Physics-guided data-driven modeling to understand complex phenomena and to solve real-world problems

UROP2100 Progress Report

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1 Abstract

Resistor networks are studied using a network science approach. We studied the change of voltages at each edge in square, hexagonal and scale-free resistor networks after removing resistors. A deep neural network is used to predict removed resistor in a resistor network with the voltage changes from a few nodes as input data.

2 Introduction

In this project, network science concepts are used to analyze complex physical systems including active matter and resistor networks. Network science is a field which studies complex systems by considering the elements in the systems as nodes and connections in the system as edges to represent the systems as networks [1].

A network science approach is used to analyze different resistor networks. Junctions and resistors in a resistor network can be represented by nodes and edges respectively. The weight of each edge is proportional to the conductance of the corresponding resistor. By solving the voltage at the nodes in the network computationally, the effects of changes in the network structure on the voltages can be examined.

In real resistor networks, monitoring the voltage changes at every nodes is often infeasible. This can make detecting a damaged resistor in a large network very difficult due to the limited information on the voltage changes. A machine learning model may be trained to identify the broken resistor better human can from these limited information. In this project, a deep neural network (DNN) is trained on the voltage change data generated by computational simulations of removing a resistor in resistor networks to predict the location of the removed resistors. This may allow us to develop an efficient way of detecting damage in a large resistor network.

3 Methodology

We studied is the effect of removing edges in a resistor network. A resistor network can be represented by an adjacency matrix C with each element C_{ij} representing weight of the edge between node i and j equal to the conductance of the resistor between the two nodes. By Ohm's Law, the current from node i to j is given by

$$I_{ij} = \frac{V_i - V_j}{R_{ij}} = (V_i - V_j)C_{ij} \tag{1}$$

where V_i is the voltage at node i. By Kirchhoff's rule, the sum of current flowing from a node i is

$$\sum_j^N I_{ij} = \sum_j^N (V_i - V_j)C_{ij} = 0 \tag{2}$$

. This can be written into matrix form $AV = \mathbf{0}$ with

$$A_{ij} = \begin{cases} -C_{ij},\ i \neq j \\ \sum_j^N C_{ij},\ i = j \end{cases} \tag{3}$$

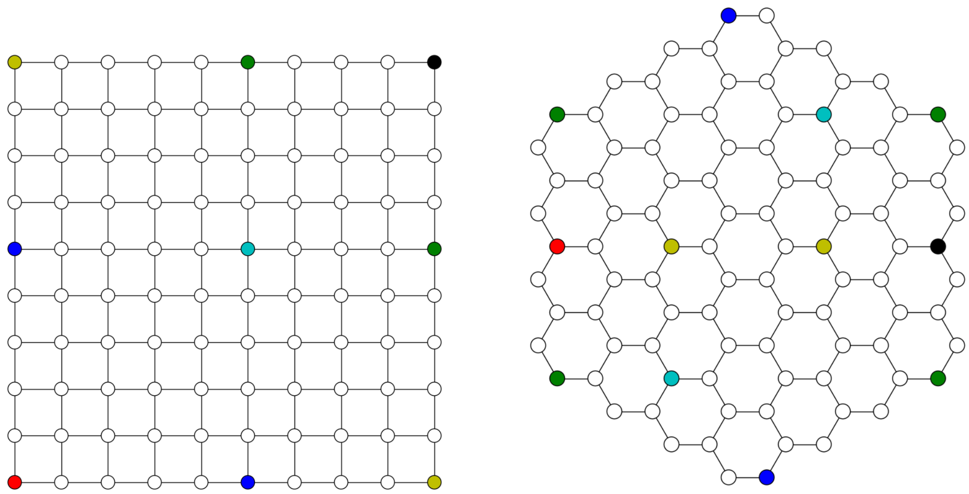
. If a voltage source V_{in} is supplied to node i ($V_i = V_{in}$), Kirchhoff's rule no longer apply to that node so all elements in the i-th row of A is 0 except $A_{ii} = 1$ and the V_{in} is added to the i-th element of the vector on the right hand side. The resulting linear equation can be solved computationally to obtain the voltage V at each node. This allows us to examine the changes in voltage at each node of the network after edge removals.

Machine learning is used to predicted the location of the removed edge with limited knowledge of the voltage changes. The voltage changes at a few selected nodes after edge removal as the input data. Each edge in the network has a unique index and the index of the removed edge is used as the output value. The index of each edge is encoded by one hot encoding into individual classes. In the data sets, multiple samples are generated from these inputs and outputs for each edge by adding a randomized noise to the input data. A classification deep neural network (DNN) with a cross entropy loss function is trained to predict the index of the removed edge. The DNN consists of 2 densely connected hidden layers which use the ReLU activation function with 128 nodes in each layer. Kaiming normalization [2] is used to improve the performance of the DNN.

In this project, three types of resistor networks including regular square network, hexagonal network and scale-free network are studied. The square and hexagonal networks used in this project are shown in fig. 1. The nodes at which the voltage changes are used as input data to train the DNN shown in fig. 1 and listed in table 1.

Table 1: Sets of selected nodes for different numbers of inputs

square network		hexagonal network	
no. of inputs	sets of nodes	no. of inputs	sets of nodes
2	blue	2	blue
4	blue, green	6	blue, green
5	blue, green, cyan	8	blue, green, yellow
6	blue, green, yellow	10	blue, green, yellow, cyan



(a) (b)

Figure 1: Resistor networks with voltage source (red), ground (black) and selected nodes for taking input data (other colored nodes)

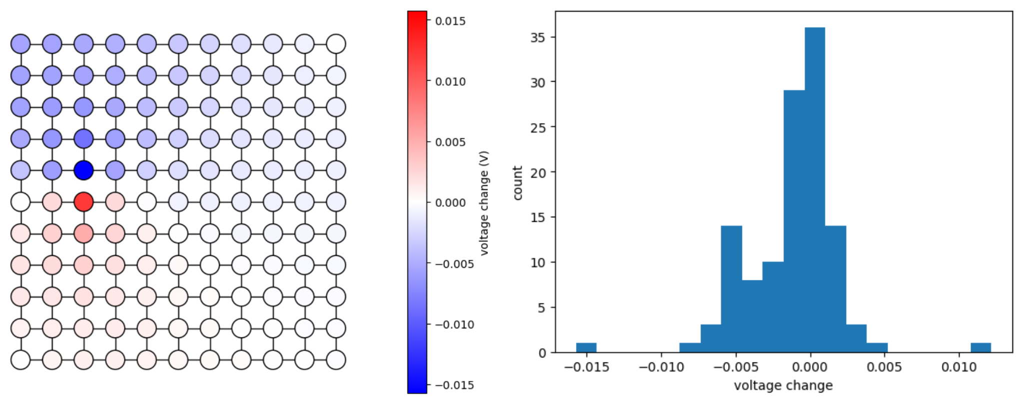
The scale-free network is generated using preferential attachment [1] starting with an initial network of three connected nodes with two of the nodes be voltage source and ground respectively. When a new node is added the network, m edges are added one by one between the t-th new node and the i-th node with probability given by

$$p = \begin{cases} \frac{k_i}{2mt+n-1}, \ i \leq t-1 \\ \frac{1}{2mt+n-1}, \ i = t \end{cases} \tag{4}$$

where n is the number of edges added for the new node and k_i is the degree of node i. The input data for the DNN is taken from N hubs in the network with the highest degree except the voltage source and ground.

4 Results

In fig. 2a, the voltage change at each node when a edge is removed is plotted. A dipole-like pattern can be seen around the removed edge with the side closer to the voltage input being positive and the side closer to the ground being negative. In fig. 2b, the voltage changes are plotted in a histogram where the voltage changes at most of the nodes are close to zero except the nodes at the ends of the removed edge.



(a) (b)

Figure 2: Voltage changes in hexagonal resistor network with one edge removed

In fig. 3, the accuracy of the prediction of the removed edges in a square network from the DNN is plotted against the percentage noise in the input data. From the figure, the DNN performs very well even with a small number of inputs at low noises but the accuracy of the prediction decreases much more quickly when the noise increase for small numbers of inputs.

Fig. 4 is plotted similarlu=y to fig. 3 but for a scale-free network instead of

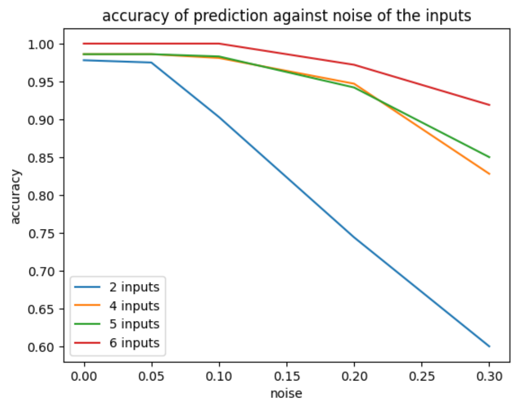


Figure 3: Change of prediction accuracy with noise for square network

a square network. For the scale-free network, the accuracy from using 4 and 5 inputs is very similar until the noise reach 30%. This shows that simply adding more inputs may no long be able to improve the accuracy of the prediction beyond around 4 to 5 nodes.

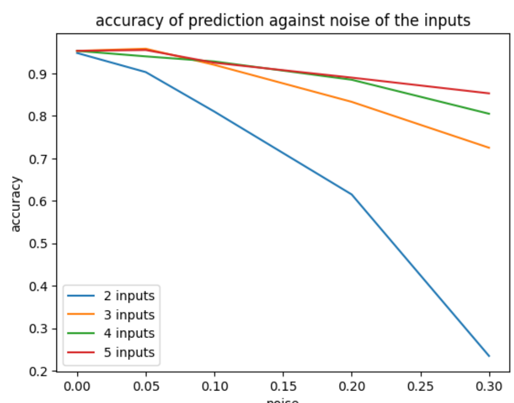


Figure 4: Change of prediction accuracy with noise for scale-free network

In fig. 5, the accuracy of the prediction of the removed edges in a hexagonal network plotted in a similar way to the last two case. For only 2 inputs, a curve similar to above two figures can be seen. However, the accuracy does not decrease significantly when the noise increases for ≥ 6 inputs. Increasing the number of inputs beyond 6 does not have significant effect of the accuracy and the accuracy stay within the 80-90% range. Changing the DNN structure may be necessary to further enhance the performance.

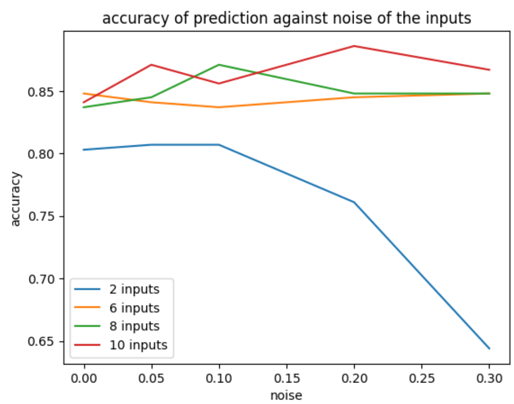


Figure 5: Change of prediction accuracy with noise for hexagonal network

In this report, the nodes selected for collecting the voltage changes as input data is selected manually, which are unlikely to be optimal, for the regular networks while selecting nodes base on the degree of the nodes requires complete information of the degree distribution for the scale-free network. Our current method of node selection limits the generalizability and effectiveness of detecting removed edges in different resistor networks. Therefore, developing a more systemic approach to finding nodes that are most sensitive to edge removal in a resistor network can be important for the further improvement of the edge detection.

5 Conclusion

In this project, we studied the effect of edge removal in square, hexagonal and scale-free resistor networks. The voltage changes after edge removal in a resistor shows a dipole-like pattern around the removed edge. We are able to use deep neural network to predict the removed edge, while only giving the model access to the voltage change at a few nodes, in different resistor at reasonable levels of accuracy. In the future, we will explore how the topology of the resistor network affects the accuracy of the prediction by testing the model on different networks and develop a better method for selecting the nodes at which voltage change data is collected.

6 Acknowledgement

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References

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[2] Wu, Y., and He, K. Group normalization. In *Proceedings of the European Conference on Computer Vision (ECCV)* (September 2018).