November 29, 2024

Chemical Engineering Science
Professor Wei Ge, Editor

Dear Prof. Ge,

Please find enclosed the revised version of our manuscript (CES-D-24-01867): "Numerical Simulation of Bubble Deformation and Breakup under Simple Linear Shear Flows". We thank the reviewers for their comments and thoughts regarding improvement of our paper. We believe that we have addressed all of the reviewers' concerns; the changes are itemized in detail below.

NOTE: all references to equations and figures below are with reference to the numbering scheme of the revised version of the paper, not the original version. Also, changes are hi-lighted in red.

Changes made in response to comments of Reviewer 2:

1. Why are there several words in italics in the Introduction section and some other parts?

   Response:
   The italic format was used to emphasize certain passages, but in retrospect, we agree that which passages deserve italics and which passages don't is open to discretion. So we have removed all the italics.

2. Page 2 Line 14: There should be an "and" before "Renardy"

   Response:
   We have corrected this mistake. Also we have made complete run-throughs of the paper checking for grammar errors and awkward sentences.

3. The authors mentioned several DNS methods in the Introduction section while choosing the CLSVOF method in this work. What are the reasons?

   Response:
   We have improved the introduction and section 3.1 in order to make it more clear about our rationale for using the CLSVOF method to capture complex deforming boundaries and also why we represent the gas-liquid interface as ideally sharp. Also, we have already applied the CLSVOF method in past work to many multiphase flow problems with complex interface deformations. Please see the top of section 3.2 in which we list past research in which we have obtained nice results for hard multiphase flow problems using the CLSVOF method.

4. Why can the effect of gravity be disregarded in this work?

   Response:
   In our computations, we set $g = 0$ because we wanted to isolate only the effects of the density and viscosity ratios and wanted to compare with previous studies dealing with the deformation and breakup of a drop with $\lambda = 1$ and $\eta = 1$. Also, in the revised paper, a brief discussion of the potential influence of gravity (Froude number) on bubble breakup dynamics has been added according to another reviewer's suggestion.

   Please see the remark in section 2.

5. Page 9: The results of D are compared with those in other literature. What are the underlying causes of these differences? It is also noticed that different literatures are used for different verification steps. Why not use the same literature for verification?

   Response:
   Table 1 shows the comparison of the deformation parameter D (Li et al., Physics of Fluids, Vol.12, 269–282, 2000) for a drop as a function of Re ($Ca = 0.3$, $\lambda = 1$ and $\eta = 1$). The D computationally obtained by Li et al. (2000) is regarded as a benchmark problem in this field. First, we compared the D by Li et al. (2000) with our computational results. Also, In Fig. 2, we compared the appearance of drop breakup (Renardy and Cristini, Physics of Fluids, Vol.13, 2161–2164, 2001) with our computational results. On the other hand, regarding quantitative physical quantities on bubble deformation and breakup, only experimental results for D exist under the condition of $Re \simeq 0$. Thus, As shown in Table 2, we performed the comparison of the deformation parameter D for a bubble as a function of Ca ($Re \simeq 0$, $\lambda \simeq 0$ and $\eta \simeq 0$). In the experiments, obtaining precise experimental data will be difficult because the dynamic motion of a bubble in a highly viscous liquid in an experimental device needs to be accurately set. We don't know the cause of the difference between both experimental results, but our numerical results were close to those of Rust and Manga (Journal of Colloid and Interface Science, Vol. 249, 476– 480 2002). For your reference, computations for bubble deformation with the condition of $Re \simeq 0$ were hard tasks and a very long computational time (one year) was needed.

6. Page 14: What are the units of the variable T?.

   Response:

T is the dimensionless time defined by $T = \Gamma t$. Please see equation (8) in the revision.

7. Figure 4: Drop deformation and breakup are compared under conditions with $Re = 1.0$ and 1.1. When $Re = 1.0$, the simulation ends at $T = 35.0$, and it is stated that drops will not break up. When $R = 1.1$, the drop also does not break up at $T = 35.0$. It breaks up at $T = 56.7$. What about also showing the status of drop under $Re = 1.0$ at $T = 56.7$? It might be a more reasonable comparison.

   Response:
   We agree with the reviewer's suggestion. We added results at $T = 40.1$, 50.0, and 56.7 to the drop case for $Re = 1.0$ in Fig. 4. As can be seen from the new Fig. 4, a drop remains a stable deformed state after $T =$ 35.0. In the revised paper, such a comment was added.

8. Figures 5 - 9: I am curious about the way that the selection of the Ts. Why are they not unified in these five figures?.

   Response:
   Regarding Figs. 5 and 6, we compared bubble dynamics for $Re = 92$ (Fig. 5) with that for $Re = 93$ (Fig. 5) at almost the same T. But, we found a mistake in Fig. 5: we replaced the figure with $T = 17.4$ with the figure with $T = 12.2$. In the same way, we replaced the figure with $T = 24.3$ with the figure with $T = 17.0$ in Fig. 10.

   Fig. 7 shows the detailed bubble breakup process. Giving the highest priority to show the detailed bubble breakup process, we selected results with T independently of T set in Figs. 5 and 6.

   Fig. 8 shows the shear stress profile around a bubble for two Reynolds numbers ($Re = 50$ and 93). In Fig. 8, the shear stress for the $Re = 50$ condition was drawn around the bubble after the bubble attained a stable deformed state. The shear stress for the case of $Re = 93$ was depicted when the bubble sufficiently elongated ($T = 14.9$). We wanted to compare the maximum shear stress value at both Re conditions.
   In the revised paper, these explanations for Fig. 8 were added.

   As for Fig. 9, we drew velocity fields around bubbles corresponding to Fig. 7.

9. Page 16 Line 395: Why does the bubble not develop the bulb-like shape like

the drop?.

Response:
In the breakup of the drop with $\lambda = 1$ and $\eta = 1$ (Fig. 4), the drop starts to break at the center of the drop without the elongation in the first place because the drop easily breaks under the low Re number. Consequently, the drop breakup is seemingly developed with a bulb-like shape because the liquid bridge between two mother drops is formed in the breakup process. On the other hand, in the case of the bubble, very large shear forces are required to deform the bubble because $\lambda \simeq 0$ and $\eta \simeq 0$. Thus, the bubble undergoing large shear forces is largely stretched along the direction of the shear flow, and the very long elongated bubble is finally formed. Accordingly, the bubble breaks up without the bulb-like shape. In the revised paper, we added these considerations.

10. Page 17 Line 424: What are the possible reasons that the bubble breakup process is different from the drop breakup process?

    Response:
    As mentioned above (Answer 9), the effect of λ and η on the breakup process is large, and especially η is a dominant factor for the breakup process. The shear stress is defined by $\tau = \mu\dot{\gamma}$ ($\dot{\gamma}$: the shear-rate (the velocity gradient)). As is obvious from this equation, since the viscosity of a gas is much less than that of a liquid($\eta \simeq 0$), so a very large velocity gradient needs to be achieved to realize τ that makes the bubble largely deformed. As a result, the bubble undergoing large shear forces (velocity gradients) is largely stretched along the direction of the shear flow, and the appearance of the bubble breakup becomes quite different from that of a drop Therefore, to be exact, the bubble breakup process is different from the drop breakup with $\lambda = 1$ and $\eta = 1$. We added "with $\lambda = 1$ and $\eta = 1$" to the revised paper. Ongoingly, we have been examining the effect of λ and η on drop breakup, and it is confirmed that large shear forces are required to deform the drop when $\lambda = 1$ and $\eta \simeq 0$ and the drop breaks up by way of forming the long elongated shape. Also, we have referred to the effect of the viscosity ratio on the deformation and breakup of the drop in the revised paper.

11. Section 4.4: What about the velocity field of the breaking drop? In the previous text and figures, it does not seem similar to that of the breaking bubble.

    Response:
    Detailed velocity fields of the deforming and/or breaking drop have already been presented in some literature. So, we have not shown the ve-

locity fields of the deforming and/or breaking drop here. The behavior of a breakup influences the velocity fields for the drop and the bubble, so the velocity fields between the drop and the bubble is not similar. In the revised paper, we added these explanations.

12. Is there any correlation that can be summarized between Rec and Ca?

    Response:
    We need to additionally investigate Re_c for a wider Ca number condition to consider correlation between Re_c and Ca, but we predict that a correlation with Re_c as a function of Ca for predicting the bubble breakup can be formulated.

On behalf of the authors,

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