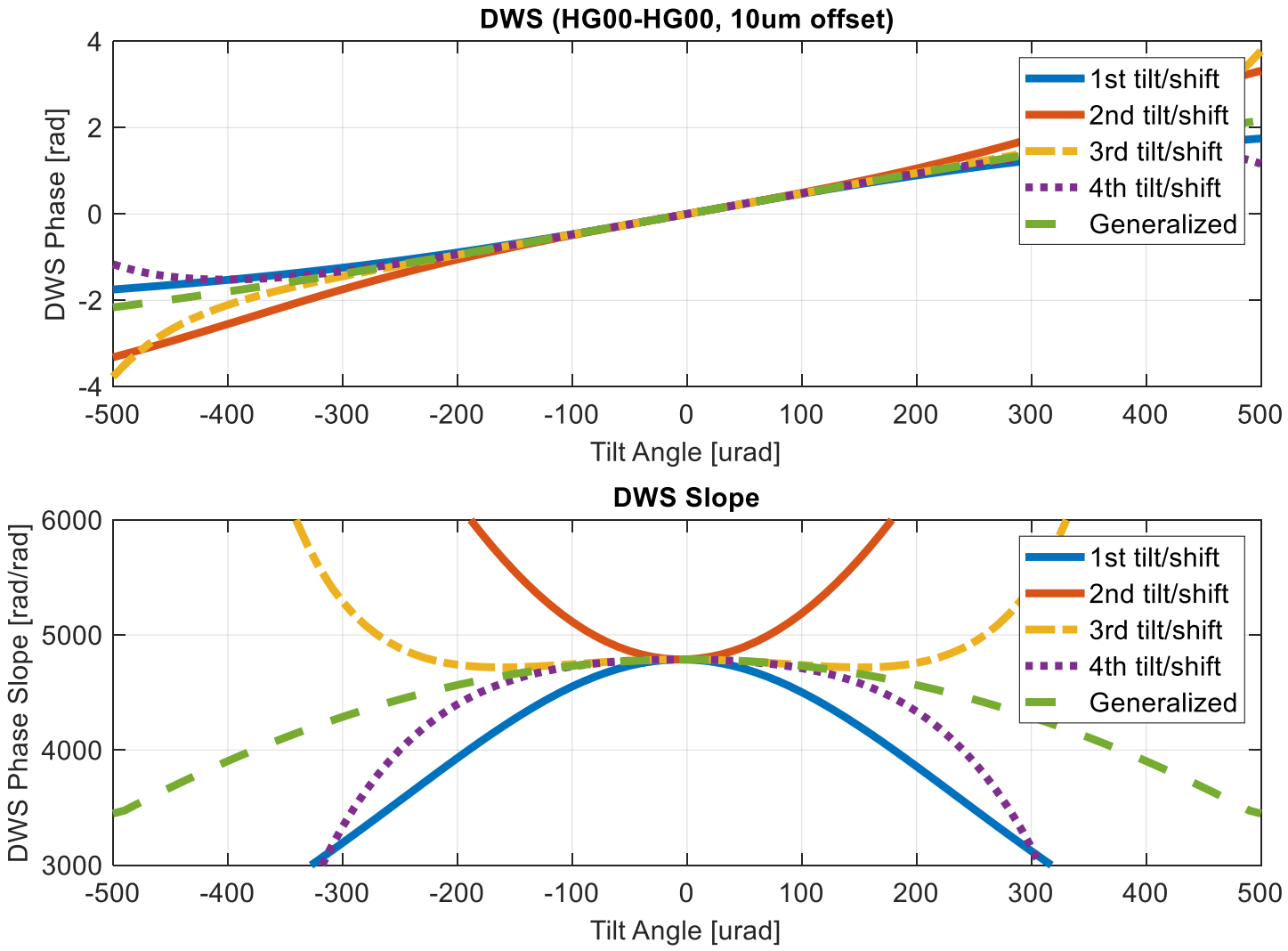
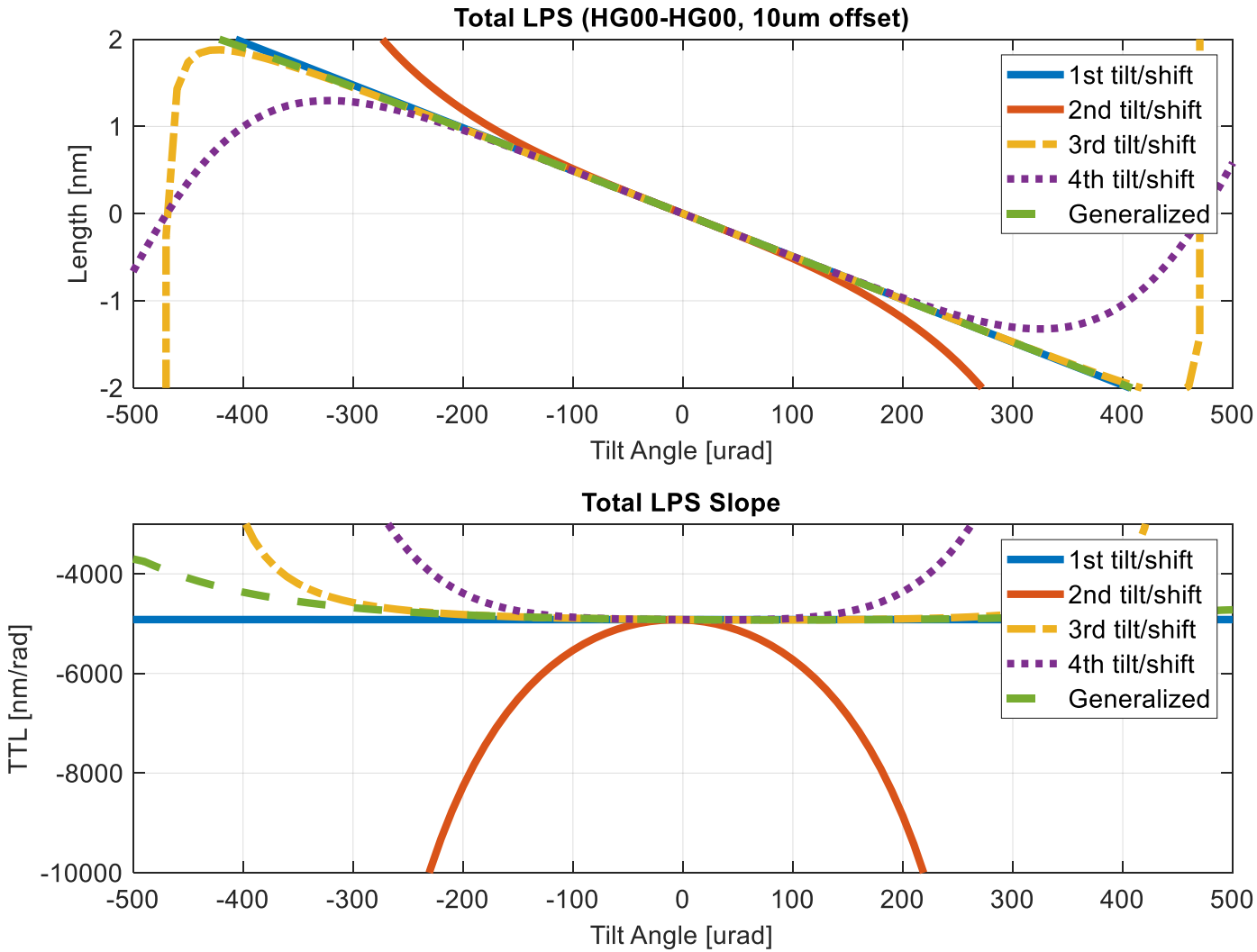
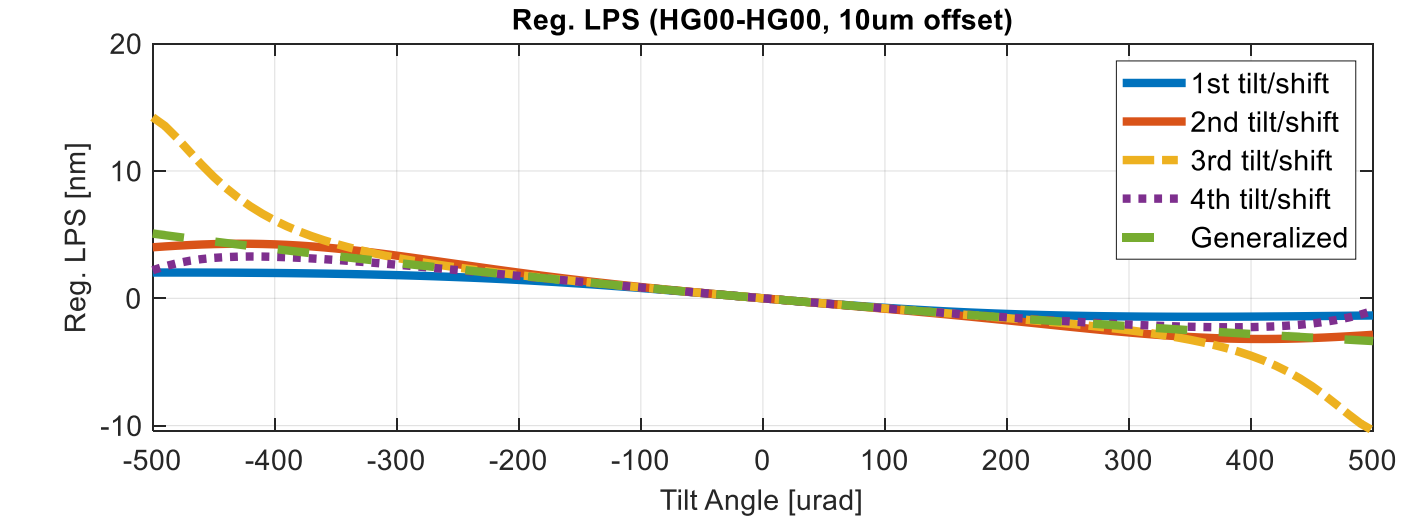
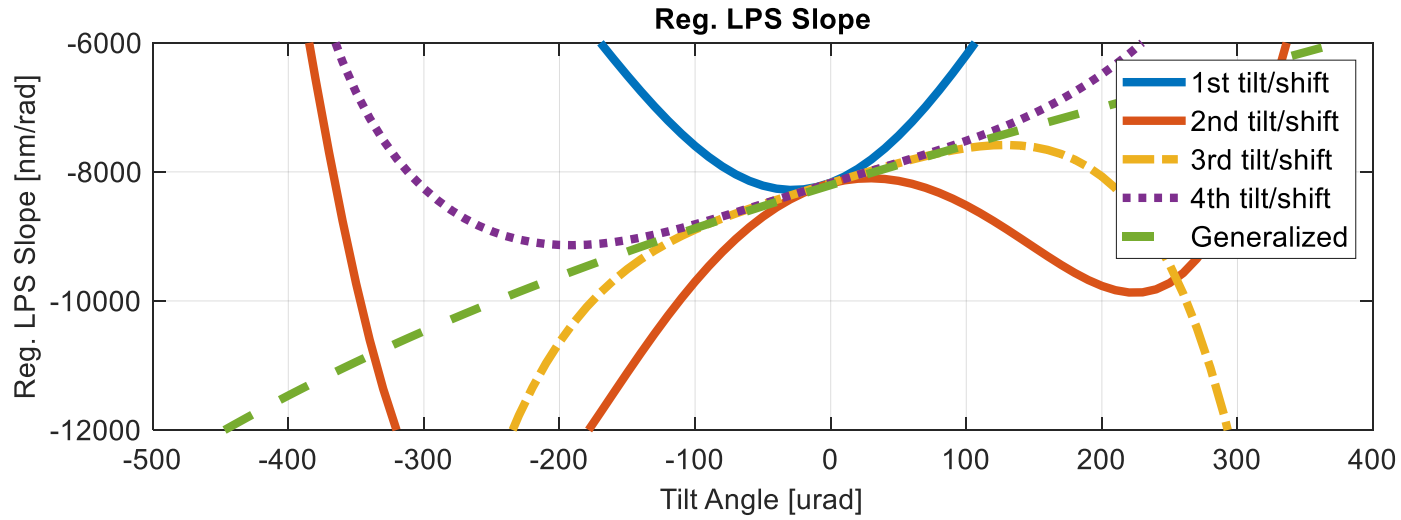
Misaligned HG00-HG00 RX TTL

Rx-Tx beam	HG00-HG00
Plots	Regular and Total LPS [nm], DWS [rad] ; Slopes [nm/rad], [rad/rad]
Longitudinal offset	**10 mm**

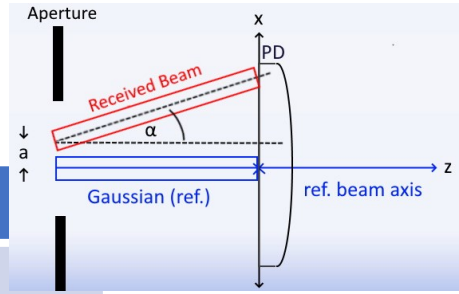

DWS (HG00-HG00, 10um offset)
DWS Phase [rad]
Tilt Angle [urad]
1st tilt/shift
2nd tilt/shift
3rd tilt/shift
4th tilt/shift
Generalized
DWS Slope
DWS Phase Slope [rad/rad]
Tilt Angle [urad]
1st tilt/shift
2nd tilt/shift
3rd tilt/shift
4th tilt/shift
Generalized


Total LPS (HG00-HG00, 10um offset)
Length [nm]
Tilt Angle [urad]
1st tilt/shift
2nd tilt/shift
3rd tilt/shift
4th tilt/shift
Generalized
Total LPS Slope
TTL [nm/rad]
Tilt Angle [urad]
1st tilt/shift
2nd tilt/shift
3rd tilt/shift
4th tilt/shift
Generalized


Reg. LPS (HG00-HG00, 10um offset)
Reg. LPS [nm]
Tilt Angle [urad]
1st tilt/shift
2nd tilt/shift
3rd tilt/shift
4th tilt/shift
Generalized


Reg. LPS Slope
Reg. LPS Slope [nm/rad]
Tilt Angle [urad]
1st tilt/shift
2nd tilt/shift
3rd tilt/shift
4th tilt/shift
Generalized

Misaligned Tophat-HG00 RX TTL



Rx-Tx beam	TH-HG00 (mode order 34)
Plots	Regular and Total LPS [nm], DWS [rad] ; Slopes [nm/rad], [rad/rad]
Longitudinal offset	**10 mm**

- How high: convergence with Alex's results
- How low: justifying 1st order shift expansion
- How fast: computation times up to 7th order tilt

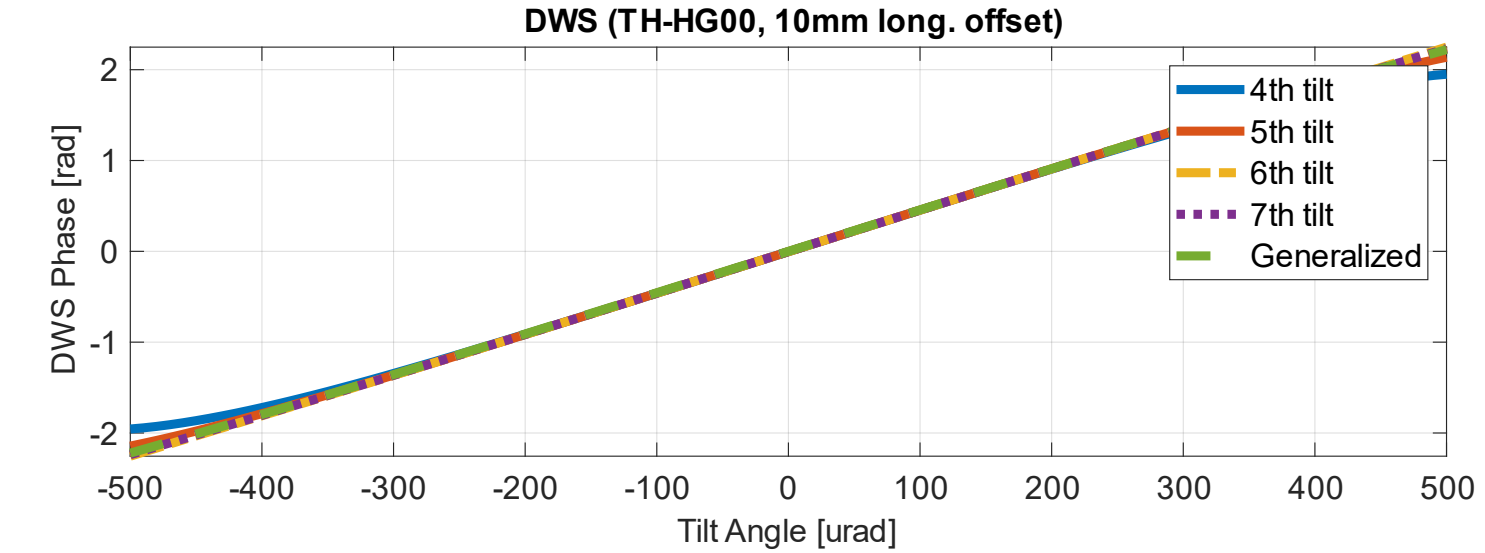
SUMMARY: Higher order results

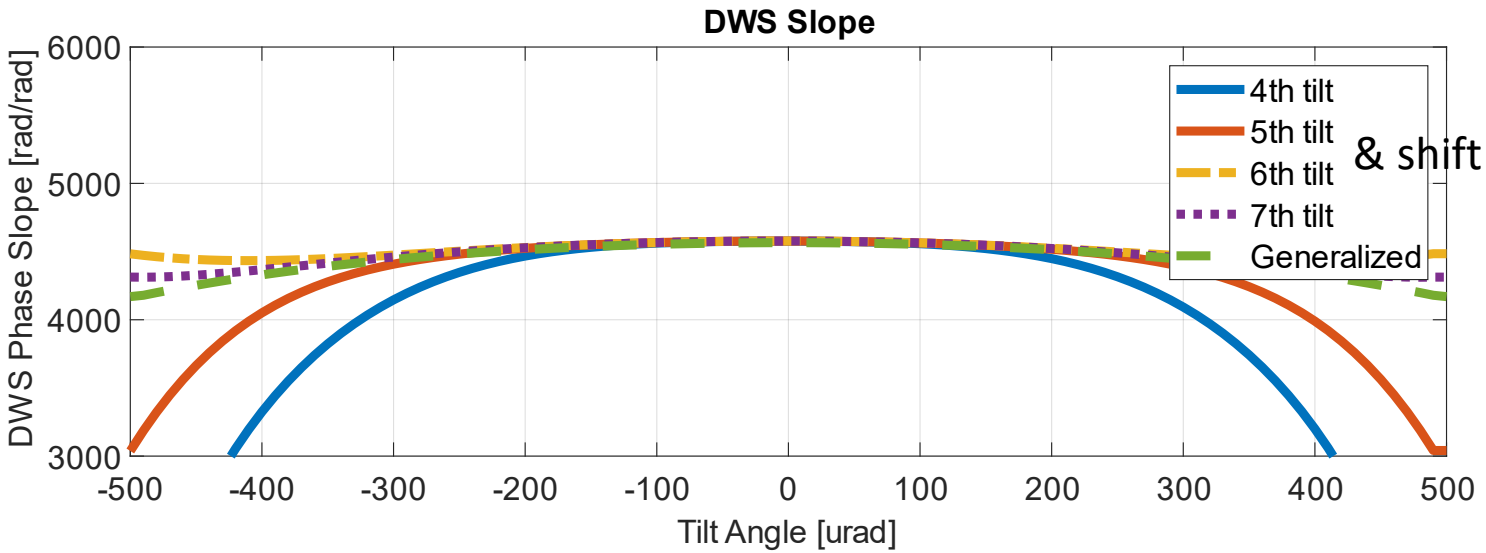
- 4^{th} order tilt,4^{th} order shift -> 7^{th} order tilt,7^{th} order shift
- **Very broad convergence with Alex at 7^{th} order**

Tilt & Shift Orders	"Accuracy" Range [urad]
4 & 4	200
5 & 5	300
6 & 6	400
7 & 7	400

Higher order results

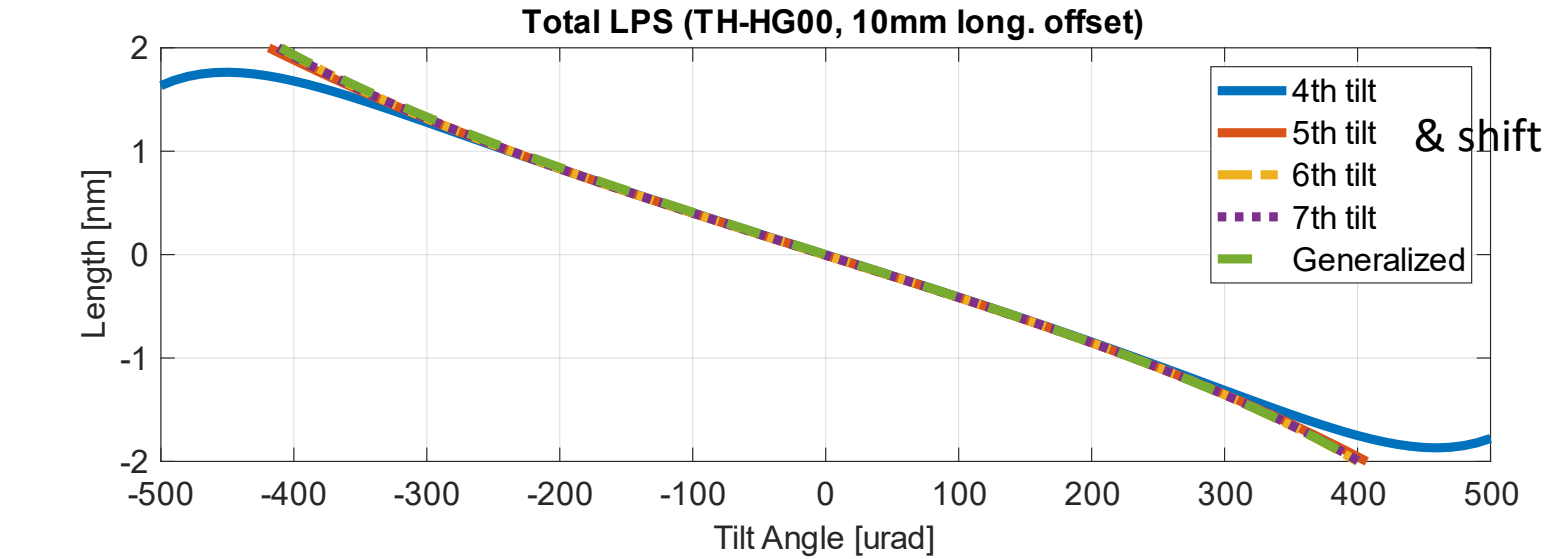
DWS

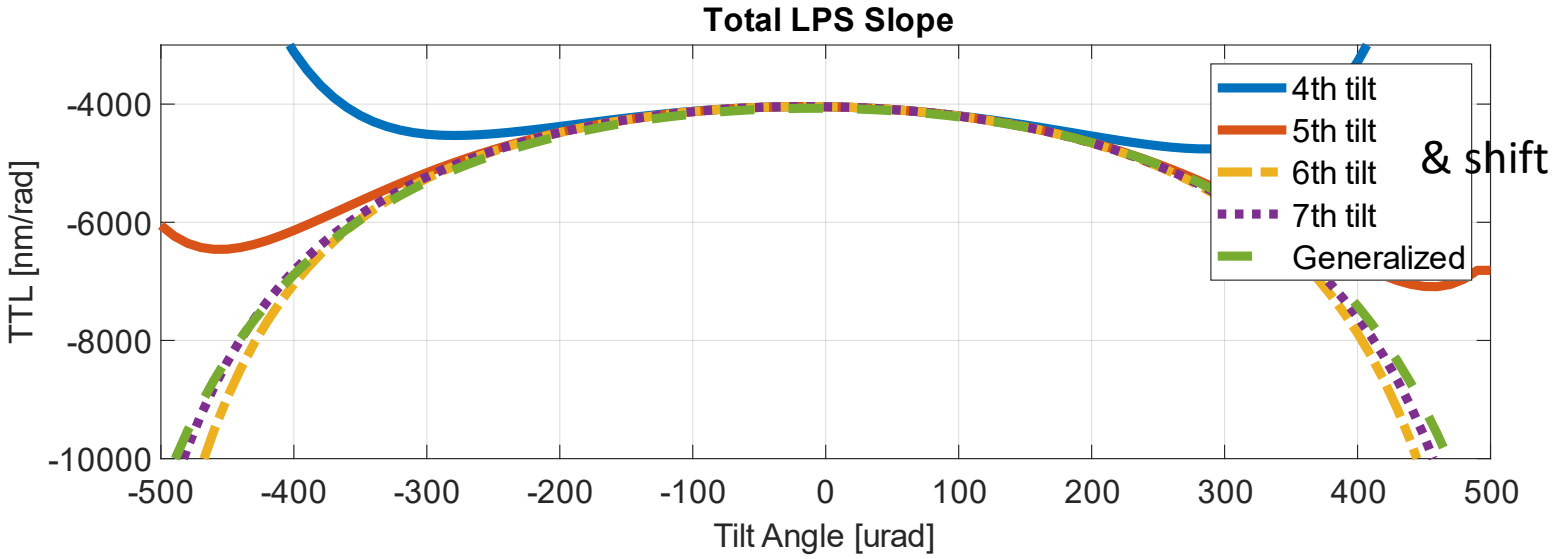




Higher order results

LPS Tot.

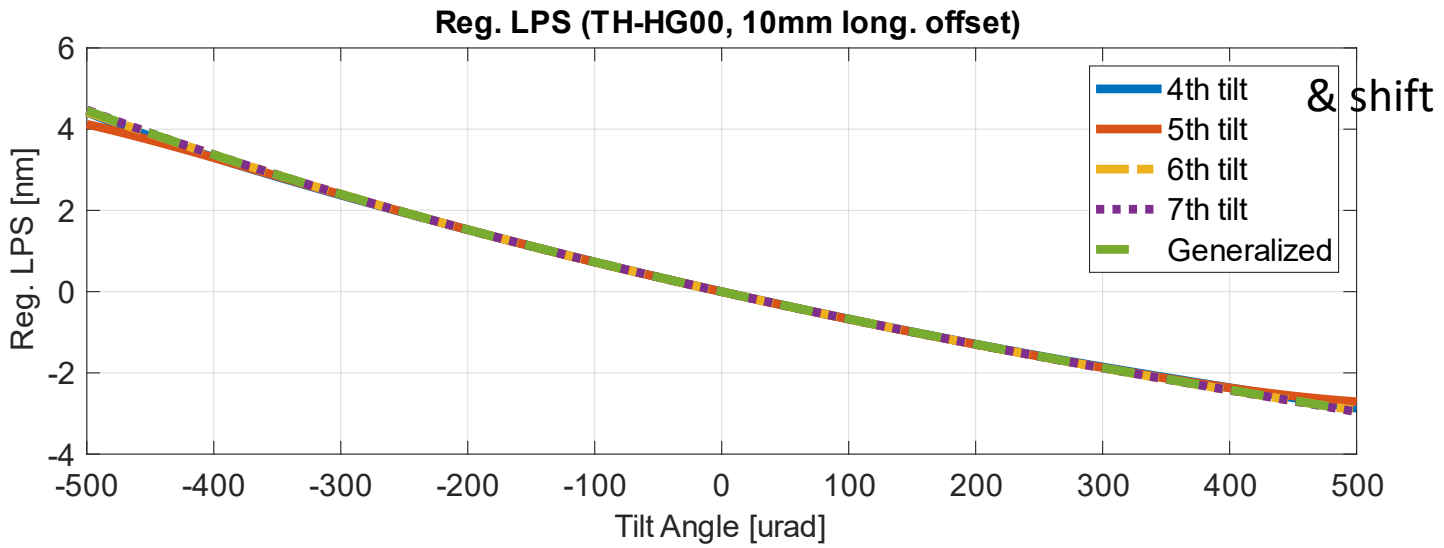




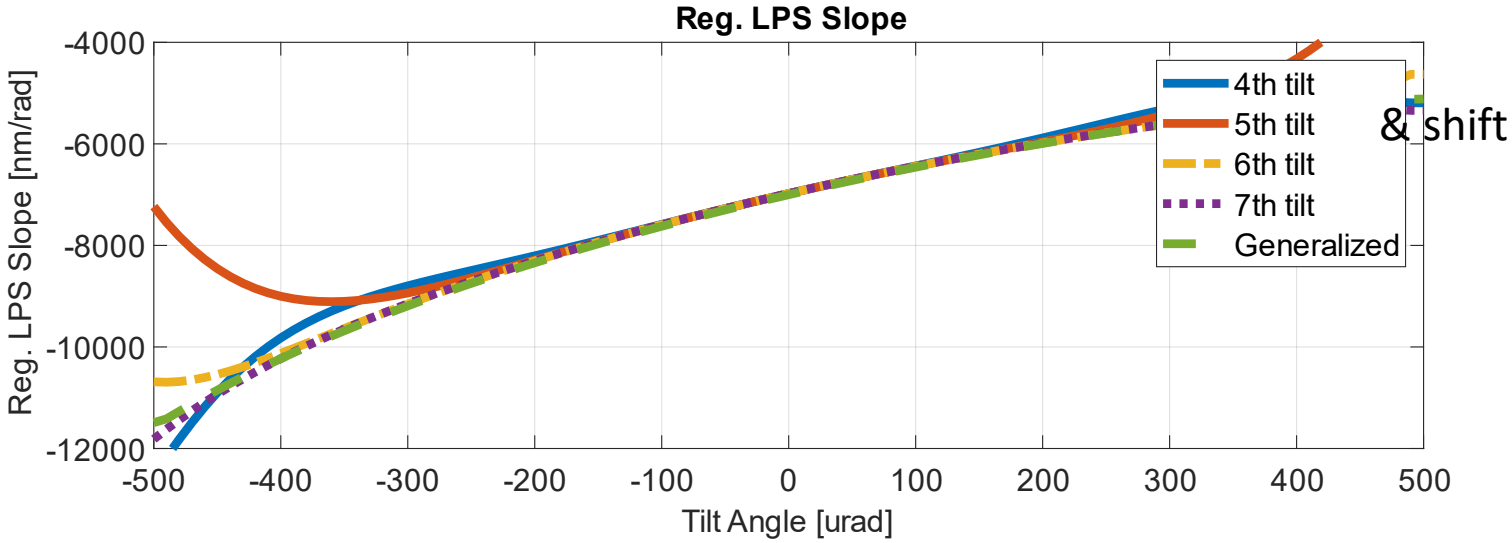
Higher order results

LPS

Reg.



& shift



& shift

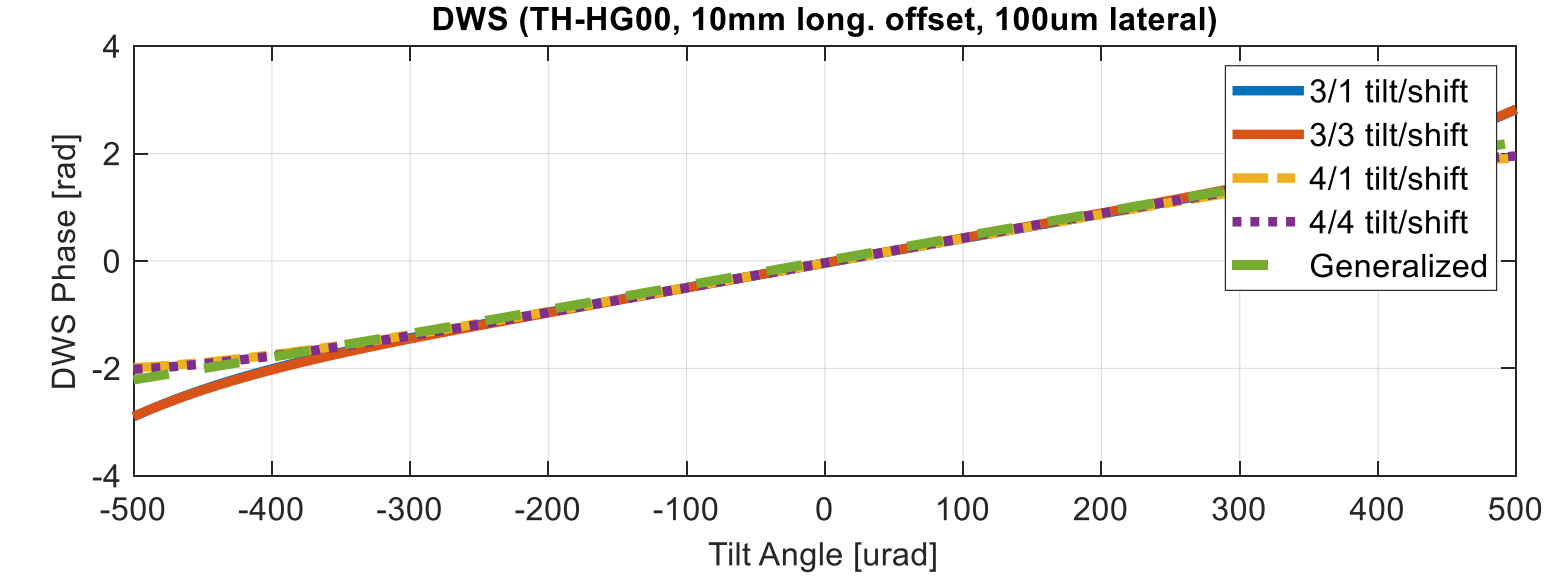
SUMMARY: Lower order shift (100 um offset)

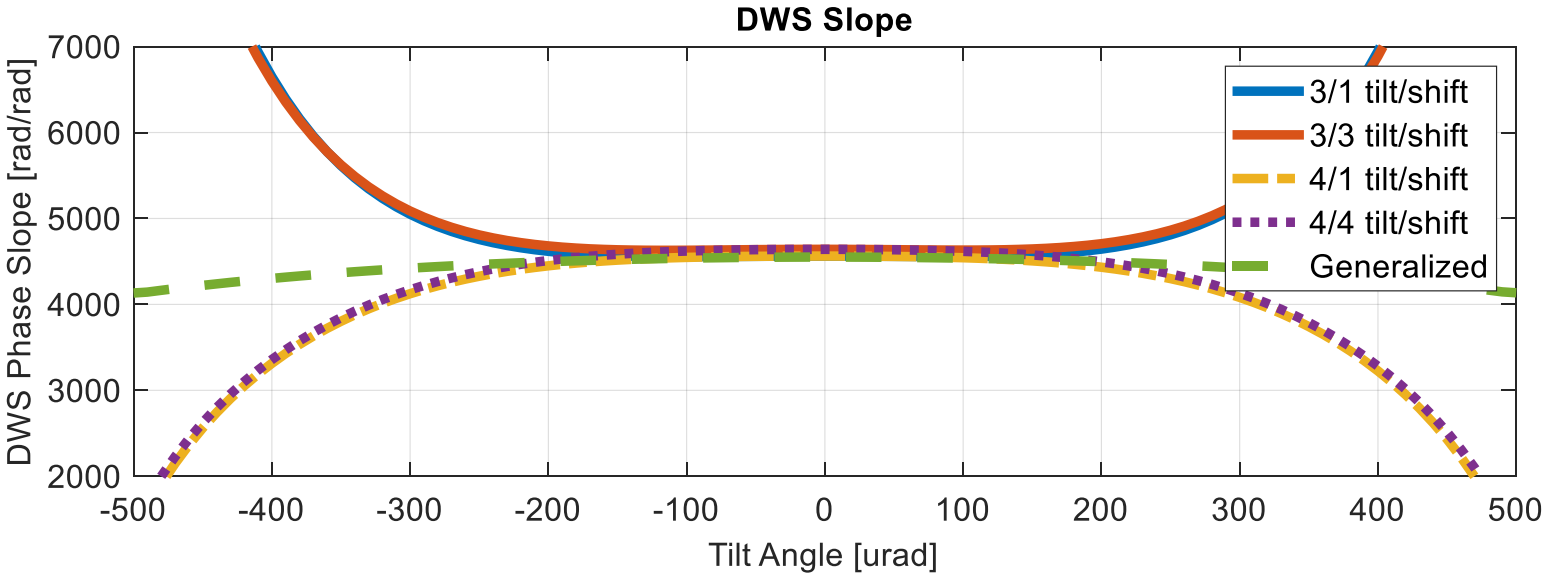
Expansion orders:

- 1^{st} shift, 3^{rd} & 4^{th} tilt
- 3^{rd} or 4^{th} shift, 3^{rd} or 4^{th} tilt
- **1^{st} order shift likely sufficient**

Lower Order Shift? 100 um offset

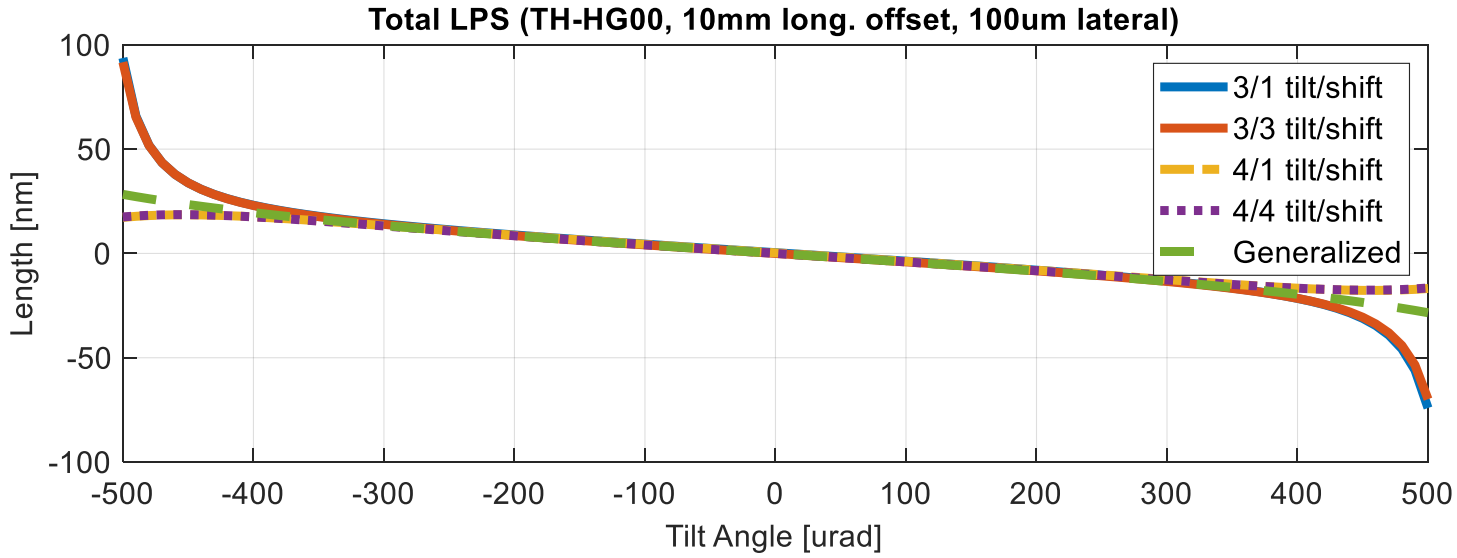
DWS

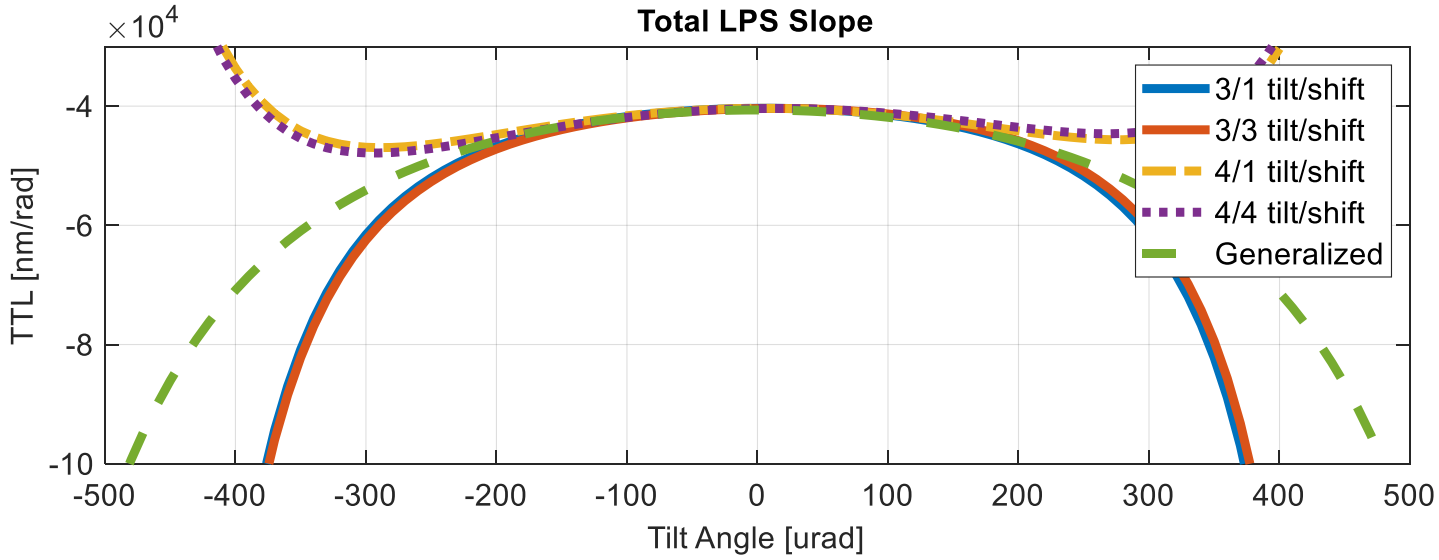




Lower Order Shift? 100 um offset

LPS Tot.

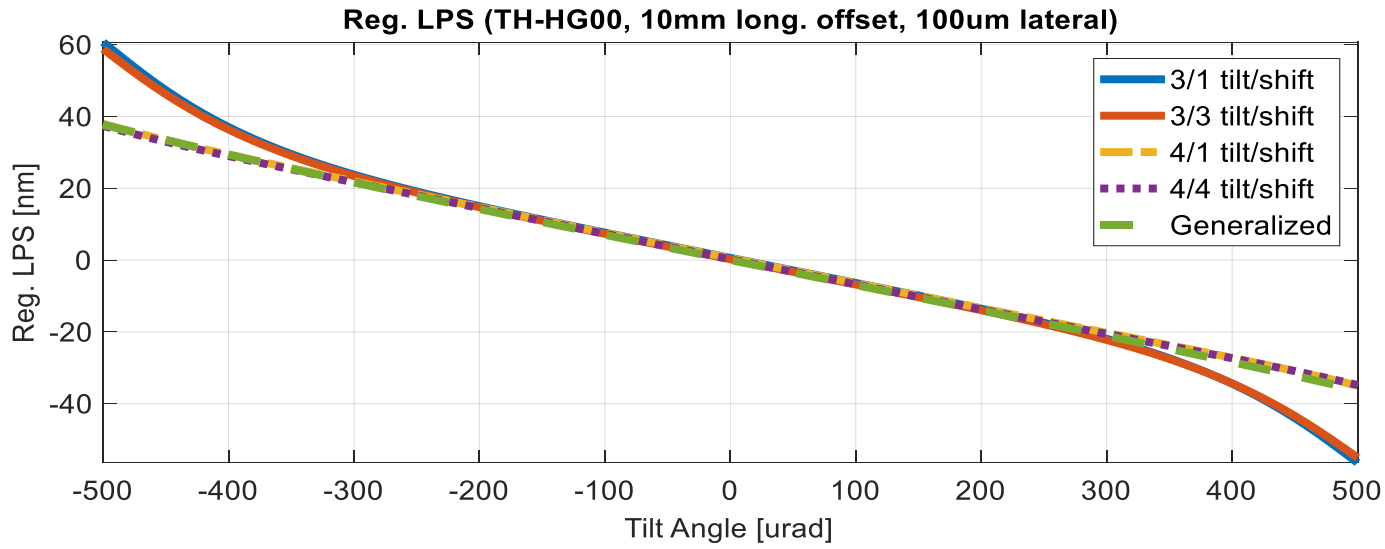


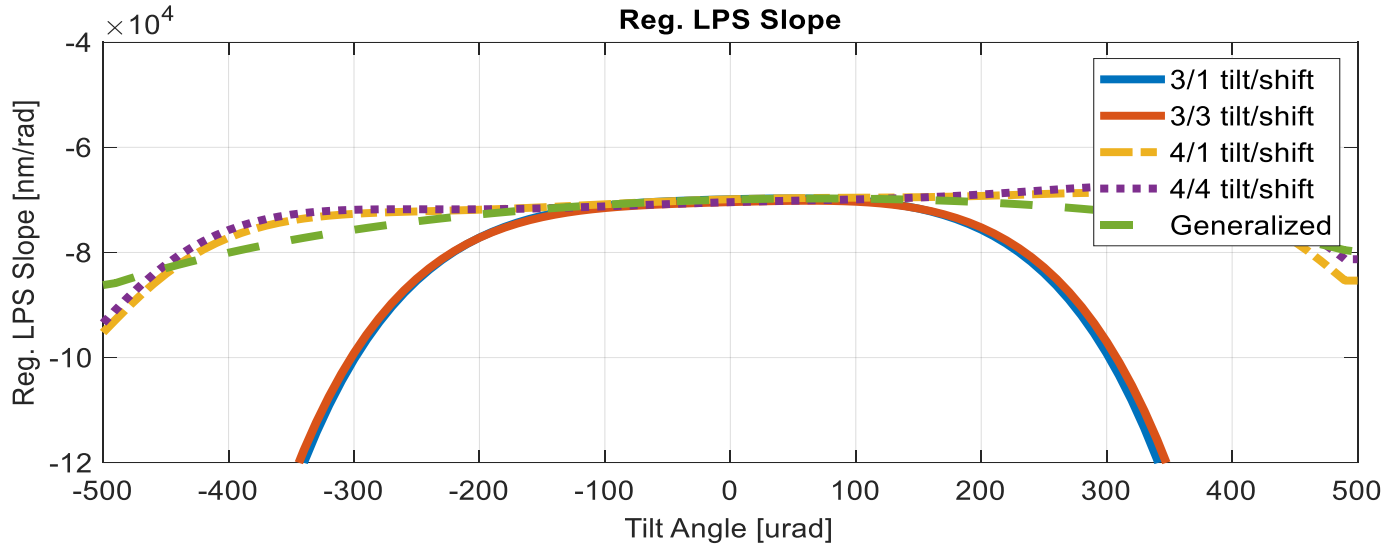


Lower Order Shift? 100 um offset

LPS

Reg.





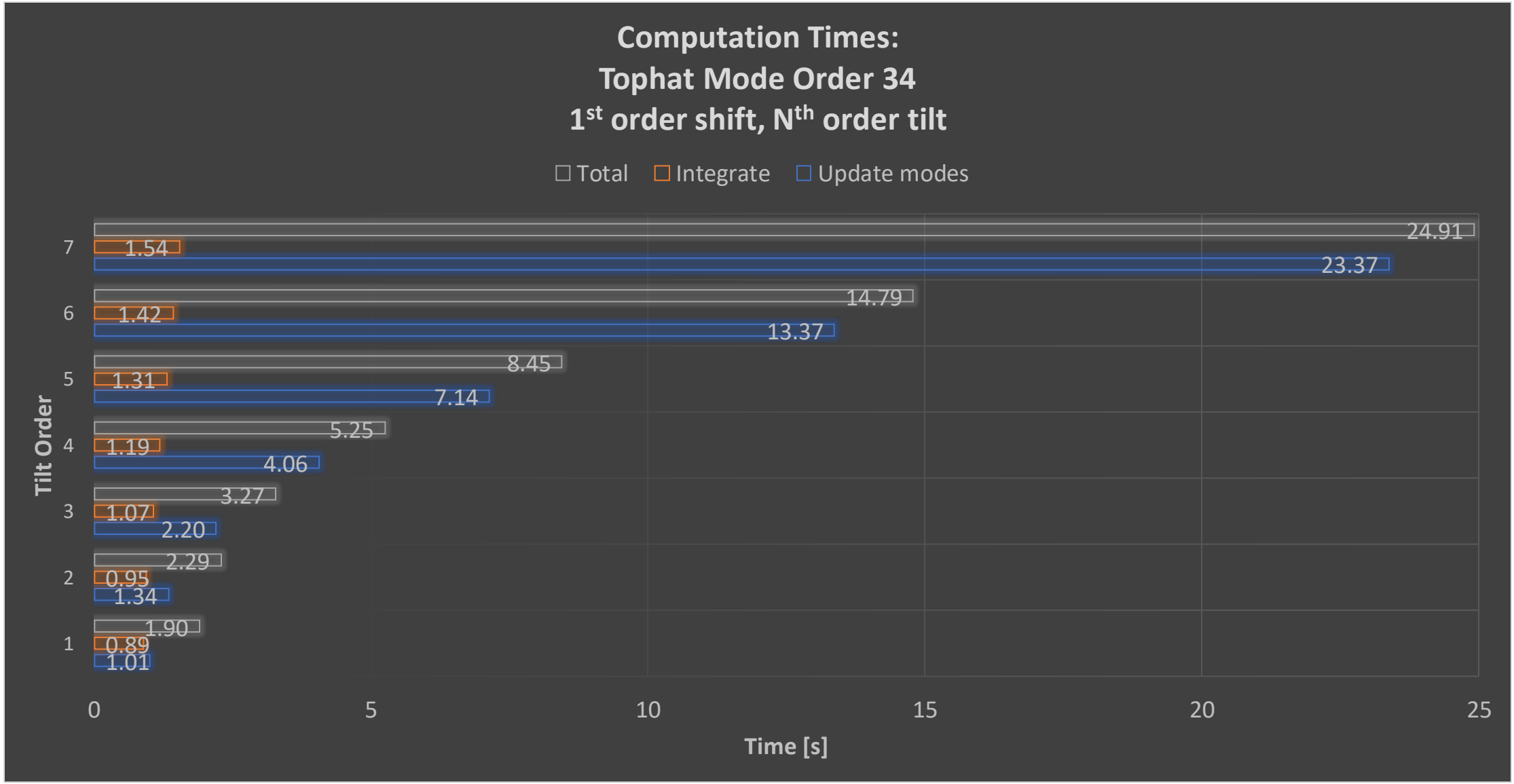
SUMMARY: Computation Times

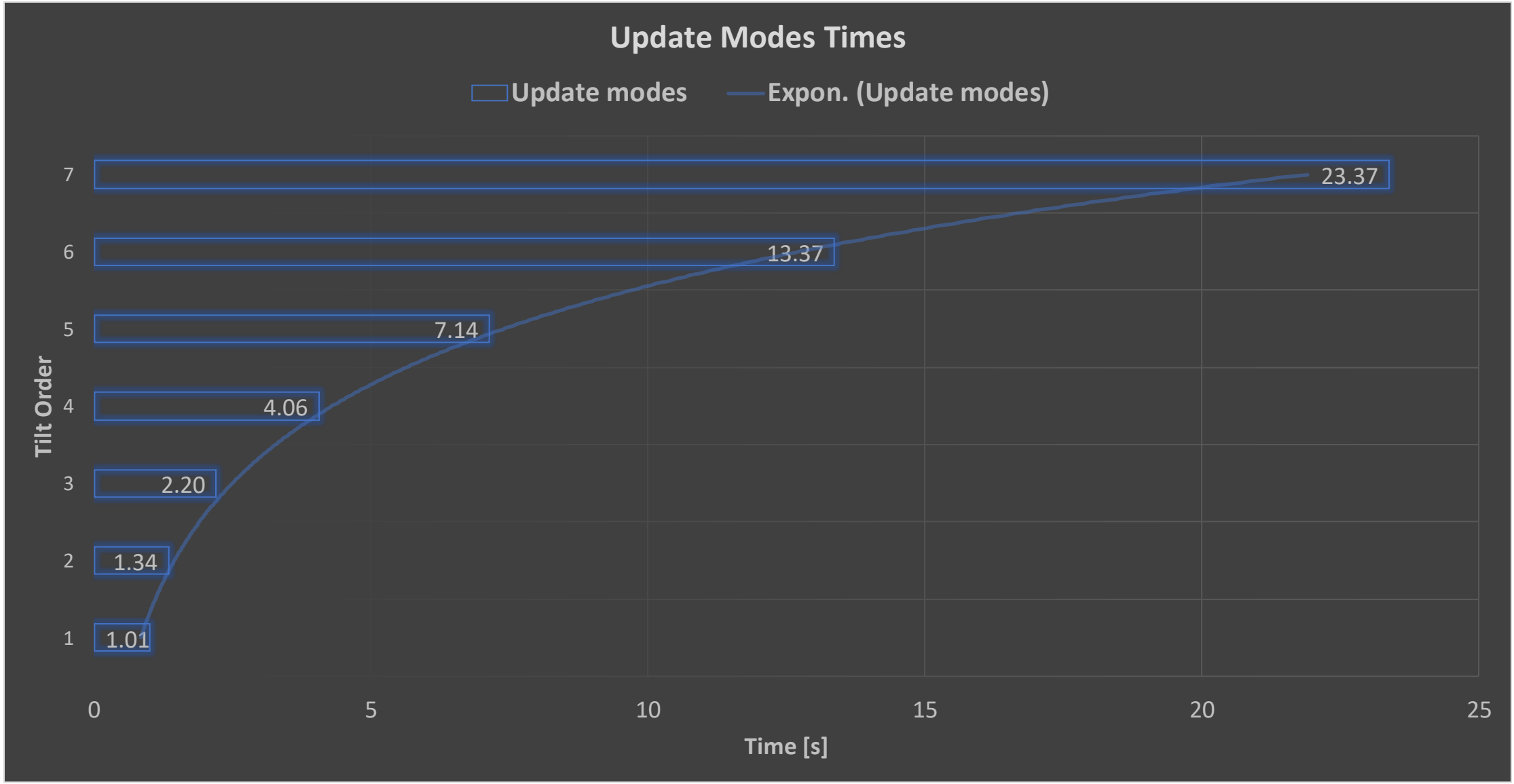
Get signals in two main steps:

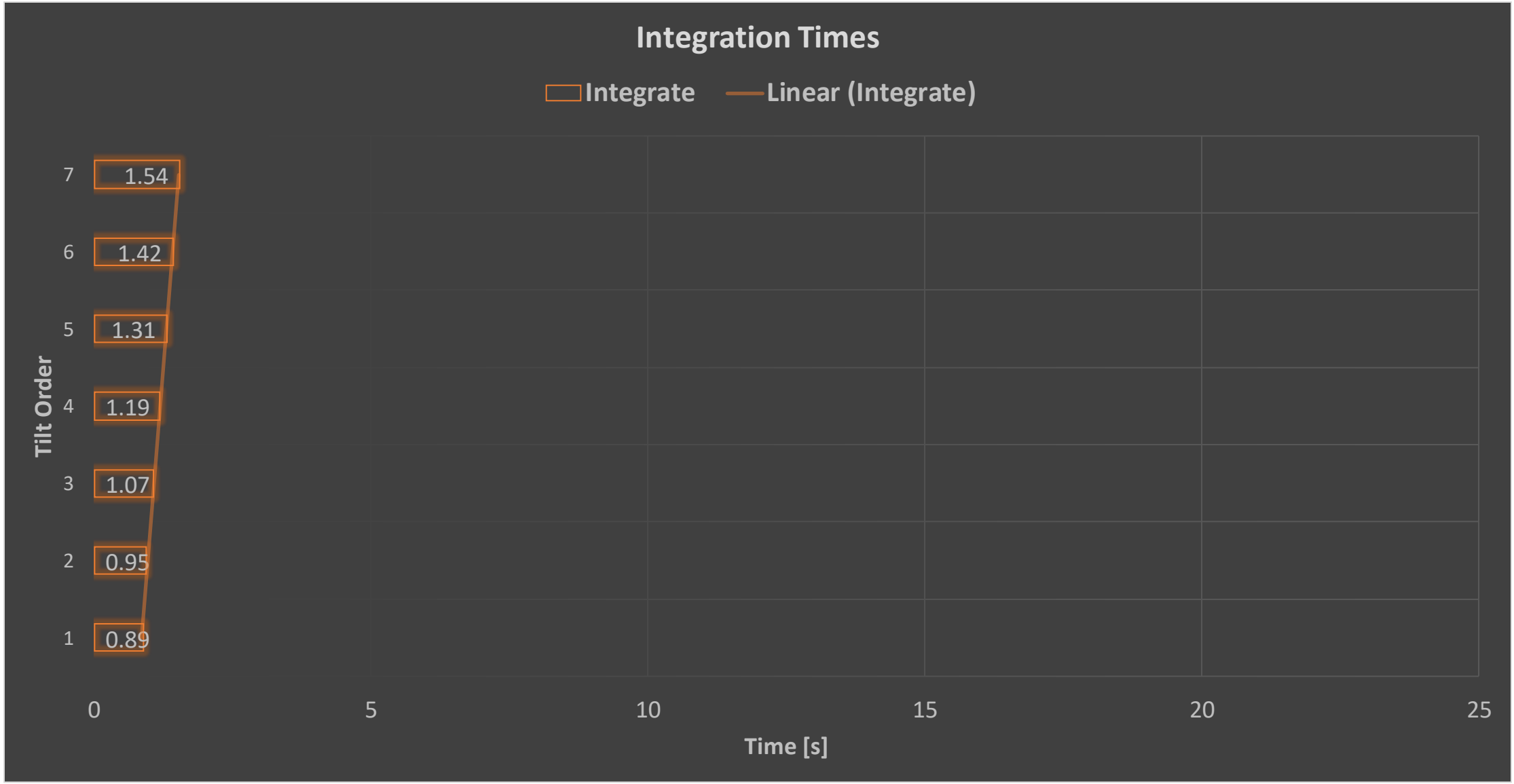
1. Update mode coefficients
2. Integrate

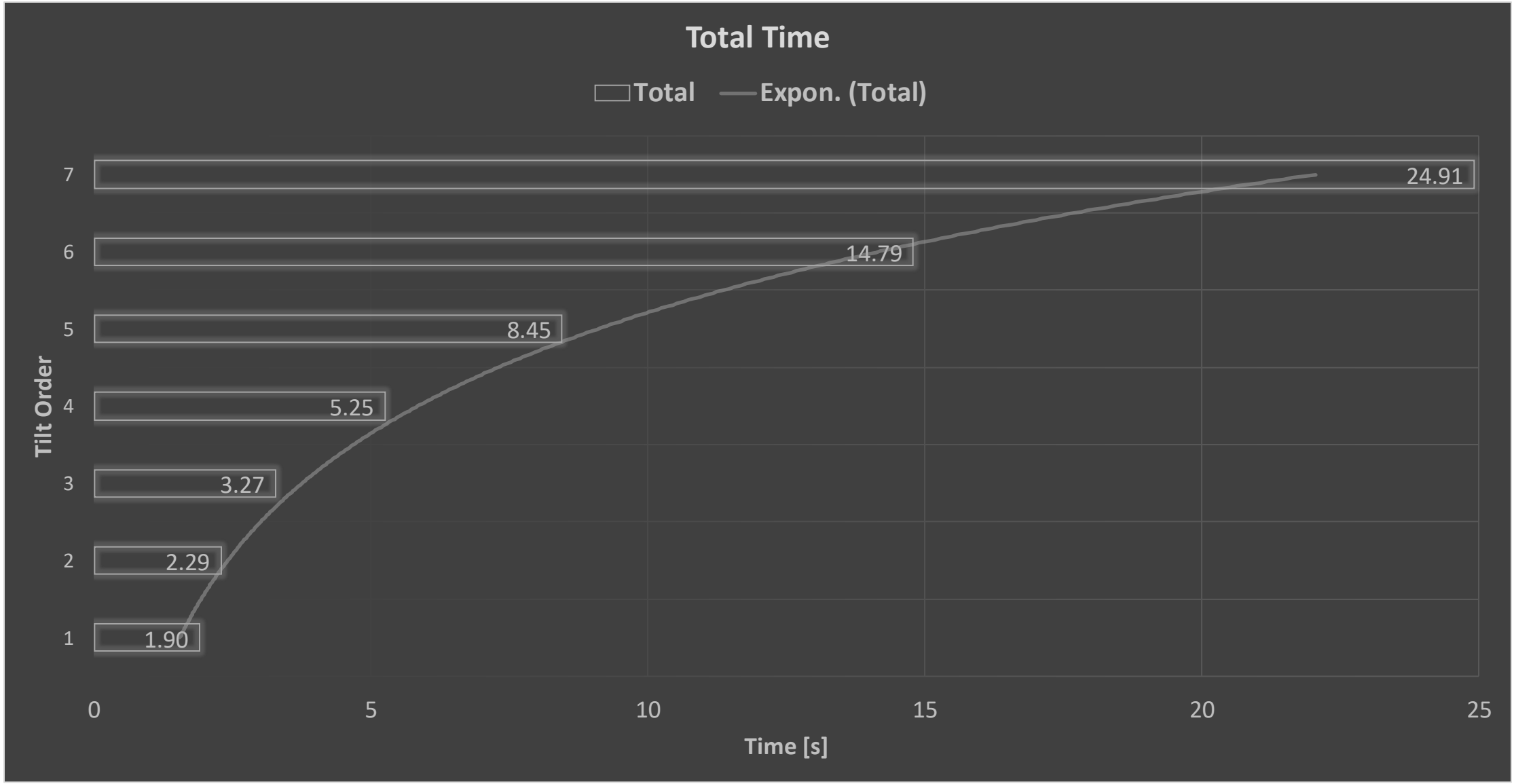
Tilt Order (shift order = 1)	Time [s]	"Accuracy" Range [urad]
3	3	200
4	5	200
5	8	300

Can reduce times by ~25%?


Computation Times:
Tophat Mode Order 34
1st order shift, Nth order tilt
Total
Integrate
Update modes
Tilt Order
7
6
5
4
3
2
1
24.91
1.54
23.37
14.79
1.42
13.37
8.45
1.31
7.14
5.25
1.19
4.06
3.27
1.07
2.20
2.29
0.95
1.34
1.90
0.89
1.01
0
5
10
15
20
25
Time [s]


Update Modes Times
Update modes
Expon. (Update modes)
Tilt Order
7
6
5
4
3
2
1
23.37
13.37
7.14
4.06
2.20
1.34
1.01
0
5
10
15
20
25
Time [s]


Integration Times
Integrate
Linear (Integrate)
Tilt Order
7
6
5
4
3
2
1
1.54
1.42
1.31
1.19
1.07
0.95
0.89
0
5
10
15
20
25
Time [s]


Total Time
Total
Expon. (Total)
Tilt Order
7
6
5
4
3
2
1
24.91
14.79
8.45
5.25
3.27
2.29
1.90
0
5
10
15
20
25
Time [s]

Speedup Structure

Mathematica strings

tilt

```
1 - a*b*j*K + b*j*K*x + (a*j*K*x)/R + (2*a*x)/w**2 + (a*b*j**2*K**2*x**2)/R + (2*a*b*j*K*x**2)/w**2 + (a*b*j*K*z)/R + (2*a*b*z)/w**2 -
(b*j*K*x*z)/R - (2*b*x*z)/w**2 - (a*b*j**2*K**2*x**2*z)/R**2 - (4*a*b*x**2*z)/w**4 - (4*a*b*j*K*x**2*z)/(R*w**2) -
(4*a*b*p**2*z*Sqrt(Factorial(n)/Factorial(-2 + n)))/w**2 - (2*a*p*Sqrt(Factorial(n)/Factorial(-1 + n)))/w -
(2*a*b*j*K*p*x*Sqrt(Factorial(n)/Factorial(-1 + n)))/w + (2*b*p*z*Sqrt(Factorial(n)/Factorial(-1 + n)))/w +
(8*a*b*p*x*z*Sqrt(Factorial(n)/Factorial(-1 + n)))/w**3 + (4*a*b*j*K*p*x*z*Sqrt(Factorial(n)/Factorial(-1 + n)))/(R*w)
```

shift

[10,10]

Python parse

p_order

```
[['+1+-a*b*j*K+(a*b*j*K*z)/R+(2*a*b*z)/w**2', '+-(2*a*p*Sqrt(Factorial(n)/Factorial(n-1)))/w+(2*b*p*z*Sqrt(Factorial(n)/Factorial
(n-1)))/w', '+-(4*a*b*p**2*z*Sqrt(Factorial(n)/Factorial(n-2)))/w**2'], ['+b*j*K*x+(a*j*K*x)/R+(2*a*x)/w**2+-(b*j*K*x*z)/R+-(2*b*
x*z)/w**2', '+-(2*a*b*j*K*p*x*Sqrt(Factorial(n)/Factorial(n-1)))/w+(8*a*b*p*x*z*Sqrt(Factorial(n)/Factorial(n-1)))/w**3+(4*a*b*j*
K*p*x*z*Sqrt(Factorial(n)/Factorial(n-1)))/(R*w)', ''], ['+(a*b*j**2*K**2*x**2)/R+(2*a*b*j*K*x**2)/w**2+-(a*b*j**2*K**2*x**2*z)/R
**2+-(4*a*b*x**2*z)/w**4+-(4*a*b*j*K*x**2*z)/(R*w**2)', '', '']]
```

x_order

[x_order,p_order]

herm. poly.

$$\sum_{n,m} C_{n,m} u_{n,m}\left(H_n(X+Y)\right) = \sum_{n,m}\sum_{K=0}^{n} Y^K [\sqrt{\frac{2^K \binom{n}{K}}{K!}} p^K] C_{n,m} u_{n-K,m}\left(H_{n-K}(X)\right)$$

x-coord.

$$X^1_{\pm} u_{n\pm1,m} = \eta^1_{\pm} (2^{\mp 1})^{-1/2} \sqrt{\frac{(n\pm1)!}{n!}} \exp(\mp i\psi(z))$$

Raw coefficients

m

n

~[n,m]