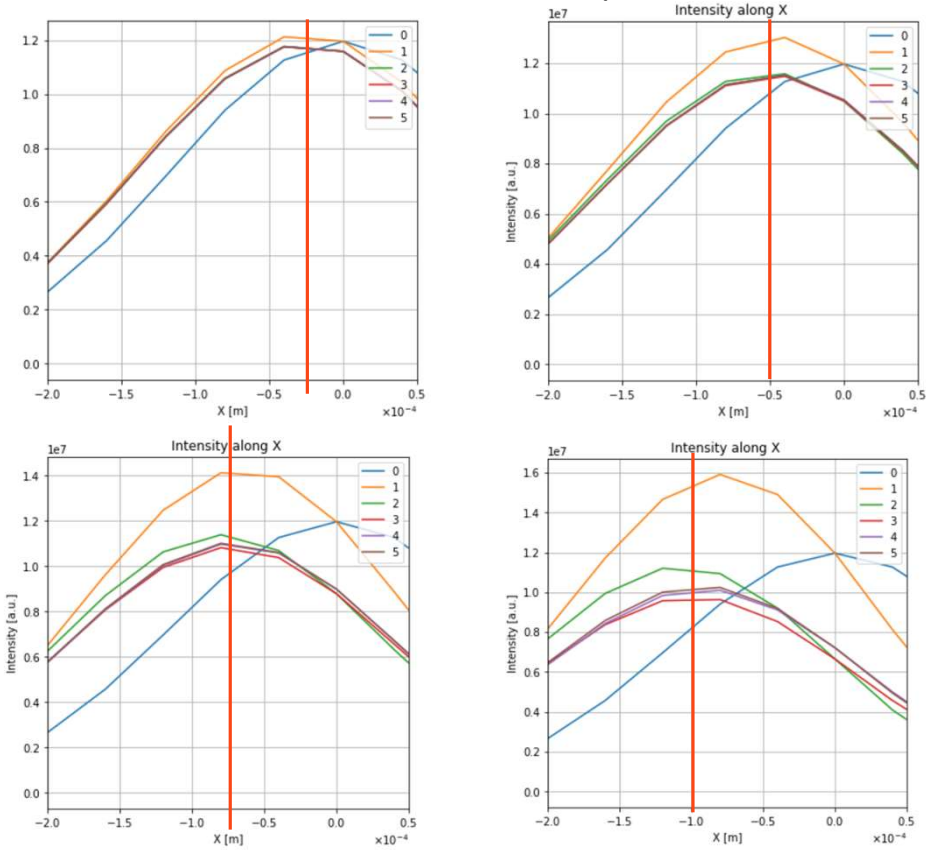
K^{th} Order Approximation: Shifted Beam

May 26 2020

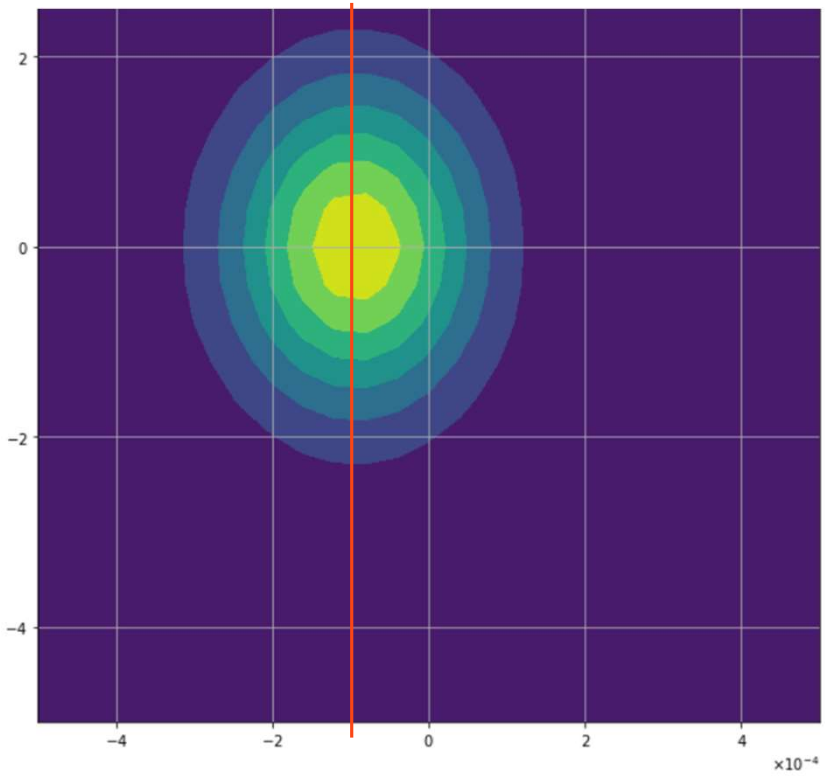
- Shift expansion analysis (at waist,*z*=0):
  - Gaussian
  - 0-100 micron offset tophat (34)
  - Computation times
    - Tophat ,varied max mode order

Shifted Gaussian

- Intensity contour (100 micron) right
- Intensity at y=0 below
- Vertical lines show expected shift
  - (L to R: 25,50,75,100um)

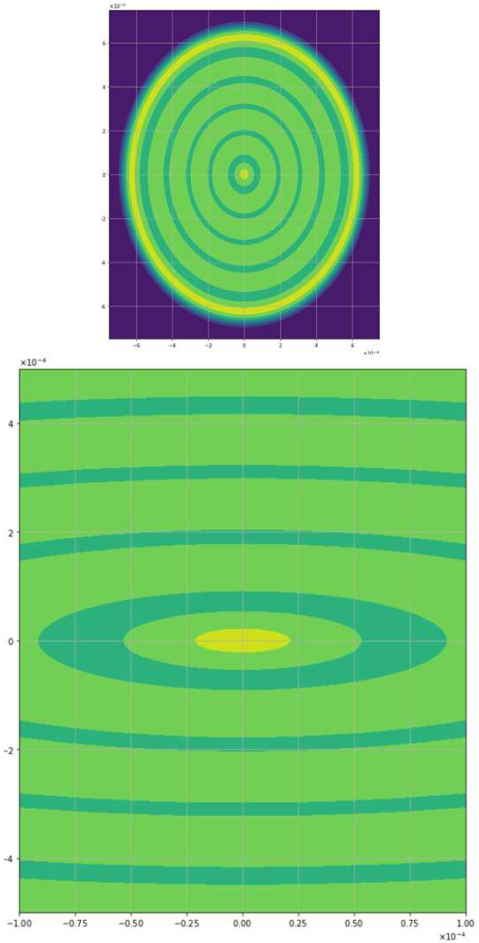


Intensity
(a=100um,5th order)
(image was low res)



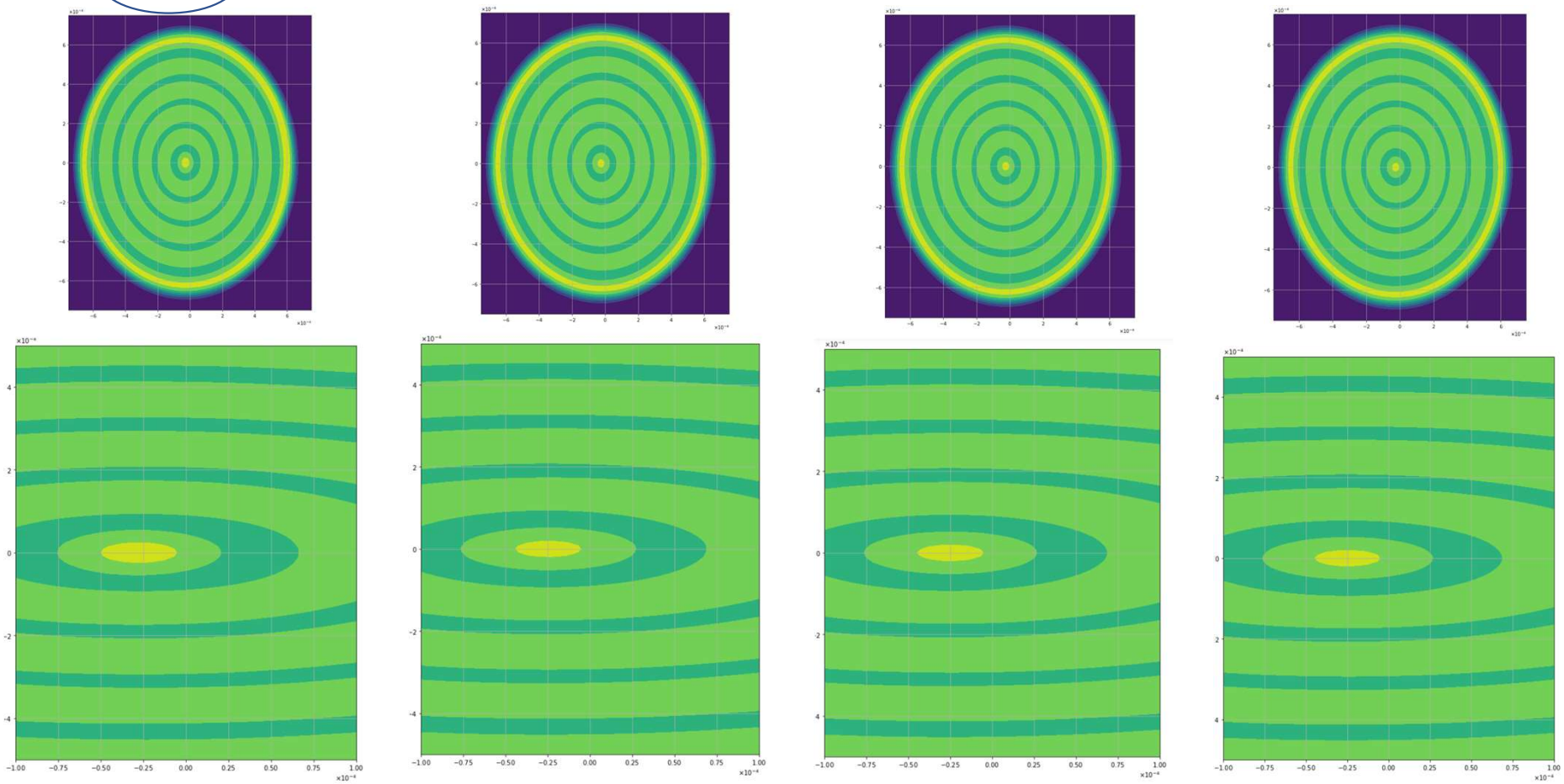
Shifted Tophat at Waist

- Shifts for 25, 50, 75, 100 micron
- Basis: beam radius = 2/3mm,w_0~ 0.2 mm
- Full mode order 34
- Intensity Resolution = 200x200
  - Full plot above zoomed plot
- *K* = approximation order
- **Convergence results:**
  - 25um at K=2
  - 50um at K=6
  - 75 um at K=8?
  - 100 um at??



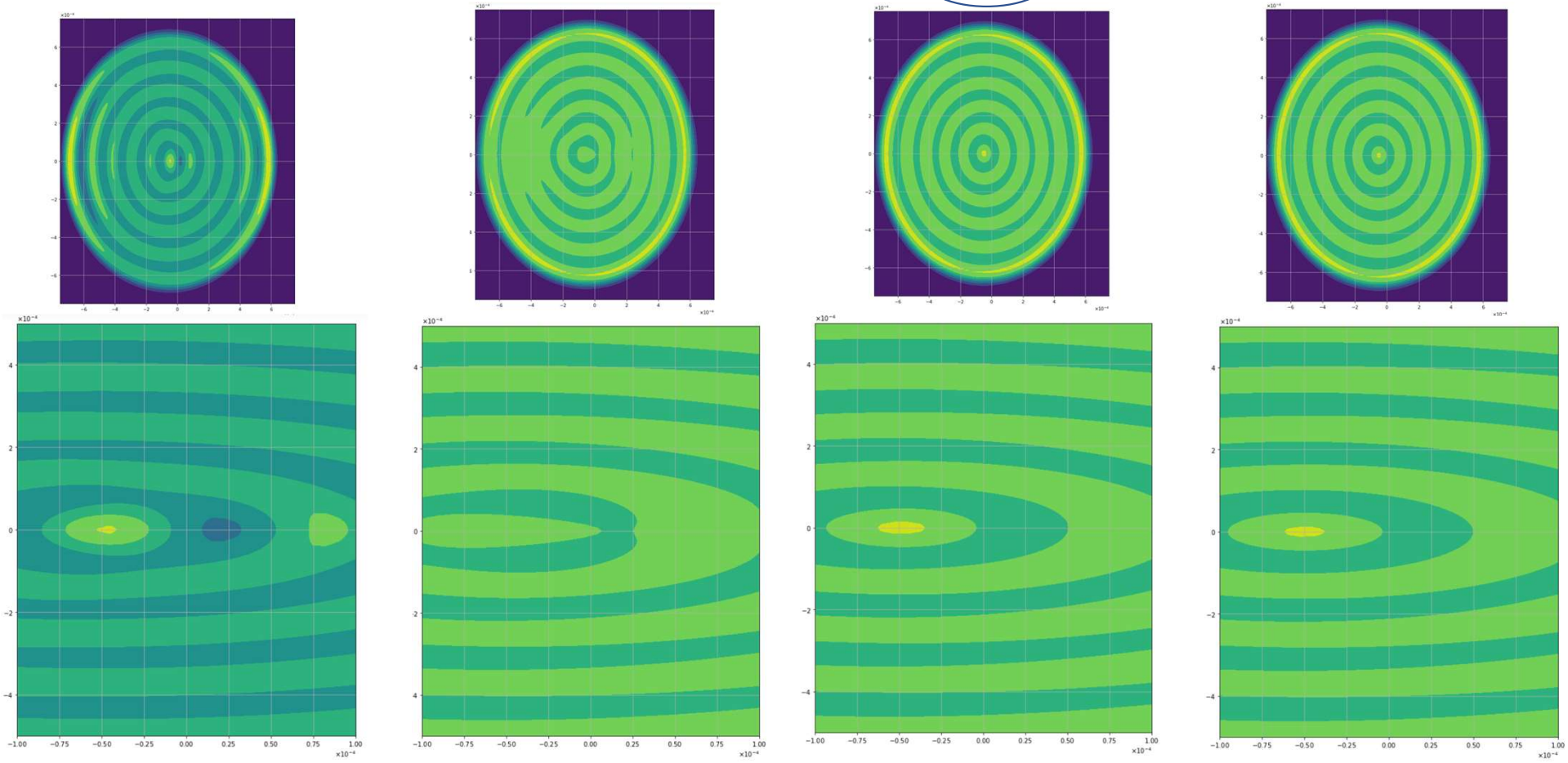
25 micron

K=2 K=4 K=6 K=8



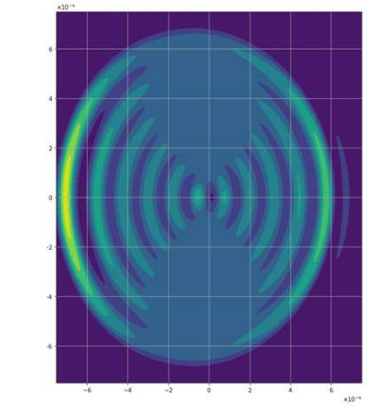
50 micron

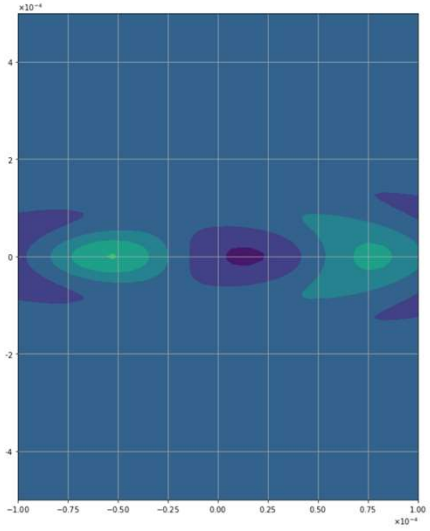
K=2 K=4 K=6 K=8



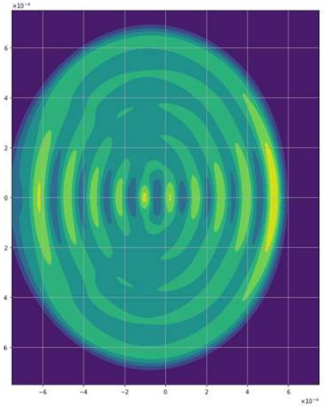
75 micron

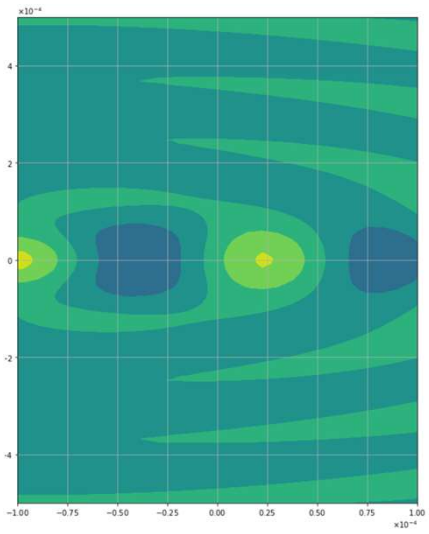
K=2



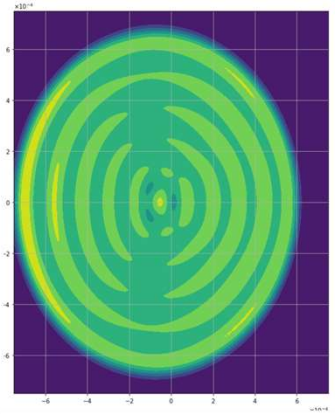


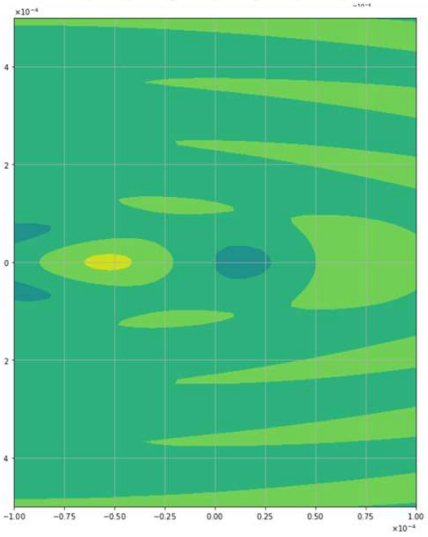
K=4



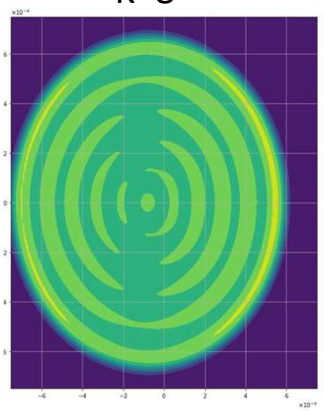


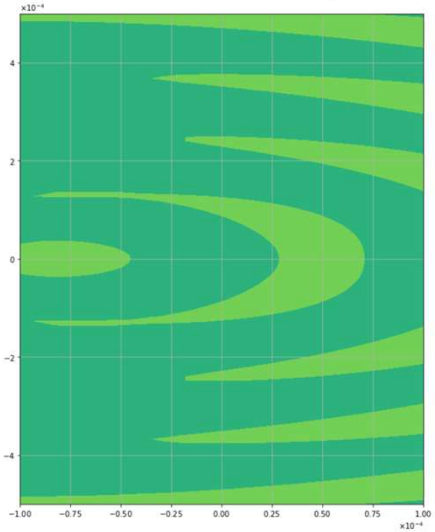
K=6





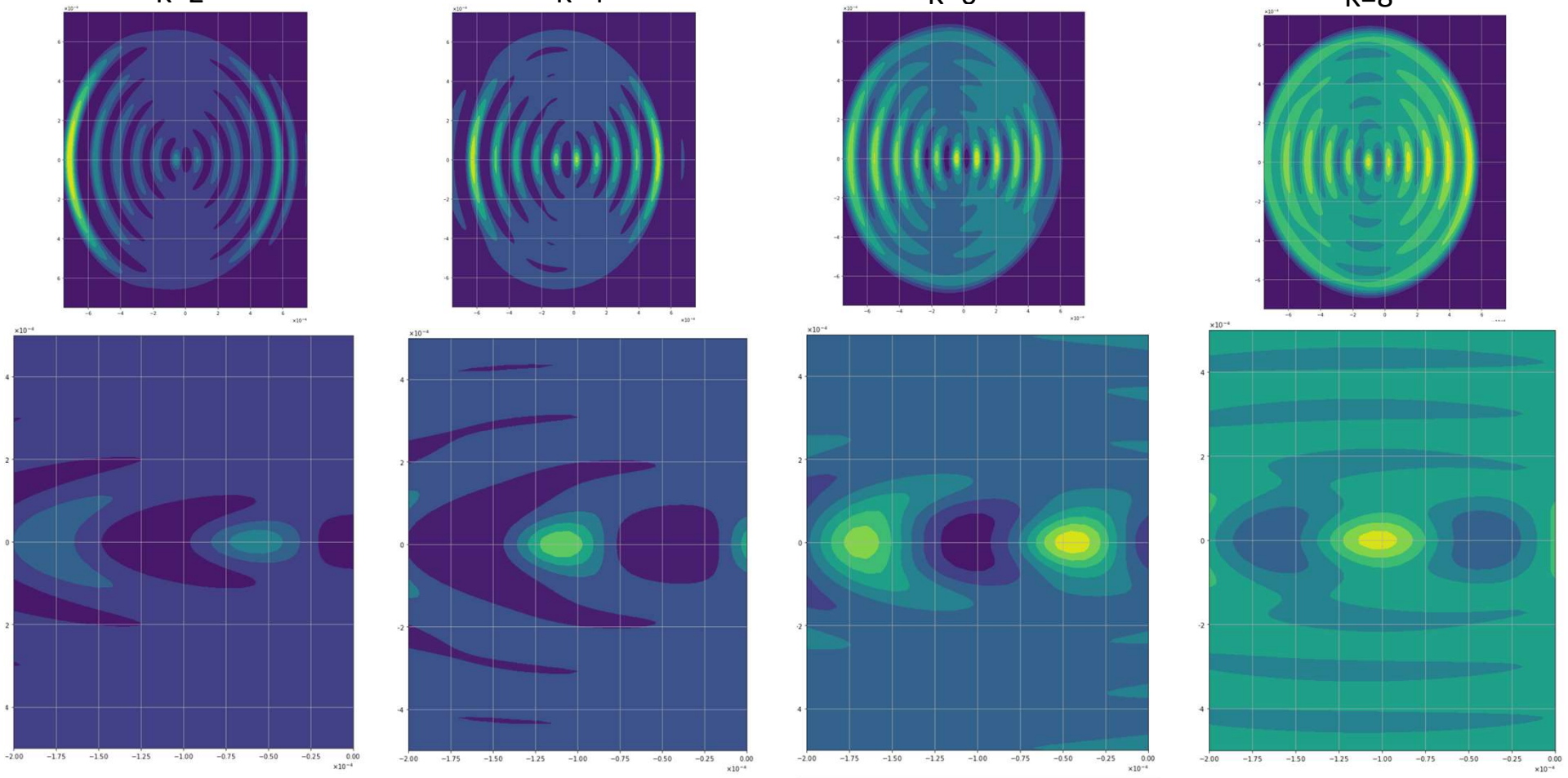
K=8





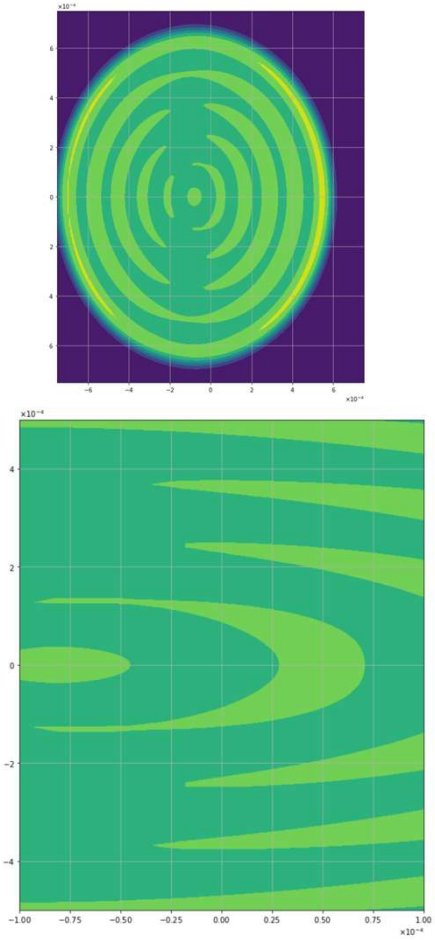
100 micron

K=2 K=4 K=6 K=8

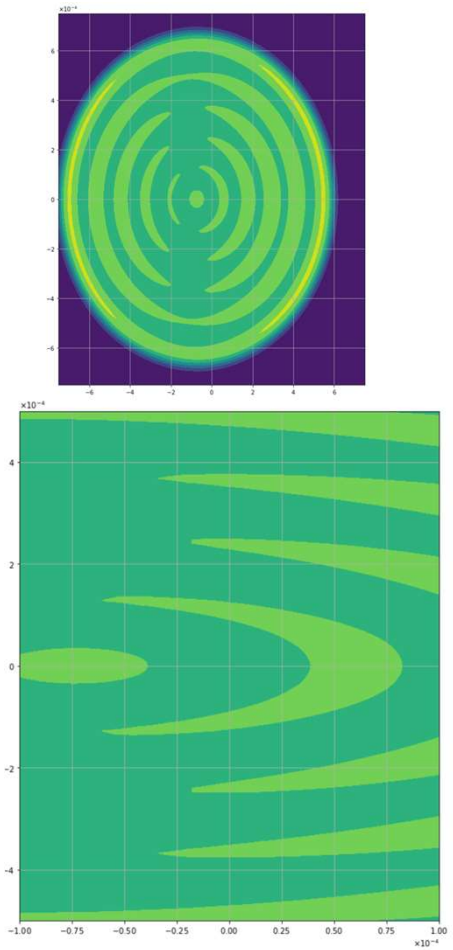


75 micron (*revisited*)

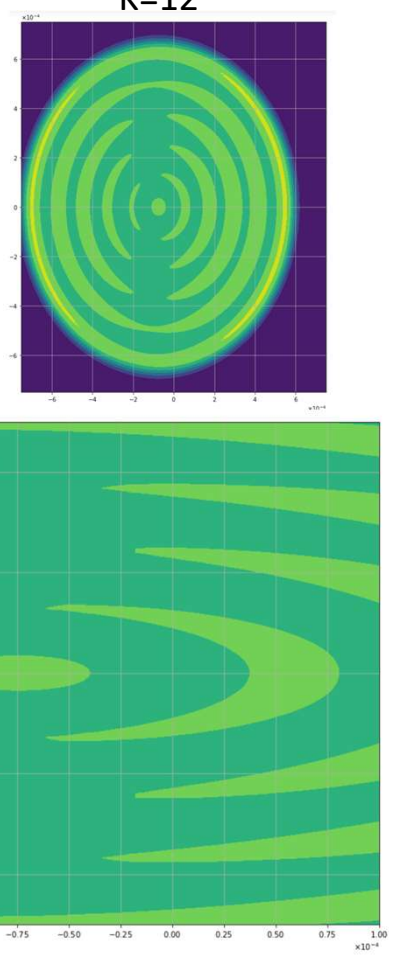
K=8



K=10

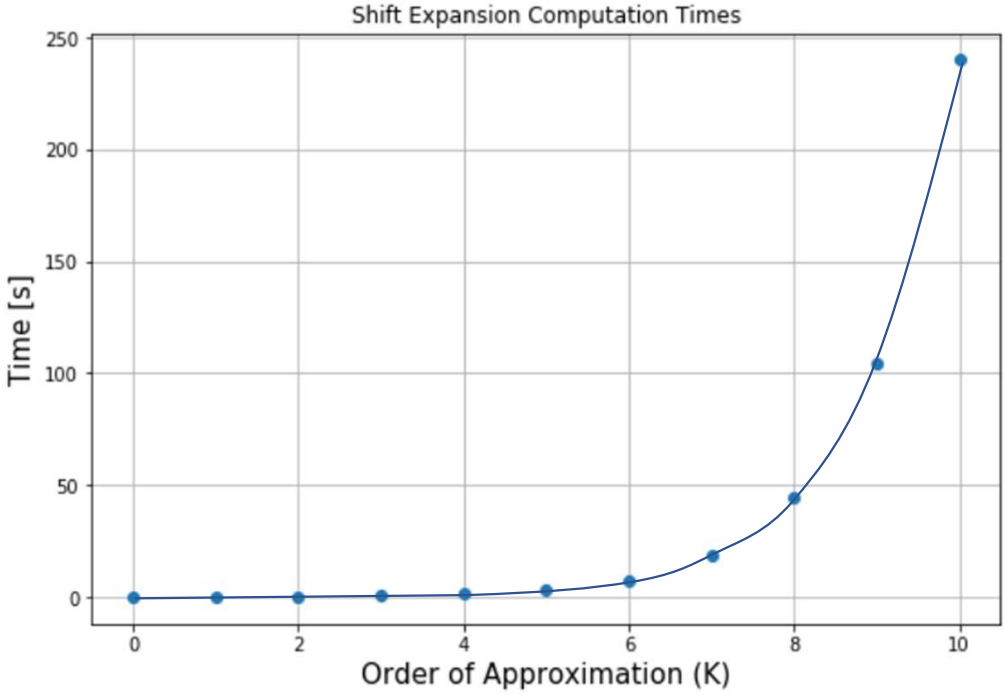


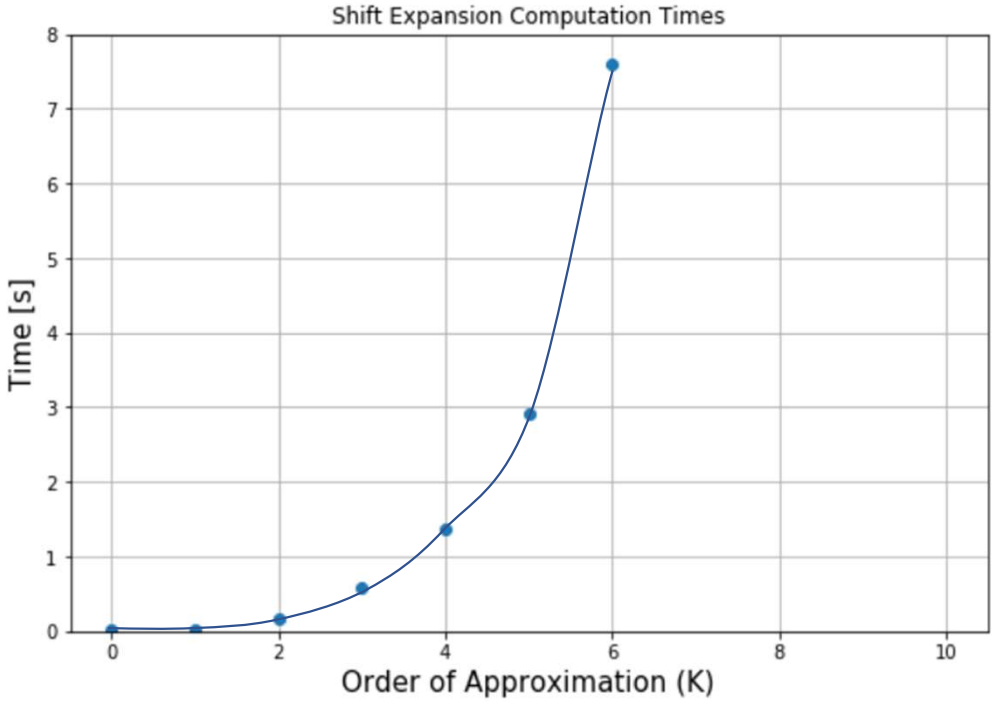
K=12



Computation Times

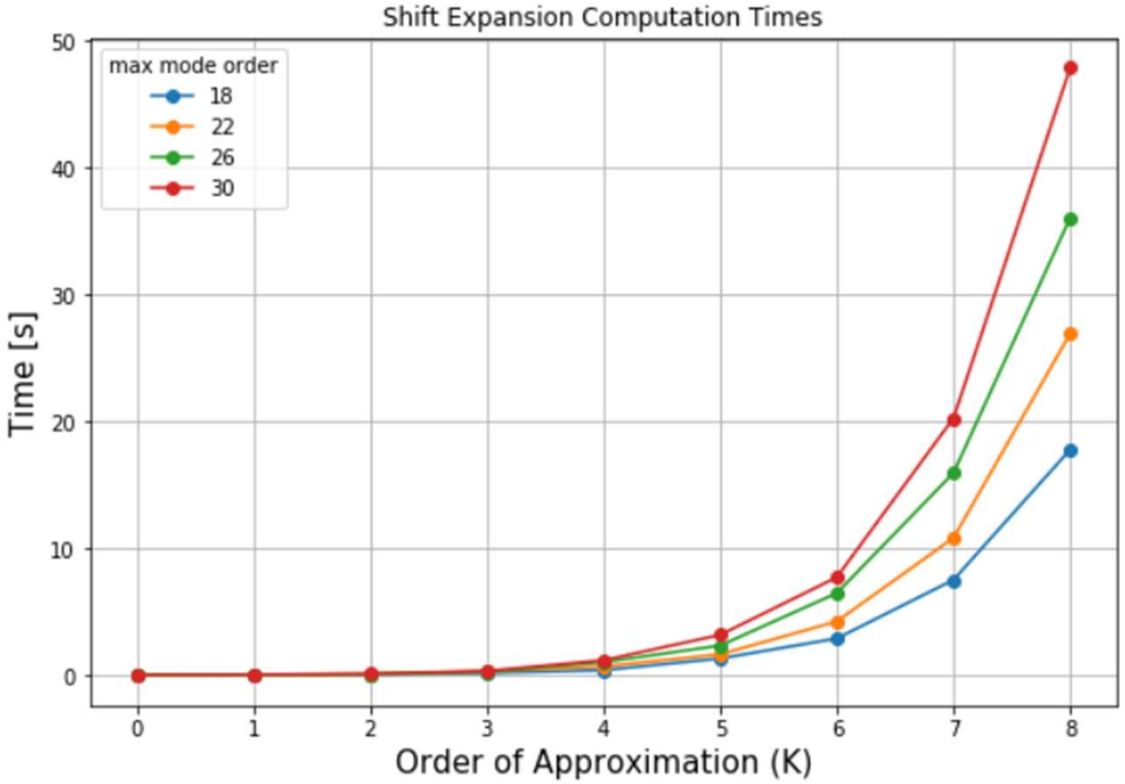
- Performing the expansion for updated C_{nm} (no signal calc. or tilt)
- Up to 10^{th} order:
  - a^{10} terms
  - x^{10} terms
- Mode order 34 tophat -> order 44 after x transformations





[0.015625, 0.015625, 0.15625, 0.59375, 1.359375, 2.921875, 7.609375, 18.953125, 44.234375, 104.390625, 240.046875]

Computation Times: Varying Tophat Max Mode Order



- Same scenario as previous slide
- Times shown below
- Time ~double from mode 18 to 26
  - ~O(n^2) …? (34 went fast)

```
18 [0.015625, 0.0, 0.046875, 0.15625, 0.375, 1.296875, 2.875, 7.5, 17.78125]
22 [0.0, 0.015625, 0.0625, 0.28125, 0.671875, 1.625, 4.21875, 10.8125, 26.921875]
26 [0.0, 0.015625, 0.0625, 0.234375, 1.015625, 2.3125, 6.453125, 15.9375, 36.0625]
30 [0.0, 0.03125, 0.09375, 0.3125, 1.15625, 3.15625, 7.703125, 20.265625, 47.890625]
```

J-th order Tilt Sneak-peek

$$\begin{pmatrix} x' \\ z' \end{pmatrix} = \begin{pmatrix} x \\ z \end{pmatrix} \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} = \begin{pmatrix} x\cos\alpha + z\sin\alpha \\ z\cos\alpha - x\sin\alpha \end{pmatrix}$$

$$u_{nm} \to (2^{n+m-1} n! m! \pi)^{-1/2} \frac{1}{w(z)} H_n\left(\frac{\sqrt{2}x'}{w(z)}\right) H_m\left(\frac{\sqrt{2}y}{w(z)}\right) \exp\left(\frac{-ik(x'^2+y^2)}{2R_c(z)} - \frac{x'^2+y^2}{w^2(z)} + i(n+m+1)\Psi(z') - ikz'\right)$$

$$e^{-ikz'} = e^{-ikz} \sum_{L=0}^{\infty} \frac{1}{L!} (-ikz[\sum_{G=1}^{\infty} \frac{(-1)^G \alpha^{2G}}{(2G)!}])^L \sum_{M=0}^{\infty} \frac{1}{M!} (-ikx \sum_{H=0}^{\infty} \frac{(-1)^H \alpha^{2H+1}}{(2H+1)!})^M$$

$$\exp[(i+n+m+1)\Psi(z')] = \exp[i(n+m+1)\arctan\frac{z}{z_R}] \exp[i(n+m+1)\left(..\right)]$$

$$H_n(X+Y) = (H(X)+2Y)^n$$

$$H_n\left(\frac{\sqrt{2}}{w}(x\cos\alpha + z\sin\alpha)\right) = H_n\left(\frac{\sqrt{2}}{w}(x\sum_{G=0}^{\infty}\frac{(-1)^G\alpha^{2G}}{(2G)!} + z\sin\alpha)\right)$$
$$= H_n\left(\frac{\sqrt{2}}{w}(x + x\sum_{G=1}^{\infty}\frac{(-1)^G\alpha^{2G}}{(2G)!} + z\sin\alpha)\right)$$

$$\sum_{R=0}^{R'} \frac{1}{R!}\sqrt{\frac{n!}{(n-R)!}}(e^{i\Psi}\frac{2}{w})^R \times \sum_{S=0}^{S'} \binom{R}{S} \sum_{H=1}^{H'} (\frac{(-1)^H\alpha^{2H}}{(2H)!})^{2R-S} \times [x^{R-S} + (z\alpha(1+\frac{1}{2H+1}))^R] u_{n-R,m}$$

What's Next?

Proposal	Time?
More 1st order tilt-shift results	?
More shift analysis, signals with 1st order tilt, Kth order shift	< 1 week
Simplified Jth order tilt solution, signals with Kth order shift	~ 1-2 weeks
Full Jth order tilt solution	~ 3-4 weeks

$$\begin{pmatrix} x' \\ z' \end{pmatrix} \approx \begin{pmatrix} x + z\alpha \\ z - x\alpha \end{pmatrix}$$