# Physics-guided data-driven modeling to understand complex phenomena and to solve real-world problems

UROP3200 Progress Report

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## 1 Abstract

Resistor networks are studied using a network science approach. We studied the change of voltages at each node in different resistor networks after removing resistors. A deep neural network is used to predict the location of removed resistors in a resistor network with the voltage changes from a few nodes as input data. A real resistor network is constructed to verify the computational results experimentally.

## 2 Introduction

Network science is a field that studies complex systems by considering the elements in the systems as nodes and connections in the system as edges to represent the systems as networks [1]. A network science approach is used to analyze different resistor networks. Junctions and resistors in a resistor network can be represented by nodes and edges respectively. The weight of each edge is proportional to the conductance of the corresponding resistor. By solving the voltage at the nodes in the network computationally, the effects of changes in the network structure on the voltages can be examined.

In real resistor networks, monitoring the voltage changes at every node is often infeasible. This can make detecting a damaged resistor in a large network very difficult due to the limited information on the voltage changes. A machine learning model may be trained to identify the broken resistor better than humans can from this limited information. In this project, a deep neural network (DNN) is trained on the voltage change data generated by computational simulations of removing a resistor in resistor networks to predict the location of the removed resistors. This may allow us to develop an efficient way of detecting damage in a large resistor network.

# 3 Methodology

We studied the effect of removing edges in a resistor network computationally. A resistor network can be represented by an adjacency matrix $C$ with each element $C_{ij}$ representing the weight of the edge between node $i$ and $j$ equal to the conductance of the resistor between the two nodes. By Ohm's Law, the current from node $i$ to $j$ is given by

$$I_{ij} = \frac{V_i - V_j}{R_{ij}} = (V_i - V_j)C_{ij} \tag{1}$$

where $V_i$ is the voltage at node $i$. By Kirchhoff's rule, the sum of current flowing from a node $i$ is

$$\sum_j^N I_{ij} = \sum_j^N (V_i - V_j)C_{ij} = 0 \tag{2}$$

. This can be written into matrix form $AV = \mathbf{0}$ with

$$A_{ij} = \begin{cases} -C_{ij},\ i \neq j \\ \sum_j^N C_{ij},\ i = j \end{cases} \tag{3}$$

. If a voltage source $V_{in}$ is supplied to node $i$ ($V_i = V_{in}$), Kirchhoff's rule no longer apply to that node so all elements in the $i$-th row of $A$ is 0 except $A_{ii} = 1$ and the $V_{in}$ is added to the $i$-th element of the vector on the right-hand side. The resulting linear equation can be solved computationally to obtain the voltage $V$ at each node. This allows us to examine the changes in voltage at each node of the network after edge removals.

In the project, we have examined different types of networks such as periodic and non-periodic regular networks, scale-free networks, and Erdős–Rényi networks. For the regular networks, we studied hexagonal, square, and triangular networks with average degree $k = 3, 4, 6$ respectively. The Erdős–Rényi network is a type of random network where edges are added between each node pair with a probability $p$ to an empty network with $n$ nodes [1]. All self-loops and isolated nodes are removed from the Erdős–Rényi networks.

Machine learning is used to predict the location of the removed edge with limited knowledge of the voltage changes. The voltage changes at a few selected nodes after edge removal as the input data. Each edge in the network has a unique index and the index of the removed edge is used as the output value. The index of each edge is encoded by one hot encoding into individual classes. In the data sets, multiple samples are generated from these inputs and outputs for each edge by adding a randomized noise to the input data. A classification deep neural network (DNN) with a cross-entropy loss function is trained to predict the index of the removed edge. The DNN consists of 2 densely connected hidden layers that use the ReLU activation function with 128 nodes in each layer. Kaiming normalization [3] is used to improve the performance of the DNN.

The effect of degree preserving randomization on the accuracy of the DNN is investigated for the regular networks by performing edge swaps on the networks. Edge swaps are performed on each edge pair with a probability $p$ where an edge pair $(n, m)$, $(u, v)$ are disconnected and reconnected as $(n, v)$, $(u, m)$ such an edge swap is performed while preserving the degrees of all nodes involved. Examples of square networks with $p = 0.5$ and $p = 1$ are shown in figure 1.

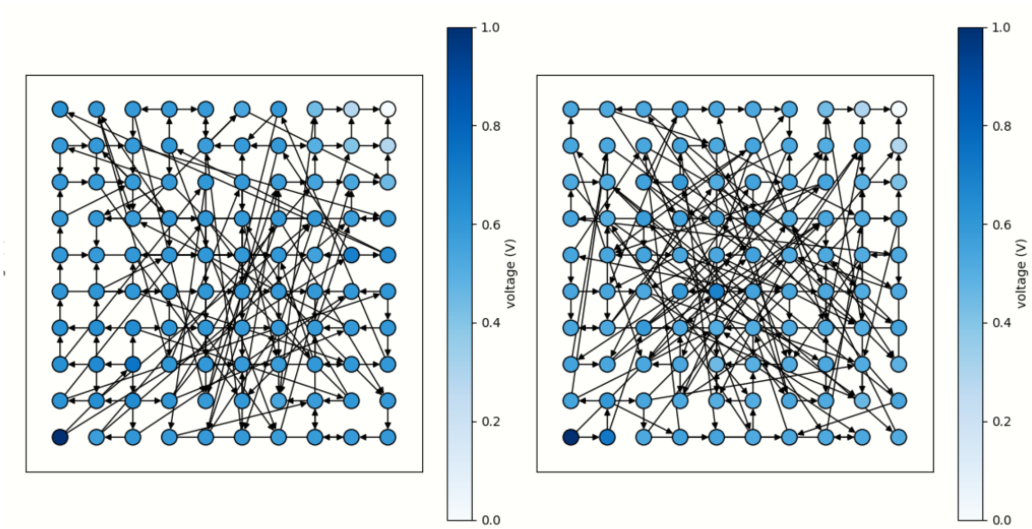


Figure 1: Square networks with edge swap probabilities of $p = 0.5$(left) and $p = 1$(right)

The dimension of a network can be defined based on the scaling exponent of the surface area

$$S(r) = kr^{d-1} \tag{4}$$

[2] where the surface area $S(r)$ is the number of node pairs with shortest pairwise distance $r$ and $d$ is the dimension of the network. By using an edge addition rule where edges $(n, n+m)$ are added to with probability $p$ each node $n$ in a simple ring network with $N$ nodes, networks with dimensions from 1 to 2 can be generated as $p$ increases. At $p = 1$, the network will become a regular $N$-by-$m$ network with degree 4. The dimension of networks with 120 nodes at different $p$ is plotted in figure 2.

For the experimental part of this project, resistor networks are constructed on a breadboard and connected to a DC power supply. The voltages of the nodes are

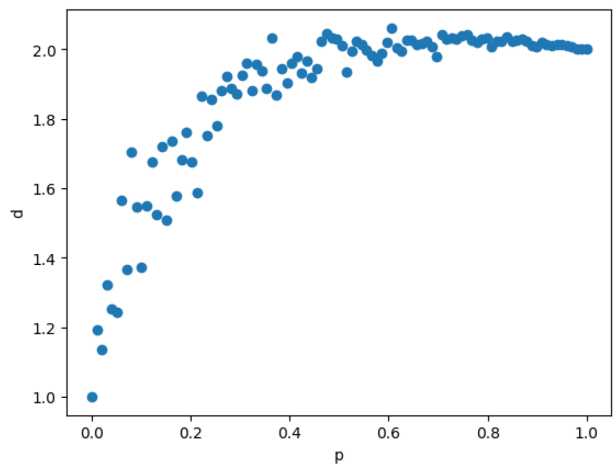


Figure 2: Dimension of networks at different edge addition probabilities

measured using a multimeter. As shown in the figure below, We constructed a simple ring with 24 nodes connected with a neighbor node on each side and added edges with the addition rule mentioned above where edges are added between node n and n+4 such that the network becomes 2 dimensional.

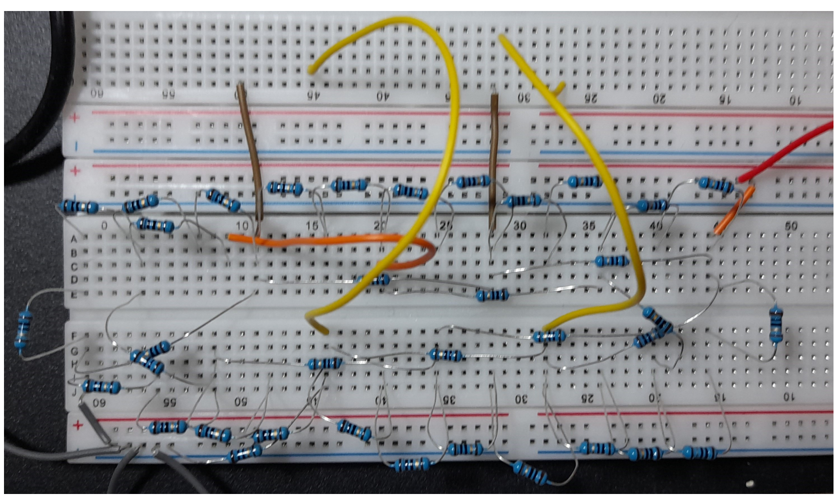

Figure 3: Constructed real resistor network

# 4 Results

Figure 4 shows the accuracy of the DNN model in predicting the location of the edge removed for various types of networks with sizes around 100 nodes while the voltage change data with 20% random noises of only 4 nodes is provided to the DNN. This demonstrates that the DNN model can predict the removed edge with high accuracy in a large range of resistor networks when only limited information on the voltage changes is available to the model.

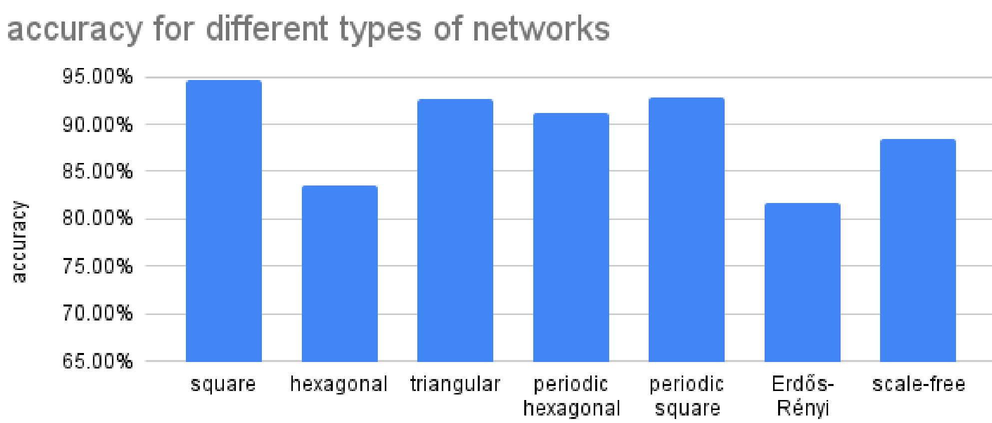


Figure 4: DNN accuracy for different types of networks

In figure 5, the accuracy of prediction of removed edges for Erdős–Rényi networks and periodic hexagonal and square networks are plotted. From the figure, the high accuracy of DNN decays slowly when the network size increases while the size of DNN and information available remains unchanged.

The robustness of the DNN model under changes in the topology of resistor networks is shown in figure 6 where the model accuracy at different edge swap

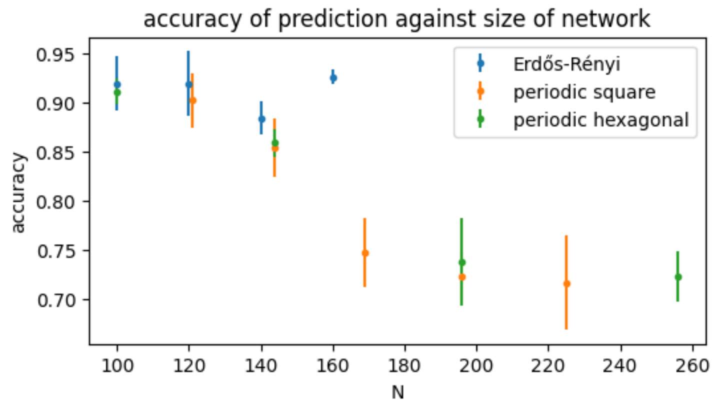


Figure 5: DNN accuracy against network size

probability $p$ is plotted for non-periodic square and hexagonal networks.

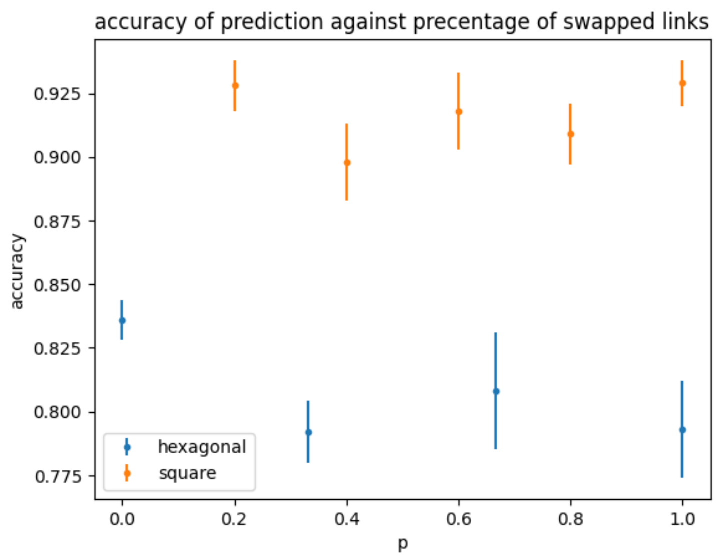


Figure 6: DNN accuracy against the probability of edge swap

In the experimental part, The measured voltages at each node of the constructed network have parentage errors of less than 1% compared to the computational values. For the voltage changes due to edge removal, the measure values have percentage errors that do not exceed 5%. These errors are well within the range of noises used for testing the DNN with computational data. The accuracy of the DNN using computational values is 89.6% while that of the experimental values is 87.5%.

## 5 Conclusion

In the project, we have examined different types of networks such as periodic and non-periodic regular networks, scale-free networks, and Erdős–Rényi networks. For most of these networks, the DNN can predict the location of the removed edge with high accuracy. This high accuracy is maintained with only slow decay when the network size increases. For the regular networks, we found no significant drop in the accuracy after degree preserving randomization. Therefore, the DNN model can still be effective when applied to larger and irregular networks. I plan to study a wider range of complex networks to confirm that the DNN would be effective for the most common types of networks regardless of their topology.

There are only a few specific networks such as ring networks that the DNN performed poorly on. We are currently investigating the characteristics of these networks using methods in network science to better understand their structures such that we may modify our method to handle this type of network. I plan to examine how the number of paths between the voltage source and the ground may affect the accuracy of the prediction of edge removal.

# 6 Acknowledgement

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# References

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# 7 Self-reflection Report

## 7.1 Lifelong Learning

I think that a project-based course like UROP 3200 allows me to understand the topics much more deeply than any lecture-based course. In this course, I found it easier to maintain my motivation, even when facing more difficulties in my research compared to regular courses, since I am working on a topic that I chose myself and am passionate about. Moreover, applying the knowledge I learned in this course in research work during the project greatly strengthened my familiarity and understanding of concepts that may be difficult to grasp through more lecture-based courses. Therefore, I can gain a much more in-depth understanding of the research topic than I may get in lecture-based courses.

My goal in this project is to use machine learning to predict the location of edge removal in different types of regular and complex networks. A major difficulty I faced during the project was isolating the many different factors that may affect the accuracy of the DNN model for any given type of resistor network. I must carefully consider all the factors that may be different for all types of networks I am examining to avoid drawing wrong conclusions about which factors are likely to cause the difference in the model accuracy. For example, I once thought that the difference in the model accuracy for the non-periodic square and hexagonal networks is due to their different average degree, but only found that this is not the case when studying the periodic networks.

In this course, I have learned to perform literature searches more efficiently than before. This will be useful for all future research projects I may work on in my academic career.

## 7.2 Ethical Awareness

The most significant ethical issue that I have faced in this project is to ensure that I spend the funding provided by the UROP project grant for conducting experiments appropriately. Since I had no prior experience buying experimental equipment and using research funding, I was worried that I might purchase equipment of inadequate quality or waste the funding on unnecessary equipment. To prevent misuse of the research funding, I asked for my supervisors' advice and approval and the help of another student in the research group to make sure that the equipment I planned to purchase was appropriate before making the purchase. I believe that I have used the funding well to purchase the necessary equipment for experiments which I am still conducting to verify my computational results in real resistor networks.

## 7.3 Others

I think the mini-conference in this course is a good opportunity for me to try presenting my research to a larger group of audience instead of my supervisors and groupmates. While I felt quite nervous when I was presenting in the mini-conference, I think that I had still performed reasonably well in my presentation which will likely improve my confidence in presenting my research in the future.

Moreover, I got to learn about other people's research projects and learn how to better present my research from their performance.