1. RATIOS, RATES AND PROPORTIONS

2.1 Ratios

Ratio –A comparison of two numbers or quantities. They are measured in the same or similar units.

Example: If the ratio of adults to children is 2 to 5, then there are two adults for every 5 children. So, if there are 50 children in attendance, then there are 20 adults.

Ratios can be written in three different ways. In the above example of 2 adults to 5 children,

- ✓ In words: 2 to 5.
- ✓ As a fraction: $\frac{2}{5}$
- ✓ Using a colon: 2 : 5

In all three cases, we read "2 to 5".

Example 2

A picture on a wall has length 32 inches and width 12 inches. Write the ratio of length to width in simplest from.

$$\frac{32}{12} = \frac{8}{3}$$

So, the ratio is:

8 to 3 or 8:3 or $\frac{8}{3}$

Q.8- There are 1029 students in a school. 504 of them are girls. Find the ratio of boys to the girls.

Solution:-

Total number of student $= 1029$

Number of girls $= 504$

Number of boys $= 1029 - 504$

$= 525$

Required ratio $=$ Number of boys : Number of girls

$= 525 : 504$

$= 175 : 168$

$= 25 : 24$ Ans.

2.2Rates

- A rate is a special type of ratio.
- Rates are used to compare different kinds of quantities.

Example: The water dripped at a rate of 2 liters every 3 hours $\rightarrow \dfrac{2\text{ L}}{3\text{ hours}}$

Unit Rate

A unit rate is a rate with a denominator of 1. A common example of a unit rate is driving speed. For example, 20 mph, read as "20 miles per hour" can be written as follows.

$$\frac{20\text{ miles}}{1\text{ hour}}$$

In order to write a rate as a unit rate, use the following steps.

Step 1: Write the rate as a fraction

Step 2: Divide the numerator by the denominator

Try this! Write each rate in simplest form, then give the unit rate.

a. 30 pencils for 12 people

b. $27 for 6 lbs of almonds

c. 46 hours in 8 weeks

d. 28,770 new jobs created in 60 months

Answer:

a. $\frac{5\text{ pencils}}{2\text{ people}}$, 2.5 pencils per person

b. $\frac{\$9}{2\text{ lbs}}$, $4.5 per pound

c. $\frac{23\text{ hours}}{4\text{ weeks}}$, 5.75 hours per week

d. $\frac{959\text{ jobs}}{2\text{ months}}$, 479.5 jobs per month

2.3Proportions

Proportions

A proportion is an equation stating that two ratios or rates are equal. It is written in the following form.

$$\frac{a}{b}=\frac{c}{d}$$

If this equation is true, than the two ratios are equivalent.

This proportion can also be read as "a is to b as c is to d." The ratios are separated by the word "as."

2.3.1Direct and Inverse Proportion

A **direct and inverse proportion** are used to show how the quantities and amount are related to each other. They are also mentioned as directly proportional or inversely proportional. The symbol used to denote the proportionality is'$\propto$'. For example, if we say, *a* is proportional to *b*, then it is represented as '$a \propto b$' and if we say, *a* is inversely proportional to *b*, then it is denoted as '$a \propto 1/b$'. These relations are governed by some proportionality rules. Now in both cases, the value of 'a' changes in terms of 'b' or when the value of 'b' changes, the value of 'a' also change. The change in both values is equated with a **constant of proportionality**. Basically, a proportion states that two ratios like a/b and c/d are equal to each other, in such a way, a/b = c/d.

Q.12- Define "Direct proportion"

Ans. The quantitative relationship between two quantities such that increase in one quanitity causes a proportional increase in the other quantity, is called direct proportion.

Q.13- Define "Inverse proportion"

Ans. The quantitative relationship between two quantities such that increase in one quantity causes a proportional decrease in the other quantity or decrease in one quantity causes a proportional increase in the other quantity, is called inverse proportion.

Q.14- What do you know about compound proportion?

Ans. When one quantity is proportional to more than one quantities either direct or inverse, then the proportion is called compound proportion.

Cross Products

A cross product, also known as cross multiplying, is a technique that can be used to determine whether a proportion is true or to solve an equation. A cross product can be performed using the following steps.

Step 1: Write out the proportion

$$\frac{a}{b} = \frac{c}{d}$$

Step 2: Find the product of "a" and "d" and set that equal to the product of "b" and "c"

$$\frac{a}{b} = \frac{c}{d}$$

$$a * d = b * c$$

Hints:

+ You can think of a cross product as multiplying on a diagonal across the equals sign.

+ If the cross products are equal, then the proportion is true

The distance between City A and City B is 270 miles. On a certain map, this distance is scaled down to 4.5 inches. If the distance between City B and City C on the same map is 12 inches, what is the actual distance between City B and City C?

Solution:
Let $\boldsymbol{x}$ be the actual distance between City B and City C in miles. Since the ratio of actual distance to the distance on the map should be the same between any two cities, we can set up a proportion as follows:

$$\frac{\mathbf{270\ miles}}{\mathbf{4.5\ inches}} = \frac{\boldsymbol{x}\ \mathbf{miles}}{\mathbf{12\ inches}}$$

By cross-multiplying, we get:

$$\mathbf{4.5}\boldsymbol{x} = \mathbf{270 \cdot 12}$$
$$\mathbf{4.5}\boldsymbol{x} = \mathbf{3240}$$

So… $\boldsymbol{x} = \mathbf{720}$. The actual distance between City B and City C is 720 miles.

EXERCISE

a) You can peel 4 potatoes in 10 minutes. How long will it take you to peel 14 potatoes?
b) You can read 45 pages of your new book in 2 hours. How many pages can you read in 3 hours?

c) Nine out of ten students prefer math class over lunch. How many students do not prefer math if 200 students were asked?

d) You estimate that you can do 12 math problems in 45 minutes. How long should it take you to do 20 math problems?

John lives with three dogs. His daughter asks him to look after her dog for a week while she goes away. John normally buys two tins of dog food a day for the three dogs. How many tins should he buy for the four dogs for a week?

Solution

There are several ways to do this. Here is one.

3 dogs eat 2 tins a day

1 dog consumes $\frac{2}{3}$ tin a day

4 dogs consume $4 \times \frac{2}{3} = \frac{8}{3}$ tins a day

4 dogs consume $7 \times \frac{8}{3} = \frac{56}{3} = 18\frac{2}{3}$ tins a week

Therefore, John should buy 19 tins for the week.

NB: It helped to simplify the problems by considering 1 dog, rather than going straight to 4 dogs.