HG00-HG00 with Waists Centered at Half-plane PD

1st and 2nd Order Approximations in Shift and Tilt

MOTIVATIONS

- Refine 1st order results
- Observe behavior of higher-order approximation
- Identify valid domain of parameters (i.e., shift and tilt terms)
- Check against Alex's generalized solution
- Validate 1st order approximations for other beam profiles and COR's

1ST ORDER APPROXIMATIONS

Shift: $a << w_0$

$$
\begin{aligned}
u_{00}(x-a,y,0) &= \left(\frac{2}{\pi}\right)^{-1/2}\left(\frac{1}{w_0}\right)\exp\left(-\frac{(x-a)^2+y^2}{w_0^2}\right) \\
&= \left(\frac{2}{\pi}\right)^{-1/2}\left(\frac{1}{w_0}\right)\exp\left(-\frac{y^2}{w_0^2}\right)\exp\left(-\frac{(x-a)^2}{w_0^2}\right) \\
&= u_{00}(x,y,0)\times exp\left(\frac{2ax+a^2}{w_0^2}\right) \\
&= u_{00}(x,y,0)\left[1+\frac{2ax}{w_0^2}+\mathcal{O}\left(\frac{a}{w}\right)^2\right] \\
&\approx u_{00}+\left(\frac{2ax}{w_0^2}\right)u_{00} \\
&= u_{00}(x,y,0)+\frac{a}{w_0}u_{10}(x,y,0)\ .
\end{aligned}
$$

Tilt: $\alpha << \frac{\lambda}{\pi w_0}$

$$
\begin{aligned}
u_{tilt(0,0)} &= u_{00}\exp(i\phi) \\
&= u_{00}\exp\left[ikx\sin(\alpha)\right] \\
&\approx u_{00}\exp\left[ikx\alpha\right] \\
&= u_{00}\exp\left[i\left(\frac{2\pi x\alpha}{\lambda}\right)\right] \\
&\approx u_{00}\left[1+i\left(\frac{2\pi x\alpha}{\lambda}\right)\right] \\
&= u_{00}(x,y,0)+i\left(\frac{\pi w_0\alpha}{\lambda}\right)u_{10}
\end{aligned}
$$

1ST ORDER DWS

$$\Delta\phi = \frac{1}{2}\left[\phi_R - \phi_L\right]$$

$$= \frac{1}{2}\left[\arctan\left[\frac{\frac{\pi\alpha}{\lambda}\left[a + \sqrt{\frac{2}{\pi}}w_0\right]}{1 + \sqrt{\frac{2}{\pi}}\frac{a}{w_0}}\right] - \arctan\left[\frac{\frac{\pi\alpha}{\lambda}\left[a - \sqrt{\frac{2}{\pi}}w_0\right]}{1 - \sqrt{\frac{2}{\pi}}\frac{a}{w_0}}\right]\right]$$

2ND ORDER APPROXIMATION IN SHIFT

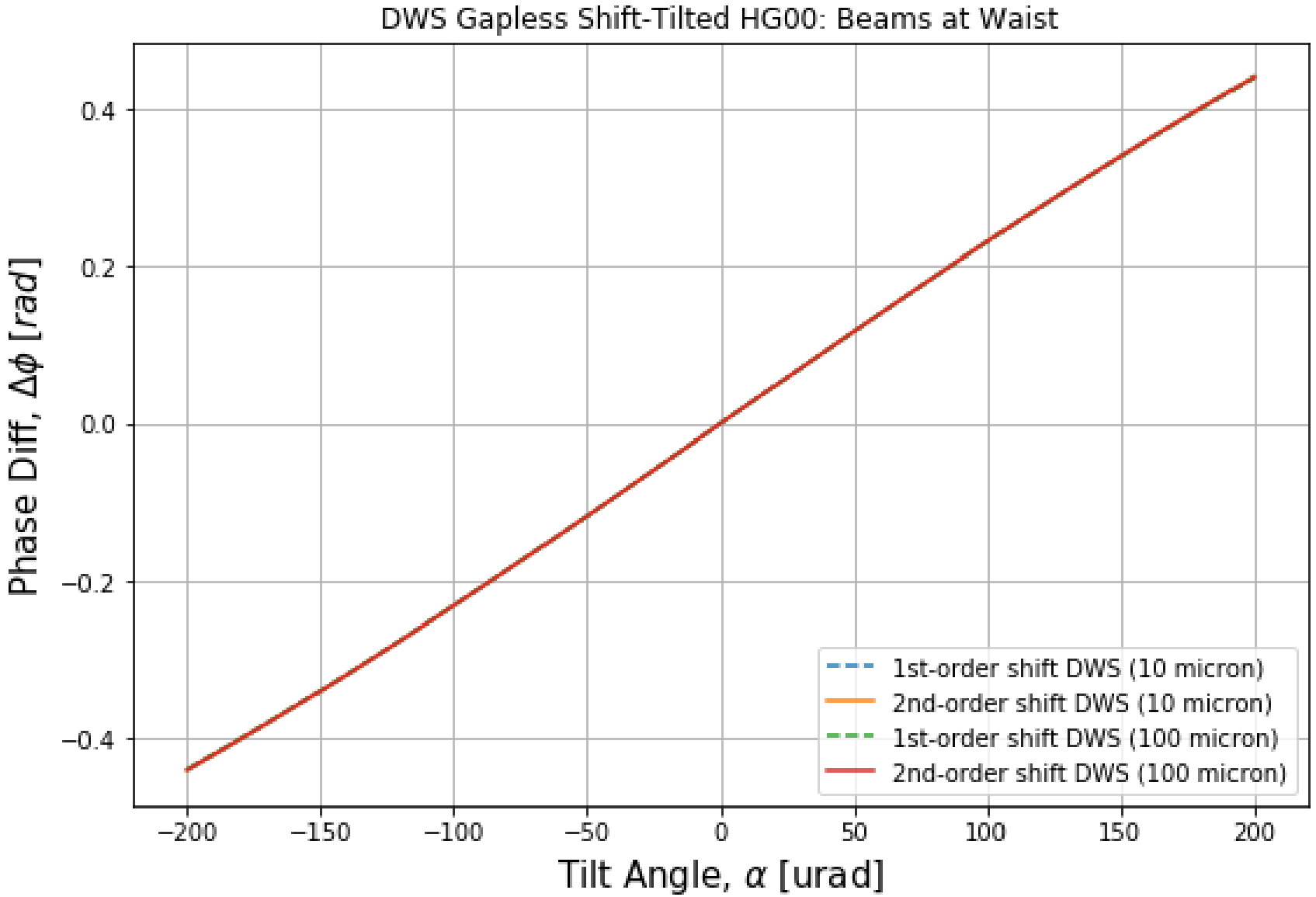
$$
\begin{aligned}
u_{00}(x-a,y,0) &= \left(\frac{2}{\pi}\right)^{-1/2}\left(\frac{1}{w_0}\right)\exp\left(-\frac{(x-a)^2+y^2}{w_0^2}\right) \\
&= \left(\frac{2}{\pi}\right)^{-1/2}\left(\frac{1}{w_0}\right)\exp\left(-\frac{y^2}{w_0^2}\right)\exp\left(-\frac{(x-a)^2}{w_0^2}\right) \\
&= \left(\frac{2}{\pi}\right)^{-1/2}\left(\frac{1}{w_0}\right)\exp\left(-\frac{y^2}{w_0^2}\right)\exp\left(-\frac{x^2}{w_0^2}\right)\exp\left(-\frac{(a^2-2ax)}{w_0^2}\right) \\
&= u_{00}(x,y,0)\times exp\left(\frac{2ax-a^2}{w_0^2}\right) \\
&= u_{00}(x,y,0)\left[1+\frac{2ax}{w_0^2}+\frac{a^2(2x^2-w_0^2)}{w_0^4}+\mathcal{O}\frac{a}{w}^3\right] \\
&\approx u_{00}+\frac{2ax}{w_0^2}u_{00}+a^2\left(\frac{2x^2}{w_0^4}-\frac{1}{w_0^2}\right)u_{00} \\
&= u_{00}(x,y,0)+\frac{a}{w_0}u_{10}(x,y,0)+a^2\left(\frac{1}{\sqrt{2}\,w_0^2}u_{20}-\frac{1}{2w_0^2}u_{00}\right)
\end{aligned}
$$

2ND ORDER SHIFT DWS

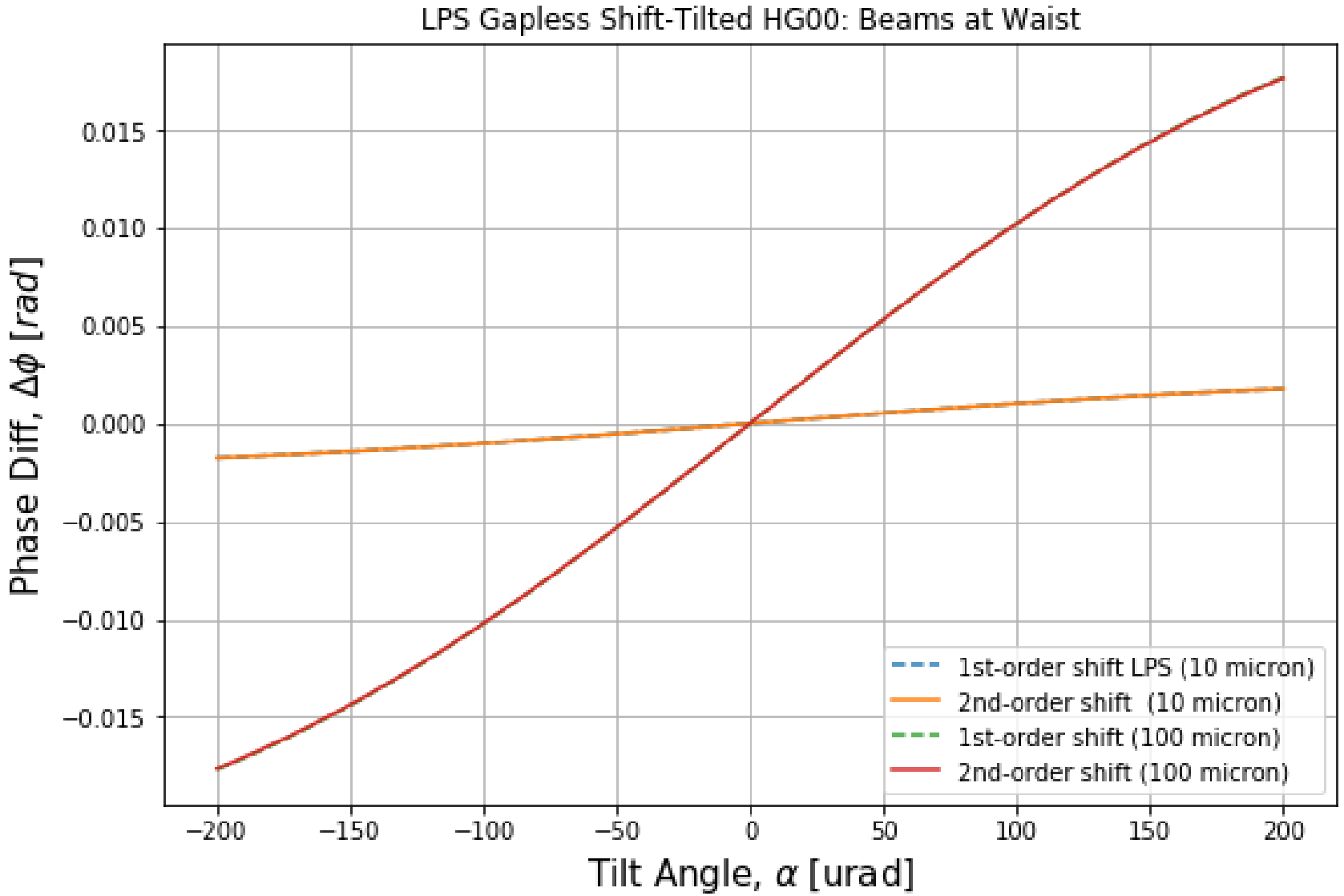
$$\Delta\phi = \frac{1}{2}\left[\phi_R - \phi_L\right]$$

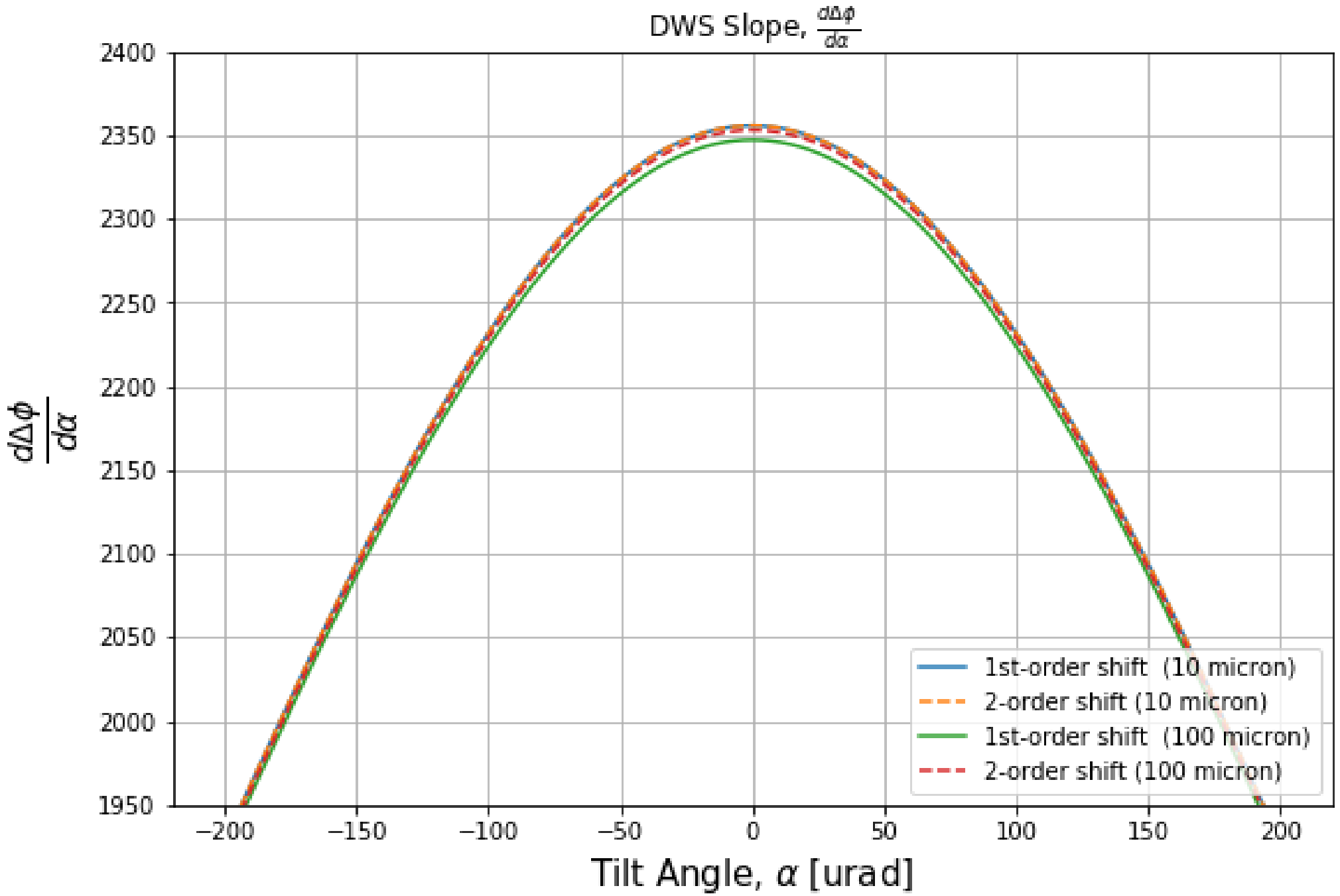
$$= \frac{1}{2}\left[\arctan\left[\frac{\frac{\pi\alpha}{\lambda}\left[a + \sqrt{\frac{2}{\pi}}w_0\right]}{1 + \sqrt{\frac{2}{\pi}}\frac{a}{w_0} - a^2\frac{1}{2w_0^2}}\right] - \arctan\left[\frac{\frac{\pi\alpha}{\lambda}\left[a - \sqrt{\frac{2}{\pi}}w_0\right]}{1 - \sqrt{\frac{2}{\pi}}\frac{a}{w_0} - a^2\frac{1}{2w_0^2}}\right]\right]$$

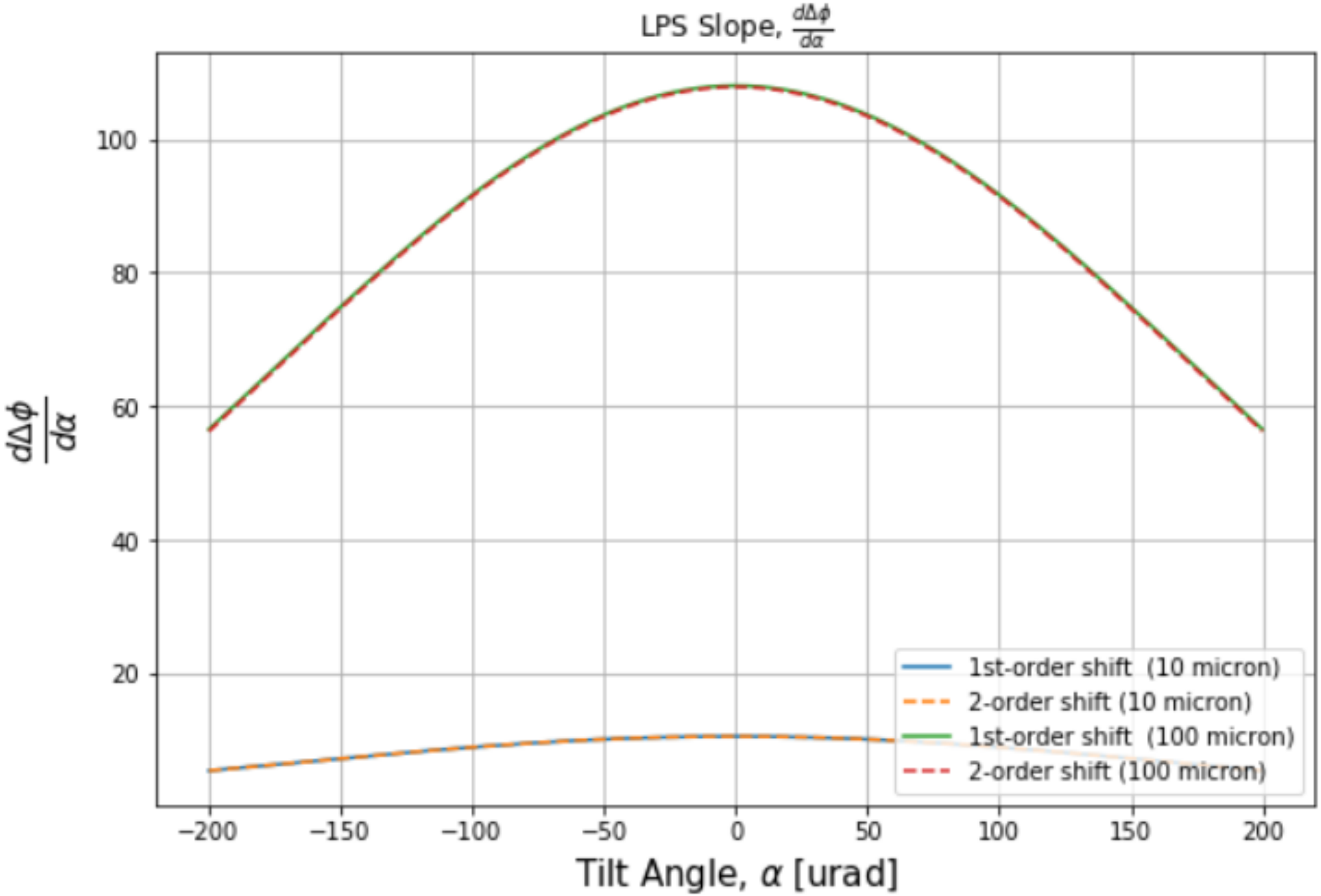
DWS (1ST AND 2ND ORDER SHIFT=10,100 MICRON)



LPS (1^{ST} AND 2^{ND} ORDER SHIFT=10,100 MICRON)




DWS Slope, $\frac{d\Delta\phi}{d\alpha}$
2400
2350
2300
2250
2200
2150
2100
2050
2000
1950
$\frac{d\Delta\phi}{d\alpha}$
−200
−150
−100
−50
0
50
100
150
200
Tilt Angle, α [urad]
1st-order shift (10 micron)
2-order shift (10 micron)
1st-order shift (100 micron)
2-order shift (100 micron)


LPS Slope, $\frac{d\Delta\phi}{d\alpha}$
$\frac{d\Delta\phi}{d\alpha}$
100
80
60
40
20
−200
−150
−100
−50
0
50
100
150
200
Tilt Angle, α [urad]
1st-order shift (10 micron)
2-order shift (10 micron)
1st-order shift (100 micron)
2-order shift (100 micron)

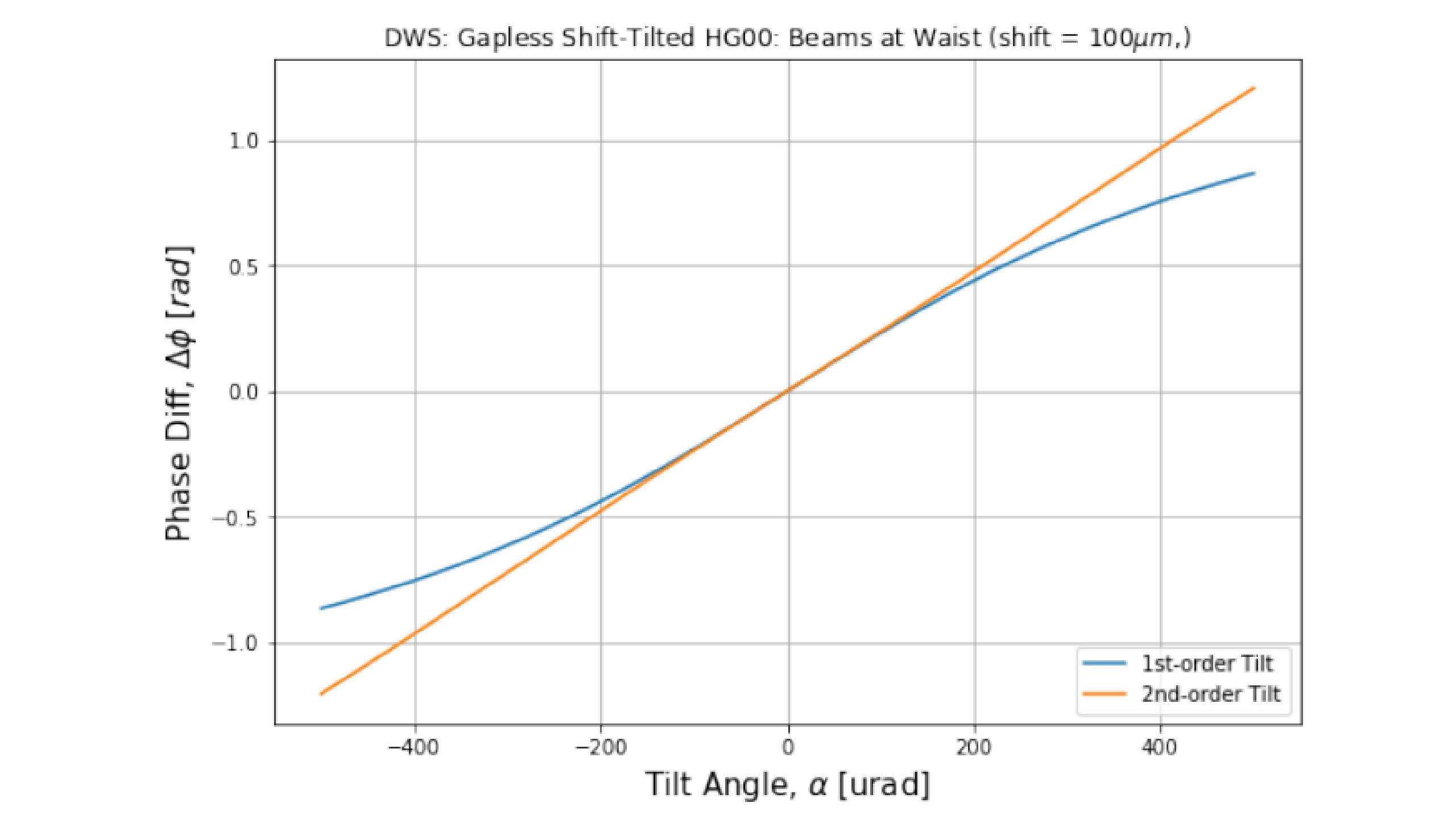
2ND ORDER APPROXIMATION IN TILT

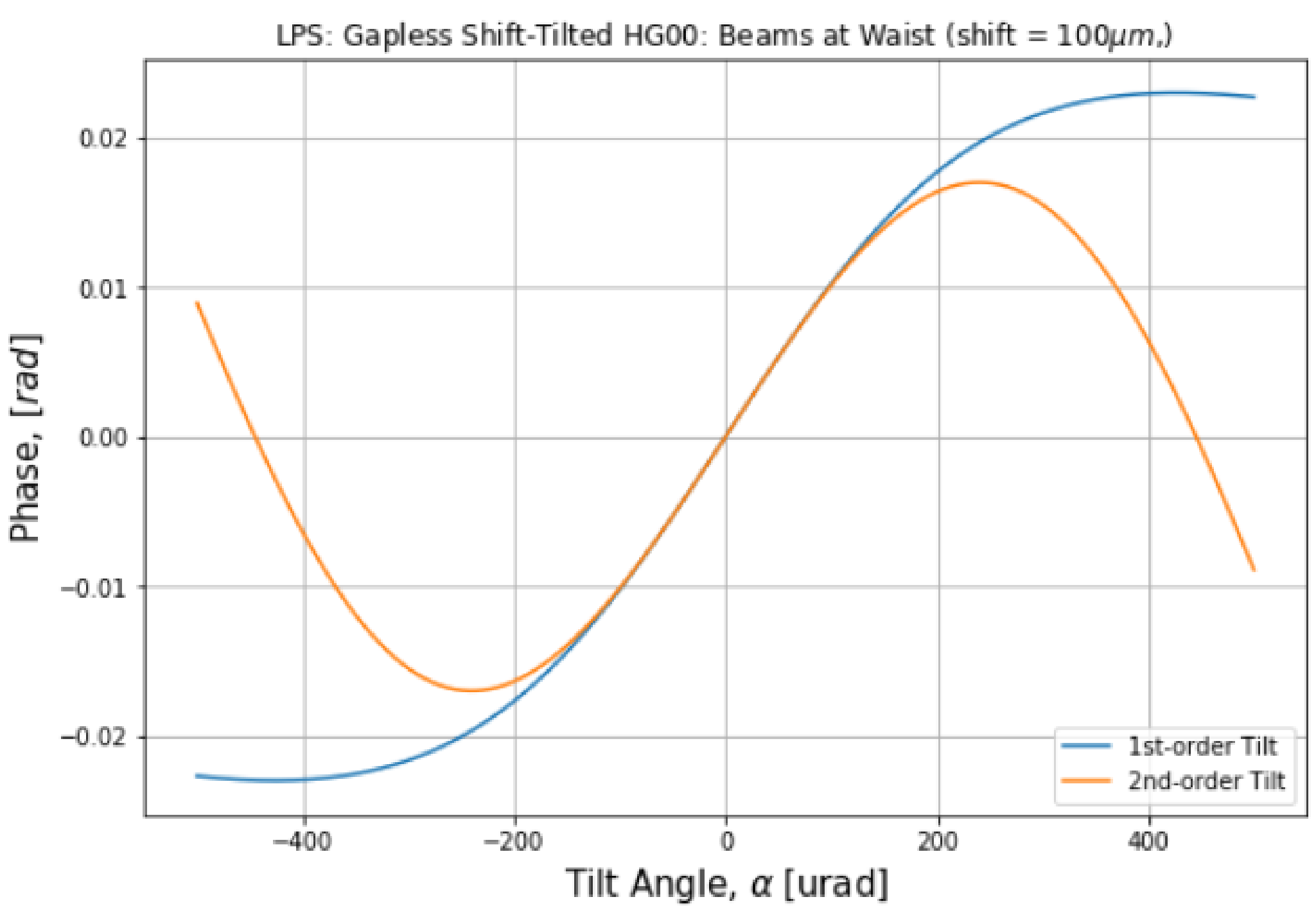
$$
\begin{aligned}
u_{00(tilt))}(x,y,0) &= u_{00}\exp(i\phi) \\
&= u_{00}\exp\left[ikx\sin(\alpha)\right] \\
&\approx u_{00}\exp\left[ikx(\alpha - \frac{\alpha^3}{6})\right] \\
&\approx u_{00}[1 + \alpha ikx - \frac{1}{2}\alpha^2(kx)^2
\end{aligned}
$$

2ND ORDER TILT DWS

$$\Delta\phi = \frac{1}{2}\left[\phi_R - \phi_L\right]$$

$$= \frac{1}{2}\left[\arctan\left[\frac{\frac{\pi\alpha}{\lambda}\left[a + \sqrt{\frac{2}{\pi}}w_0\right]}{1 + \sqrt{\frac{2}{\pi}}\frac{a}{w_0} - \frac{1}{16}\alpha^2k^2\left[w_0^2 + aw_0\left(\frac{1}{4}\sqrt{\frac{2}{\pi}}\right)\right]}\right] - \arctan\left[\frac{\frac{\pi\alpha}{\lambda}\left[a - \sqrt{\frac{2}{\pi}}w_0\right]}{1 - \sqrt{\frac{2}{\pi}}\frac{a}{w_0} - \frac{1}{16}\alpha^2k^2\left[w_0^2 - aw_0\left(\frac{1}{4}\sqrt{\frac{2}{\pi}}\right)\right]}\right]\right]$$


DWS: Gapless Shift-Tilted HG00: Beams at Waist (shift = 100μm,)
Phase Diff, $\Delta\phi$ [rad]
1.0
0.5
0.0
−0.5
−1.0
−400
−200
0
200
400
Tilt Angle, α [urad]
1st-order Tilt
2nd-order Tilt


LPS: Gapless Shift-Tilted HG00: Beams at Waist (shift = 100μm,)
Phase, [rad]
0.02
0.01
0.00
−0.01
−0.02
−400
−200
0
200
400
Tilt Angle, α [urad]
1st-order Tilt
2nd-order Tilt