB+A'B$$

$$=A+B(A+A')=A+B$$

iv.  $(A+B)(A+B')(A'+C)=AC$

Let  $y=(A+B)(A+B')(A'+C)$

$$=(AA+AB'+BA+BB')A'+C=(A+AB+AB')(A'+C)$$

$$=[A(1+B)+AB'](A'+C)$$

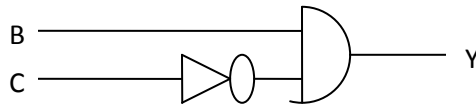
$$= (A+AB')(A'+C)=A(1+B')(A'+C)=A.1(A'+C)=A(A'+C)=AA'+AC=AC$$

- v. Simplify the expression and show minimum gate implementation  
 $y=ABC'D'+A'BC'D'+BC'D$

Since  $A+A'=1$  and  $A.1=A$

$$=BC'D'(A+A') + BC'D = BC'D'.1 + BC'D$$

$$= BC'D' + BC'D = BC'(D+D') = BC'.1 = BC'$$



## 7. Dedmorgans theorems

The theorems are useful in simplifying expressions in which a product or sum of variables is complimented or inverted.

The two theorems are

a)  $(A+B)'=A'B'$

When the OR sum of two variables  $(A+B)$  is complimented, this is same as if the 2 variable's compliments were ANDed.

i.e. – compliment of an OR sum is AND product of the compliment.

b)  $(AB)'=A'+B'$

Compliment of an AND product is equal to OR sum of its compliment

## Karnaugh maps (K-maps)

K-maps/ vetch diagram is a method to simplify Boolean expressions. The maps reduce the need for extensive calculations by taking advantage of human pattern-recognition capability.

In K—map, the Boolean variables are transferred (generally from a truth table) and ordered according to the principles of gray code in which only one variable changes in between squares.

Once the table is generated and the output possibilities transcribed, the data is arranged into the largest possible groups containing  $2^n$  cells ( $n=0, 1, 2, 3...$ ) and the minterms generated through the axiom laws of Boolean algebra

### Note

**A minterm** is a product (AND) of all variables in the function, in directs or complemented form. A minterm has the property that it is equal to 1 on exactly one row of the truth table.

A **maxterm** is a sum (OR) of all the variables in the function, in direct or complemented form. A maxterm has the property that it is equal to 0 on exactly one row of the truth table.

**Don't care** conditions are represented by X in the K-Map table. A don't-care term for a function is an input-sequence (a series of bits) for which the function output does not matter (0,1).

AB CD	00 (A'B')	01 (A'B)	11 (AB)	10 (AB')
00 (C'D')	M0	M4	M12	M8
01 (C'D)	M1	M5	M13	M9
11 (CD)	M3	M7	M15	M11
10 (CD')	M2	M6	M14	M10

### Procedure

K-map method may theoretically be applied to simplify any Boolean expression through works well with  $\leq 6$  variable.

- Each variable contributes two possibilities. The initial value and its inverse.

The variables are arranged in gray code in which only one variable changes between two adjacent grid boxes.

- Once the variables have been defined, the output possibilities are transcribed according to the grid location provided by the variables. Thus for every possibility of a Boolean input or variable the output possibility is defined.

When the K-map has been completed, to derive a minimized function the one's or desired outputs are grouped into the largest possible rectangular groups in which the num of grid boxes (output possibilities) in the groups must be equal to power of two.

- Don't care(s) possibilities (generally represented by X) are grouped only if the group created is larger than the group with minterms.

The boxes can be used more than once if they produce the least number of groups and each desired output must be contained within at least one grouping.

- The groups generated are then converted to a Boolean expression by locating and transcribing the variable possibility attributed to the box, and by the axiom laws of Boolean algebra – in which, if the initial variable possibility and its inverse are contained within the same group the variable term is removed.

### Note

Each group provides a “product” to create a “SOP” in the Boolean expression. To determine the inverse of the K-map, the 0’s are grouped instead of the 1’s. the two expressions are non-complementary.

Each square in a K-map corresponds to a minterm and maxterm in the venn diagram.

### Example

Following is an unspecified Boolean algebra function with Boolean variables ABC and D and their inverses

They can be represented in two different ways

- $F(A,B,C,D)=\sum(6,8,9,10,11,12,13,14,15)$
- $F(A,B,C,D)=A'BCD' + AB'C'D' + AB'C'D + AB'CD' + AB'CD + ABC'D' + ABCD'$

### Truth table

Using the defined minterms the table can be created as follow.

M#	ABCD(bin)	ABCD(gray)	F
0	0000	0000	0
1	0001	0001	0
2	0010	0011	0
3	0011	0010	0
4	0100	0110	0
5	0101	0111	0
6	0110	0101	1
7	0111	0100	0
8	1000	1100	1
9	1001	1101	1
10	1010	1111	1

11	1011	1110	1
12	1100	1010	1
13	1101	1011	1
14	1110	1001	1
15	1111	1000	1

### K-MAP

The input variable can be combined in 16 different ways, so the K-map has 16 positions and thus is arranged in a 4×4 grid.

AB \ CD	00	01	11	10
00	M0	M4	M12	M8
01	M1	M5	M13	M9
11	M3	M7	M15	M11
10	M3	M6	M14	M10

AB \ CD	00	01	11	10
00	0	0	1	1
01	0	0	1	1
11	0	0	1	1
10	0	1	1	1

The bin digits in the map rep the function output for any given combination of inputs.

After K-map is constructed, now find the minimal terms to use in the final expression.

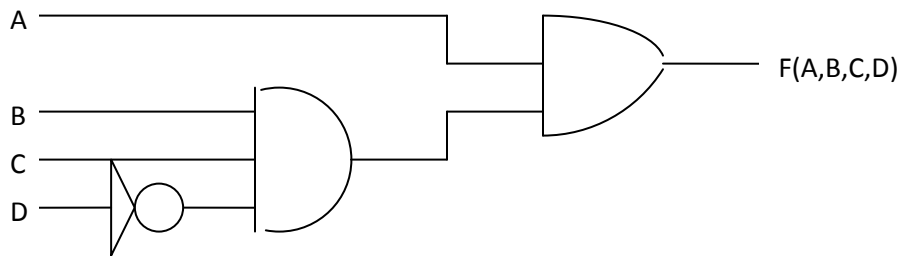
AB \ CD	00	01	11	10
00	0	0	1	1
01	0	0	1	1
11	0	0	1	1

10	0	1	1	1
----	---	---	---	---

Therefore the expression is:  $F(A,B,C,D)=A+CD'B$

#### Note

- Encircled groups may overlap.
- The grid is toroidally connected which means the grouping may wrap around edges.



Sum of products (SOP) – Summation of the minterm multiplicities

- It uses the minterms or variables that are high

$$F(A,B,C,D)=ABC+CD'B$$

Products of Sums (POS) – Multiplication of the maxterm summations

- It uses the maxterms or variables that are low

$$F(A,B,C,D)= (A+B)(A+C)(B'+C'+D')$$

#### **DON'T CARES**

The variables with a don't care may assume minterms or maxterms so long as they produce the most simple circuit.

$$F(W,X,Y,Z)=YZ+XY+XZ \text{ - SOP}$$

$$F(W,X,Y,Z)=(X+Z)(X+Y)(X+Z) \text{ - POS}$$

WX \ YZ	00	01	11	10
00	0	0	X	0
01	0	1	X	0
11	1	1	X	X
10	0	1	x	X

# TOPIC 9: EMERGING TRENDS

## The 7 Biggest Trends and Challenges in the Electronics Manufacturing Industry

The era of electronics began with the invention of the transistor in 1947 and silicon-based semiconductor technology. Seven decades later, we are surrounded by electronic devices and, much as we try to deny it, we rely on them in our everyday lives.

The performance of silicon-based devices has improved rapidly in the past few decades, mostly due to novel processing and patterning technologies, while nanotechnology has allowed for miniaturization and cost reduction.

For many years silicon remained the only option in electronics. But recent developments in materials-engineering and nanotechnology have introduced new pathways for electronics. While traditional silicon electronics will remain the main focus, alternative trends are emerging. These include:

### 1. 2-D electronics

Interest in the field started with the discovery of graphene, a structural variant of carbon. Carbon atoms in graphene form a hexagonal two-dimensional lattice, and this atom-thick layer has attracted attention due to its high electrical and thermal conductivity, mechanical flexibility and very high tensile strength. Graphene is the strongest material ever tested.

In 2010, the Royal Swedish Academy of Sciences decided to award the Nobel Prize in Physics to Andre Geim and Konstantin Novoselov for their “groundbreaking experiments” in graphene research.

Graphene may have started this 2D revolution in electronics, but silicene, phosphorene and stanene, atom-thick allotropes of silicon, phosphorus and tin, respectively, have a similar honeycomb structure with different properties, resulting in different applications.

All four have the potential to change electronics as we know it, allowing for miniaturization, higher performance and cost reduction. Several companies around the globe, including Samsung and Apple, are developing applications based on graphene.

### 2. Organic electronics

The development of conducting polymers and their applications resulted in another Nobel prize in 2000, this time in chemistry. Alan J. Heeger, Alan G. MacDiarmid and Hideki Shirakawa proved that plastic can conduct electricity.

Unlike conventional inorganic conductors and semiconductors, organic electronic materials are constructed from organic (carbon-based) molecules or polymers using chemical synthesis. Organic electronics is not limited to conducting polymers, but includes other organic materials that might be of use in electronics. These include a variety of dyes, organic charge-transfer complexes, and many other organic molecules.



In terms of performance and industrial development, organic molecules and polymers cannot yet compete with their inorganic counterparts. However, organic electronics have some advantages over conventional electronic materials. Low material and production costs, mechanical flexibility, adaptability of synthesis processes and biocompatibility make organic electronics a desirable choice for certain applications.

Commercially available high-tech products relying on organic semiconductors, such as curved television screens, displays for smartphones, coloured light sources and portable solar cells, demonstrate the industrial maturity of organic electronics. In fact, several high-tech companies, including LG Electronics and Samsung, have invested in cheap and high-performance organic-electronic devices. It is expected that the organic electronics market will grow rapidly in the coming years.

### **3. Memristors**

In 1971 Leon Chua reasoned from symmetry arguments that there should be a fourth fundamental electronic circuit-board element (in addition to the resistor, capacitor and inductor) which he called memristor, a portmanteau of the words memory and resistor. Although Chua showed that memristors have many interesting and valuable properties, it wasn't until 2007 that a group of researchers from Hewlett Packard Labs found that the memristance effect can be present in nanoscale systems under certain conditions. Many researchers believe that memristors could end electronics as we know it and begin a new era of "ionics".

While commonly available transistor functions use a flow of electrons, the memristor couples the electrons with ions, or electrically charged atoms. In transistors, once the flow of electrons is interrupted (for example by switching off the power) all information is lost. Memristors "memorize" and store information about the amount of charge that has flowed through them, even when the power is off.

The discovery of memristors paves the way to better information storage, making novel memory devices faster, safer and more efficient. There will be no information loss, even if the power is off. Memristor-based circuits will allow us to switch computers on and off instantly, and start work straight away.

For the past several years, Hewlett Packard has been working on a new type of computer based on memristor technology. HP plans to launch the product by 2020.

### **4. Spintronics**

Spintronics, a portmanteau word meaning "spin transport electronics", is the use of a fundamental property of particles known as "electron spin" for information processing. Electron spin can be detected as a magnetic field with one of two orientations: up and down. This provides an additional two binary states to the conventional low and high logic values, which are represented by simple currents. Carrying information in both the charge and spin of an electron potentially offers devices with a greater diversity of functionality.

So far, spintronic technology has been tested in information-storage devices, such as hard drives and spin-based transistors. Spintronics technology also shows promise for digital electronics in general. The ability to manipulate four, rather than only two, defined logic states may result in greater information-processing power, higher data transfer speed, and higher information-storage capacity.

It is expected that spin transport electronic devices will be smaller, more versatile and more robust compared with their silicon counterparts. So far this technology is in the early development stage and, irrespective of intense research, we have to wait a couple of years to see the first commercial spin-based electronic chip.

## **5. Molecular electronics**

The ultimate goal of electrical circuits is miniaturization. Also known as single molecule electronics, this is a branch of nanotechnology that uses single molecules or collections of single molecules as electronic building blocks.

Molecular electronics and the organic electronics described above have a lot in common, and these two fields overlap each other in some aspects. To clarify, organic electronics refers to bulk applications, while molecular-scale electronics refers to nano-scale, single-molecule applications.

Conventional electronics are traditionally made from bulk materials. However, the trend of miniaturization in electronics has forced the feature sizes of the electronic components to shrink accordingly. In single-molecule electronics, the bulk material is replaced by single molecules. The smaller size of the electronic components decreases power consumption while increasing the sensitivity (and sometimes performance) of the device. Another advantage of some molecular systems is their tendency to self-assemble into functional blocks. Self-assembly is a phenomenon in which the components of a system come together spontaneously, due to an interaction or environmental factors, to form a larger functional unit.

Several molecular electronic solutions have been developed, including molecular wires, single-molecule transistors and rectifiers. However, molecular electronics is still in the early research phase, and none of these devices has left the laboratory.

## **5 Challenges Electronics Manufacturers Face**

The world is quickly and constantly transforming as new technologies continue to enter the market. This rapid pace of change is perhaps felt most heavily in the electronics industry which brings in the biggest and newest innovations every year. Look back just a decade and you'll see how the industry has shifted, with completely new heavyweights leading the field. The evidence is clear that a company must consistently evolve with the times to remain an industry leader in electronics.

Following, we'll look at the five foremost challenges said company must look forward to.

### **Brief Product Life Cycles**

Technology isn't evolving for its own sake. It's responding to the wants and needs of consumers hungry for products that perfectly suit their day to day lives. Thus, companies in Electronics Manufacturing Services and contract manufacturers are required to have quality processes in place for new product introduction. To make sure product launches hit set goals on quality, volume and release, it's important to use closed-loop communication concepts between engineering, sales and manufacturing.

### **Intricate International Supply-Chain**

It is obvious now that we live in a global economy. Those who are positioned best to deal with the complexities of international sales are those best positioned to succeed in the long run. Now, it is common for components to skate across multiple continents—sometimes more than three—before arriving at their end point. Companies must be prepared to deal with varying international standards along with the twin issues of compliance and traceability that are prone to raise operational problems.

### **Demand**

The worst of the global economic crisis is fortunately in the rear view mirror, so it is expected that the demand for electronics should continue to rise. While cyclical fluctuations and economic dips are responsible for large shifts in demand, on a smaller level technology is highly susceptible to changing local conditions because tech is now so heavily tied to consumer demand. Consumer demand is an uncertain thing, determined by the vagueness of perceived value and swiftly fleeting tastes. For that reason, production capabilities must remain lean and able to shift quickly with uncertain demand.

### **Environmental Issues**

This is no longer a world where companies' margins are freely raised above the concerns of the environment. New standards and regulations are pushing electronic manufacturers to consider their 'social responsibility' when making decisions both small and large. A manufacturing consultant says some Electrical Engineering Master's programs are now including sustainable engineering strategies in their curriculum to accommodate the growing trend of environmental awareness. The entire life cycle of a product must be considered; from manufacturing, with the use of harmful chemicals and human exposure; to consumer use, with the consumption of energy; to the end of its life, with waste disposal and complex disassembly.

### **Tighter Margins**

Consumers have benefited from a global marketplace that has emphasized competition to bring in the latest and greatest innovations and lower prices. On the supply side, however, this has to lead to shrinking margins. Gains in efficiency and organization have slowed and there is not enough differentiation between products to stave off this growing trend of commoditization. Electronic manufacturers must deal with this downward pressure on operating margins as lights continue to turn on across the world.