Physics Principles with Applications - Chapter 1

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Contents

1 Introduction, Measurement, Estimating

Physics is the most basics of sciences, divided into **classical physics** (ex: motion, fluids, light) and **modern physics** (ex: relativity, quantum theory, astrophysics). It is important to review how science is done before getting into it.

1.1 The Nature of Science

- **Observations -** Things we notice, can include experiments, require imagination
- **Theories -** Formed Around observations in an attempt to explain phenomena
- **Testing -** Experiments or other assessments to determine if predictions based off of theories are true
- Theories are accepted or rejected based off of data from testing

1.2 Physics and its Relation to Other Fields

- Over time as science developed, different fields broke away from physics and established themselves (ex. life sciences and chemistry which are not defined under physics but were once part of it)
- Other sciences can apply physics, but that does not mean they are the same field (ex. architecture uses physics but is its own field)

1.3 Models, Theories, and Laws

- **Model -** Analogy used to approximate a mental image
- **Theory -** More broad than a model, gives more detail and testable predictions for phenomena
- **Law -** Used for general statements about nature's behavior, often in the form as a relation between 2 quantities
- **Principle -** Used for less general, more precise statements, such as Archimedes' Principle
- Scientific laws describe how nature *does* behave rather than how it *should* behave, and are subject to change if new evidence is presented.

1.4 Measurement and Uncertainty; Significant Figures

Uncertainty

- **Estimated Uncertainty -** Gives a range of values that a measurement could fall between
  - Example: A ruler's who's smallest measurement is 1mm could have an uncertainty of ±0.1cm and a measurement of 5.3cm could have a true value between 5.2cm and 5.4cm
- **Percent Uncertainty -** The ratio between the uncertainty and measured value expressed as a percent. Can be found with
$$\frac{Uncertainty}{MeasuredValue} * 100\%$$

Significant Figures

- SigFigs (or significant digits) Represent reliably known digits in measurements and calculations
- To count SigFigs, there are several rules:
  - Any digits from 1-9 are significant
  - Any 0's between digits from 1-9 are significant
  - Any 0's to the right of a sigfig and to the left of a decimal point are significant
  - Any 0's trailing after a decimal point are significant
- Examples -
  - **505** has **3** significant digits
  - **1**00 has **1** significant digit
  - **1200.** has **4** significant digits
  - 0.0**16000** has **5** significant digits

Scientific Notation

- Used to display very large or very small numbers using powers of 10
  - Example: 45,000 in scientific notation is expressed as $4.5 * 10^4$
- Remember to always have one significant figure to the left of the decimal

Percent Uncertainty vs. Significant Figures

- There are exceptions to the significant figures rules
- When calculating percent uncertainty display the digits to the same precision as the uncertainty
- Add an extra digit if it gives a more realistic % uncertainty

Approximations

- We often cannot solve problems precisely
- Be aware an approximation will be nowhere near as precise as a true result

Accuracy vs. Precision

- **Accuracy** - How close a value is to the "true" or "accepted" value
- **Precision** - how close different measured values are to each other

1.5 Units, Standards, and the SI System

- **Unit** - Tells what is being measured and must be stated along with values (ex. kilograms tells mass is being measured)
- **Standard** - A defined quantity of a unit that can be reproduced easily for use in experiments and testing

Length

- International standard unit is the **meter**
- Defined as how long light travels in 1/299,792,458 seconds.

Time

- Standard is the **second**
- Currently defined as the amount of time a cesium atoms takes to oscillate 9,192,631,770 times.

Mass

- Standard is the **kilogram**
- Defined a platinum-iridium cylinder whose mass is exactly 1kg

Unit Prefixes

- Used to quantify very large or very small amounts of a unit
- Example
  - Kilo- means 1000 so 1 kilometer is 1000 meters

Systems of Units

- Used to make equation writing easy by having consistent units
- **Systeme International** - French standardized system of units used globally

Base vs. Derived Quantities

- **Base Units** - Fundamental units of measurement used to derive other units, listed as:
  - Meter (length)
  - Second (time)
  - Kilogram (mass)
  - Ampere (electricity)
  - Kelvin (temperature)
  - Mole (amount of a substance, similar to a dozen)
  - Candela (luminous intensity)
- **Derived Quantity** - Quantities defined in terms of the 7 base quantities listed before
  - Example - The newton is defined as:

$$\frac{kg * m}{s^2}$$

- **Operational Definition** - The rule or procedure used to define a base or derived unit

1.6 Converting Units

- Measurements consist of a value and a unit, to go from one unit to another, use a conversion factor
- **Conversion Factor** - Ratio between two units that can be used to go from one to the other
  - Example - The conversion factor between inches and centimeters is $\frac{2.54cm}{in}$, to determine how many cm are in 8.00 inches, use the conversion factor:

$$8.00in * \frac{2.54cm}{in} = 20.3cm$$

  - Note that the inches units cancel out leaving only centimeters, and that the answer was adjusted to 3 significant figures

1.7 Order of Magnitude: Rapid Estimating

- **Order-of-Magnitude Estimating** - A technique of estimating that rounds the answer to a single sigfig and its power of 10 (order of magnitude)
  - Example - Estimate the thickness of a sheet of paper from a book that is 300. pages long and 2.40cm thick
  - To solve, divide the thickness by the number of pages:

$$\frac{2.40cm}{300.sheets} = 0.00800\frac{cm}{sheet} = 8.00 * 10^{-3}\frac{cm}{sheet}$$

1.8 Dimensions and Dimensional Analysis

- **Dimensions** - "Type" of quantity being measured
  - Kilograms measure the dimension of mass, abbreviated as $[M]$
  - Square meters measure the dimension of area, abbreviated as $[L^2]$
- **Dimensional Analysis** - Used to verify a relationship, both sides of an equation must have the same dimensions
  - Example - To verify a relationship between Newtons ($\frac{kg*m}{s^2}$) vs mass (kg) and acceleration ($\frac{m}{s^2}$), write the units in their base dimensions and compare:

$$\frac{kg * m}{s^2} = kg * \frac{m}{s^2} \Rightarrow \frac{kg * m}{s^2} = \frac{kg * m}{s^2}$$

  The dimensions are correct, the left and right sides are equal

- Dimensional analysis can also be used to verify derived equations you are unsure about
- Note that some units, such as radians, are dimensionless and should therefore not be included in dimensional analysis

2 Describing Motion: Kinematics in One Dimension

The study of motion, forces, and energy form the field of mechanics, the movement of stuff. Mechanics is divides into kinematics, how objects move, and dynamics, why objects move. Motion without rotation is called translational motion. Point particles, objects with no mathematical size, will be used to simulate translational motion.

2.1 Reference Frames and Displacement

- **Frame of Reference** - Location from which measurements are made
  - Example - If you are in a moving vehicle, observations made about stationary objects passing by would make it seem like they are moving
- **Coordinate Axes** - Used to represent frames of reference and directions. The origin and directions of the axes can be placed anywhere for convenience
- **Origin** - Center of a coordinate plane, where both x and y directions equal 0, graphed as (0,0)
- **Position** - Where an object is with respect to its reference frame
- In one dimensional motion, if an object is moving horizontally, it is usually graphed on the x-axis, but if it is falling then it is graphed on the y-axis
- **Displacement** - Different from distance traveled, this is how far an object has moved from its starting point
- An object can move a greater distance than its displacement, it can go in a circle around its point of origin but end up in the same spot which would give it some distance but no displacement
- **Vector** - Quantities representing both magnitude and direction
  - Example - An object's displacement can be represented with a vector, if it moves 40 meters up, the coordinate plane would show an arrow 40 meters long pointing up
- **Change in** - Used often in physics, the change in a value is the final value minus the initial value, shown below:
  $$x_f - x_i = \Delta x$$
  Where the f and i subscripts mean final and initial respectively and Δ represents "change in"

2.2 Average Velocity

- **Average Speed** - Distance traveled by an object divided by the time it takes to travel that distance
  $$average\ speed = \frac{distance\ traveled}{time\ elapsed}$$
- **Average Velocity** - A vector that quantifies average speed along with direction, it can be negative if the object is moving in a negative direction
  $$average\ velocity = \frac{displacement}{time\ elapsed} = \frac{final\ position - initial\ position}{time\ elapsed}$$
- It is important to note that average speed is defined in terms of **distance** while average velocity is defined in terms of **displacement**
- **Time Elapsed** - Defined as change in time or Δtime
- **Time Interval** - Points in time between which time elapsed is measured, represented as
  $$t_f - t_i = \Delta t$$
- Average velocity is more formally represented as
  $$\vec{v} = \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}$$
- Note that average velocity is just that, the *average*, even if an object changes velocity during the time interval, the average between the two velocities is what's important

2.3 Instantaneous Velocity

- **Instantaneous Velocity** - The average velocity over an infinitely short time interval, represented by
  $$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$
- While it seems counter intuitive, instantaneous velocity is just the velocity an object is traveling at at any given point in time
  - Example - an object increasing in velocity at a rate of $1\frac{m}{s}$ every second would have an instantaneous velocity of $2\frac{m}{s}$ after 2 seconds
- Graphs could also be used to determine instantaneous velocity, in a velocity vs time graph, the instantaneous velocity at any point in time would be given by its corresponding y-coordinate

2.4 Acceleration

- **Acceleration** - A change in either the magnitude or direction of velocity, represented by

$$average\ velocity = \frac{change\ of\ velocity}{time\ elapsed}$$

- **Average Acceleration** - Change in velocity over a given time interval, represented by

$$\vec{a} = \frac{v_f - v_i}{t_f - v_i} = \frac{\Delta v}{\Delta t}$$

- **Instantaneous Acceleration** - Similar to instantaneous velocity, this is the acceleration of an object at any given point in time, represented by

$$v = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

- Acceleration is measured in meters per second per second or

$$\frac{m}{s^2}$$

  - Example - An object is travelling at $3\frac{m}{s}$ accelerates to $27\frac{m}{s}$ in 8 seconds, its averave acceleration is

$$\vec{a} = \frac{27\frac{m}{s} - 3\frac{m}{s}}{8s} = \frac{24\frac{m}{s}}{8s} = 3\frac{m}{s^2}$$

- **Deceleration** - When an object is decreasing in velocity
- Note that deceleration does not necessarily mean acceleration is negative, if an object is moving in the negative direction and slows down, the acceleration is actually positive

2.5 Motion and Constant Acceleration

- **Constant Acceleration** - The same acceleration over a given time period
- To solve for final velocity after constant acceleration, the formula for average acceleration can be manipulated to solve for final velocity:

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t}$$
$$a\Delta t = v_f - v_i$$
$$v_f = v_i + a\Delta t$$

- To solve for final position after constant acceleration, the following formula is used

$$x_f = x_i + v_o\Delta t + \frac{1}{2}a\Delta t^2$$

- To solve for final velocity when time is not given, the following formula is used

$$v_f^2 = v_i^2 + 2a\Delta x$$

- The four most important equations for objects in constant acceleration are
  - $v_f = v_i + a\Delta t$
  - $x_f = x_i + v_o\Delta t + \frac{1}{2}a\Delta t^2$
  - $v_f^2 = v_i^2 + 2a\Delta x$
  - $\vec{v} = \frac{v_f - v_i}{2}$

2.6 Solving Problems

- There are many strategies students can use to determine what a word problem is asking and what formulas can be used to solve it
- For problems concerning constant acceleration, determine the object studied and time interval during which it is moving
- Draw a diagram of the problem with coordinate axes, determine which quantities are "known" and which are unknown
- Determine which equations for the unknowns can be solved using current known values, this may require some manipulation to solve for the unknown
- Calculate the answer and determine if it is reasonable to the problem, would a snail be able to accelerate to $100\frac{m}{s}$?
- Make sure the units of your answer are correct with dimensional analysis

2.7 Freely Falling Objects

- Common example of constant acceleration in one dimension, free-fall problems analyze the acceleration of objects, well, falling freely
- It is important to note that the distance travelled by objects in free-fall is proportional to the square of the time during which they fell, represented by

$$d \propto t^2$$

- Air resistance is not considered since it is beyond the scope of this unit, therefore all objects no matter how heavy fall with the same acceleration
- The acceleration due to Earth's gravity is $9.80\frac{m}{s^2}$, sometimes rounded to $10.0\frac{m}{s^2}$
- Not every problem will have an object that is dropped, sometimes the object may be thrown up before coming down or thrown down, in these cases the initial velocity is crucial
- Sometimes, problems involving constant acceleration will involve quadratic equations with more than one solution, in these cases the "unphysical" answer is ignored, such as a negative time interval
- The acceleration of some objects may be given in multiples of Earth's acceleration, called g's
  - Example - A space shuttle travelling at 6.00g's would have an acceleration of

$$6.00g's * \frac{9.80\frac{m}{s^2}}{g} = 58.8\frac{m}{s^2}$$

2.8 Graphical Analysis of Linear Motion

Velocity as Slope

- In a position vs time graph, the slope is

$$slope = \frac{\Delta x}{\Delta t}$$

- Note that it is equivalent to the definition for velocity, therefore the slope of a position vs time graph is the velocity of the object
- When the slope is not linear, the average velocity between a given time period can be found the exact same way,

$$v = \frac{\Delta x}{\Delta t}$$

  Where the positions are the y values of the time inputs
- The line connecting the points graphed by (t_1, x_1) and (t_2, x_2) is called a chord and has the same slope as the average velocity
- Moving the point of (t_2, x_2) infinitely close to the point on (t_1, x_1) will turn the chord into a tangent to the curve
- The slope at the tangent would be the instantaneous velocity at that point, look into differentiation in calculus to learn more about how to do this

Slope and Acceleration

- Similar to velocity, acceleration is the slope of a velocity vs time graph
- Average acceleration between 2 points is given by

$$a = \frac{\Delta v}{\Delta t}$$

- Instantaneous acceleration is given by the tangent to the graph at any point

3 Kinematics in Two Dimensions; Vectors

Motion of objects is usually considered in multiple dimensions, one such example is projectile motion where objects are projected outwards near Earth's surface

3.1 Vectors and Scalars

- **Vector** - A quantity with direction and magnitude
- **Scalar** - A quantity with only magnitude
- Vectors are represented by arrows in diagrams modelling problems
  - Example - A car's velocity as it changes may be represented by an arrow whose length represents the magnitude of velocity
- Vector quantities are written in boldface with a small arrow, scalars are written in italics
  - Vector for velocity: $\vec{\mathbf{v}}$
  - Scalar for speed: v

3.2 Addition of Vectors - Graphical Methods

- Vector additions is more tricky since direction must be added as well
- **Tail to Tip Method** - Drawing the tail of one vector on the tip of the other, the resultant vector is
- To add vectors in direction perpendicular to each other, the Pythagorean theorem is used by treating the two vectors as sides a and b and their sum as side c
- The direction of the sum is determined uring trigonometry
  - Example - A car moves 30 km east and 40km north, what is the resultant vector of its displacement?

$$(30km)^2 + (40km)^2 = 2500km^2$$

$$\sqrt{2500km^2} = 50km\ displaced$$

$$\arctan\frac{40km}{30km} = 53°$$

Therefore, the car moved 50km at 53° north of east

3.3 Subtraction of Vectors, and Multiplication of a Vector by a Scalar

- The negative of a vector $\vec{\mathbf{v}}$ has the same magnitude but opposite direction
- Subtracting a vector from another has the same effect as adding its negative
- Multiplying a vector by a scalar increases its magnitude by the factor of the scalar

3.4 Adding Vectors by Components

Adding vectors by components is much more accurate and applicable in multiple dimensions **Components**

- A vector $\vec{\mathbf{v}}$ on a plane is the sum of two smaller **component** vectors, one on each axis
- To determine the magnitude of each component vector is known as resolving it into its components
- Trigonometry can be used to resolve vectors, pretend the vector is the hypotenuse of a right triangle
- **Sine** - The sine of an angle of a right triangle is

$$\frac{opposite\ side}{hypotenuse}$$

- **Cosine** - The cosine of an angle of a right triangle is

$$\frac{adjacent\ side}{hypotenuse}$$

- **Tangent** - The tangent of an angle of a right triangle is

$$\frac{opposite\ side}{adjacent\ side}$$

- If an angle and a component vector are known, trig can solve for the other component vector
- If the component vectors are known, inverse trig can be used to solve for the angle

Adding Vectors

- To add vectors using components, resolve each one into its components, add the x and y components individually, and combine the resultant components
- Equations used

$$\vec{v_{RX}} = v_{1x} + v_{2x}$$
$$\vec{v_{RY}} = v_{1y} + v_{2y}$$
$$\vec{v_R} = \sqrt{v_{RX}^2 + v_{RY}^2}$$

3.5 Projectile Motion

- Objects moving in the air near Earth's surface are projectiles, their motion is described by **projectile motion**
- In many cases we do not consider air resistance as its effect is minimal
- An object in projectile motion maintains constant velocity in the x direction but accelerates negatively in the y direction
- Displacement in the x direction is given by

$$\vec{d_x} = v_{xi}t$$

- Displacement in the y direction is given by

$$\vec{d_y} = -\frac{1}{2}gt^2$$

- After a given amount of time, the displacements are calculated and added vertically to determine overall displacement

3.6 Solving Projectile Motion Problems

- Equations for projectile motion:
  - Horizontal motion

$$\vec{v_x} = v_{x0}$$
$$x = x_0 + v_{x0}t$$

  - Vertical motion

$$v_y = v_{y0} - gt$$
$$y = y_0 + v_{y0} - \frac{1}{2}gt^2$$
$$v_y^2 = v_{y0}^2 - 2g(y - y_0)$$

- Equations for magnitude of initial velocity based off of angle of launch
  - Horizontal velocity

$$v_{x0} = v_0 cos\theta$$

  - Vertical velocity

$$v_{y0} = v_0 sin\theta$$

- Equation for determining range of a projectile (only if $y_f = y_0$)

$$R = \frac{v_0^2 sin2\theta_0}{g}$$

Where θ_0 is the angle of launch

3.7 Projectile Motion is Parabolic

- Simplifying projectile motion by ignoring air resistance, it is parabolic, or, a projectile moves in a parabola
- The basic form of a parabola is

$$y = Ax + Bx^2$$

Where A and B are constants, which is very similar to the equation for vertical displacement in projectile motion

3.8 Relative Velocity

- Relative velocity is the sum of the vector velocities acting on an object from a frame of reference
  - Example - If Car A is travelling $75\frac{km}{h}$ and Car B is travelling $100\frac{km}{h}$, the relative velocity of Car B to Car A is
$$100\frac{km}{h} - 75\frac{km}{h} = 25\frac{km}{h}$$
- If the velocities are in two different directions, then they can be added/subtracted like any vector
- The velocity of object A relative to object B is the opposite of the velocity of object b relative to object A, represented by
$$\vec{v}_{BA} = -\vec{v}_{AB}$$

4 Dynamics: Newton's Laws of Motion

Dynamics Deals with *Why* objects move, what makes them start to move? What causes them to accelerate or decelerate? We will introduce force and the connection between force and motion

4.1 Force

- **Force -** A push or pull that causes an object to move
- An object at rest requires force to start to move, to change velocity, or to change direction
- Forces have magnitude and direction, they are vectors
- The same vector math for velocity applies to forces

4.2 Newton's First Law of Motion

- **Newton's First Law of Motion -** Every object continues in its state of rest, or of uniform velocity in a straight line, as long as no net force acts on it.
- Basically, an object with no force acting on its will remain at rest or in motion
- **Inertia -** An object's tendency to maintain its state of rest or uniform velocity
- Thus, Newton's First Law is often called the **Law of Inertia**

Inertial Reference Frames

- Newton's first law depends on the frame of reference
  - Example - To an outside observer the driver of an accelerating car is accelerating, but from inside the car, you are at rest
- **Inertial Reference Frames -** A frame of reference from which Newton's first law holds
- For most purposes a fixed point on Earth serves as an inertial frame of reference
- **Noninertial Reference Frames -** A frame of reference that does not adhere to Newton's First Law, such as a frame of reference that is acceleration

4.3 Mass

- **Mass -** Measure of inertia of an object, the more mass something has, the more it will resist a force
- Also known as the quantity of matter an object has
- The SI unit for mass is the kilogram (kg)
- Weight is NOT the same as mass, weight measures the force of gravity an object experiences, weight is dependent on mas

4.4 Newton's Second Law of Motion

- **Newton's Second Law of Motion -** The acceleration of an object is directly proportional to the net force acting on it, and is inversely proportional to the object's mass. The direction of the acceleration is in the direction of the force acting on the object
- In equation form:

  $$\vec{a} = \frac{\Sigma\vec{F}}{m}$$

  Where $\vec{a}$ is acceleration, $\Sigma\vec{F}$ is the sum of the forces acting on an object, and m is the mass of the object.
- Manipulating the equation to solve for the total force acting on an object gives

  $$\Sigma\vec{F} = m\vec{a}$$

- The sum of force on an object is also known as the *net force*
- Net forces cause acceleration
- **Force -** An action capable of accelerating an object
- Force can be written in component form as

  $$\Sigma\vec{F_x} = m\vec{a_x}$$

  $$\Sigma\vec{F_y} = m\vec{a_y}$$

4.5 Newton's Third Law of Motion

- **Newton's Third Law of Motion** - Whenever one object exerts a force on a second object, the second object exerts an equal force in the opposite direction of the first
- A force always causes an equal and opposite force from the object being acted on
  - Example - A hammer on a nail causes the nail to exert an equal force in the opposite direction of the hammer
- The equation for the force of object B on object A if A acts on B is

$$\vec{F_{AB}} = -\vec{F_{BA}}$$

4.6 Weight - the Force of Gravity; and the Normal Force

- Earth exerts on all objects on it, gravity, which is the same for all objects and acts downwards
- The force of gravity of Earth on an object is represented by

$$\vec{F_g} = m\vec{g}$$

  Where $\vec{g}$ is the acceleration due to gravity of object on Earth, typically accepted to be $9.80\frac{m}{s^2}$
- WHen two objects are in contact, they exert a *contact force* on each other
- **Normal Force** - A contact force that is perpendicular to the common surface of two objects
- When an object is on a surface, the object exerts a downward gravitational force on the surface and the surface exerts an upward *normal* force on the object
- The normal force exerts by a surface on an object has the same magnitude as the gravitational force from the object, given by

$$F_N = mg$$

  where m is the mass of the object and g is the acceleration due to gravity
- Normal forces can also be cause when an object pushes on a surface horizontally instead of vertically

4.7 Solving the Problems with Newton's Laws; Free-Body Diagram

- **Free=body Diagram** - Also called a force diagram, this is a simple way to represent the forces acting on an object in order to determine the net force on it
- **Net Force** - To review, this is a the vector sum of all the forces on an object
- The object in question in a fre body diagram is treated as a *point particle*. That is, its size and dimensions are not taken into account

Tension in a Flexible Cord

- **Tension** - The force a flexible cord experiences when it is pulled on
- We treat flexible cords as having "negligible mass" or mass so little that it does not have ot be taken into account

4.8 Problems Involving Friction, Inclines

Friction

- The surfaces of objects are not perfectly smooth, they have small imperfections and bumps that cause them to catch on each other when they slide together, the resistance of movement caused by surfaces is called friction and is a force
- **Kinetic Friction** - Friction between two surfaces in motion, represented by

$$F_k = \mu_k F_N$$

  Where F_k is the force of kinetic friction, μ_k is the coefficient of kinetic friction between the two surfaces,and F_N is the normal force between the two surfaces
- **Static Friction** - Friction between two surfaces that are not in motion, represented by

$$F_s \leq \mu_s F_N$$

  Where F_s is the force of static friction and μ_s
- Note that the coefficients of kinetic and statics friction for the same surfaces can and most likely will be different, usually μ_s will be greater than μ_k

- The force of static friction scales with your push until the maximum force is reached, represented by

$$F_s = \mu_s F_N$$

, once this limit is surpassed, kinetic friction will take over

Inclines

- Objects on an incline experience the force of gravity as a pushing force resisted by friction
- The normal force experienced by an object on an incline is given by

$$F_N = mg\cos\theta$$

WHere θ is the angle of the incline. Since the force of gravity is at an angle relative to the normal force from the incline

- The pushing force experienced by an object on an incline is given by

$$F_p = mg\sin\theta$$

Which is the other component of gravitational force

5 Circular Motion; Gravitation

An object that experiences a force at an angle to its direction of motion, it is moving in a curved path, called circular motion. In this section, all object are assumed to be in circular motion unless otherwise stated

5.1 Kinematics of Uniform Circular Motion

- An object in circular motion with a constant velocity is said to be in **uniform circular motion**
- **Centripetal Acceleration** - Also called *radial acceleration* this is acceleration experienced by an object in circular motion, even if velocity is constant, the object is constantly changing direction and therefore accelerating. This is calculated by

  $$a_R = \frac{v^2}{r}$$

  Where a_R is centripetal acceleration, v^2 is the square of the linear velocity, and r is the radius of the circular path
- **Frequency** - The number of revolutions in a circle an object makes per second
- **Period** - The amount of time it takes an object to make a single revolution, calculated with

  $$T = \frac{1}{f}$$

  Where T is period and f is frequency
- The velocity of an object in motion can be calculated with

  $$v = \frac{2\pi r}{T}$$

5.2 Dynamics of Uniform Circular Motion

- The centripetal force experienced by an object is calculated with

  $$\Sigma F_R = m\frac{v^2}{r}$$

- A net force must be directed towards the center of the circle if an object is to continue circular motion
- Centrifugal (or outward rotational force) is *not* a true force, the outward feeling is caused by the momentum of the object making it want to continue in a straight line instead of a curved path

5.3 Highway Curves: Banked and Unbanked

- A common application of circular dynamics is simulating a car on a curved path, called a curve. A banked curve is a curve at an angle relative to the ground
- A car in motion on the road without slipping tires has its tires experiencing static friction, if the centripetal force of friction is too great then they will slip and experience kinetic friction
- Banking curves reduces reliance on friction, the case in which the centripetal acceleration is equal to the pushing force caused by gravity is when the following is true

  $$F_N \sin\theta = m\frac{v^2}{r}$$

  Which is called the **design speed** of the banked curve

5.4 Nonuniform Circular Motion

- Sometimes, objects in circular motion also accelerate their *tangential* velocities
- The acceleration of an object in motion has two components, its *tangential* acceleration and its *centripetal* acceleration, they are perpendicular so the total acceleration of an object in circular motion can be found by

  $$a = \sqrt{a_{tan}^2 + a_{cen}^2}$$

5.5 Newton's Law of Universal Gravitation

- Newton discovered that the Force of gravity on an object is proportional to its distance from Earth, or anything else attracting it, represented by
$$F_G = \propto \frac{1}{r^2}$$
Where r is the distance from the center of the object to the center of the Earth
- Force of gravity is also represented by
$$F_G \propto \frac{m_g m_{obj}}{r^2}$$
Where m_g is the mass of the Earth and m_{obj} is the mass of the object
- **Law of Universal Gravitation -** Every particle in the universe attracts every other particle with a force that is proportional to the product of their masses and inversely proportional to the square of the distance between them. This force acts along the line joining the 2 particles
- Gravitational force between 2 objects is calculated by
$$F_G = G\frac{m_1 m_2}{r^2}$$
Where m_1 and m_2 are the masses of the objects, and G is the universal gravitational constant
- The magnitude of gravitational force is calle an *inverse square law* because the force of gravity is inversely proportional to r^2
- The universal gravitational constant is defined as
$$G = \frac{6.67 * 10^{-11} N * m^2}{kg^2}$$

5.6 Gravity Near the Earth's Surface

- Comparing the two ways to calculate the force of gravity of Earth gives
$$mg = G\frac{m m_E}{r_E^2}$$
Thus
$$g = G\frac{m_E}{r_E^2}$$
Therefore an object's distance from Earth's center affects the force of gravity on it
- Note that Earth is not a perfect sphere and so the previous equation does not give perfectly precise answers, but the impact is negligible

5.7 Satellites and "Weightlessness"

Satellite Motion

- For a satellite to successfully orbit Earth, its centripetal force must be equal to the force of gravity from Earth on it, represented by
$$G\frac{m m_E}{r^2} = m\frac{v^2}{r}$$
Solving for velocity gives
$$v = \sqrt{\frac{G m_E}{r}}$$
Which gives the velocity a satellite must travel to remain in orbit at a given distance

Weightlessness

- **Apparent Weightlessness -** When an object relative to a reference point appears to not have weight but gravity still exists
- Weightlessness can be experienced in a freely falling elevator or satellite, since they are technically falling towards Earth, just fast enough so that they miss Earth

5.8 Planets, Kepler's Laws, and Newton's Synthesis

- **Geocentric View** - The belief that Earth is at the center of the Universe
- **Heliocentric View** - The belief that the Sun is at the center of the solar system

Kepler's Laws

- **Kepler's First Law** - The path of each planet around the sun is an ellipse with the sun at one focus
- **Kepler's Second Law** - Each planet moves so that an imaginary line drawn from the Sun to the planet sweeps out equal areas in equal periods of time
- **Kepler's Third Law** - The ratio of the squares of the periods of any two planets revolving around the Sun is equal to the ratio of the cubes of their mean distances from the sun, represented by

$$(\frac{T_1}{T_2})^2 = (\frac{s_1}{s_2})^3$$

which can rewritten as

$$\frac{s_1^3}{T_1^2} = \frac{s_2^3}{T_2^2}$$

Kepler's Third Law Derived, Sun's Mass, Perturbations

- Kepler's third law can be rearranged to give

$$(\frac{T_1}{T_2})^2 = (\frac{r_1}{r_2})^3$$

- **Perturbation** - Deviations in the orbit of objects caused by other objects in orbit, such as the effect of planets on each other

Other Centers for Kepler's Laws

- The equation derived above can be applied to other situations such as satellites orbiting Earth

Distant Planetary Systems

- Planets around distant stars were predicted to exist because of the "wobble" effect they induced on their stars due to gravity

Newton's Synthesis

- **Causal Law** - Sometimes Newton's laws are labelled this because they *cause* each other
- **Deterministic View** - A way of looking at the universe as a machine whose parts' movement can be determined

Sun/Earth Reference Frames

- The heliocentric and geocentric views described earlier are different reference frames, but there does not exist a single preferred reference frame as zooming out always introduces new interactions and movements

5.9 Moon Rises an Hour Later Each Day

- The moon's orbit takes 24hr50min meaning it does not align exactly with Earth's rotation
- Because of this, the moon needs more time to completely align with Earth and the Sun and thus new moons do not occur every day

5.10 Types of Forces in Nature

- The four fundamental forces are the
  - Gravitational Force
  - Electromagnetic Force
  - Strong Nuclear Force
  - Weak Nuclear Force
- Attempts to unify these forces into a single theory have been attempted and given mixed results
- Other forces like friction are due to the electric repulsion between objects' electrons when they touch

6 Work and Energy

New concepts explored here are energy and momentum, as well as their laws of conservation. Work is also explored.

6.1 Work Done by a Constant Force

- Work is done on an object when a force is applied wo move it a certain distance, calculated by

  $$W = Fd\cos\theta$$

  Where W is work, F is force, d is distance, and $\cos\theta$ is the angle between the force and direction of displacement, this is included because work only take into account the component of force parallel to displacement

- Work is measured in Joules (J)

  $$1J = 1N * m$$

- A force can be exerted on an object but if it does not move then no work was done on it

- Important to specify if work is being done *by* or *on* an object

6.2 Work Done by a Varying Force

- Work done by a varying force can be calculated graphically, plot force as a function of distance, work is the area under the graph

- Dividing the graph into smaller and smaller segments makes the calculation more precise

6.3 Kinetic Energy, and the Work-Energy Principle

- Defined traditionally as "the ability to do work"

- Note that total energy before and after a process remains the same, it is only converted from one form to another

- **Kinetic Energy** - The energy of motion, calculated by

  $$KE = \frac{1}{2}mv^2$$

- **Work-energy principle** - The net work on an object is equal to the change in the object's kinetic energy, represented as

  $$W_{net} = \Delta KE = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

- Net work means the work done by every force on the object

6.4 Potential Energy

- **Potential Energy** - Energy waiting to be used

Gravitational Potential Energy

- An object raised above the ground has PE_G and has the ability to work, as it falls it converts the PE_G into KE

- Formula for GPE

  $$PE_G = mpy$$

  Where PE_G is gravitational potential energy, m is the mass of the object, g is the acceleration due to gravity, and y is its height above the ground

- PE_G is the negative of the work done by gravity on an object

Potential Energy Defined in General

- Each type of potential energy is associoated with a force and is the negative of the work done by that force

Potential Energy of Elastic Spring

- Extending a spring so that it is stretched or compressed requires a force, calculated by

  $$F_{ext} = kx$$

  Where k is the spring constant (unique to each material) and x is the distance the spring is extended

- The force a spring extends on the item extending it is given by

$$F_s = -kx$$

Which is the negative of the force required to extend the spring

- The elastic potential energy of a spring is given by

$$PE_{el} = \frac{1}{2}kx^2$$

- Note that the distance a spring is compressed of stretched is relative to its position when no force is applied, called its natural position

Potential Energy as Stored Energy

- Potential energy is an object's potential to do work, thus it is stored energy

6.5 Conservative and Nonconservative Forces

- Potential energy depends on the positions of objects thus it must be stated uniquely for each point
- Potential energy can only be defined for a conservative force, thus not every force has a potential energy

Work-Energy Extended

- The work done by a nonconservative force is equal to the sum of the change in kinetic and potential energies, represented by

$$W_{NC} = \Delta KE + \Delta PE$$

- Note that *all* forces must be included, either in the potential or kinetic terms, but not both

6.6 Mechanical Energy and its Conservation

- For a conservative force, the following relation remains true

$$KE_2 + PE_2 = KE_1 + PE_1$$

or

$$E_1 = E_2$$

Where E is the total mechanical energy of a system

- **Principe or Conservation of mechanical energy -** If only conservative forces do work, the total mechanical energy of a system neither increases nor decreases in any process. It stays constant - it is conserved

6.7 Problem Solving Using Conservation of Mechanical Energy

- To find the total mechanical energy of a system involving PE_G, the following equation is used

$$E = \frac{1}{2}mv^2 + mgy$$

Where v is the velocity of a falling object at any point

- Just before a dropped object hits the ground, its PE will be 0 as all of it has converted to KE
- Problems involving spring energy are similar, the total energy in a spring system remains the same if other forces such as friciton are ignored, therefore

$$E = \frac{1}{2}mv_1^2 + \frac{1}{2}kx_1^2 = \frac{1}{2}mv_2^2 + \frac{1}{2}kx_2^2$$

6.8 Other Forms of Energy and Energy Transformations, the Law of Conservation of Energy

- Energy can be transformed, such as from one form to another or from one object to another
- Work is done when energy is transferred from one object to another
- **Law of Conservation of Energy -** The total energy is neither increased nor decreased in any process. Energy can be transferred from one form to another, and transferred from one object to another, but the total amount remains constant

6.9 Energy Conservation with Dissipative Forces: Solving Problems

- Forces that reduce the mechanical energy (but not total energy) of a system are called dissipative forces, which include friction
- Taking friction into account gives

  $$KE_2 + PE_2 + F_{fr}d = KE_1 + PE_1$$

  Which includes the work done by the force of friction over a distance moved

Work-Energy versus Energy Conservation -

- If you study a system where external forces are at play then use the work energy principle to determine change in kinetic energy
- If the system has no external forces at play then use the law of conservation of energy
- If forces are not constant, then energy may be easier to use than Newton's laws
- Use the information you are given to decide on the simplest approach to a problem

6.10 Power

- **Power -** Rate at which work is done, average power is work divided by time, represented by

  $$P = \frac{Work}{Time} = \frac{EnergyTransformed}{Time}$$

- The SI unit of power is watts, defines as

  $$1W = \frac{1J}{s}$$

- For example, a person walking one mile would feel less tired than someone running the same distance because it is easier for the body to transform chemical potential energy to mechanical energy over a longer length of time
- Vehicles are limited by the rate at which they can do work
- Power can also be calculated by

  $$P = Fv$$

  Where F is force and v is velocity
- **Efficiency -** The ratio of useful power output to the power input, represented by

  $$e = \frac{P_{out}}{P_{in}}$$

7 Linear Momentum

There exist other laws of conservation in physics, such as linear momentum discussed in this chapter. Momentum is useful in analyzing a system where multiple objects interact with each other

7.1 Momentum and its Relation to Force

- **Linear Momentum** - The product of an object's mass and velocity, represented by

  $$\vec{p} = m\vec{v}$$

  Where p is momentum

- Forces are required to change an object's momentum, the net force on an object is its change in momentum over time, represented by

  $$\Sigma F = \frac{\Delta p}{\Delta t} = m\frac{\Delta v}{\Delta t}$$

- Note that the quotient of momentum divided by time takes into account possible changes in mass of objects studied

7.2 Conservation of Momentum

- If no external forces act on a system, total momentum remains constant, momentum before = momentum after, represented by

  $$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B$$

  Which is the sum of the momentums of objects A and B before and after the event denoted by the apostrphe

- If two objects collide then the force exerted on the first is the negative of the force exerted on the second, by Newton's third law, thus

  $$F = \frac{\Delta p_B}{\Delta t}$$

  and

  $$-F = \frac{\Delta p_A}{\Delta t}$$

- **Law of Conservation of Momentum** - The total momentum of an isolated system of objects remains constant

- **System** - Set of objects chosen to study

- **Isolated System** - A system in which the only forces are those between the objects in the system

7.3 Collisions and Impulse

- Two objects that are moving relative to each other and touch are said to collide

- When two objects collide, their velocities change and thus their momentums change

- **Impulse** - The change in momentum of an object after a collision, shown by

  $$Impulse = F\Delta t$$

- Used mainly when dealing with small amounts of time, force is likely to vary during the short time and so the average force is used

7.4 Conservation of Energy and Momentum in Collisions

- **Elastic Collision** - A collision in which the total kinetic energy is conserved before and after two objects collide, represented by

  $$KE_A + KE_B = KE'_A + KE'_B$$

- Most collisions that can be seen are not elastic as kinetic energy is lost externally due to sound or friction

- **Inelastic Collision** - A collision during which kinetic energy is not conserved, represented by

  $$KE_A + KE_B = KE'_A + KE'_B + otherformsofenergy$$

7.5 Elastic Collisions in One Dimension

- Rearranging conservation of momentum gives

  $$v_A + v'_A = v_B + v'_B$$

  Which can be rewritten as

  $$v_A - v_B = -(v'_A - v'_B)$$

7.6 Inelastic Collisions

- **Completely Inelastic** - A collision during which two objects stick together
- KE is not conserved but total energy remains constant

7.7 Collisions in Two Dimensions

- An elastic collision with a moving and stationary object can be modeled by

$$m_A v_A = m_A v'_A \cos\theta'_A + m_B v'_B \cos\theta'_B$$

Where θ is the angle between the path of each objects and the original path of object A

- The other component of object A's momentum is 1, thus

$$0 = m_A v'_A \sin\theta'_A + m_B v'_B \sin\theta'_B$$

- Applying the conservation of kinetic energy gives

$$\frac{1}{2}m_A v_A^2 = \frac{1}{2}m_A v_A'^2 + \frac{1}{2}m_B v_B'^2$$

7.8 Center of Mass (CM)

- Until now, objects have been assumed to be point particles with no shape, only mass, but in real life objects may have many parts that move relative to each other
- The point in an object that moves in the same path a point particle would is its *center of mass* (CM)
- The general motion of an extended object is the sum of the translational motion of its CM plus other types o motion around the CM such as vibrational
- The center of mass of a two-object system is calculated with

$$x_{CM} = \frac{m_A x_A + m_B x_B}{M}$$

Where x is the position of each object on a coordinate system chosen such that each object is on the x-axis, m is the mass of each object, and M is the total mass of the two objects

- The center of mass of a multidimensional system requires coordinates in more than one dimension, the y coordinate of the CM of a system can be calculated with

$$y_{CM} = \frac{m_A y_A + m_B y_B}{M}$$

- **Center of Gravity** - The point at which gravity can be considered to act on a system/object
- Note that gravity actually acts on all points of a system and it is usually easier to determine CM and CG through experimentation rather than calculation

7.9 CM for the Human Body

- As an example of calculating CM, each of a person's major body parts (upper and lower limbs, neck back, head) are assigned a CM and mass
- These assigned values are used to determine the person's CM

7.10 CM and Translational Motion

- The formula for center of gravity can be rewritten as

$$Mx_{CM} = m_A x_A + m_B x_B$$

- The velocity of a system's CM when the objects have different velocities is

$$Mv_{CM} = m_A v_A + m_B v_B$$

- The acceleration of a system's CM when the objects have different accelerations is

$$Ma_{CM} = m_A a_A + m_B a_B$$

- The net force on all the objects on a system is equal to the product of the total mass of the system and the acceleration of its CM, represented by

$$Ma_{CM} = F_{net}$$

This is called *Newton's second law for a system of particles*

- The CM of a system of particles moves as if all of its mass is centered at one point and forces are applied only to that point
- One application of this is in analyzing nearby stars that seem o wobble in their movement, such wobbles can be caused by other stars or by planets orbitting it

8 Rotational Motion

For our purposes, we will consider the rotational motion of rigid objects around a fixed (non-moving) axis. Rigid objects are objects with a definite shape that does not change.

8.1 Angular Quantities

- **Axis of rotation -** The point at which all other points of an object rotate around
- **Angular position -** How far a point or object has travelled around its axis of rotation, measured in radians
- **Radian -** Unit of angular position, defined as the length of the arc an object travels in rotation divided by the radius, or the distance an object is from the axis of rotation, represented as

  $$\theta = \frac{l}{r}$$

  or

  $$l = r\theta$$

  Where θ is radians, l is length, and r is radius
- Note that for any radius, a full rotation around an axis of rotation is 2πradians
- **Average Angular Velocity -** Like average velocity, but it measures the change in angular position over time, represented by

  $$\omega = \frac{\Delta\theta}{\Delta t}$$
- **Instantaneous Angular Velocity -** The rotational analogue to instantaneous linear velocity, this is the velocity of rotation at a specific point in time, represented by

  $$\omega = \lim_{\Delta \to 0} \frac{\Delta\theta}{\Delta t}$$
- The tangential (linear) velocity of an object in rotational motion is calculated with

  $$v = r\omega$$
- The tangential (linear) acceleration of an object in rotational motion is calculated with

  $$a_{tan} = r\frac{\Delta\omega}{\Delta t} = r\alpha$$
- Recalling centripetal acceleration, it can also be solved as

  $$a_R = \omega^2 r$$
- Frequency can be calculated by

  $$f = \frac{\omega}{2\pi}$$
- Period can be calculated by

  $$T = \frac{2\pi}{\omega}$$

8.2 Constant Angular Acceleration

- The equations for linear acceleration are analogous to their rotational (angular) counterparts

Linear	Angular
$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
$x = v_0 t + \frac{1}{2}at^2$	$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 = v_0^2 + 2ax$	$\omega^2 = \omega_0^2 + 2\alpha\theta$
$v = \frac{v+v_0}{2}$	$\omega = \frac{\omega+\omega_0}{2}$

8.3 Rolling Motion (Without Slipping)

- Rolling without slipping depends on an object's static friction without the ground since at each moment in time, the point of contact with the ground is at rest
- The velocity of a rolling object on the ground is represented by

  $$v = r\omega$$

  , but this is ony valid if the object rolls without slipping

8.4 Torque

- The force to rotate an object about its rotational axis is dependent on the direction and location of the force
- **Lever arm -** The distance from the axis of rotation a force acts upon, also called moment arm
- Angular acceleration is proportional to the product of force and lever arm, the product is called moment of force or *torque*, thus
$$\alpha \propto \tau$$
- The equation for torque is
$$\tau = rF\sin\theta$$
Where r is the distance from the axis of rotation and θ is the angle between the distance of the lever arm and the direction of force, that is, if the force is applied perpendicular to the lever arm then
$$\tau = rF$$

Forces that act to Tilt the Axis

- Sometimes, forces are applied parallel to the axis of rotation and will cause it to tilt, though we consider the axis to remain fixed so these forces are ignored

8.5 Rotational Dynamics; Torque and Rotational Inertia

- The angular acceleration of an object is proportional to the net torque applied to it, thus
$$\alpha \propto \Sigma\tau$$
- Since tangential acceleration is the product of radius and angular acceleration, the following is true
$$F = mr\alpha$$
- Plugging this into the equation of torque gives
$$\tau = mr^2\alpha$$
- **Moment of Inertia -** An object's rotational inertia,equal to
$$mr^2$$
- The moment of inertia of every point on an object is summed to get the object's overall moment of inertia, represented by
$$I = \Sigma mr^2$$
WHere I is moment of inertia
- Plugging moment of inertia into toqrue gives
$$\Sigma\tau = I\alpha$$

8.6 Solving Problems in Rotational Dynamics

- The units for torque are
$$m * N$$
and the units for moment of inertia are
$$kg * m^2$$
- like other types of problems, to determine how to solve a problem, analyze the information you do know and the information you want to know to determine which equations to use, and never forget to check with dimensional analysis

8.7 Rotational Kinetic Energy

- **Rotational Kinetic Energy -** The kinetic energy of an object in rotationa motion, calculated by
$$rotationalKE = \frac{1}{2}I\omega^2$$
with units of Joules
- Some objects have both translational and angular kinetic energy, so their total kinetic energy is
$$KE = \frac{1}{2}Mv_{CM}^2 + \frac{1}{2}I_{CM}\omega^2$$

Work done by Torque

- Work done by torque is calculated with
$$W = \tau\Delta\theta$$
- The power of a torque is calculated with
$$P = \tau\omega$$

8.8 Angular Momentum and Its Conservation

- **Angular Momentum -** The angular analogue of linear momentum, calculated by

$$L = I\omega$$

  Where L is linear momentum with units $\frac{kg*m^2}{s}$

- Torque can be calculated with linear momentum, shown below

$$\Sigma\tau = \frac{\Delta L}{\Delta t}$$

  Which also implies

$$I\frac{\Delta\omega}{\Delta t} = I\alpha$$

- **Law of conservation of angular momentum -** The total angular momentum of rotating object remains constant if the net torque acting on it is zero

8.9 Vector Nature of Angular Quantities

- Since the only direction unique to rotational motion is the axis of rotation, it is the direction of the angular velocity vector
- The right hand rule is used to determine the direction of the vector, which is: When the fingers of the right hand are curled around the axis of rotation and point in the direction of rotation, then the thumb points in the direction of angular velocity
- The right hand rule is also used to determine the direction of angular acceleration and angular momentum
- Another interesting way of observing motion is with *rotating frames of reference*, but they are outside of the scope of this chapter

9 Static Equilibrium; Elasticity and Fracture

Static equilibrium is the analysis of objects with no net force or net torque and no linear or rotational motion. Though objects with no net force can still experience internal forces and when these forces get too great they may deform or fracture **Statics** - The study of forces acting on structures that are not in motion

9.1 The Conditions of Equilibrium

- **Equilibrium** - The state of begin at rest

The First Condition for Equilibrium

- The forces acting on an object must add up to zero, an object experiences no net force

The Second Condition for Equilibrium

- The sum of the torques acting on an object must be zero

9.2 Solving Statics Problems

- Statics allows us to calculate forces while others are already known
- In most problems a surface is considered where only 3 equations are need, one for each dimension and one for torque
- The strategy for statics problems consists of drawing a free body diagram, defining a coordinate system, write down the force and torque equations, and solving
- **Cantilever** - A beam that extends beyond its support, such a beam experiences both force and torque

9.3 Applications to muscles and joints

- Bones in the human body connect at *joints* and attach at *insertions*
- The human body contains many joints and rotating parts that can be used as examples for statics problems

9.4 Stability and balance

- When an object is displaced, there are 3 possible outcomes:
  - **Stable Equilibrium** - the object returns to its original position
  - **Unstable Equilibrium** - THe object moves even farther from its original position
  - **Neutral Equilibrium** - The object remains in its new position
- Stable equilibrium is also called *balance*
- An object that is difficult to make fall over is said to be more stable than another object
- The wider an object's base is the more stable it is

9.5 Elasticity; Stress and Strain

- Elasticity studies the effects of forces on objects

Elasticity and Hooke's Law

- **Hooke's Law** - Force exerted on an object will cause its length to change slightly, represented as

$$F = k\Delta l$$

  Where F is the force applies, Δl is the change in length, and k is a proportionality constant

- Hooke's law remains true for most materials up to the *proportional limit*, after which the relationship between F and Δl is not easily predicted
- **Elasticity Limit** - The point up to which an object will return to its original shape, elongating it beyond this point will deform the object permanently
- **Ultimate Strength** - Also called *breaking point*, this is the maximum force that can be applied to a material before it breaks

Young's Modulus

- The elongation of an object also depends on the material it is made of
- The elongation of an object can be calculated with

$$\Delta l = \frac{1}{E}\frac{F}{A}l_0$$

  Where l_0 is the original length, A is the cross-sectional area, F is the force applied, and E is the constant of proportionality, also called *elastic modulus* or *Young's modulus*

- The young's modulus of a material has units $\frac{N}{m^2}$, the greater the young's modulus is, the more it resists elongation

Stress and Strain

- **Stress** - The force per unit area in an elongating object, calculated with

$$stress = \frac{F}{A}$$

  Where F is force and A is area, with units N/m^2

- **Strain** - The ratio between an object's change in length and its original length, calculated with

$$strain = \frac{\Delta l}{l_0}$$

- Another important equation when dealing with stress and strain is

$$\frac{F}{A} = E\frac{\Delta l}{l_0}$$

- From this, the following can be derived

$$E = \frac{stress}{strain}$$

Tension, Compression, and Shear Stress

- An object being pulled on is under tension and experiencing *tensile stress*
- An object being compressed is under *compressive stress*
- An object with equal and opposite forces across its opposite faces is under *shear stress*
- Shear strain can be calculated with

$$\Delta l = \frac{1}{G}\frac{F}{A}\Delta l_0$$

  Where G is the shear modulus

Volume change - Bulk Modulus

- **Pressure** - Force per unit area on an object, equivalent to stress
- An object experiencing inward forces from all sides will decrease in volume
- The change in volume has its own constant of proportionality called the bulk modulus, B, which can be calculate with

$$B = -\frac{\Delta P}{\Delta V/V_0}$$

  Where ΔP is the change in pressure, ΔV is the change in volume, and V_0 is the original volume

- For liquids and gases, only the bulk modulus applies

9.6 Fracture

- **Fracture** - Occurs when a solid object is under too much stress and breaks
- **Safety Factor** - A factor that maintains the safety of materials by not allowing them to be subject to a certain fraction of their ultimate strengths
- Sometimes materials are combined to combine their ultimate strengths, such as reinforces concrete which combines the compressive strength of concrete with the tensile strength of steel

9.7 Spanning a Space: Arched and Domes

- **Arch** - An architectural design made to make its components experience compressive strength in order to avoid excessive deformation
- **Dome** - An arch spanning 3-dimensional space for the same purpose
- Each design is not stable until all the stones or other elements are in place

10 Fluids

The focus is now on materials with no definite shape that flow, called fluids, which include liquids and gases

10.1 Phases of Matter

- **Liquid** - Retains its volume but changes shape to fits its container
- **Gas** - Same as a liquid but changes volume to fill its container

10.2 Density and Specific Gravity

- **Density** - Defined as mass per units volume, calculated with

$$\rho = \frac{m}{V}$$

  Where ρ is density, m is mass, and V is volume, units are usually either kg/m^3 or g/cm^3

- Mass can be calculated from volume and density with

$$m = \rho V$$

- **Specific Gravity** - The ratio between the density of a substance and the density of water at 4.0°C

10.3 Pressure in Fluids

- **Pressure** - Force per unit area, where F is the magnitude of force perpendicular to the surface area

$$P = \frac{F}{A}$$

  Pressure has units pascals, Pa, defined as

$$1Pa = \frac{1N}{m^2}$$

- Fluids exert pressure in every direction
- At rest, fluid pressure acts perpendicular to the surface is is resting on
- The pressure due to the weight of a liquid is calculated with

$$P = \rho g h$$

  Where h is the depth of the liquid

- Pressure varies across the depth of a liquid, the height in the previous equation is the height of the liquid above the point in question
- Liquids may also change in height, the change in pressure is given by

$$\Delta P = \rho g \Delta h$$

10.4 Atmospheric pressure and Guage Pressure

Atmospheric Pressure

- The Earth's atmosphere exerts pressure, one atmosphere of pressure is defined as

$$101.3kPa$$

- Another unit of pressure is the bar, defined as

$$1bar = 1.000 * 10^5 N/m^2$$

Gauge Pressure

- **Gauge Pressure** - The pressure registered y measuring devices
- These devices do not account for atomspheric pressure
- **Absolute Pressure** - The sum of gauge pressure and atmospheric pressure, calculated by

$$P = P_G + P_0$$

  Where P_G is gauge pressure and P_0 is atmospheric pressure

10.5 Pascal's Principle

- **Pascal's Principle** - If an external pressure is applied dot a confined fluid, the pressure at every point within the fluid increases by that amount
- The force input to a fluid F_{in} increases pressure equally throughout a fluid, thus in a hydraulic press,

  $$P_{out} = P_{in}$$

  Or,

  $$\frac{F_{out}}{F_{in}} = \frac{A_{out}}{A_{in}}$$

- **Mechanical advantage** - Ratio of the areas of a hydraulic device, calculated by F_{in}/F_{out}

10.6 Measurement of Pressure: Gauges and Barometer

- **Manometer** - Simplest device used to measure pressure, where pressure is

  $$P = P_0 + \rho g \Delta h$$

- **Torr** - Unit of measuring pressure, called mm-Hg or "millimeters of Mercury", defined as roughly 133Pa since that is the pressure of Mercury at a depth of one millimeter
- **Aneroid Gauge** - Another type of pressure gauge, uses deformation to measure pressure and a pointer to indicate measurement
- **Barometer** - Modified mercury manometer, uses mercury's properties to make measurement easier
- A common misconception is that vacuums and other suction devices move fluids by actively moving them but this is not the case, what is actually happening is a drop in pressure that causes the atmosphere to push the fluid up into the suction device

10.7 Buoyancy and Archimedes' Principle

- **Buoyancy** - An upward force that causes objects to float on/in fluids, calculated by

  $$F_B = m_F g$$

  Which is also equal to the weight of the fluid with the volume an object displaces

- **Archimedes' Principle** - The buoyant force on an object immersed in a fluid is equal to the weight of the fluid displaced by the object
- **Apparent weight** - Weight of an object in a fluid, such as water, which can be used to determine an object's density
- Note that the specific gravity of an object multiplied y the density of water gives its density
- The following relationship is true,

  $$\frac{V_{displ}}{V_O} = \frac{\rho_0}{\rho_F}$$

  Where V_{displ} - is volume displaced by an object, V_O, is its full volume, ρ_O, is its density, and ρ_F is the density of the fluid it is in

10.8 Fluids in motion; FlowRate and the Equation of Continuity

- **Fluid Dynamics** - The study of fluids in motion
- **Hydrodynamics** - Fluid dynamics for water
- **Laminar Flow** - Layers of a flowing fluid slide by each other smoothly
- **Turbulent Flow** - Layers of a flowing fluid collide and form small whirlpools called *eddies*
- **Viscosity** - The internal friction of a fluid, the higher viscosity it has, the slower it flows
- **Mass Flow Rate** - The mass of a fluid that passes a point per unit time, calculated by

  $$Masflowrate = \frac{\Delta m}{\Delta t} = \rho_1 A_1 V_1$$

- **Equation of Continuity** - The flow rate at two points of the same ube with different areas is the same, given by

  $$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

- If a fluid is incompressible, that is its density does not change with pressure, its density remains constant and the equation of continuity becomes

  $$A_1 V_1 = A_2 V_2$$

10.9 Bernoulli's Equation

- **Bernoulli's Principle -** Where the velocity of a fluid is high, the pressure is low, and where the velocity is low, the pressure is high
- The work done by a force to displace a fluid is given by

  $$W = P_1 A_1 l_1$$

  The work done on the other end of the fluid is

  $$W = -P_2 A_2 l_2$$

  which is negative because the force exerted is opposite to displacement, the work done by the force of gravity on a fluid is

  $$W = -mg(y_2 - y_1)$$

- **Bernoulli's equation -** Combining the three previous equations gives

  $$P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2 = P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1$$

- Points 1 and 2 can be along any tube flow, therefore

  $$P + \frac{1}{2}\rho v^2 + \rho g y = constant$$

10.10 Applications if Bernoulli's Principle Torricelli, Airplanes, Baseballs, Blood Flow

- **Torricelli's Theorem -** If pressure is equal at both points, Bernoulli's equation can be rearranged to solve for velocity,

  $$v_1 = \sqrt{2g(y_2 - y_1)}$$

- If a fluid is flowing horizontally, Bernoulli's equation simplifies to

  $$P_2 + \frac{1}{2}\rho v_2^2 = P_1 + \frac{1}{2}\rho v_1^2$$

Airplane Wings and Dynamic Lift

- **Dynamic Lift -** An upward force caused on wings with an upper rounded surface, which causes air to travel faster over the top of the wing and thus have lower pressure than air travelling under the wing

Sailboats

- Sailboats apply a similar concept to their sails, they cause a lower pressure on their fronts which causes the sail to be pushed by higher pressure air behind it

Baseball Curve

- Spinning baseballs curve because they cause one side of the air surrounding them to have a higher pressure than the other and thus be pushed by it

Lack of blood to the Brain

- TIA is a temporary lack of blood to the brain
- This is caused when a sudden pressure change in the body combined with a blockage in the arteries, this causes low pressure on one side of the body and the other side tries to divert blood to it, but accidentally ignores the brain in the process

Other Applications

- **Venturi Tube -** A pipe with a narrow constriciton in the middle
- **Venturi meter -** Used to measure flow speed across a venturi tube using the differences in area and pressure across the wider and narrower sections of the venturi tube

10.11 Viscosity

- **Viscosity -** Internal friction between a fluid
- Each fluid has its own viscosity, represented by its coefficient of viscosity, η, with units Pa*s
- The force required to move a plate over a fluid is given by

  $$F = \eta A \frac{v}{l}$$

  Where A is the area of the plate in contact with the fluid, v is the velocity of the plate, and l is the distance between the moving plate and stationary surface

10.12 Flow in Tubes: Poiseuille's Equation, Blood Flow

- **Poiseuille's Equation -** Used to determine volume rate of flow of an incompressible fluid in a tube in laminar flow, mainly blood, which is

  $$Q = \frac{\pi R^4 (P_1 - P_2)}{8\eta l}$$

  Where R is the radius of the tube, l is the length of the tube, and Q is the volume rate of flow

10.13 Surface Tension and Capillarity

- **Surface tension -** Fores between the molecules on the surface of a liquid that keep it together, calculated by

  $$\gamma = \frac{F}{l}$$

  Defined as Force per unit length, l, that acts perpendicular to any line in a liquid surface

- Soaps lower the surface tension of water which allows it to penetrate small crevices in objects being cleaned

Capillarity

- **Cohesion -** Bonding forces between molecules of the same type
- **Adhesion -** Bonding forces between molecules of different types
- **Capillarity -** The phenomenon where liquids in thin tubes rise and fall relative to the level of the surrounding liquid

10.14 Pumps and the Heart

- **Vacuum Pump -** Moves fluids by reducing pressure
- **Force Pump -** Moves fluids by increasing pressure
- **Circulating Pump -** Moves fluids by moving them in a closed path
- The heart moves blood by acting as a circulating pump, moving blood throughout two main paths in the body

11 Oscillators and Waves

Physical object such as springs and elastic solids as well as electrical components can oscillate. These oscillations usually follow wave patterns which is cause by vibrational energy

11.1 Simple Harmonic Motion - Spring Oscillations

- **Periodic** - Describe motion of objects that move back and forth
- **Equilibrium position** - The position of an oscillator where it exerts no force, such as a spring that remains still
- Springs automatically want to return to their equilibrium position and exert a force to do so, called the equilibrium force

  $$F = -kx$$

  Where k is spring constant and x is distance extended
- The force required to extend a spring is

  $$F_{ext} = kx$$

- **Simple Harmonic Motion (SHM)** - Describes a system where restoring force is proportional to the negative of the displacement
- **Amplitude** - The greatest distance from the equilibrium point of a simple harmonic oscillator, represented by A

11.2 Energy in Simple Harmonic Motion

- Elastic potential energy of a spring is given by

  $$PE = \frac{1}{2}kx^2$$

- The total energy in a spring is given by

  $$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

- The total mechanical energy of a simple harmonic oscillator is porportional to the square of the amplitude

  $$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$$

- The maximum velocity of an oscillator is calculated by

  $$v_{max} = \sqrt{\frac{k}{m}}A$$

- The the velocity at any point in an oscillation is given by

  $$v = \pm v_{max}\sqrt{1 - \frac{x^2}{A^2}}$$

11.3 The Period and Sinusoidal Nature of SHM

- The period of a simple harmonic oscillator is given by

  $$T = 2\pi\sqrt{\frac{m}{k}}$$

- The frequency is the reciprocal of the period

  $$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Period and Frequency Derivation

- Note that maximum velocity is equal to

  $$v_{maz} = \frac{1\pi A}{T} = 2\pi Af$$

- Also note that

  $$\frac{A}{v_{max}} = \sqrt{\frac{m}{k}}$$

- Thus, solving for period and frequency gives the equations in the beginning of this section

Position as a Function of Time

- The equation for the position of a mass undergoing SHM is

$$x = A\cos\theta$$

- The same equation in terms of period is

$$x = A\cos(\frac{2\pi t}{T})$$

- The same equation in terms of frequency is

$$x = A\cos(2\pi f t)$$

Sinusoidal Motion

- The equations in the previous section assume an object is starting from rest, at its maximum displacement, and at t=0
- Another equation for positions in SHM is

$$x = A\sin(\frac{2\pi t}{T})$$

  Where x starts at 0 instead of A

- **Sinusoidal** - Having the shape of a sine function, simple harmonic motion is sinusoidal

Velocity and Acceleration as a Function of Time

- Graphing velocity in SHM as a function of time gives

$$v = -v_{max}\sin(\frac{2\pi t}{T})$$

- The maximum velocity of a mass in SHM is given by

$$v_{max} = A\sqrt{\frac{k}{m}}$$

- Graphing acceleration in SHM as a function of time gives

$$a = -a_{max}\cos(\frac{2\pi t}{T})$$

11.4 The Simple Pendulum

- **Simple Pendulum** - A small object on a cord that does not stretch and has negligible mass
- Simple pendulums swing and move in SHM
- The force on a pendulum at any point of its path is

$$F = -mg\sin\theta$$

- The period of a simple pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

  Where l is the length of the cord and g is the acceleration due to gravity

- The frequency of a simple pendulum is given by

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

11.5 Damped Harmonic Motion

- **Damped Harmonic Motion** - Harmonic motion whose amplitude decreases as number of swings increases, such as by friction
- **Underdamped** - Describes a damped system that oscillates multiples times before stopping
- **Overdamped** - Describes a damped sysem that does not oscillated at all before stopping
- **Critical Damping** - Damping that causes motion to stop in the shortest amount of time
- **Shock Absorbers** - An example of a critically damped system, but as they wear out, underdamping occurs

11.6 Forced Oscillations; Resonance

- **Forces Oscillation** - A situation where which an object is forced to oscillate at a frequency other than its naturl frequency
- **Natural Frequency** - Also called *resonant frequency*, this is the frequency at which an a spring oscillates when no force is applied to it during oscillation, given by

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

- **Resonance** - Occurs when forced frequency and natural frequency are equal and act at the same time ($f = f_0$)
- Most materials are elastic in some form and have their own resonant frequency which is important to consider in construction; if a building has a force applied near its resonant frequency it is at risk of collapse

11.7 Wave Motion

- **Mechanical Waves** - Oscillations of matter
  - Examples include water waves and waves in a rope
- Not particles in a mechanical eaves do not actually move with the wave but rather they move around an equilibrium point, ultimately they stay in the same position
- The velocities nor directions of particles in a wave and the wave itself very different, a wave can move right while its particles can move up and down
- **Pulse** - A single bump in a wave

11.8 Types of Waves and their Speeds: Transverse and Longitudinal

- **Transverse Wave** - A wave whose particles move perpendicular to the waves, such as an ocean wave where the water moves vertically but the wave travels horizontally
- **Longitudinal Wave** - A wave whose particles move parallel to the direction of the wave, such as a sound wave moving horizontally, its particles will also oscillate horizontally
- **Medium** - The material or space through which a wave travels
- Typically, the medium of a longitudinal wave moves very little relative to the wave itself, their oscillations are fast

Speed of Transverse Waves

- Speed of a transverse wave ina string or cord is calculated by

$$v = \sqrt{\frac{F_T}{\mu}}$$

Where F_T is the force of tension within the string and μ is the mass by length ($\mu = m/l$) of the cord

Speed of Longitudinal Waves

- The speed of a longitudinal wave travelling down a solid rod is

$$\sqrt{\frac{E}{\rho}}$$

Where E is the elasticity modulus (Chapter 9)

- The speed of a longitudinal wave travelling in a fluid is

$$\sqrt{\frac{B}{\rho}}$$

Where B is the bulk modulus (Chapter 9)

Other Types of Waves

- Earthquakes produce both types of waves
  - Transverse - Also called *shear* or S waves
  - Longitudinal - Also called *pressure* or P waves
  - Longitudinal waves caused by earthquakes have led experts to infer that part of the Earth's inside is made of liquid

11.9 Energy Transported by Waves

- **Intensity -** Measure how strong a wave is, calculated by
$$I = \frac{power}{area}$$
Where area is is perpendicular to the direction of energy flow
- Intensity is also proportional to the amplitude squared, thus
$$I \propto A^2$$
- A wave that flows from its source in all directions, its is 3-dimensional and its area is the surface area of a sphere, thus
$$I = \frac{P}{4\pi r^2}$$
Thus
$$I \propto \frac{1}{r^2}$$
Which is an *inverse square law* For example, a wave twice as far from its source has half the amplitude and one quarter the intensity
- The following relationship is true
$$\frac{I_2}{I_1} = \frac{r_1^2}{r_2^2}$$
- The previous relations give
$$A \propto \frac{1}{r}$$
- The following relationship is true
$$\frac{A_2}{A_1} = \frac{r_1}{r_2}$$

Intensity Related to Amplitude and Frequency

- For sinusoidal waves, particles move in SHM, thus each particle has energy
$$E = \frac{1}{2}kA^2$$
Where k can be written in terms of frequency, $k = 4\pi^2 mf^2$ Where m is the mass of a particle in a medium
- Expanding the previous equation to include the expanded k gives
$$E = 2\pi^2 mf^2 A^2$$
- The following expansions can be made to the previous equation:
  - $m = \rho V$
  - V=Sl, we are using S to represent surface area since A is being used for amplitude
  - l is the distance the wave travels and can be represented by l=vt
  - Thus $m = \rho Svt$
- Applying this to the equation gives
$$E = 2\pi^2 \rho Svtf^2 A^2$$
- The average power transported by a wave is given by
$$P = \frac{E}{t} = 2\pi^2 \rho Svf^2 A^2$$
- Intensity is thus
$$I = \frac{P}{S} = 2\pi^2 \rho vf^2 A^2$$

11.10 Reflection and Transmission of Waves

- Waves in a cord can be reflected once they reach the end of the cord
- If the end is fixed to a support, the wave reflect inverted (upside down)
- If the end is free, the wave reflects right side up
- **Wave Front -** All the points along the wave that form its crest
- **Ray -** A line in the direction of wave motion, perpendicular to its front
- **Plane Waves -** Wave fronts that have lost their curvature and are straight
- **Law of Reflection -** The angle of reflection equals the angel of incidence
- **Angle of Incidence -** The angle between the direction of a wave and the perpendicular of the surface it reflects off of
- **Angle of Reflection -** The angle of incidence reflected across the perpendicular to the surface of reflection

11.11 Interference; Principle of Superposition

- **Interference** - Occurs when two waves pass through the same region of space
- **Principle of Superposition** - The resultant displacement is the algebraic sum of their separate displacements
- Crests are considered positive and troughs are considered negative
- **Destructive Interference** - Occurs when waves have opposite displacements and cancel out
- **Constructive Interference** - Occurs when waves have equal displacements and combine
- **Phase** - Used to describe the relative position of the crests of two waves
- Waves whose crests are aligned are called *in phase*
- Waves where a crest and a trough are aligned are called completely *out of phase*

11.12 Standing Waves; Resonance

- Oscillating a cord at the right frequency creates a *standing wave* with a large amplitude
- **Node** - Point at which a standing wave is still
- **Antinode** - Point at which a cord oscillates with maximum amplitude
- **Natural Frequency** - A frequency at which a cord forms a standing wave
- Standing waves are the result of two opposite waves colliding
- **FUndamental Frequency** - Lowest natural frequency of a wave, one antinode
- **Overtones** - Other natural freuqncies, these are multiples of the fundamental, also called *harmonics*
- The length of a string can be calculated by

  $$l = \frac{n\lambda_n}{2}$$

  Where n is the harmonic and *lambda* is wavelength
- Solving for wavelength gives

  $$\lambda_n = \frac{2l}{n}$$
- Solving for frequency gives

  $$f = \frac{v}{\lambda_n} = n\frac{v}{2l}$$

11.13 Refraction

- **Refraction** - Occurs when a wave crosses into a new medium, its speed and direction change
- **Law of Refraction** - Useful in determining the angle of refraction of a wave,

  $$\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1}$$

11.14 Diffraction

- **Diffraction** - The bending of a wave when it encounters an obstacle
- An area of medium behind an object where a wave does not penetrate is called a shadow
- Rule of thumb: There will only be a significant shadow if the wavelength is smaller than the obstacle
- Rough equation

  $$\theta(radians) \approx \frac{\lambda}{l}$$

  Where θ is the angular spread of a wave, and l is the width of the opening or object it passes around

11.15 Mathematical Representation of a Travelling Wave

- Expressing a sinusoidal wave mathematically, th follwoing equation is used

  $$y = A\sin\frac{2\pi}{\lambda}x$$

  Where y is the displacement, A is the desired amplitude, λ is wavelength, and x is simply the x axis
- Waves move, and the distance after moving over a specified amount of time is given by vt, inserting this into the previous equation gives

  $$y = A\sin[\frac{2\pi}{\lambda}(x - vt)]$$

  Thus the wave's shape moves along with time

12 Sounds

Sound refers to the physical phenomenon that stimulates our ears in longitudinal pressure waves. Aspects of sound are there must a be source of the sound, energy is transferred with sound, and sound is detected.

12.1 Characteristics of Sound

- Sound must have a medium to travel through, it cannot travel where there is not matter
- **Speed of Sound** - Different in different materials, in air of pressure 1tm at 0°C is defined as 331m/s
- Speed of sound increases roughly with temperature, represented by

$$v \approx (331 + 0.60T)m/s$$

  Where T is the temperature in degrees celcius
- Room temperature is assumed to be 20°C, thus

$$v = (331 + 0.60(20))m/s = 343m/s$$

- **Loudness** - Refers to the intensity of a sound wave
- **Pitch** - Refers to the frequency of a sound wave, a high pitch sound has a high frequency and vice versa
- **Audible Range** - The range of sound frequencies most humans can hear, around 20Hz-20,000Hz
- **Ultrasonic** - Describes sound waves whose pitch is above the human hearing range (¿20,000Hz)
- **Infrasonic** - Describes sound waves whose pitch is below the human hearing range (¡20Hz)
- **Pressure Wave** - Another name for longitudinal waves, means that longitudinal waves are variations in pressure that travel through a medium

12.2 Intensity of Sound: Decibels

- Intensity has units of $\frac{W}{m^2}$
- To produce a sound with twice the sound level requires 10 times the intensity

Sound Level

- **Bel** - Unit used to measure sound level on a lograrithmic scale
- **Decibel** - 1/10th of a bel, abbreviated as dB, 1dB = 1 bel
- **Sound Level** in terms of intensity is defined as

$$\beta(in\ dB) = 10 \log \frac{I}{I_0}$$

  Where β is sound level, I is intensity, and I_0 is the lower threshold of hearing, defined as

$$I_0 = 1.0 * 10^-12 \frac{W}{m^2}$$

- Note that the threshold of hearing in decibels is 0dB
- Over long distance, the rate at which intensity decreases becomes faster than the inverse square relation between distance and intensity as some energy is transfered to irregular motion of air molecules
- Note that this loss occurs sooner for higher frequencies than lower frequencies

Intensity Related to Amplitude

- Amplitude can be calculated by

$$A = \frac{1}{\pi f} \sqrt{\frac{I}{2\rho v}}$$

12.3 The Ear and its Response; Loudness

- The ear directs sound to the ear drum which detects incoming sound waves
- Th ear is an extremely complex amplifier of sound and has several delicate mechanisms

The Ear's Response

- The ear is not equally responsive to all frequencies, averaged curves are used to determine which frequencies are heard louder than others at the same loudness level
- Note that the ear is most sensitive to sound between 2000Hz and 4000Hz while much lower pitches sounds require significantly more intensity to be audible

12.4 Sources of Sound; Vibrating Strings and Air Columns

- The source of any sound is a vibrating object
- When musical instruments are played, they produce standing waves at their resonant frequencies which travel through the air
- **Octave** - A distance between musical notes, correspond to a doubling or halving of frequency

Stringed Instruments

- Pitch is normally determined by the lowest resonant frequency or *fundamental* frequency
- The fundamental frequency of a string, denoted by f_1 is given by

$$f_1 = v/\lambda = v/2l$$

  Where v is the velocity of the wave on the string

- The other possible frequencies of a standing wave on a stretched string are whole number multiples of the fundamental, thus

$$f_n = nf_1 = n\frac{v}{2l}$$

- Changing the length of a string changes the velocity of the wave on it,

$$v = \sqrt{F_T/\mu}$$

  Where μ is the mass per unit length on the string and F_T is the force of tension within the string

Wind Instruments

- The basic structure of wind instruments is they produce a vibrating air column
- Higher frequency standing waves above the fundamental are called *overtones* or *harmonics*
- A single node in a sound wave within an air column corresponds to the fundamental frequency
- Another way of looking at sound wave within an air column is node represent variations in air pressure that travel throughout
- **Open tube** - A tube for air vibration open at both ends
- **Closed tube** - A tube for air vibration closed at one end
-

12.5 Quality of Sound, and Noise; Superposition

- **Quality** - Used to describe sound, this is a distinct difference in sounds from different sources, also called *timbre* or *tone color*
- Quality of sound depends on overtones and their relative amplitudes
- **Principle of Superposition** - The simultaneous presence of multiple waves
- **Waveform** - The overtones of a sound use the principle of superposition to add together to produce a *composite waveform*
- Each musical instrument has different proportions of amplitudes in its harmonics, leading to different tone qualities

12.6 Interference of Sound Waves; Beats

Interference in Space

- The same sound produced by two different speakers will have different intensities depending on where the listener is, some places have constructuve interference and the sound is louder while others have destructive interference and the sond is muted
- When the waves are in phase, constructive interference occurs
- When the waves are out of phase, destructive interference occurs

Beats - Interference in Time

- **Beats** - A phenomenon that occurs when two sound of similar frequencies are played and the sounds interfere, the relative sound level fluctuates as the frequencies fall in and out of phase
- The sum of the waves of the sounds produces a composite wave with its own crests and troughs
- The frequency of these composite crests is known as the *beat frequency*

12.7 Doppler Effect

- **Doppler Effect -** The change in pitch of a sound when its source is in motion
- This is because as the sound source moves, it "catches up" to the crests it has already emitted in its paths and moves farther away from the crets it leaves behind, thus the frequency of sound in front of it is slightly higher than behind it
- The new frequency when a source is moving toward a stationary observer is given by

$$f' = \frac{f}{1 - \frac{v_{source}}{v_{sound}}}$$

- The new frequency when a source is moving away from a stationary observer is given by

$$f' = \frac{f}{1 + \frac{v_{source}}{v_{sound}}}$$

- The new frequency when an observer is moving away from a stationary source is given by

$$f' = (1 - \frac{v_{source}}{v_{sound}})f$$

- The new frequency when both the observer and source are moving is given by

$$f' = f(\frac{v_{sound} \pm v_{obs}}{v_{sound} \mp v_{source}})$$

  Use the upper signs when the source and observer are moving closer and the lower signs when they are moving farther apart

Doppler Effect for Light

- The doppler effect applies to light as well, it causes a shift in the color of light observed
- **Redshift -** Occurs when objects move away from the observer, the wavelengths become longer and thus move down the spectrum to red

12.8 Shock Waves and the Sonic Boom

- **Supersonic -** Describes an object moving through a medium faster than the speed of sound in that medium
- **Mach's Number -** The ratio between an object's speed and the speed of sound in the medium
- **Shock Wave -** Occurs when an object moves at the speed of sound in its medium, the crests of the sound wave pile up in front of it and are released as a single larger crest
- **Sonic Boom -** The sound produced by a shockwave
- **Sound Barrier -** Another name for the speed of sound in a medium, the barrier needs to be broken to go past the speed of sound
- Shock waves take the shape of a cone and the angle of this cone is given by

$$\sin\theta = \frac{v_{sound}}{v_{object}}$$

12.9 Applications: Sonar, Ultrasound, and Medical Imaging

Sonar

- **Sonar -** Also called *pulse-echo*, this technique uses the speed of sound in a medium to determine the distance of an object by measuring how long it takes a sound to reflect back to the source
- Used to determine the depths of object under the sea, usually uses ultrasonic frequencies

Ultrasound Medical Imaging

- Similar to the pulse echo technique, but sound waves are much higher, ranging from 1 to 10mHz
- Used to detect the boundaries of surfaces within the body to image it
- The time between each pulse is the quotient of the desired depth to measure within the body and the speed of sound in the body, given by

$$t = \frac{d}{v}$$

- The strength of reflected pulses depends on the different in density of the two materials on either side of the interface

13 Temperature and Kinetic Energy

This section focuses on atoms and their continuous random motion

13.1 Atomic Theory of Matter

- **Atom** - Means indivisible, this is the smallest piece of matter
- **Atomic and Molecular Mass** - The mass of an atom or molecule in unified atomic mass units (u), defined as
$$1u = 1.6605 * 10^-27kg$$
- **Element** - A substance that cannot be broken down into smaller parts
- **Compound** - Substances made of elements
- **Molecule** - The smallest piece of a compound
- **Brownian Motion** - Describes the erratic, random motion of small particles, later used by Einstein to determine the average size of an atom to be around 10^-10m
- **Macroscopic** - Meaning large scale
- **Microscopic** - Meaning small scale
- Atoms and molecules naturally repel each other, leading to constant motion as they try to push away from each other

13.2 Temperature and Thermometers

- **Temperature** - A measure of how hot or cole something is
- Objects tend to expand when they are at a higher temperature
- Some objects emit light at extreme temperatures
- Many thermometers depend on the expansion caused by heat to measure temperature
- **Bimetallic Strip** - A thermometer made by binding two different metals with different rates of expansion, as the temperature increases, the different expansions cause the strip to bend

temperature Scales

- Most temperature scales use readily reproducible temperatures as points of reference
- To convert from Celsius to Fahrenheit
$$T(^\circ C) = \frac{5}{9}[T(^\circ F) - 32]$$
- To convert from Fahrenheit to Celsius
$$T(^\circ F) = \frac{9}{5}T(^\circ F) + 32$$

Standard Temperature Scale

- Materials usually do not expand at the same rate throughout a wide range of temperaturers
- To improve precision, a *constant-volume gas thermometer* is used which expands a gas at the same rate
- The thermometer mentioned above is used as the basis for the *standard temperature scale*

13.3 Thermal Equilibrium and the Zeroth Law of Thermodynamics

- Two objects of different temperatures placed in thermal contact will eventually reach the same temperature or *thermal equilibrium*
- **Thermal Contact** - A connection between two objects that allows thermal energy to transfer between them

The Zeroth Law of Thermodynamics

- **Zeroth Law of Thermodynamics** - If two systems are in thermal equilibrium with a third system, then they are in thermal equilibium with each other
- When systems are in thermal equilibrium with each other there is no net thermal energy exchanged

13.4 Thermal Expansion

- The rate of thermal expansion of an ibject is dependent on its material

Linear Expansion

- Thermal expansion is proportional to the original length of the object, represented by

$$\Delta l = \alpha l_0 \Delta T$$

Where l_0 is the original length, α is the coefficient of linear expansion, and ΔT is change in temperature

- This can be rewritten as

$$l = l_0(1 + \alpha\Delta T)$$

Volume Expansion

- Objects also expand in volume when they gain heat energy, represetned by

$$\Delta V = \beta V_0 \Delta T$$

Where V_0 is the original volume, $\beta isthecoefficientofvolumeexpansion$, and ΔT is the change in temperature

- The equations fof change in length and volume due to thermal expansion are only accurate for small changes and β varies significantly with temperature

Anomalous Behavior of Water Below $4°$

- Water has its greatest density at 4°C
- Because of this, ice is less dense than water and bodies of water at freezing temperatures freeze on the surface, leaving the rest at around 4°C

Thermal Stresses

- **Thermal Stresses** - Tensile or compressive stresses within a material due to thermal expansion
- The thermal stresses within an object is calculate by

$$\frac{F}{A} = \alpha E \Delta T$$

Where A is the cross sectional area of the object, α is the coefficient of linear expansion, E is the young's modulus, and ΔT is the change in temperature

13.5 The Gas Laws and Absolute Temperature

- **Equation of State** - A relation between the volume pressure, temperature, and quantity of a gas
- **Equilibrium State** - A state of a system where the variables that describe is do not change in time
- **Boyle's Law** - Typically, the volume of a gas is inversely proportional to its pressure at a constant temperature,

$$V \propto \frac{1}{P}$$

Therefore

$$PV = constant$$

- **Absolute Zero** - The lowest temperature possible, the theoretical temperature where a gas would have no volume **Absolute Scale** - ALso called the *Kelvin Scale*, this is a temperature scale with the same distance between temperatures as Celsius but 0 is defined as −273.15.15°C
- TO convert from Celsius to Kelvin,

$$T(K) = T(°C) + 273.15$$

- **Charles's Law** - The volume of a fixed quantity of a gas is directly proportional to the absolute temperature when the pressure is kept constant,

$$V \propto T$$

- **Gay-Lussac's Law** - At constant volume, the absolute pressure of a fixed quantity of a gas is directly proportional to the absolute temperature

$$P \propto T$$

13.6 The Ideal Gas Law

- **Ideal Gas Law** - Combines Boyle's, Charles's, and Gay-Lusac's Laws, note that changing two of the variables will change the other
$$PV \propto T$$
- Including the mass of a gas gives
$$PV \propto mT$$
- **Mole** - SI unit for the amount of a substance, defined as $6.02 * 10^23$ objects of something, used to determine how much a substance there is
- Number of moles of a substance is calculated by
$$n(mole) = \frac{mass(grams)}{molecular\ mass(g/mol)}$$
- The proportion can be rewritten as
$$PV = nRT$$
Where R is the *universal gas constant*, defined as
$$R = 1.99 calories(mol * K)$$
- **Ideal Gas Law** - Real gases do not follow it precisely, but close enough at low pressures
$$PV = nRT$$

13.7 Problem Solving with The Ideal Gas Law

- **Standard Conditions** - Also called *standard temperature and pressure* (STP), defines the temperature and pressure for most problems,
$$T = 273K\ (0^{\circ})C$$
and
$$P = 1.00atm = 101.3kPa$$
- Problems that involve a change in pressure, volume, or temperature but no change in the volume of gas can be solved with
$$\frac{P_1V_1}{V_1} = \frac{P_2V_2}{V_2}$$

13.8 Ideal Gas Law in Terms of Molecules; Avogadro's Number

- **Avogadro's Hypothesis** - Equal volumes of gas at the same pressure and temperature contain equal numbers of molecules, consistent with R being the same for all gases
- **Avogadro's Number** - $N_A = 6.02 * 10^{23}$
- The ideal gas law can be rewritten in terms of numbers of molecules as
$$PV = NkT$$
Where N is the total number of molecules present and k is the *Boltzmann Constant*, defined as
$$k = \frac{R}{NN_A} = 1.38 * 10^{-23} J/K$$

13.9 Kinetic Theory and the Molecular Interpretation of Temperature

- **Kinetic Theory** - The study of matter in terms of atoms in continuous random motion
- Kinetic Theory uses statistical approached to predict the behavior of atoms on macroscopic scales
- Postulates of the Ideal Gas
  - Our assumption is that gases fill the volume of their container, in the atmosphere's case, it is kept from escaping by Earth's gravity
  - Molecules are on average far apart from each other, the distance between them is greater than the diameter of each molecule
  - Molecules are assumed to obey classical mechanics and collide with each other accordingly, molecules exert weak attractive force on each other, the potential energy caused by this is small compared to the kinetic energy
  - Collisions are assumed to be perfectly elastic, also the time of a collision is very short compared to the time between collisions, thus potential energy is ignored
- The pressure on the vessel of a gas is caused by the collisions of the molecules on its walls

- The average force exerted by one molecule of a gas is

$$F = \frac{mv_x^2}{l}$$

Where m is the mass, v_x is the x component of its velocity, and l is the length is the vessel

- The pressure of a gas on the wall of its vessel is

$$P = \frac{1}{3}\frac{NMv^2}{V}$$

- The kinetic energy of objects in motions in an ideal gas is directly proportional to the absolute temperature of the gas,

$$KE = \frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$$

Kinetic Energy Near Absolute Zero

- As temperatures approach absolute zero, molecular motion does not approach zero, instead it approaches a very small non-zero value
- Molecular motion does not cease even at absolute zero

13.10 Distribution of Molecular Speeds

- **Maxwell Distribution of Speeds -** A graph that shows the relative number of molecules at a certain speed
- The Maxwell Distribution for the same gas changes depending on its temperature
- At higher temperatures, the proportion of molecules with high speed increases

13.11 Real gases and Changes of Phase

- When temperatures and pressures of gases are high, the ideal gas law begins to break down
- **PV Diagram -** Relates the Pressure and volume of a gas at a constant temperature
- At lower temperatures, it is possible for a gas to liquefy at certain volumes or pressures, shown as the liquid-vapor region
- **Critical Temperature -** The temperature at which the PV diagram of a gas has a horizontal curve that touched the liquid-vapor region
- **Critical Point -** The point at which the critical temperature touches the liquid-vapor region
- **PT Diagram -** also called a *phase diagram*, shows the phase of a substance at temperature and pressure combinations
- **Sublimation -** The process by which a material converts from solid directly to gas
- **Triple Point -** The point at which the three curves on a PT Diagram intersect, all three phases can exist in equilibrium
- **Superfluidity -** Describes a liquid that has no viscosity
- **Liquid Crystals -** Can be considered a phase between liquid and solid

13.12 Vapor Pressure and Humidity

Evaporation

- **Evaporation -** The process by which a liquid converts to gas
- Caused when the fastest molecules in a liquid escape its surface
- Evaporation is a cooling process

Vapor Pressure

- **Condensation -** The process of vapor reentering its liquid
- **Saturated Vapor Pressure -** The pressure of a vapor when it is saturated
- **Saturation -** When the amount of vapor in the surrounding air has reached the maximum the air can hold and the number of molecules exiting and entering is equal

Boiling

- A liquid boils when its saturated vapor pressure equals the external pressure
- Saturated vapor pressure increases with temperature

- High elevations cause boiling points to lower as the surrounding air presure is lower and thus the saturated vapor pressure does not have to reach as high

Partial Pressure and Humidity

- **Partial Pressure** - The pressure exrted by each individual gas in air if it were alone
- **Relative Humidity** - THe ration between the partial pressure of water and its saturated vapor pressure in an environment,

$$Relative\ Humidity = \frac{partial\ pressure\ of\ H_2O}{Saturated\ vapor\ pressure\ of\ H_2O} * 100\%$$

- **Supersaturaed** - Describes when the partial pressure of water in air exceeds is saturated vapor pressure, can occur when a temperature decrease occurs, when this happens water can condense back into clouds or dew
- **Dew Point** - The temperature at which the partial pressure of water exceeds its saturated vapor pressure for a given humidity

13.13 Diffusion

- **Diffusion** - The natural mixing of fluids until they become unifmr
- Occurs due to the random motion of molecules explained by kinetic theory
- Diffusing substances move from a region where its concentration is high to a region where its concentration is low
- **Fick's Law** - The rate of diffusion for a substance is given by

$$J = DA\frac{C_1 - C_2}{\Delta x}$$

  Where D is the diffusion constant of the fluid, A is the area of the cross section of the vessel, C_1 is the concentration in the area of high concentration, C_2 is the concentration in the area of low concentration, and Δx is the distance between the two regions
- Diffusion is essential for many biological processes as it allows fluids to be exchanged without the organism itself expending energy

14 Heat

Heat is said to "flow" from hot objects to cold objects, this flow eventually causes the objects to be in thermal equilibrium, when there is no more heat flow. Note that heat and temperature are NOT the same.

14.1 Heat as Energy Transfer

- **Calorie** - The amount of heat necessary to raise the temperature of 1 gram of water by 1 degree Celsius.
- **BUT** - Stands for british thermal unit, this is the amount of heat required to raise the temperature of 1lb of water by 1 degree Fahrenheit
- **Mechanical Equivalent of Heat** - An amount of work always has an equivalent amount of heat, the conversion between Joules and Calories is given by

$$1.186J = 1cal$$

$$1.186kJ = 1kcal$$

- **Heat** - The energy transferred from one object to another because of a difference in temperature

14.2 Internal Energy

- **Internal Energy** - The sum of the energy of all the moleculaes in an object, also called *thermal energy*

Distinguishing Temperature, Heat, and Internal Energy

- Temperature is the average kinetic energy of individual molecules
- Internal Energy is the total energy of all molecules
- Heat is the transfer of energy from one object to another

Internal Energy of an Ideal Gas

- The internal energy of the ideal non atomic gas is

$$U = \frac{3}{2}nRT$$

Where n is the number of moles, R is the universal gas constant, and T is temperature

- If molecules contain more than one atom, their vibrational and rotational energies must also be accounted for
- The internal energy of liquids is not as simple as therre are electrical potential energies between molecules that must be accounted for

14.3 Specific Heat

- The amount of heat, Q, required to change the temperature of an object is calculated by

$$Q = mc\Delta T$$

Where m is mass, ΔT is the intended change in temperature, and c is the *specific heat*

- **Specific Heat** - Unique to each material, this is used to determine how much temperature will change with a given amount of heat, units of $\frac{kcal}{kg*^\circ C}$ or $\frac{cal}{g*^\circ C}$
- Water has one of the highest specific heats of all substances, which gives it many useful applications in insulating or maintaining heat

Specific Heats for Gases

- Since gases change in volume due to changes in temperatures, specific heat is not very applicable to them
- Instead, constant pressure, c_p and constant volume, c_v, are used, with the same units as specific heat

14.4 Calorimetry - Solving Problems

- **System** - Object(s) considered in a proble
- **Open System** - A system where mass not leave, if energy does not leave either it is said to be *isolated*
- **Closed System** - A system where mass and energy may leave
- **Thermal Equilibrium** - A state that describes when an entire system has reached the same temperature
- For an isolated system,
  - heat lost = heat gained
  - energy *out* one part = energy *into* another part
  - The sum of all the heat transfers is equal to zero,

    $$\Sigma Q = 0$$

    where each Q represents heat entering or leaving a part of the system
- **Calorimetry** - A technique that measures heat exchange,
- **Calorimeter** = A device that measures heat exchange, used to determine the specific heats of substances

Bomb Calorimeter

- **Bomb Calorimeter** - Used to measure thermal energy released from substances burning, also used to measure the caloric content of foods
- The caloric content determined from this method may be unreliable as our bodies cannot metabolize all the available energy in a food

14.5 Latent Heat

- Changes of phase always involve energy
- **Heat of fusion** - Refers to the heat required to convert 1.0kg of a substance from solid to liquid
- **Heat of Vaporization** - Refers to the heat required to convert 1.0kg of a substance from liquid to vapor
- Heats of fusion and vaporization are also called *latent heats*, which refers to heat released by a substance when it changes from gas to liquid or liquid to solid
- Latent heats can be modeled by

  $$Q = mL$$

  Where m is the mass of the substance, L is its latent heat, and Q is the heat added or released

Evaporation

- **Evaporation** - The process of a liquid converting to a gas at room temperature
- Evaporation is a cooling process and is used by our bodies to keep cool

Kinetic Theory of Latent Heats

- At the point of fusion, latent heat overcomes the attractive forces between molecules and allows them to become liquid
- The same at the point of vaporization, where the latent heat allows molecules to spread farther apart

14.6 Heat Transfer: Conduction

- **Conduction** - A method of heat transfer through direct contact with a heat course
- Can be visualized as molecular collisions
- Heat flow is given by

  $$\frac{Q}{t} = kA\frac{T_1 - T_2}{l}$$

  Where $\frac{Q}{t}$ is the rate of heat flow over time, k is the thermal conductivity of the material, A is the cross sectional area, l is the distance between the hot and cold regions, T_1 is the temperature of the hotter region, and T_2 is the temperature of the cooler region
- Thermal conductivity is unique to each material and has units of $\frac{J}{s*m*°C}$
- Materials with high thermal conductivities are said to be good thermal conductors, they can transfer heat easily

- Materials with low thermal conductivities are said to be good thermal insulators, they transfer heat slower

R-Value for Building Materials

- R-values specify the insulating properties of materials, units of ft^2*
- Larger R values mean a material is a better insulator

14.7 Heat Transfer: Convection

- **Convection** - Process by which heat flows by bulk movement of molecules from one place to another
- For example, hot water expands and is less dense than cold water, it rises, but once it reaches the surface it cools down, gets denser and sinks
- Convection is important in home heating, some systems use it to transfer heat from loawr levels of the house to higher ones

14.8 Heat Transfer: Radiation

- **Radiation** - Heat transferred as radiant energy
- **Stefan-Boltzmann Equation** - The heat flow radiated by a source is calculated by

$$\frac{Q}{t} = \epsilon\sigma AT^4$$

  Where ϵ is the emissivity, σ is the Stefan-Boltzmann constant, A is the surface area of the object, and T is its temperature

- Emissivity is a value that ranges from 0 to 1 and depends on the surface of the radiating material, if it is darker in color it has an emissivity close to 1 and vice versa
- The Stefan-Boltzmann constant is defined as

$$5.67 * 10^-8W/m^2K^4$$

- Good absorbers are good emitters
- The net rate of radiant heat flow is given by

$$\frac{Q}{t} = \epsilon\sigma A(T_1^4 - T_2^4)$$

- Since the sun is a point source, its radiant heat cannot be calculated using the above equations, instead, the following is used

$$\frac{Q}{t} = (1000W/m^2)\epsilon A\cos\theta$$

  WHere θ is the angle between the direction of the sun's rays and a line perpendicular to the surface of the object being hit

- **Themography** - A technique that measured the intensity of infrared rays from many points on the body to form a picture

15 The Laws of Thermodynamics

Thermodynamics is the study of processes in which energy is transferred as heat and as work.

15.1 The First Law of Thermodynamics

- **The First Law of Thermodynamics** - The change in internal energy of a closed system, ΔU, will be equal to the energy added to the system by heating minus the work done by the system on the surroundings,
$$\Delta U = Q - W$$
- This is a general statement of the law of conservation of energy
- **State Variables** - Variables that describe the state of a system, these are energy U, pressure P, volume V, temperature T, and mass M or number of moles n

The First Law of Thermodynamics Extended

- In a system with kinetic and potential energy, the first law would be rewritten as
$$\Delta KE + \Delta PU + \Delta U = Q - W$$

15.2 Thermodynamic Processes and the First Law

Isothermic Processes

- **Isothermic Process** - A process that occurs at a constant temperature
- An isothermic ideal gas system would follow
$$PV = constant$$
The curves describing this state are called isotherms
- We assume the gas is in contact with a *heat resevoir* and that changes in volume occur very slowly
- Internal energy does not change, thus
$$\Delta U = \frac{3}{2} nR\Delta T = 0$$

Adiabatic Processes

- **Adiabatic Process** - A process where heat does not flow in or out of a system, or it is so well insulated that heat does not get the chance to flow out, Q=0
- Adiabatic systems have PV curves similar to those of isotherms, but not the same as heat is still allowed to flow within the system and thus changes in pressure and volume change the temperature

Osibaric and Isovolumetric Processes

- **Isobaric** - A process where pressure is kept constant
- **Isovolumetric** - A process where volume is kept constant

Work done in Volume Changes

- The work done in a volume change is given by
$$W = P\Delta V$$
- Only the volume change in an isotherm is used to determine work done

15.3 Human Metabolism and the First Law

- **Metabolism** - The energy changing processes in an organism
- The human body does work all the time, it gets energy for this by eating food and lets it out by releasing it into the surrounding area

15.4 The Second Law of Thermodynamics - Introduction

- **Second Law of Thermodynamics** - Heat can flow spontaneously from a hot object to a cold object; heat wil not flow spontaneously from a cold object to a hot object
- The above statement is specific to certain processes but implies that many processes are not reversible

15.5 Heat Engines

- **Heat Engine -** a device that changes thermal energy to mechanical work
- Heat engines that cycle have an internal energy change of 0 since they alwasy return to their starting point
- **Operating Temeratures -** The high temperature input and lower temperature output of a heat enginer

Steam Engine and Internal Combustion Engine

-
- These engines use thermal energy to heat a substance to produce work
- **Working Substance -** The work heated to work, usually either steam or fuels such as gasoline

Why ΔT is Needed to Drive a Heat Engine

- Heat engines need some form of energy loss from intake to exhaust in order to use that energy loss as work
- If there is no change in temperature, pressure remains the same on both sides and thus work cannot be done

Efficiency

- **Efficiency -** The ratio of the work a heat engine does and the input at the high temperature,

$$e = \frac{W}{Q_H}$$

Carnot Engine

- **Carnot Engine -** Named after French scientist Sadi Carnot, this is the ideal engineer but it does actually exist
- Carnot Engines do work reversibly, that is they can be done in reverse with no change in work done or heat exchanged
- Real life processes have many factors that interrupt reversibility and are thus *irreversible*
- **The Third Law of Thermodynamics -** No device is possoble whose sole effects is to transform a given amount of heat completely into work

15.6 Refrigerators, Air Conditioners, and Heat Pumps

- **Clausius statement of the second law of thermodynamics -** No device is possible whose sole effect is to transfer heat from one system at a temperature T_L into a system at a higher temperature T_H
- All refrigerators exert some work to move heat outside of themselves, in most cases it is a motor
- **Coefficient of Performance -** The heat removed from the lower temperature area divided by the work done to do so

  $$COP = \frac{Q_L}{W}$$

  Where Q_L is the heat removed from the inside of the refrigerator
- The COP for a heat pump is

  $$COP = \frac{Q_H}{W}$$

  Where Q_L is the heat delivered to the inside of the house

SEER Rating

- SEER stands for seasonal energy efficiency ratio, it is similar to the COP of a device, defined as

$$SEER = \frac{heat\ removed\ in\ BTU}{electrical\ input\ in\ watt-hours}$$

15.7 Entropy and the Second Law of Thermodynamics

- **Entropy** - A function of the state of a system, it goes along with temperature, volume, pressure, and mass
- Change in entropy is

$$\Delta S = \frac{Q}{T}$$

  Where Q is the heat added and T is a constant temperature in a system in kelvin
- Since processes are not reversible, Q in the equation cannot be negative and thus change in entropy is always positive
- The second law of thermodynamics in terms of entropy is: The entropy of an isolated system never decreases. It can only stay the same or increase
- **General Statement of the Second Law of Thermodynamics** - The total entropy of any system plus that of its environment increases as a result of many natural processes

15.8 Order to Disorder

- Entropy can be considered the measure of disorder of the system, thus the second law of thermodynamics can be expressed as: natural processes tend to move to a state of great disorder
- An example of order to disorder is dropping a rock, some of its kinetic falling energy is converted to thermal energy which adds to the random disorderly movement of molecules

Biological Development

- As organisms develop over time, they seem to become more orderly
- This does not violate entropy as waste molecules without order are produced by their metabolism as they grow

Time's Arrow

- The second law also tells us which direction processes go, you can tell a movie is played backwards when you see it
- A decrease in entropy indicates a process occurred backwards in time, thus entropy is called *time's arrow*

15.9 Unavailability of Energy; Heat Death

- Another aspect of the second law: in any natural process, some energy becomes unavailable to do useful work
- As time goes on, energy is degraded and eventually converts to less useful forms such as thermal or internal enegry
- **Heat Death** - The prediction that all energy in the universe will, at some point, convert to heat energy

15.10 Statistical Interpretation of Energy and the Second Law

- **Microstate** - Specifies the position and velocity of every particle in a system
- **Macrostate** - Gives visible scale states of the system, such as temperature and pressure
- It is assumed that statistically, every microstate is equally probable
- The second law of thermodynamics implies that *those processes occur which are most probable*
- The second law does not *forbid* entropy decreasing, but makes it extremely unlikely
- On the macroscopic scale, there are so many molecules that deviating from what is expected is extremely unlikely

15.11 Thermal Pollution, Global Warming, and the Energy Resources

- **Thermal Pollution** - The thermal energy output by every heat engine, must be absorbed by the environment, which alters the Earth's ecology
- **Air Pollution** - Chemicals put into the air by burning fuels, mainly in the form of Carbon Dioxide, CO_2
- Carbon dioxide in the atmosphere absorbs some of the Earth's infrared radiation and keeps it from escaping, causing *global warming*
- **Carbon Footprint** - Refers to the negative impact of an activity by how much carbon dioxide it releases

16 Electric Charge and Electric Field

Atomic theory tells us liquids and solids are formed by the electric bonds between molecules. Many forces, including elastic and friction, result from the electric forces acting at the atomic level. Note that gravity is a separate force

16.1 Static Electricity; Electric Charge and its Conservation

- **Static Electricity** - An electric charge caused by friction
- There are two types of electric charge, positive and negative
- Unlike charges attract, like charges repel
- **Law of Conservation of Electric Charge** - The net amount of electric charge produced in any process is zero, no net charge can be created or destroyed

16.2 Electric Charge in the Atom

- Atoms are made of a positively charged nucleus surrounded by negatively charged electron cloud
- Nucleus is made of protons (positive charge), neutrons (no charge, "neutral") and the electron cloud is made of electrons (negative charge)
- Atoms can lose or gain electrons, in this case their charges are imbalanced and they become an ion
- **Polar** - Describes an object whose net charge is 0, but the charge is not distributed evenly
- Water is polar and can attract electrons that have "leaked off" of objects

16.3 Insulators and Conductors

- **Conductor** - An object that can transfer electricity easily
- **Insulator** - An object that does not transfer electricity well
- **Semiconductor** - A material that falls between being a conductor ans insulator
- Good conductors have loosely bound electrons that can move freely within a material, such electrons are called *free electrons* or conduction electrons
- Free electrons in a conductor can be attracted or repelled by a charge coming near the conductor

16.4 Induced Charge; the Electroscope

- **Charging by COnduction** - THe process of causing a charge in an object by touching with a charged object
- **Charge by induction** - Bringing a charged object close to a neutral object will separate the charges inside and make the two sides have opposite charges
- **Grounded** - Describe an object connected to the Earth, which is used as a reservoir of electric charge
- **Electroscope** - A device used to measure electric charge, uses two small metal leaves to show the magnitude of an object's charge, if it has a high charge, the leaves separate further apart
- **Electrometers** - More precise, sensitive electroscopes used in actual measurements

16.5 Coulomb's Law

- **Coulomb's Law** - Relates the force between two charges with the distance between them

  $$F = k\frac{Q_1Q_2}{r^2}$$

  Where Q_1 is the first charge, Q_1 is the second charge, r is the distance between the two charged, and k is a proportionality constant
- Only gives the magnitude, the direction of the force is along the line connecting the two charges and follows the rules of attraction between like or opposite charges
- **Coulomb** - The SI unit of charge
- k in Coulomb's Law is defined as

  $$k = \frac{8.988 * 10^9 N * m^2}{C^2}$$
- 1C of charge placed on two objects 1.0M apart would produce a force of 10^9N (a lot!)
- **Elementary Charge** - Smallest observable charge, defined as

  $$e = 1.6022 * 10^-19C$$

- Any electric charge can be given in terms of elementary charges, since the elementary charge the equivalent to the charge given by a single electron or proton
- Similar to the law of universal grvitatio, Coulomb's Law is an inverse square law $F \propto 1/r^2$
- **Permitivity of free space** - Another constant that can be used for Coulomb's Law,

$$\epsilon = \frac{1}{4\pi k}$$

- Coulomb's law is best used when determining the force between two objects whose size is much smaller than the distance between them, if this is not the case then r is defined as the distance betwene their centers
- **Point Charges** - Charges whose spacial size is negigible
- **Electrostatics** - The study of electrical charges at rest, Coulomb's Law gives the electrostatic force
- **Principle of Superposition** - The net force on any one charge will be the vector sum of the forces due to each of the others

16.6 Solving Problems Involving Coulomb's Law and Vectors

- The electrostatics force is a vector and follows the same vector math rules as any other

Vector Addition Review

- Vectors in multiple dimensions are added via their components,

$$F_x = F\cos\theta$$

$$F_y = F\sin\theta$$

- The directional components of each vector are added to get the component of the resultant vector

$$F_x = F_1x + F_2x$$

$$F_y = F_1y + F_2y$$

- The magnitude of the resultant vector is

$$F = \sqrt{F_x^2 + F_y^2}$$

- The direction of the resultant vector is

$$\tan\theta = \frac{F_y}{F_x}$$

Adding Vector FOrces; Principle of Superposition

- To determine the net force on an object with multiple charges acting upon it,
  - Use vector addition to determine the magnitude of the forces
  - Determine the direction of each force graphically
  - Combine the forces with their directions to determine the net force

16.7 The Electric Field

- An electric field, extends outward from every charge and permeates space
- **Electric Field** - E, at any point in space is defined as the force exerted on a tiny positive test charge placed at that point divided by the magnitude of the test charge

$$E = \lim_{q\to 0}\frac{F}{q}$$

- Electric field at a point is also equal to

$$E = k\frac{Q}{r^2}$$

- The force on a charge is thus

$$F = qE$$

- **Superposition Principle** - FOr an electric field, the electric field at a point is the sum of all the electric fields exrted at that poit

16.8 Electric Field Lines

- **Electric FIeld Lies -** Also called lies of force, these are used to visualize the direction of the electric field at different points in space, start at positive charges and end at negative charges
- Lines are drawn so that the number of lines attached to a charge is proportional to the magnitude of the charge
- The more lines there are in a region, the stronger the electric filed is at that region
- **Electric Dipole -** A system that contains two charges of opposite charge but equal magnitude
- For a charge between two closely spaces, oppositely charged flat parallel plates, the electric field is constant,
$$E = constant$$
- **Properties of Field lines**

Gravitational Field

- **Gravitational FIeld -** Aplies to every object that has mass, defined as force per unit mass
- Fields can be tought of as areas of possibility, a force CAN be applied there but only if something that can be influenced by that force is present

16.9 Electric Field and Conductors

- The electric field inside a conductor is zero in the static situation
- Any net charge on a conductor distributes itself on the surface
- The electric field is always perpendicular to the surface outside of a conductor
- These rules only apply to conductors, not nonconductors

16.10 Electric and Molecular Biology: DNA Structure and Replication

- Cellular processes are considered to be a result of random molecular motion plus the ordering effect of the electrostatic force
- The compounds inside of DNA are held together by electrostatic bonds since the molecules within them are polar
- **Hydrogen Bond -** Weak bond formed between a hydrogen ion and a negatively charged ion
- DNA shapes are so specific that small changes can make the electrostatic bonds between molecules impossible to form

16.11 Photocopy Machines and Computer Printers Use Electrostatics

- **Photoconductvitiy -** The propoerty of being nonconductive in the dark but conductive when exposed to light
- **Photocpiers -** Use the property of conductivity to replicate an image
- **Laser Printers -** Similar to photocopiers except they use a laser as their light source
- **Inkjet Printer -** Use tiny jets to spray small droplets of ink onto paper, so not use electric propoerties

16.12 Gauss's Law

- **Electic Flux -** The electric field passing through a given area, defined as
$$\phi_E = EA\cos\theta$$
Where E is electric field, A is area, and θ is the angle between the electric field direction and a line perpendicular to the area
- The number of electric field lines in an area is proportional to electric flux
$$N \propto \phi$$
- The total flux in a given area is defined as
$$\phi_e = \Sigma E_\perp \Delta A$$
Where $E_\perp$ is equal to $E\cos\theta$

- **Coulomb's Law -** The total flux in an enclosed area is proportional to the net charge encolsed by the surface,

$$\Sigma E_{\perp}\Delta\frac{Q_{encl}}{\epsilon_0}$$

  Where Q_{encl} is the net charge enclosed by a surface and ϵ_0 is the constant of proportionality

- Normally, surfaces with symmetry are chosen

- The electric field between two equally spaced, oppositely charged, parallel plates is

$$E = \frac{Q}{\epsilon_0 A}$$

17 Electric Potential

This section covers the use of electricity for energy, and applies the law of conservation of energy

17.1 Electric Potential Energy and Potential Difference

Electric Potential Energy

- The work done by an electric field to move a charge a distance is

$$W = Fd = qEd$$

Where F is force of the electric field, d is the distance moved, q is the charge, and E is the electric field

- Change in electic potential energy is the negative of the work done by electric force

$$PE_b - PE_a = -qEd$$

- Note the previous two equations apply only to a uniform electric field

Electric Potential and Potential Difference

- **ELectric Potential -** ELectric potential energyper unit charge, calculated by

$$V = \frac{PE_a}{q}$$

Which represents the potential energy of a charge at a point a divided by the magnitude of the charge

- **Difference in Potential -** Only *differences* in potential energy are maeningful, this is the difference in potential energy of two points a and b
- When a force does mechanical work, the change in potential between points a and b is

$$V_{ba} = V_b - V_a = \frac{PE_b - PE_a}{q}$$

- **Voltage -** Another name for potential difference, measured in *volts*
- Change in potential energy is equal to the product of the charge and volatge

$$\Delta = qV_{ba}$$

17.2 Relation Between Electric Potential and Electric Field

- Work done by an electric field is the negative of the charge times voltage,

$$W = -qV_{ba}$$

- Solving for electric field gives

$$E = -\frac{V_{ba}}{d}$$

- Notes that units for electric field can be volts per meter (v/m)or newtons per coulombs (N/C)

General Relation Between E and V

- It can be said that the electric field in a direction at any point is equal to the rate at which the electric potential decreases over distance in that direction
- FOr example, the electric field in the x direction over a small distance x is

$$E_x = -\frac{\Delta V}{\Delta x}$$

Breakdown Voltage

- **Breakdown -** Occurs in air when electric field exceeds about $3 * 10^6 V/m$, electrons are knocked out of molecules in air

17.3 Equipotential Lines and Surfaces

- **Equipotential Surface -** A surface where all points are at the same potential
- An equipotential surface must be perpendicular to the electric field at any point
- The entire volume of a conductor must be entirely at the same potential in the static case, since there is no electric field in the conductor

17.4 The Electron Volt, a Unit of Energy

- **Electron VOlt** - Symbol is eV, unit of energy equivalent to one elementary charge moving through 1V of potential difference

$$1eV = 1.6022 * 10^-19J$$

- Not a proper unit, should be converted to J

17.5 Electric Potential Due to Point Charges

- **Coulomb's Potential** - Electric potential at a distance from a point charge is

$$V = k\frac{Q}{r} = \frac{1}{r\pi\epsilon_0}\frac{Q}{r}$$

  For a single point charge, V=0 and r=∞

- Finding the electric field due to multiple point charges, can be found by adding each electric field vectorially, all that is needed is the charge and position of each point charge

17.6 Potential Due to Electric Dipole; Dipole Moment

- **Electric Dipole** - Two point charges of opposite charge but equal magnitude, divided by a distance
- The potential difference at a point due to a dipole is the sum of the potentials due to each charge,

$$V = \frac{kQ}{r} + \frac{k(-Q)}{r+\Delta r} = kQ(\frac{\Delta r}{r(r+\Delta r})$$

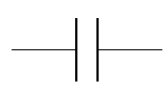
- **Dipole Moment** - Potential difference cause by a dipole at an arbitrary point, equal to distance between charge times magnitude of charge,

$$V \approx \frac{kp\cos\theta}{r^2}$$

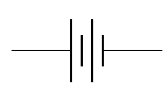
- **Polar Molecules** - Molecules that have a dipole moment

17.7 Capacitance

- **Capacitor** - A device that can store electrical charge, consisting of two conducting objects placed near each other
- Also called condensers as they condense large amounts of electrical energy
- In circuit diagrams, the symbol for capacitors is



- IN circuit diagrams, the symbol for a voltage source is



  Where the larger end represents the positive end and the smaller end represents the negative

- The amount of charge held by each plate in the capacitor is calculated by

$$Q = CV$$

  Where V is the potential difference bteween the plates and C is the capacitance

- **Capacitance** - Measured in coulombs per volt or *farads*
- Capacitance between two parallel plates can be calculated by

$$C = \epsilon_0\frac{A}{d}$$

  Where A is the area of the plates and d is the distance between the plates, ϵ_0 is the permittivity of free space

- **Condenser Microphone** - Uses a capacitor in the microphone to detect changed in air pressure

Derivation of Capacitance for Parallel Plate Capacitor

- Plugging the equation for voltage into $C = \frac{Q}{V}$ gives

$$C = \frac{Q}{(Q/A\epsilon_0)d} =$$

  Which simplifies to

$$C = \epsilon_0\frac{A}{d}$$

17.8 Dielectrics

- **Dielectric** - Materials that break don less readliy than air, usually placed between plates in a capacitor
- **Dielectric Constant** - Unique to various materials, this increases the capacitance of a capacitor depending on the dielectric
- **Dielectric Strength** - The maximum electric field before breakdown occurs

Moleculr Description of Dielectric -

- Electric field within the dielectric of a capacitor is less than it would be in air, which causes a voltage decrease
- To keep Q constant in $Q = CV$, capacitance must increase

17.9 Storage of Electric Energy

- The work done to move the total charge Q from one plate to another is

$$W = Q\frac{V_1}{2}$$

- Thus the potential energy stored in a capacitor is

$$PE = \frac{1}{2}QV$$

- The potential can also be calculated as

$$PE = \frac{1}{2}\epsilon_0 E^2 Ad$$

Health Effects

- Ventricular Fibrillation occurs when the heart beats fast irregular rates
- This can be solved by a defribillator which is a capacitor charged to a high voltage, which stops the heart and is normally followed by regular heart rhythm

17.10 Digital; Binary Numbers; Signal Voltage

- **Supply Voltage** - A constant voltage used to power devices
- **Signal Voltage** - A variable voltage used to affect something else
- **Analog** - Describes signal voltages that vary continuously
- **Digital** - Voltages with only two possible values, on or off
- **Decimal** - Latin for 10
- **Binary** - Describes a number system where each digit or *bit* has two possibilities, 1 or 0
- Each place in binary increases in value by a power of 2
- Digital information is contained in a *byte* which contains 8 bits allowing for $2^8 = 256$ possible values
- **Analog to Digital Converter** - A device that converts analog voltages to digital
- **Digital to Analog Converter** - A device that converts digital voltages to analog
- **Quantization Error** - Loss in the original analog signal by an analog to digital converter, can be mitigated by increasing bit depth and sampling rate
- **Bit Depth** - Number of bits for a given voltage
- **Sampling Rate** - Number of times per second the original signal is measured
- Digital data can be *compressed* in order to tak eup less storage space
- **Bandwidth** - Fixed range of frequencies allotted to a radio or tv station or internet connection

Noise

- **Noise** - Unwanted electrical signals from external sources

17.11 TV and Computer Monitors; CRTs, Flat Screens

CRT

- **CRT** - Stands for Cathode Ray Tube, a device that depends on thermionic emission, allows for deflection of an electron beam
- The electrodes in a CRT are the cathode, or negative, and anode, or positive
- CRTs emit rays of negative charge called cathode rays but now known as electrons
- **Grid** - Component of CRTs that limits how many electrons can escape by producing a negative voltage
- **Color Screens** - May also use CRTs but each pixel has red, blue, and green phsophors that glow when hit by an electron

Flat Screens and Addressing PIxels

- **Pixel** - Short for picture elements, consist of 3 subpixels, one red, blue, and green, to produce a picture
- **Addressing** - The process of providing each pixel in a display with the correct voltage, done by providing voltage to only one row of pixels at a time

Active Matrix

- **Active Matrix** - Used in displays, each pixel has a thin-film transistor which can block or allow a voltage

Oscilloscopes

- **Oscilloscope** - A device used to amplify, measure, and display electrical signals

17.12 Electrocardiogram

- **Electrocardiogram (EKG)** - Used to record tha potential changes in a person's heart
- Cells are polar, when muscle cells contract they "depolarize" which emits a voltage
- When the heart beats the tiny voltages produced by each muscle cell add up and can be measured

18 Electric Currents

When charges move along a conductor, an electric field is needed. Current requires a potential difference to flow

18.1 The Electric Battery

- Contact between dissimilar metals with some moisture can produce a voltage

Electric Cells and Batteries

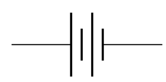
- **Electrodes** - Two plates of dissimilar metals in a battery
- **Electrolyte** - A solution or acid the electrodes are submersed in
- **Electric Cell** - A combination of electrodes and electrolytes, multiples connected cells form a battery
- **Terminal** - The part of each electrode outside the electrolye

Electric Cars

- **Lithium-ion** - A battery type used by electric cars whic uses lithium as its anode and carbon as its cathode
- **Range** - How far an electric vehicle can travel on a single charge

18.2 Electric Current

- The symbol for a battery is



- **Electric Current** - Any flow of charge, defined as the net charge that passes through a wire per unit time,
$$I = \frac{\Delta Q}{\Delta t}$$
- **Ampere** - Unit of current defined as coulombs per second
$$1A = 1C/s$$
- Current can only flow when there is a continuous path or *complete circuit*
- **Closed Circuit** - Describes a circuit with a break, such as a cut wire
- **Conventional Current** - The direction where positive charge flows
- **Electron Current** - The direction where negative charge flows
- **Ampere-Hour** - Unit of charge, defined as
$$1A * h$$

18.3 Ohm's Law: Resistance and Resistors

- Current is proportional to voltage
$$I \propto V$$
- **Resistance** - Caused by collisions within a conductor, defines as the proportionality factor between V and I
- **Ohm's Law** - Relates voltage, current, and resistance
$$V = IR$$
- **Ohm** - Unit of resistance, defined as
$$1.0\Omega = 1.0V/A$$
- The circuit symbol for a resistor is

- **Voltage Drop** - The voltage decrease across a resistor

Some Helpful Clarification

- Voltage is applied across a wire
- Current flows through the wire
- Current is not a vector
- Input and output charge and current are always the same

18.4 Resistivity

- The resistance of a conductor is calculated with

$$R = \rho \frac{l}{A}$$

WHere ρ is the constant of proportionality of the material or *resistivity*, l is the length of the conductor, and A is the cross-sectional area

Temperature Dependence of Resistivity

- Resistance of metals generally increases with temperature
- The change in resistivity due to a change in temperature can be calculated by

$$\rho_T = \rho_0[1 + \alpha(T - T_0)$$

Where T_0 is the reference temperature and α is the temperature coefficient of resistivity which is unique to each material

- Very large temperature variances may require equations different than the previous one as α itself can vary with temperature

18.5 Electric Power

- Convenctional filament light bulbs only release about 10
- The power transformed by an electrical device is calculated by

$$P = \frac{energy\ transformed}{time} = \frac{QV}{t}$$

- Charge that flows per second, Q/t, is current, I thus power is

$$P = IV$$

WIth the same units as mechanical power, Watts

- Power in terms of resistance is

$$P = I^2R$$

or

$$P = \frac{V^2}{R}$$

18.6 Power in Household Circuits

-
- WHen a wire carries ore current than is safe, it is considered *overloaded*
- **Circuit Breaker** - Switches on circuits that open the circuit when it exceeds a safe value
- **Short** - An instance in which two wires in a circuit touch when they are not supposed to
- **Parallel Circuits** - CIrcuits Designed so that each connected device is individually connected to the voltage source

18.7 Alternating Current

- **Direct Current** - Electricity that moves steadily in one direction, called DC
- **ALternating Current** - Electric current that reverses directions in a sinusoidal fashion
- The voltage and current produced by an AC source is sinusoidal, the voltage as a function of time is

$$V = V_0 \sin 2\pi f t = V_0 \sin \omega t$$

Where f is the oscillations made per second and ω is equal to $2\pi f$

- **Peak Voltage** - The highest magnitude of voltage in AC, represented by V_0
- Current can be calculated by

$$I = I_0 \sin \omega t$$

- **Peak Current** - The maximum magnitude of current in AC, calculated by

$$I_0 = V_)/R$$

- Average power can be calculated by
$$P = \frac{1}{2} I_0^2 R$$
or
$$P = \frac{1}{2} \frac{V_0^2}{R}$$
- **Root Mean Square** - Also called *rms*, this is the average value of voltae or current in AC, calculated by
$$I_{rms} = 0.707 I_0$$
and
$$V_{rms} = 0.707 V_0$$

18.8 Microscopic View of Electric Current

- **Drift Velocity** - The average velocity of electrons moving in a conductor
- The magnitude of current can be calculated by
$$I = neAv_d$$
Where n is the total number of electrons, e is the fundamentl charge, A is the cross sectional area through which current flows, and v_d is the drift velocity
- Electric fields travel at the speed of light

18.9 Superconductivity

- **Superconducting** - Occursat very low temperatures, this is when the resistivity of a material becomes nearly 0
- Occurs at a critical temperature, T_C usually within a few degrees of absolute 0
- Current work is focused on making materials with higher critical temperature to remove the need for intense cooling

18.10 Electrical Conduction in the Human Nervous System

- **Neuron** - the most basic element of the nervous system, transmits electricl signals
- **Dipole Layer** - Difference in charge across the cell membrane of a neuron, resting potential across the layer is
$$V_{inside} - V_{outside}$$
- **Action Potential** - The voltage across a neuron when it is "fired"
- Ions are the main driver behind neuron activation

19 DC Circuits

Electrical circuits are fundamental to modern life, visual representations of electrical circuits use symbols for each component called schematics

19.1 EMF and Terminal Voltage

- **Source -** Also called *electromotive force*, this is a device that provides electrical energy
- **EMF -** The potential difference between two terminals of a source
- **Internal Resistance -** The resistance within a source
- **Terminal Voltage -** The voltage between two terminals, calculated by

$$V_{ab} = V_a - V_b$$

- Chemical reactions in a battey produce a voltage, represtned by ξ if there is no current being drawn, then

$$V_{ab} = \xi - Ir$$

- Internal resistance is assumed to be negligible

19.2 Resistors in Series and Parallel

- **Series -** Describe the connection between resistors when they are connected in a single path, end to end
- Total voltage is equal to the sum of the voltages across each resistor

$$V = V_1 + V_2 + ... = IR_1 + IR_2 + ...$$

or

$$V = IR_{eq}$$

- R_{eq} is the sum of the resistances, for resistors in series this is

$$R_{eq} = R_1 + R_2 + ...$$

- **Parallel -** Describes connection in a circuit where the current branches off into multiple paths
- Splits in current cause it to break into components, so the total current is

$$I = I_1 + I_2 + ...$$

Where I_n is

$$\frac{V}{R_n}$$

- The sum of resistance in resistors in parallel is

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + ...$$

19.3 Kirchhoff's Rules

- **Junction Rule -** at any junction point, the sum of all currents entering the junction must equal the sum of all currents leaving the junction
- **Loop Rule -** The sum of the changes in potential around any closed loop of a circuit must be zero
- Roller Coaster analogy - As a roller coaster mves along the track, energy is converted between kinetic and potential (junction rule) and once it returns, its potential energy is the same as it started (loop rule)

19.4 EMFs in Series and in Parallel; Charging a Battery

- Sum of EMFs from sources in series is the sum of the individual EMFs
- Batteries in parallel with the same EMF will only have to produce a fraction of the current needed for the circuit, so they wear out less

Safety when Jump Starting

- When jump starting a car, connect the negative terminal of the good battery with a piece of bare metal on the dead car and connect the positive terminals of each battery
- Do not connect the negative terminal of the good battery to that of the bad one as it may spark hydrogen gas leaking from the bad battery

19.5 Circuits Containing Capacitors in Series and in Parallel

- The equation for capacitors in parallel is

$$C_{eq} = C_1 + C_2 + ...$$

- The equation for capacitors in series is

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + ...$$

19.6 RC Circuits - Resistor and Capacitor Series

Capacitor Charging

- **RC Circuit** - A circuit containing both resistors and capacitors
- The voltage across a capacitor is given by

$$V_C = \xi(1 - e^{-t/RC})$$

  Where ξ is the voltage of the source, e is the base of the natural log, R is the resistance of the resistor in the RC circuit, and C is the capacitance

- The charge within a capacitor is given by

$$Q = Q_0(1 - e^{-tRC})$$

- **Time COnstant** - The product of resistance and capacitance in an RC circuit, represented by τ

$$\tau = RC$$

- The voltage across a resistor in an RC circuit is calculated by

$$V_R = \xi e^{-t/RC}$$

  Thus current is

$$I = \frac{\xi e^{-t/RC}}{R}$$

Capacitor Discharging

- The voltage in a capacitor as it discharges is

$$V_C = V_0 e^{-t/RC}$$

- The charge as it discharges is

$$Q = Q_0 e^{-t/RC}$$

Medical and Other Applications of RC Circuits

- **Sawtooth Voltage** - A voltage whose graph resembles a saw blade
- **Electronic Pacemaker** - A medical device that uses an RC circuit to deliver evenly spaced electrical pulses to the heart

19.7 Electric Hazards

- Electric current running through the body may cause burns or stimulate nerves annd muscles
- **Ventricular Fibrillation** - Occurs when a high current passes trhough the heart, it will cause irregular rhythms and blood is not properly pumped

Safe Wiring

- Power outlets contain 3 wires, the hot wire from which current flows, the neutral wire which carries away current, and the ground wire that connects to ground
- **Circuit Breakers** - Devices that dtecte when too much current flows thruogh a wire, if the current passes over the threshold then it stops current from flowing
- **Leakage Current** - Current that flows along an unintended path

19.8 Ammeters and Voltmeters - Measurement Affects the Quantity being Measured

- **Ammeter** - A device that measures current
- **Voltmeter** - A device that measures voltage
- **Galvanometer** - An analog ammeter or voltmeter that works by using the force between a magnetic field and a current carrying coil

How to Connect Meters

- Ammeters must be connect directly within a circuit
- Voltmeters are connect in parallel with the points between which voltage is being measured

Effects of Meter Resistance

- Sometimes, a meter can give a misleading reading
- Voltmeters with high resistances compared to the circuit they are measuring have more accurate readings
- The more sensitive a galvanometer is, the less its effect on the circuit is

Other Meters

- **Multimeter** - A meter that measure multiple electrical units
- **Ohmeter** - A meter that measures resistance

Digital Meters

- Digital meters do not use a galvanometer but rather use semiconductor devices to measure
- Their precision is extremely high, often around 0.01%

20 Magnetism

Electricity and magnetism are closely related, electric currents produce a magnetic field

20.1 Magnets and Magnetic Field

- **Poles** - The two ends/faces of a magnet, do not confuse them with electric charge
- **Magnetic Monopole** - A magnet with a single poles instead of two
- **Ferromagnetic** - Metals and alloys that are not iron but are influence by magnets
- **Magnetic Field** - The magnetic analog of an electric field surrounding an electric charge
- **Magnetic Field Lines** - Used to display the direction and magnitude of a magnetic field at any point

Earth's Magnetic Field

- Earth's geographic poles do not coincide with its magnetic poles, the north ones are about 1000km apart
- **True North** - The geographic north pole
- **Magnetic Declination** - The angle between the direction of a compass at a point and true north
- **Magnetic Dip** - Angle the Earth's magnetic field makes with the horizontal at a point

Uniform Magnetic Field

- Simplest type of magnetic field
- Magnetic field between two flat parallel poles is nearly uniform if their surface area is much greater than their separation

20.2 Electric Currents Produce Magnetic Fields

- An electric current produces a magnetic field
- **Right Hand Rule** - Used to determine the direction of a magnetic field, grasp wire with your right hand with your thumb pointed in the direction of positive current, the direction of your fingers is the direction of the magnetic field

20.3 Force on an Electric Current in a Magnetic Field; Definition of B

- The direction of the force is always perpendicular to the direction of the current and also perpendicular to the direction of the magnetic field, also given by the right hand rule
- The force on a wire in a uniform magnetic field is given by

  $$F = IlB\sin\theta$$

  Where I is current, l is the length of the wire in the magnetic field, B is the strength of the field, and θ is the angle between the flow of current and magnetic field
- The maximum force on a wire is given by

  $$F_{max} = IlB$$

- Magnetic fields have units of *Teslas* which is

  $$1T = \frac{1N}{A * m}$$

- **Gauss** - Another unit, equal to 10^-4 Teslas

20.4 Force on an Electric Charges Moving in Magnetic Field

- The force on a single charge moving through an electric field is

  $$F = qvB\sin\theta$$

  Where q is the charge, v is the velocity of the charge, and B is the strength of the magnetic field
- Anothe rright hand rule is used to determine the direction of force for a positive charge, point your fingers in the direction of the charge's velocity, bend them so they point in the direction of the magnetic field, the direction of your thumb is the direction of the force

- The time needed for a charge moving a constant speed to make one revolution in a uniform magnetic field is

$$T = \frac{2\pi m}{qB}$$

Frequency, called *cyclotron frequency* is

$$f = \frac{2\pi m}{qB}$$

Aurora Borealis

- **Aurora Borealis** - Phenomenon caused by charged ions entering the atmosphere near the poles
- As a particle approaches the north pole, its magnetic field gets stronger, enough particles with strong magnetic fields ionize the air, creating light

The Hall Effect

- **Hall Effect** - The potential difference across the sides of a conductor in a magnetic field, as charges move, they are influenced by the outside magnetic field
- **Hall EMF** - The potential difference caused by the Hall effect

20.5 Magnetic Field Due to a Long Straight Wire

- The magnetic field produced by a wire is proportional to the current it carries and inversely proportional to the distance from the wire

$$B \propto \frac{I}{r}$$

- The magnetic field can be calculated by

$$B = \frac{\mu_0}{2\pi}\frac{I}{r}$$

Where μ_0 is the permeability of free space, equal to $\mu_0 = 4\pi * 10^-7T * m/A$

20.6 Force Between Two Parallel Wires

- The force exerted by a magnetic field, B_1, on another wire, l_2 is given by

$$F_2 = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d} l_2$$

Definition of the Ampere and the Coulomb

- One ampere is defined as the current flowing in each of two long parallel wires 1m apart, which results in a force of exactly $2 * 10^-7N$ Per meter of length of each wire
- Coulombs, C=A*s, are defined using this definition of the ampere

20.7 Solenoids and Electromagnets

- **Solenoid** - A coil of wire consisting of many loops
- The magnetic field produced by a solenoid is

$$B = \frac{\mu_0 N I}{l}$$

Where N is the number of turns in the coil and l is the length of the coil

- **Electromagnet** - A solenoid with an iron core
- Solenoids can have an iron core placed partially within the coil, so running a currnet through it would exert a force on it

Magnetic Circuit Breakers

- These circuit breakers work by pulling an iron plate away from the circuit once enough current passes through to exert enough force to do so

20.8 Ampere's Law

- **Ampere's Law** - Used to determine the magnetic field around a wire of any shape and length,

$$\Sigma B \Delta l = \mu_0 I_{encl}$$

Where ΣB is the sum of the magnetic field parallel to length and I_{encl} is the current in a closed path

- Ampere's law agrees with the equations for magnetic field around a long straight wire, and field inside a solenoid

20.9 Torque on a Current Loop; Magnetic Moment

- The torque on a current loop caused by current flow is

$$\tau = IaB\frac{b}{2}$$

  Where a is the length of the vertical arm o the coil and b is the width of the coil

- The sum of the torques acting on each level arm is

$$\tau = IAB$$

  Where A=a*b

- Torque for a coil with multiple loops is

$$\tau = NIAB$$

- For coils that make an angle with the magnetic field, torque is

$$\tau = NIAB\sin\theta$$

- **magnetic Dipole Moment -** M=NIA

20.10 Applications: Motors, Loudspeakers, Galvanometers

Galvanometer

- Galvanometers contain a coil with a pointer attached to a spring, when a magnetic field is applied the coil produces a torque and pointer rests at the point where the torque in the spring and coil are equal,

$$\tau = NIAB\sin\theta = \tau_s = k\psi$$

  Where

$$\psi = \frac{NIAB\sin\theta}{k}$$

Electric Motors

- **Electric Motors -** Convert electrical energy to rotational mechanical energy
- The coil of a motor is mounted on an iron cylinder called ther rotor
- To spin continuously in one direction, motors use brushes and commutators to alternate the current flow
- The brush causes the commutator to reverse current direction and continue rotating

Loudspeakers and Headsets

- Speaker wires are connected to a coil of wire, attached to a speaker cone
- The coil is place within a permanent magnet, when it receives an alternating current flow, it moves within the magnet, causing the cone to move and produce sound

20.11 Mass Spectrometer

- **Mass Spectrometer -** Device that measures the masses of atoms
- Charges moving at a velocity of

$$V = \frac{E}{B}$$

  will passe throug the spectrometer's slits

- The mass of a charge is given by

$$m = \frac{qB'r}{v}$$

  WHere B' is the magnetic field, and r is the radius of the circular path charges take to reach the detector in a mass spectrometer

20.12 Ferromagnetism: Domains and Hysteresis

- **Ferromagnetic** - Describes amterials that can be made into strong magnets

Sources of Ferromagnetism

- Iron is made of tiny domains that behaves like its own tiny magnet
- In a magnet, all the domains are aligned om pme direction
- **Curie Temperature** - The temperature above which a given material cannot be a magnet, 1043K for iron

Magnetic Permeability

- When an iron core is placed inside a solenoid, its magnetic field increases significantly
- The total field can be calculated with
$$B = \mu NI/l$$
Where μ is the magnetic permeability of the magnetic material

Hysteresis

- Measurements on magnetic materials are often made using a torus
- **Saturation** - Occurs when every domain in a piece of iron aligns with each other
- **Hysteresis** - The phenomenon that the plot between B_0 and B does not retrace itself
- **Hysteresis Loop** - The graph of a hysteresis plot, electromagnets circulating through the hysteresis loop produce friction as the domains repeatedly align and unalign

21 Electromagnetic Induction and Faraday's Law

After the discovery that electric currents produce a magnetic field, people discovered the opposite was also true and that magnetic fields can produce an electric field. This has has world changing applications, including the electric generator.

21.1 Induced EMF

- A constant magnetic field does not produces an electric current, but a changing one does, called an *induced emf*
- **Electromagnetic Induction** - The process through which electric current is produced from changing magnetic waves

21.2 Faraday's Law of Induction; Lenz's Law

- Emf is proportional to the rate of change of the magnetic flux, ϕ_0, defined as

$$\phi_0 = BA\cos\theta$$

  Where B is the magnetic field, A is the area of the loop the flux is passing through, and θ is the angle between B and a line perpendicular to the area of the loop

- **Weber** - Units of magnetic flux, defined as

$$1Wb = 1T * m^2$$

- **Faraday's Law of Induction** - The emf induced in a circuit is calculated by

$$1\xi = -\frac{\Delta\phi_B}{\Delta t}$$

- A current produced by an induced emf moves in a direction so that the magnetic field created by that current opposes the original change in flux
- An induced emf is always in a direction that opposes the original change in flux that caused it

21.3 EMF Induced in a Moving Conductor

- The emf induced in a moving conductor, also called $motionalemf$ is given by

$$\xi = Blv$$

  Where v is the velocity of the conductor and l is the width of the loop

21.4 Changing Magnetic Flux Produces Electric Field

- A changing magnetic flux produces an electric field
- The effective field in a moving in a magnetic field rod is equal to

$$E = vB$$

  Where v is the velocity of the rod and B is the magnetic field

21.5 Electric Generators

- **Electric Generator** - A device that converts mechanical energy into electrical energy
- Works by rotating a coil in a magnetic field to induce a current in it

Alternators

- **Alternators** - Replace DC generators in car generators
- Alternators work by having the electromagnet being fed by the car battery and rotates by a belt from the engine

Deriving the Generator Equation

- The radians per second output of a generator is

$$V_{rms} = \frac{NB\omega A}{\sqrt{2}}$$

  Where $\omega = 2\pi f$, N is the number of turns in the coil, and A is the area of the loop

21.6 Back EMF and Counter Torque; Eddy Currents

Back EMF in a motor

- As a motor turns, it produces a counter emf that acts to oppose motion
- As motor speed increases, so does back emf

Counter Torque in Generator

- **Counter Torque -** Torque that opposes the rotation of a generator
- Caused by the magnetic field surrounding a current carrying coil
- Strength of counter torque increases with electrical load

Eddy Currents

- **Eddy Currents -** Curents caused within a conductor from an outside that opose changes in the field they move through
- **Magnetic Damping -** The use of eddy currents to decrease the vibrations in a vibrating system
- Eddy currents can waste the energy in a motor as they may oppose some of its motion

21.7 Transformers and Transmission of Power

- **Transformer -** Device consisting of a primary and secondary coil of wire, desgined to transfer magnetic flux from primary to secondary coil
- When ac is applied to primary coil, voltage is induced in the secondary coil, calculated by

$$V_S = N_S \frac{\Delta \phi_B}{\Delta t}$$

  Where $\frac{\Delta \phi_B}{\Delta t}$ is the rate of change of magnetic flux

- The following are true

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}$$

  and

$$\frac{I_S}{I_P} = \frac{N_P}{N_S}$$

  Where S means secondary and P means primary

- **Step-Up Transformer -** A transformer where the secondary coil gives a higher voltage than the primary coil
- **Step-Down Transformer -** A transformer where the primary coil gives a lower voltage than the primary coil
- Car ignitions use step-up transformers to convert the battery voltage to an extremely high one
- Transformers are used in energy transmission as they can produce the high voltages required to move electricity long distances

WIreless Tranmission of Power-Inductive Charging

- Wireless chargers use a primary coil in the charger that induces a current in the secondary coil of the device being charged
- The induced current recharges batteries, but it must be done over a very short distance to maintain efficiency

21.8 Information Storage: Magnetic and Semiconductor; Tape, Hard Drive, RAM

Magnetic Storage: Read/Write on Tape and Disks

- Digital information is written onto disks by heads that act as tiny electromagnets and interact with ferromagnetic surfaces of the disks
- Some signals written may be analog, which is converted to digital using bits to store magnitude of information
- **Optical Drive -** A device that uses a laser to reflect off of surfaces instead of electricity to read data

Semiconductor Memory: DRAM, Flash

- **Random Access Memory -** ALso called RAM, this is a method of storage that stores what your computer is working on for quick access

- RAM stores bits as electrical signals in semiconductor devices, flash memory does the same but for long term storage
- **Dynamic RAM** - Uses arrays of MOSFETs as on/off switches to store bit values
- Cells in DRAM consist of a transistor and capacitor, they are written on by having the capacitor provide a high enough voltage to turn the transistor on
- Cells are read by detecting a change in voltage across their capacitor
- **MRAM** - Stands for Magnetoresistive RAM, this is RAM that does not required power or refresh to maintain storage

21.9 Applications of Induction: Microphone, Seismograph, GFCI

Microphone

- Some microphones act opposite of a loudspeaker, they have a small coil connected to a membrane that vibrates with sound
- The coil vibrates with the membrane and induces an emf which is converted to a signal

Credit Card Reader

- Swiping a credit card passes its magnetic strip across a device that connects the information contained within the strip to your credit card account

Seismograph

- Measures the intensity of an earthquake using a magnet and coil, when the ground shakes an emf is induced

Ground Fault Circuit Interrupter

- A device that is meant to protect humans
- It detects an imbalance in the hot and neutral wires of a circuit, after which it trips and stops current flow

21.10 Inductance

Mutual Inductance

- When 2 coils are near each other, the change in flux in coil 2 is given by

$$\epsilon = -M\frac{\Delta I_1}{\Delta t}$$

  Where I_1 is the current running through the first coil, and M is the constant of proportionality called mutual inductance
- Mutual inductance has units in Henrys $1H = 1\Omega * s$
- Note that M is not universally constant, but rather it does not depend on the current flowing through coil 1

Self-Inductance

- A changing magnetic flux in a coil produces an emf that opposes change in flux, the reverse emf is calculated by

$$\epsilon = -L\frac{\Delta I}{\Delta t}$$

  Where L is the constant of proportionality called self inductance, also measured, and depends on shape and size, as does mutual inductance
- **Inductor** - A coil that produces self inductance

21.11 Energy Stored in a Magnetic Field

- The energy carried in an inductance carrying a current is

$$U = \frac{1}{2}LI^2$$

  Where L is inductance and I is current
- **Energy Density** - Energy stored per unit volume, given by

$$u = \frac{1}{2}\frac{B^2}{\mu_0}$$

21.12 LR Circuits

- **LR Circuit -** A circuit that contains an inductor and resistor, the current running through an LR circuit is given by

$$I = (\frac{V_0}{R})(1 - e^{-t/\tau})$$

  Where τ is the time constant given by

$$\tau = \frac{L}{R}$$

- After enough time, the current in an inductor will reach a steady value and

21.13 AC Circuits and Inductance

- AC power sources produce sinusoidal voltages with frequency f, the current generated is given by

$$I = I_0 \cos 2\pi f t$$

  Where I_0 is peak current

Resistor

- The voltage produced by an AC power source is given by

$$V = V_0 \cos 2\pi f t$$

  Where $V_0 = I_0 R$ is peak voltage

- Because voltage and current move in the same direciton at the same time, they are said to be in phase

Inductor

- The following is true

$$\frac{\Delta I}{\Delta t} = \frac{V}{L}$$

- Current lags voltage by 90° for an inductor
- **Inductive Reactance -** Constant of proportionality that is used to calculated current and voltage in an inductor circuit

$$X_L = 2\pi f L$$

- The voltage in an inductor circuit is calculated by

$$V = IX_L$$

Capacitor

- Current leads the voltage by 90° for a capacitor
- Only a resistance will dissipate energy as thermal energy in an ac circuit
- **Capacitive Reactance -** Similar to inductive reactance, calculated by

$$X_C = \frac{1}{2\pi f C}$$

  and is used in

$$V = IX_C$$

- Capacitors impede low frequency signals and inductors impede high frequency signals, they "filter" what they impede

21.14 LRC Series AC Circuit

- **LRC Circuit -** A circuit thay contains a resistor (R), inductor (L), and capacitor (C)
- Let V_R, V_L, V_C represent the voltage across a resistor, inductor, and capacitor respectively, the total voltage at a given instance in time is the sum of these,

$$V = V_R + V_L + V_C$$

- Sometimes a subscript 0 is used to identify the peak R L or C in a circuit

Phasor Diagram

- **Phasor Diagram -** Visualization of AC circuits, use arrows to represent each voltage with length representing magnitude and angle represents direction
- The sum of the vectors in a phasor diagram show the total voltage

- As time passes, the vectors in the phasor diagram rotate and so does the peak voltage, denoted by V_0,
- The angle the peak voltage forms with $V_R 0$ is represented by ϕ
- The voltage at a point in time is calculated by

$$V = V_0 \cos 2\pi f t + \phi$$

- **Impedance -** Analogous to resistance and reactance, used in

$$V_0 = I_0 Z$$

or

$$V_{rms} = I_{rms} Z$$

- The total impedance in a circuit is calculated by

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

- The phase angle ϕ is calculated by

$$\tan phi = \frac{X_L - X_C}{R}$$

or

$$\cos \phi = \frac{R}{Z}$$

- The average power in an RLC circuit is given by

$$P = I_{rms} V_{rms} \cos phi$$

Where $\cos \phi$ is referred to as the power factor in the circuit

21.15 Resonance in AC Circuits

- The rms current in an LRC circuit is given by

$$I_{rms} = \frac{V_{rms}}{Z}$$

- The maximum frequency of an LRC circuit is given by

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{1}{LC}}$$

- When the true frequency of an LRC circuit is equal to its maximum frequency, that is $f = f_0$, the circuit is said to be *in resonance*
- When $X_C = X_L$ impedance is only caused by resistance
- A circuit with only a capacitor and inductor will oscillate at f_0 and is called an electromagnetic oscillation

22 Electromagnetic Waves

The greatest discovery of 19th century electromagnetic theory was that waves of EM fields can travel through space. Which led to an increase in communications worldwide with the advent of the telegraph, radio,and cellphones

22.1 Changing Electric Fields Produce Magnetic Fields; Maxwell's Equations

- **Maxwell's Equations -** The basic equations for all electromagnetism
- The equations summarized in words are
  1. A general form of Coulomb's law that relates electric field to its source
  2. A similar law for the magnetic field, except that magnetic field lines are always continuous, they do not begin or end
  3. An electric field is produced by a changing magnetic field
  4. A magnetic field is produced by an electric current or by a changing electric field

Maxwell's Fourth Equation (Ampere's Law Extended)

- Two different surfaces bound by the same enclosed path and the same magnetic field will have the same current passing through them
- **Displacement Current -** Represented by I_D, this is the changing electric field between two plates of a capacitor
- I_D is given by
$$I_D = \epsilon_0 \frac{\Delta \phi E}{\Delta t}$$

22.2 Production of Electromagnetic Waves

- **Electromagnetic Waves -** Waves produced y oscillations in electric and magnetic fields
- **Radiation Field -** The electromagnetic field far away from an antenna, where the fields for loops
- The energy carried by EM waves follows a reverse square law
- The electric and magnetic fields at any point are perpendicular to each other, and to the direction of wave travel
- **Plane Waves**Farfrom an antenna, a wave is very flat and spread over a large area
- EM waves are waves of fields, they can move through empty space
- Accelerating electric charges gives rise to EM waves
- The speed of EM waves is given b
$$v = c = \frac{E}{B}$$
Where c is defined as
$$\frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

22.3 Light as an Electromagnetic Wave and the Electromagnetic Spectrum

- Light is a form of EM waves travels at
$$c = \ 3.00 * 10^8$$
m/s
- The freqeuncy and wavelength of EM waves are modeled by
$$c = \lambda f$$
- The ranges of frequencies of EM waves along with their respective wavelengths are shown in the *electromagnetic spectrum*
- If EM waves travel through materials with an electric permittivity and magnetic permeability different that that of free space's, the speed of the waves is given b
$$v = \frac{1}{\sqrt{\epsilon \mu}}$$

22.4 Measuring the Speed of Light

- One of the most precise methods of determining the speed of light involved hitting a rotating eight-sided mirror with light
- The light would reflect and hit another mirror a large distance away and back
- The rotating mirror had to move at a specific rate in order for the reflected light to be measured, using this rate of rotation and the distance of the far mirror was used to determine the speed of light
- The speed of light in a vacuum is given by

$$c = 2.99792458 * 10^8$$

  m/s

22.5 Energy in EM Waves

- The energy stored per unit volume, or energy density, where an EM wave is present is given by

$$u = u_E + u_B = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0}$$

  or

$$u = \sqrt{\frac{\epsilon_0}{\mu_0}}EB$$

- Note that the energy due to the electric and magnetic fields are equal
- **Intensity** - The enegry an EM wave transfers per unit time per unit area
- The energy passing through an area at a time due to an EM wave is given by

$$\Delta U = (\epsilon_0 E)(Ac\Delta t)$$

- The average intensity of an EM waves is given by

$$I = \frac{E_0 B_0}{2\mu_0}$$

22.6 Momentum Transfer and Radiation Pressure

- Maxwell predicted that EM waves exerts a change in momentum when they hit a surface
- **Radiation Pressure** - The force exerted by an EM wave when it encounters a surface
- The momentum transferred when radiation is fully absorbed is given by

$$\Delta \rho = \frac{\Delta U}{c}$$

- The momentum transferred when radiation is fully reflected is given by

$$\Delta \rho = \frac{2\Delta U}{c}$$

- The radiation pressure when radiation is fully absorbed is given by

$$P\frac{I}{c}$$

- The radiation pressure when radiation is fully reflected is given by

$$P\frac{2I}{c}$$

- **Optical Tweezers** - Devices that use EM waves to manipulate extremely small objects such as individual components within a living cell

22.7 Radio and Television; Wireless Communication

- **Audiofrequency** - The frequency at which the audio signal being tranmistted oscillates
- **Carrier Frequency** - The frequency at which the EM waves carrying the audio frequency oscillates
- **Amplitude Modulations (AM)** - A method used to mix aurdio and carrier frequencies by having the amplitude (height) of the EM waves vary with proportion to the audio signal
- **Frequency Modulation (FM)** - Another method of mixing the signals, which is done by varying the frequency of the carrier signal with proportion to the frequency of the audio signal

- Devices receive signals of specific frequencies by adjusting the resonant frequency of an LC cicuit to equal that of a station's carrier frequency
- The signal then goes through the demodulator where the audio frequency is separated from the carrier frequency

Other EM Wave Communications

- Government agencies assign different frequency ranges to different functions such as telecommunications
- EM waves can be transmitted through the "air" or through coaxial cables that run through the ground

23 Light: Geometric Optics

AN object can be seen by either seeing light directly from it or seeing a reflection. The interactions between light and object are discussed in this section.

23.1 The Ray Model of Light

- **Ry Model of Light** - A model that assumes light travels in stright lines called rays
- When we see an ibject, light travels in countless directions when it hits the object and reflects back
- **Geometric Optics** - The subject that studies the ray nature of light and how it interacts with objects

23.2 Reflection; Image Formation by a Plane Mirror

- **Angle of Incidence** - The angle between a ray of light striking a surface and a line normal to the surface being hit
- **Angle of Reflection** - The angle between a reflected light ray and a line normal to the surface being reflected
- **Law of Reflection** - Angle of reflection equals angle of incidence

$$\theta_r = \theta_i$$

- **Diffuse Reflection** - Occurs when light hits a rough surface and reflects in many directions since the angle of the surface is different across
- **Specular Reflection** - Reflection on a mirror, light reflects at the same angles and will only hit your eyes at the right angle
- **Image** - A reflection of objects in a mirror
- **Plane Mirror** - A mirror with a single flat reflective surface
- **Image Point** - THe point from which a set of diverging rays appears to come from when an object is reflected in a mirror
- **Image Distance** - The distance at which an image in a mirror appears to be from the mirror
- Image distance = Object distance
- **Virtual Image** - The image seen and reflected from a plane mirror
- **Real Image** - An image where light passes through the image and can appear on a white surface, or on film

23.3 Formation of Images by Spherical Mirrors

- **Spherical Mirror** - A mirror whose surface is curved i the shape of a sphere or a section of one
- **Convex** - Describes a spherical mirror where the reflective surface is on the outside of the spherical shape
- **Concave** - Describes a spherical mirror where reflection takes place on the inside

Focal Point and Focal Length

- Light rays from a distant object will strike a concave mirror precisely parallel
- **Focus** - The point at which rays that strike a concave mirror will cross
- **Principal Axis** - The straight line perpendicular to the curved surface at its center, light rays must be parallel to this axis in order to have a focus
- **Focal Point** - The point where reflected rays come to a focus
- **Focal Length** - The length between the focal point and center of a concave mirror
- **Paraxial Rays** - Rays that make small angles with the principal axis
- Focal length is half the radius of curvatuce

$$f = \frac{r}{2}$$

- Focii are approximations, defects in focus on a spherical mirror are called *spherical aberrations*
- **Parabolic Reflector** - A mirror shape that will reflect rays at a perfect focus
- In diagrams of light reflections, the focal point is labelled F and the center of curvature is labelled C

Image Formation - Ray Diagrams

- For an object at infinity, the image is locaed at that focal point of a concave spherical mirror
- Constructing a real image of an object requires having a point on it be parallel to the principal axis bet wen points C and F
- A point, O' is used to project rays and form a real image and is right above point O, where the top of the object at O is
- To form an image from O', draw 3 rays:
  - A ray leaving O' parallel to the principal axis
  - A ray leaving O' and crossing through point F
  - A ray leaving O' along the radius of the spherical surface
- These three rays then converge at a point, I', which is the real image

Mirror Equation and Magnification

- **Object Distance -** The distance an object is from the center of a spherical mirror, represented by d_o
- **Image Distance -** The distance an object's virtual image is from the center of a spherical mirror, represented by d_i
- The height of the object is represented by h_o
- The height of the image, $I'I$ is represented by h_i
- **Mirror Equation -** Relates the object and image distances to the focal length,

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

- **Magnification -** Defined as the height of the image divided by the height of the object

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

- Two sign conventions when dealing with magnification are:
  - The image height is positive if the image is upright and negative if it si s inverted relative to the object
  - $d-i$ or d_o is positive if the image or object is in front to of the mirror, negative if either is behind

Seeing the Image; Seeing Yourself

- Your eyes must intercept the rays appraoching you in order fo ryou to see an image f yourself in a mirror
- For a concave mirror, placing your eyes between points O and I, *converging* rays would hit your eyes and the image would be diffifcult to discern
- If you are behind point C then you will see a clear, inverted image of yourself

Convex Mirrors

- Analysis of concave mirrors can be applied to convex mirrors, even the mirror equation
- Any reflected rays diverge but seem to com from a point F behind the mirror, called its focal point
- **Focal Length -** The distance an focal point is from the center of the mirror
- The mirror equations hold but focal length and radius of curvature are considered to be negative
- To solve mirror problems:
  - Draw a ray diagram
  - Apply the mirror and magnifications equations
  - Check for sign conventions

23.4 Index of Refraction

- The speed of light in air of other materials is slower than it is in a vacuum
- **Index of Refraction -** The ration between the speed of light in a vacuum and a given material, never less than 1, given by

$$n = \frac{c}{v}$$

- Light travels slower in materials than in a vacuum due to absorption and re emission of light by atoms and molecules in the material

23.5 Refraction: Snell's Law

- **Refraction** - The change in direction of a light ray due to it entering a new medium
- **Angle of Incidence** - The angle at which a ray strikes a new medium and a line normal to its surface
- **Angle of Refraction** - The angle between a ray's new direction in the new medium and a line normal to its surface

Snell's Law

- **Law of Refraction** - Also called Snell's law, used to relate the angles of incidence and refraction in optics, given by
$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$
Where θ_1 is the angle of incidence and θ_2 is the angle of refraction and n_1 and n_2 are the indices of refraction of the materials in question
- Note that when $n_2 > n_1$, light bends towards the normal, and vice versa

23.6 Total Internal Reflection; Fiber Optics

- At a certain angle of incidence, the angle of refraction will be $90°$ and the ray would skim the surface instead of refracting out into the new medium
- **Critical Angle** - The angle of incidence at which this occurs, given by
$$\sin \theta_c = \frac{n_2}{n_1}$$
- **Total Internal Reflection** - Occurs when all light is reflected back into the medium it originated from, can only occurs when $n_2 < n_1$

Fiber Optics; Medical Instruments

- **Fiber-Optic Cable** - A bundle of plastic or blass fibers that use total internal reflection to transmit signals in the form of light
- Light only glances off the boundaries of the cable and thus is maintained within it
- This has been used in transmitting data extremely quickly such as for high-res medical imaging

23.7 Thin Lenses; Ray Tracing

- The axis of a lens is a straight line passing through its center
- **Focal Point** - THe point at which rays travelling parallel to the axis of a lens will intersect after passing through it
- Thins lenses are those whose diameter is small relative to its radius of curvature
- **Focal Length** - Distance between the focal point and the center of the lens, the same on both sides of a double-convex lens
- **Focal Plane** - The plane of a double convex lens containing all of its focal points
- **Converging Lens** - A lens that is thicker in the center than at its edges, they focuses light
- **Diverging Lens** - A lens that is thinner in the center than at its edges, they spread light out
- **Lens Power** - Used to determine the strength of a lens, given by
$$P = \frac{1}{f}$$
Where f is focal length
- **Diopeter** - A unit for lens power, defined as the inverse of a meter, $1D = 1M^{-1}$
- **Real Image** - Formed by a double convex lens, since rays actually pass through the image

Diverging Lens

- The image formed by a diverging lens is *virtual*

23.8 The Thin Lens Equation

- **Thin Lens Equaiton -** Relates image distance to object distance and focal length, given by

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

  Where d_o is object distance and d_i is image distance

- f is negative for a diverging lens
- d_i is negative when the image i on the same side of the lens as the light comes from
- **Magnification -** Same as the magnification of a mirror, given by

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

23.9 Combinations of Lenses

- When analyzing multiples lenses, the image formed by the first lens then becomes the object of the second lens, the image of the second lens becomes the object of the third, and so on
- The total magnification of multiple lenses is the product of the individual magnifications

23.10 Lensmaker's Equation

- **Lensmaker's Equation -** Relates focal length of a lens to its radii of curvature, and index of refraction, given by

$$\frac{1}{f} = (n-1)(\frac{1}{R_1} + \frac{1}{R_2})$$

  Where n is the index of refraction and R_1 and R_2 are the radii of curvature of both sides

24 The Wave Nature of Light

Light can also be observed as a particle carrying energy. Its wave-particle duality has been used to explain a wide variety of phenomena.

24.1 Waves vs. Particles; Huygen's Principle and Diffraction

- **Wave Front** - All points along a multi-dimensional wave that form its crest
- **Huygen's Principle** - Every point on a wave front can be considered as a force of tiny wavelets that spread out in the forwards direction at the speed of the wave itself. The new front is the envelope of all the wavelets
- **Diffraction** - The bending of waves behind obstacles into the shadow region (the area where the wave is not present)

24.2 Huygen's Principle and Law of Refraction

- Snell's law is derived from Huygen's principle
- When a light wave travels from one medium to another, its frequency remains constant and its wavelength changes, given by

$$\lambda_n = \frac{\lambda}{n}$$

  Where λ is the wavelength in the first medium and n is the index of refraction of the second medium
- Note that the previous equation is consistent with $c = f\lambda$
- Wave fronts can be used to describe mirages, on hot days the hot air has a lower refraction index and thus light is bent towards it and creates an illusion

24.3 Interference - Young's Double-Slit Experiment

- **Monochromatic** - Meaning of one color, this describes plane waves with one frequency
- When light waves pass through double-slits, they are diffracted ad interfere with each other on the other side
- **Constructive Interference** - Occurs when waves are *in-phase*, the waves and crests combine to become greater in magnitude
- **Destructive Interference** - Occurs when waves are *out-of-phase*, the waves and crests cancel out and decrease in magnitude
- **Fringe** - A dark or bright line on a screen caused by constructive and destructive interference
- To determine where fringes are, the following is used

$$d \sin\theta = m\lambda$$

  Where θ is the angle the wave makes with the horizontal, d is the distance between the slits, and m is the order of the fringe
- The order of a fringe is how many fringes away a fringe is from the central fringe

Coherence

- **Coherent Sources** - Sources of light that produce the same wavelength and frequency
- Slits in the double-slit experiement are coherent sources

24.4 The Visible Spectrum and Dispersion

- **Intensity** - How much energy light carries per unit area
- **Color** - Related to frequency or wavelength **Visible Spectrum** - The wavelengths of light that humans can observe
- **Ultraviolet** - The wavelengths of light too long for humans to detect
- **Infrared** - The wavelengths of light too short for humans to detect
- **Dispersion** - The spreading of white light into the full spectrum

24.5 Diffraction by a Single Slit or Disk

- **Diffraction Pattern** - The light pattern formed by a sharply-edged object illuminated by a point source
- The intensity at light on a surface is graphed by

$$\sin\theta\frac{\lambda}{D}$$

  Where $\lambda$is wavelength and D is the width of the slit
- The graph of the previous function looks like a sine wave whose magnitude decreases dramatically
- The intensity is at a maximum at 0 and a minimum at $\sin\theta$

24.6 Diffraction Grating

- **Diffration Grating** - A large number of equally spaced parallel slits
- **Tranmission Grating** - A diffraction gratins with its own slits
- **Reflection Grating** - Made by ruling lines into a reflective surface
- **Order** - An integer value that tells how many fringes away from the central fringe a fringe is
- Maxima and Minima are much sharper and narrower for a grating than for a double-slit
- When light strikes a grating and is not monochromatic, each wavelength will produce maxima at different angles

24.7 The Spectrometer and Spectroscopy

- **Spectrometer** - A deice used to measure wavelength of light, using the following equation

$$\lambda = \frac{d}{m}\sin\theta$$

  Where m is the order of the maxima and d is the distance between grating slits
- **Line Spectrum** - Specific wavelengths emitted when a gas is heated
- **Absorption Lines** - The specific wavelengths of light a material absorbs

24.8 Interference in Thin Films

- **Thin-Film Interference** - The constructive interference formed between light reflected from the two surfces of a thin film
- This occurs because at a certain angle, light from only one wavelength is reflected and the entire surface produces several wavelengths across the spectrum
- **Newton's Rings** - Concentric rings of light formed when a curved glass comes in contact with a flat glass, caused by the widening air gap between the two surfaces leading to interference of light waves coming from different areas
- **Phase Shift** - A change in the cycle of a wave (ex. is flips and crests become troughs)
- A beam of light, reflected by a material with index of refraction greater than that of the material in which it is traveling, changes phase by $180°$ or $1/2$ cycle
- When the air gap is an odd multiple of the wavelength, the waves undergo constructive interference and form the rings

Colors in a Thing Soap Film

- In a soap film, gravity makes the film thicker at the bottom than at the top, thus it shifts phase more and reflects into different wavelengths across the film
- Sometimes, the soap film is thinner than the wavelength of light, so it does not change phase, and remains the same

Lens Coatings

- Optical instruments use thin lenses to reduce the percentage of light reflected from the surface of the glass
- A single coating cannot eliminate all wavelengths, so instruments use several to cover the spectrum

24.9 Michelson Interferometer

- **Michelson Interferometer -** A device that uses a beam splitter in order to split a beam of light in half
- The interferometer reflects light to a detector and the intensity of light is dependent on the distance an object is from the source, in multiples of wavelength
- The device is used for finding very small distances between objects as the wavelengths of light used are also very small

24.10 Polarization

- **Polarized -** Describes a wave that oscillates in a single plane
- **Linearly Polarized -** Describes a wave that oscillates in a single plane
- If a polarized wave oscillating on the vertical axis passes through a horizontal slit, it will be stopped
- **Unpolarized -** Describes a wave that oscillates in multiple planes

Polazroids

- **Polaroid Sheet -** A sheet composed of long molecules parallel to each other, acts as a series of parallel slits, the axis of the molecules is called the transmission axis
- A plane-polarized wave passing through a polaroid sheet with transmission axis at an angle will exit plane-polarized to the sheet and its new intensity is

$$I = I_0 \cos^2 \theta$$

- **Polarizer -** Light passes through this device and comes out plane-polarized
- **Analyzer -** Determines is light is polarized and the plane of polarization

Polarization by Reflection

- Reflected light is preferentially in the plane parallel to the surface it reflects off of
- This explains why most polaroid sunglasses have their axes vertical, most surfaces in nature are horizontal and thus the sunglasses block more light
- The amount of polarization is dependent on the *polarization angle*
- **Polarization Angle -** Related to the index of refraction of the two materials on either side of a change in medium, given by

$$\tan \theta = \frac{n_2}{{}_1}$$

  Also known as Brewster's Angle
- Brewster's angle is used to determine the angle at which a reflection is perfectly plane polarized, by substituting it into Snell's law

24.11 Liquid Crystal Displays

- **Liquid Crystal Display (LCD) -** Use polarization to display images on cell phones
- The pixels in an LCD use rotation of the polarized axis of light to increase the intensity of light and make it seem brighter
- Smaller displays use ambient light to display y reflecting it off the inside of the inside of the pixel
- Larger displays use subpixels of red, blue, and green light to make colors

24.12 Scattering of Light by the Atmosphere

- As light from the sun hits the atmosphere, some of it is absorbed by air molecules which reemit it
- The reemitted light is plane polarized and someone viewing it at a right angle to the direction of sunlight will see polarized light
- Scattering of light depends on the wavelength of light and size of molecules, the shorter the wavelength, the more light is scattered
- This is why the sky is blue as blue light has shorter wavelengths

25 Optical Instruments

This section discusses some more applications of optics and the instruments that use them.

25.1 Cameras: Films and Digital

- **Camera** - Basics elements are a lens, light-tight box, shutter, and light sensor
- The shutter allows light into the box for a short amount of time, the image is captured by the sensor and creates a pictrue
- **Charge-Coupled Device** - Uses a sensor made of semiconductor pixels that use capacitors to charge capacitors and detect light
- **Complementary Metal-Oxide Semiconductor** - This sensor uses newspaper to carry sound

Digital Cameras, Electronic Sensors

- **Digital Camera** - Uses a semiconductor sensor to detect light
- **Exposure time/shutter speed** - how long the sensor can make a reading, faster shutter means less shutter
- **F-stop** - The size of the opening of the camera, given by the focal length, f, and lens diameter, D

$$f - stop = \frac{f}{D}$$

- **Focusing** - Placing the lens in the correct position relative to the sensor to create a sharp image

Picture Sharpness

- Smaller pixels require more exposure time
- Digital cameras tend to reduce the quality of pictures by compressing them to reduce the amount of memory they use
- *Resolution* - Measures the sharpness a lens can produce

Telephotos and Wide-Angles

- **Telephoto Lens** - acts as a telescope to magnify images by having a longer focal length
- **Wide-Angle Lens** - A shorter focal length, makes a wider field of view
- **Optical Zoom** - A change in a lens' focal length that maintains resolution
- **Digital Zoom** - Artificial enlargement of pictures but loses sharpness

25.2 The Human Eye; Corrective Lenses

- **Cornea** - Acts as the lens of the eyes, refraction index of n=1.386 to 1.406
- **Pupil** - The hole through which light is let through
- **Retina** - Acts as the sensor of the eye
- **Accommodation** - The adjustment of the focal length of the eye to focus on objects at different distances
- **Myopia** - A condition where the eye can only focus on near objects, usually caused by an eyeball that is too long
- **Hyperopia** - A condition where the eye can only focus on far objects, usually caused by an eyeball that is too short

Contact Lenses

- Used to correct errors in how the eye focuses light
- Main process is by modifying the near point of the eye

25.3 Magnifying Glass

- **Simple Magnifier** - A converging lens
- **Magnifying Power** - Shown by M, this is the ratio between the angle of an object through a lens and the angle using the unaided eye
- Can also be calculated by

$$M = \frac{N}{f} + 1$$

  Where N is the near point of the eye, usually around 25cm, and f is the focal point

25.4 Telescopes

- **Keplerian Type** - A type of telescope made of two converging lenses at opposite ends of a long tube
- **Total Magnifying Power** - Calculated by

  $$M = -\frac{f_o}{f_e}$$

  Where f_o is the focal length of the objective lens and f_e is the focal length of the eyepiece
- **Reflecting Telescopes** - The largest type of telescope, use a curved mirror as the objective
- Light from the object being observed is reflected off of a large mirror which focuses light on a smaller mirror into the eyepiece

25.5 Compound Microscope

- **Compound Microscope** - Uses both objective and eyepiece lenses to magnify an object
- The total magnification is the product of the magnifications of the two lenses, calculated by

  $$M = M_e m_o$$

  Where M_e is the magnification of the eyepiece and m_o is the magnification of the objective lens

25.6 Aberrations of Lenses and Mirrors

- **Lens Aberrations** - Deviations of lenses from simple theory
- **Spherical Aberrations** - Aberrations caused by a spherical lens, where light rays intersect at many different points instead of a single point
- **Curvature of Field** - Another type of aberration, a problem in cameras, caused by the curvature of a lens causing an image to not be perfectly square on a sensor
- **Distortion** - Results from variation in magnification at different distances from a lens axis
- **Chromatic Aberration** - Caused by the dispersion of light, some colors of light are bents more than others and are more concentrated in different areas

25.7 Limits of Resolution; Circular Apertures

- Lens aberrations and diffraction are the key limiters of resolution
- Fringes of light are formed by diffraction around objects, when two objects are near these fringes interfere with each other and reduce resolution
- **Rayleigh Criterion** - Two images are just resolvable when the center of the diffraction disk of one image is directly over the first minimum in tge diffraction pattern of the other
- The angle at which two images are just resolvable is given by

  $$\theta = \frac{1.22\lambda}{D}$$

  Where D is the diameter of the lens

25.8 Resolution of Telescopes the λ Limit

-
- An increase in magnification above a certain point results in magnification of the diffraction patterns
- **Resolving Power (RP)** - Property of a microscope, the minimum separation of two object points that can be resolved, given by

  $$RP = f\theta = \frac{1.22\lambda f}{D} \approx \frac{\lambda}{2}$$

  Where f is the focal length of the lens
- Thus, it is not possible to resolve detail of objects smaller than the wavelength of the radiation being used

25.9 Resolution of the Human Eyes and Useful Magnification

- The resolution of the human eye is roughly $5 * 10^{-4}$ radians
- The maximum useful magnification of a microscope is about 500X, any higher would make the diffraction pattern of the lens visible

25.10 Specialty Microscopes and Contrast

- **Contrast** - The difference in brightness between the image of an object and the image of its surroundings
- **Interference Microscope** - Makes use of the wave nature of light to increase contrast in a transparent object
- This works by transmitting light through a medium and an object surrounded by the medium, the light exiting the medium alone will be slightly different than the light exiting the medium as well as the object, which our eyes can detect
- Variations in the thickness of the object will appear as variations in brightness in the image
- **Phase-Contrast Microscope** - Another type of microscope, uses contrast to show objects
- Does this by having light travel through an object and a medium alone, the lone light passes through thicker glass than the light that passes through the object and thus the two lights can be out of phase and be seen clearly apart from each other

25.11 X-Rays and X-Ray Diffraction

- After X-Rays were discovered, it was shown that their wavelengths are extremely short, on the order of spacing between atoms
- Thus, crystal structures can be used as diffraction gratings for X-Rays

X-Ray Diffraction

- **X-Ray DIffraction** - Also called crystalography, this is a method of using X-rays in order to observe the atomic scale
- **Bragg Equation** - Gives the distance a second ray will travel to cause constructive interference when reflecting off of a crystal, given by

$$m\lambda = 2d \sin \phi$$

  Where m is any integer, d is the distance between molecules or atoms in a crystal and ϕ is the angle between an X-ray and a line parallel to the surface of a crystal
- If the angle is known, then the distance between particles in a crytsal can be determined

25.12 X-Ray Imaging and the Computed Tomography (CT Scan)

Normal X-Ray Image

- X-Rays do not detect refraction of light through the body but rather the absorption of light
- X-Rays show a shadow of the light that flows through the body, the more that is absorbed in the body, the lighter the image on film

Tomography Images (CT)

- **Computed Tomography (CT)** - An imaging technology that takes images of slices of the body
- The apparatus continually takes slice images at intervals of about $1°$ until the whole body is scanned

Image Formation

- Slices from CT scans are divided into pixels
- **False-Color** - A method of adding color to CT scans, does so by adding color to pixels relative to how much light was absorbed

Tomographic Image Reconstruction

- **Iterative Technique** - A technique used in image reconstruction, compares the absorption at each pixel in a CT scan to form an image
- This process is very math intensive, I suggest reading more on your own if you are interested!

26 The Special Theory of Relativity

By the beginning of the 20th century, it seemed as if only a few mysteries were left in the realm of physics. This was not the case and the physics discovered later on turned out to be much more complex. From this, the *theory of relativity* and *quantum theory* were developed. Physics discovered before the 20th century is known as classical physics while anything after is usually referred to as modern physics

26.1 Galilean-Newtonian Relativity

- **Inertial Reference Frame** - A reference frame where Newton's First Law is valid, can move at a constant velocity
- **Relativity Principle** - The basic laws of physics are the same in all inertial reference frames
- **Absolute** - Describes measurements that do not change from one reference frame to the next, includes space and time
- Position is not absolute, the speed of different objects relative to the Earth is different from that of the same objects with respect to each other
- All inertial reference frames are equivalent
- **Ether** - The assumed medium light travels through in space, the velocity of light given by Maxwell's Equations was assumed to be respect to this ether
- **Null Result** - The failure to detect a difference in the speed of light when it travels in different directions relative to the ether

26.2 Postulates of the Special Theory of Relativity

- **First Postulate** - The laws of physics have the same form in all inertial frames
- **Second Postulate** - Light propogates through empty space with a definite speed, c, independent of the speed of the source or observer
- These two postulates form the basis of the *special theoery of relativity*
- **Thought Experiments** - Simple experimental situations which can be thought about and used to see the consequences of relativity theory

26.3 Simultaneity

- **Event** - Something that happens at a particular place at a particular time
- Two events are said to occur simultaneously if they occur at the same time
- Two events which take place at different locations and are simultaneous to one observer are not simultaneous to a second observer in a different location
- There is not "best observer", evenn if bot observers get different results, they are both right, thus *simultaneity is not an absolute concept*

26.4 Time Dilation and the Twin Paradox

- Einstein's theory of relativity predicts time itself is not absolute
- **Time Dilation** - Clocks moving relative to an observer are measured to run more slowly, as compared to clocks at rest
- Time is measured to pass more slowly in any moving reference frame relative to your own
- Time dilation is due to the fact that as an object moves, light has to "cath up" to it before it can reflect back to an observer
- Time dilation is given by

  $$\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}}$$

  Where Δt is the interval of time dilated, Δt_0 is the interval of time undilated, v is the velocity of the reference frame, and c is the speed of light
- The factor, $\frac{1}{\sqrt{1-v^2/c^2}}$ occurs often and is simplified as γ, thus time dilation can be rewritten as

  $$\Delta t = \gamma \Delta t_0$$

- Δt_0 is more regularly defined as the time interval between two events in a reference frame where an observer at rest sees the two events occurs at the same point in space ad is also known as proper time
- Δt represents the time interval between two events as measured in a reference frame moving with speed v with respect to the first

Space Travel

- All processes including agin and other life processes, run more slowly for the astronaut as measured by an Earth observer. But to the astronaut, time would pass in a normal way.
- Therefore, an astronaut travelling at extremely high speeds would come back to Earth to find more time had passed on Earth than they experienced on the ship

Twin Paradox

- If one of a pair of twins travelled in a spaceship at very high speeds to a distant star and the other stayed on Earth, the travelling twin would return to see their Earthbound twin had aged much more than them
- The reason the travelling twin does not observe the same events but in reverse is because they are not observing from an intertial reference frame

Global Positioning System

- Satellites compare the time differences between other satellites in order to determine your position
- It does this by determining its own position and the angle from which it receives your signal

26.5 Length Contraction

- **Length Contraction** - The length of an object relative to an observer is measure to be shorter along its direction of motion than when it is at rest
- Length contraction is given by

$$l = \frac{l_0}{\gamma}$$

  where l_0 is proper length or the length of the object at rest, and l is the observer length
- Note that length contraction only occurs along the direction of motion, something moving up has no length contraction to the left or right

26.6 Four-Dimensional Space-Time

- **Four-Dimensional Space-Time** - The idea that space takes up three dimensions and time is the fourth dimension
- **Space-time interval** - The quantity of four-dimensional space-time between two events, given by

$$(\Delta 2)^2 = (c\Delta t)^2 - (\Delta x)^2$$

26.7 Relativistic Momentum

- **Relativistic Momentum** - Redefining the law of conservation of momentum in relativity gives

$$\rho = \gamma m v$$

  Where m is mass of a particle, v is its velocity

Rest Mass and Relativistic Mass

- **Relativistic Mass** - The mass of an object with reference to its velocity, given by

$$m_{rel} = m\gamma$$

- The mas of an object appears to increase as its speed increases
- Note that as an object's relativistic mass increases, it does not gain particles

26.8 The Ultimate Speed

- Special relativity gives that the speed of an object cannot exceed the speed of light
- Accelerating the velocity of an object up to c would make its momentum infinite and require infinite energy, thus it is not possible

26.9 $e = mc^2$; Mass and Energy

- The total energy of a particle at rest is given by

$$E = mc^2$$

- The above equation relates energy and mass and suggetss that it might be possible to convert between energy and mass
- A change in total energy of a system is given by

$$\Delta E = (\Delta m)(c^2)$$

- The mass of particles can be related to the speed of light and their energy, the mass of an electron can be defined as

$$0.511MeV/c^2$$

- Total energy can also be defined in terms of momentum

$$E^2 = \rho^2 c^2 + m^2 c^4$$

Invariant Energy - Momentum

- The previous equation can be rewritten as

$$E^2 - \rho^2 c^2 = m^2 c^4$$

- Since mass is conserved the difference remains constant and the quantity $E^2 - \rho^2 c^2$ is *invariant*

When Do We Use Relativistic Formulas?

- Relativistic formulas are not practical in most applications as they give results that vary very slightly at even extreme speeds
- If the ratio KE/mc^2 is less than a given threshold, it is safe to use classical formulas instead of relativistic ones

26.10 Relativistic Addition of Velocities

- If an object is travelling at a speed v and another object is travelling in the same direction at another speed relative to the first, u', then the addition of these velocities is given by

$$u = \frac{v + u'}{1 + vu'/c^2}$$

  Where u is the sum of the velocities

26.11 The Impact of Special Relativity

- **Correspondence Principle** - The insistence that a more general theory can give the same results as a more restricted one
- An example of the correspondence principle is the hopes that relativity and classical mechanics overlap, which they do when dealing with speeds much lower than the speed of light

27 Early Quantum Theory and Models of the Atom

Quantum theory took nearly three decades of effort from many scientists and it began with Planck's quantum hypothesis

27.1 Discovery and Properties of the Electron

- **Cathode Rays -** Rays that are emitted by cathode tubes
- Cathode rays were discovered to be made of electrons using the below equation

$$\frac{e}{m} = \frac{E}{B^2 r}$$

  Where r is the radius of the cathode tube, B is the magnetic field, and E is the electric field within the tube

- The ratio given by the above equation is

$$\frac{e}{m} 1.76 * 10^{13} C/kg$$

  and describes the electron

Electron Charge Measurement

- The charge of an electron is measured to be $e = 1.6 * 10^{-19} V$
- The mass of an electron is accepted as $m_e = 9.11 * 10^{-31} kg$
- All other charges are a multiple of e, thus electric charge is *quantized*

27.2 Blackbody Radiation; Plank's Quantum Hypothesis

Blackbody Radiation

- **Blackbody -** A body that, when cool, would absorb all the radiation falling on it
- **Blackbody Radiation -** Radiation emitted by a blackbody when hot and luminous, used to approximate radiation emitted from real objects
- **Wien's Law -** The peak wavelength emitted by an object at a temperature, T in Kelvin, given by

$$\lambda_p T = 2.90 * 10^{-3} m * K)$$

Planck's Quantum Hypothesis

- **Planck's Quantum Hypothesis -** Max Planck related the energy in an atom to the product between its frequency and some minimum value, shown below

$$E = nhf$$

  Where h is Planck's constant, n is a quantum number and f is frequency

- **Planck's Constant -** Common in quantum physics, defined as

$$h = 6.626 * 10^{-34} J * s$$

- **Quantum Number -** Meaning discrete amount, this is a positive integer
- **Quantum of energy -** The smallest amount of energy that can exist, defined as the quantity hf
- Energy is also quantized, that is, it increases like stairs rather than a ramp

27.3 Photon Theory and Light and the Photoelectric Effect

- Light contains energy in packets or *quanta*, each with energy

$$E = hf$$

- **Photons -** Particles of light, these carry quanta
- **Photoelectric Effect -** The emission of electrons when light hits a surface
- The first prediction from the Photoelectric Effect is if light intensity increases, the number of electrons ejected and their maximum KE should be increases because higher intensity means greater electric amplitude which should eject electrons farther
- The second assumption is the frequency of light should not affect the KE of the ejected electrons, only intensity

- Some work, W_0 must be done to eject an electron from the surface of a metal, thus the input energy of a photon is equal to
$$hf = KE + W$$
- Photon theory makes three predictions, the first is: an increase in intensity of the light beam means more photons are incident so more electrons will be ejected; but since eney of each photon is not changes, the maximum KE of electrons is not changes
- The second is: if the frequency of the light is increased, the maximum KE of the electrons increases linearly, that is
$$KE_{max} = hf - W_0$$
- The third is: If the frequency is less than the cutoff frequency, f_0, where $hf_0 = W_0$, no electrons will be ejected, no matter the intensity of light

Applications of the Photoelectric Effect

- The effect is used in electronic motion sensors, as an object interrupts a beam of light, electrons in a sensor circuit stop being ejected and trigger the sensor
- Any circuit or device that uses light uses the photo electric effect to gain information from light
- **Photodiode** - Device that detects when an electron is ejected by detecting a change in conductivity

27.4 Energy, Mass, and the Momentum of a Photon

- Photons always travel at the speed of light and are thus relativistic particles
- Calculating the relativistic momentum of a photon gives a denominator of 0, thus we assume they have no mass, which is consistent with $E = hf$
- The momentum of a photon is calculated instead by
$$\rho = \frac{h}{\lambda}$$

27.5 Compton Effect

- **Compton Effect** - The phenomenon that photons tend to lose energy after passing through a material, indicating a loss in wavelength
- The new wavelength is given by
$$\lambda' = \lambda + \frac{h}{m_e c}(1 - \cos\phi)$$
Where λ' is the new wavelength, λ is the original wavelength, ϕ is the change in angle of the photon, and m_e is the mass of the electron the photon collides with to lose energy
- **Compton Wavelength** - The quantity $\frac{h}{m_e c}$
- The Compton effect has been used to detect the density of electrons in bone material which in turn can be used to detect bone density

27.6 Photon Interactions; Pair Production

- There are four interactions a photon can undergo, the first is: The photoelectric effect. A photon may knock an electrons out of an atoms and in the process the photon disappears
- The second is: the photon may knock an atomic electron to a higher energy state in the atom if its energy is not sufficient to knock the electron out altogether. In this process the photon also disappears. and all its energy is given to the atoms. The atoms is said to be in an excited state
- The third is: The photon can be scattered from an electron and lose energy; this is the Compton effect
- The fourth is: Pair Production: A photon ca actually create matter, such as the creation of an electron and a positron
- In pair production, the photon disappears and the two particles annihilate each other, releasing their energy as more photons
- Note that pair production cannot occur in empty space as momentum would not be conserved

27.7 Wave-Particle Duality; the Principle of Complementarity

- **Wave-Particle Duality** - The fact that light acts as both a wave and a particle
- **Principle of Complementarity** - The notion that in order to understand an experiment, we might have to interpret the results as light behaving as a wave or particle, therefore the two aspects compliment each other
- Einstein's equation $E = hf$ itself refers the two sides of light, the E refers to the energy of a particle and the f refers to the frequency of a wave

27.8 Wave Nature of Matter

- **de Broglie Wavelength** - The wavelength of a particle in linear motion given by

$$\lambda = \frac{h}{\rho}$$

electron Diffraction

- Electrons have wavelengths on the scale of $10^{-10}m$ which can be the distance between atoms in a crystal that can serve as a diffraction grating
- Using crystals to diffract electrons forms a diffraction pattern
- Thus, wave-particle duality applies to both light and matter

27.9 Electron Microscopes

- **Electron Microscope** - Developed using the wave aspects of electrons, this device can produce images with greater magnification than standard light microscopes
- Electrons are focused using electric fields from wires rather than lenses

27.10 Early Models of the Atom

- **Plum-Pudding Model** - Early ideas of atoms imagined them as dots of negative charge floating in a sea of positive charge
- **Planetary Model** - The gold foil experiment led to the conclusion that atoms are made of massive positively charged centers and surrounded by smaller negatively charged electrons

27.11 Atomic Spectra: Key to the Structure of the Atom

- It was discovered that excited gases emit light, but only at certain wavelengths unique to each gas
- **Line Spectrum** - The specific wavelengths of light an element of compound emits when it is excited
- **Emission Spectrum** - Similar to line spectrum, this serves as a "fingerprint" for the material
- **Absorption Spectrum** - Shows the wavelengths of light that a gas absorbs, this spectrum is the inverse of an emission spectrum
- The spacing between lines on a hydrogen emission spectrum decreases regularly and can be modeled by the Balmer series
- **Balmer Series** - The formula used to determine the spacing between hydrogen's emission spectrum in the visible spectrum, given by

$$\frac{1}{\lambda} = R(\frac{1}{2^2} - \frac{1}{n^2}), \qquad n = 3, 4, ...$$

  Where R is the Rydberg constant and n is any integer greater than 2
- **Rydberg Constant** - Defined as $R = 1.0974 * 10^7 m^{-1}$
- **Lyman Series** - Similar to the Balmer series, this contains lines in the UV spectrum, given by

$$\frac{1}{\lambda} = R(\frac{1}{1^2} - \frac{1}{n^2}), \qquad n = 3, 4, ...$$

- **Paschen Series** - Similar to the Balmer series, this contains lines in the IR spectrum, given by

$$\frac{1}{\lambda} = R(\frac{1}{3^2} - \frac{1}{n^2}), \qquad n = 3, 4, ...$$

27.12 The Bohr Model

- Niels Bohr proposed that electrons orbit the nucleus but only in discrete orbits and an electron would move in each orbit without radiating energy
- **Stationary States** - The discrete orbits electrons can move about in
- In the *Bohr Model*, electrons only emit light when they jump from a higher to lower energy level, a single level jump releases a single photon,

$$hf = E_u - E_l$$

  Where u indicates upper and l indicates lower

- **Bohr's Quantum Condition -** Gives the angular momentum of an electron,

$$L = mvr_n = n\frac{h}{2\pi}, \qquad n = 1, 2, 3, ...$$

  Where n is an integer, called a *quantum number* and r_n is the radius of the electron's orbit

- n labels both the orbit radii and *energy level* of an electron

- The lowest energy level, E_1 is called the ground state and higher levels are called excited states

Spectra Lines Explained

- The different series shown before model electrons jumping to an energy state from another n levels higher

- A general formula for the wavelengths of light emitted by a material is given by

$$\frac{1}{\lambda} = \frac{2\pi^2 Z^2 e^4 k^2}{h^3 c}(\frac{1}{n'^2} - \frac{1}{n^2})$$

  WHere Z is the charge on the nucleus o the atom, e is the charge on the proton, k is Coulomb's Constant, and n' corresponds to the series in question (n'=2 for the Balmer Series)

- **Correspondence Principle -** The use of classical mechanics in the quantum world, still yields precise results

27.13 de Broglie's Hypothesis Applied to Atoms

- Applying de Broglie's Principle to electrons proposes the notion that electron orbits are standing waves that can only exist in a whole number of orbits

- Another quantum condition is the follwing:

$$mvr_n = \frac{nh}{2\pi}$$

  which gives discrete orbits and energy levels of electrons

28 Quantum Mechanics of Atoms

Bohr's model could not predict the spectra for complex atoms and could not explain why emission lines consist of multiple, very closely spaced lines. It was not theoretically sufficient and showed a more comprehensive theory was needed.

28.1 Quantum Mechanics- A New Theory

- **Quantum Mechanics** - Unifies wave-particle duality into a single theory and successfully deals with spectra emitted by cmplex atoms
- Quantum mechanics satisfies the correspondence principle as it would give the classical formulas if applied to the macroscopic world

28.2 The Wave Function and Its Interpretation; the Double-Slit Experiment

- **Wave Function** - Represents EM wave amplitude as a function of time and position, represented by ψ
- When applied to an electron in an atom, ψ^2 at a certain point in space and time represents the probability of finding the electron at a given position and time, thus it is referred to as the *probability density* of the electron

Double-Slit Interference Experiment for Electrons

- Applying ψ^2 to the double slit experiment gives a maximum where the electrons are more concentrated and a minimum where they are sparse

28.3 The Heisenberg Uncertainty Principle

- Quantum mechanics puts a limit on how precise measurements can get as measuring an object without disturbing it is impossible
- Using photons to measure small objects like electrons causes uncertainty in both the electron's position and momentum
- **Heisenberg Uncertainty Principle** - Tells that measuring the position and momentum of an object at the same time is impossible, given by

$$(\Delta x)(\Delta \rho_x) \geq \frac{h}{2\pi}$$

  Where Δx is uncertainty in position and $\Delta \rho_x$ is uncertainty in momentum
- The principle can also be stated in terms of energy and time

$$(\Delta E)(\Delta t) \geq \frac{h}{2\pi}$$

  Where ΔE is uncertainty in energy and Δt is uncertainty in time

28.4 Philosophic Implications: Probability Versus Determinism

- The states of extremely small particles are determined with probability rather than exact calculations
- Because of this, and the fact that objects are made of these tiny particle, the states of macroscopic objects are expected to be governed by probability
- **Copenhagen Interpretation** - A way of looking at probability as an inherent nature of quantum mechanics

28.5 Quantum-Mechanical View of Atoms

- **Cloud** - Describes the position of electrons around an atom from a particle view, electrons exist as a cloud of probability rather than a set position
- Clouds can be though of as probability distributions of electrons

28.6 Quantum Mechanics of the Hydrogen Atom; Quantum Numbers

- Four quantum numbers specify each electron state in an atom
- The first is doneted by n, and is called the *principle quantum number*, from the Bohr model, gives the energy state
- The second is the *orbital quantum number*, l, and ranges from 0 to $n-1$, gives the angular momentum of an electron by

$$L = \sqrt{l(l+1)}h$$

- The third is the *magnetic quantum number*, m_t and is related to the direction of the electron's angular momentum, ragning from $-l$ to $+l$ in integers
- The fourth is the *spin quantum number*, m_s and can have a value of $\pm 1/2$ for an electron
- **Orbitals -** Different shapes of probability distributions for electrons at different energy levels

Selection Rules

- Selection rules limit what electrons can do, one such rule is an electron can only transition between states with values of l that differ by one unit, $\Delta l = \pm 1$
- **Forbidden Transition -** A transition that violates a selection rule
- **Allowed Transition -** Transitions that do not violate selection rules
- Forbidden transitions can actually occur but they are rare

28.7 Multielectron Atoms; the Exclusion Principle

- **Atomic Number -** The number of protons or electrons in a neutral atom
- **Pauli Exclusion Principle -** no two electrons in an atom can occupy the same quantum state, thus they cannot have the exact same 4 quantum numbers
- All protons neutrons and electrons are identical, thus if two of the same particle had the same quantum numbers they would be the same particle

28.8 The Periodic Table of Elements

- **Periodic Table -** A method of organizing elements by their properties
- Electrons with the same value of n are referred to as being in the same shell
- Electron with the same n and l are referred to as being in the same subshell
- The Pauli exclusion principle limits how many electrons can be in each shell, this number increases each shell
- The electron configuration of an element is given by assigning a sperscript to each subshell that tell show many electrons are in it, for example, the ground state configuration for sodium (11 electrons) is $1s^2 2s^2 2p^6 3s^1$
- Groups (row) in the periodic table have different numbers of electrons in their outermost shells but the transition metals are irregular

28.9 X-Ray Spectra and the Atomic Number

- X-rays are produced when electrons are accelerated by a high voltage and strike a metal target in the x-ray tube
- The graph of wavelength and intensity of x-rays produced gives a minimum wavelength λ_0 depend dent on the voltage used, and two peaks, K_alpha, K_beta which are dependent on the target material
- The characteristic x-rays produced are from electrons dropping into lower energy levels, the peaks are produced by electrons dropping into the first shell
- **Bremsstrahlung -** The emission of radiation by a decelerating electron as it loses momentum
- The minimum wavelength of x-rays in a vacuum tube is given by

$$\lambda_0 = \frac{hc}{eV}$$

  Where eV is the elctron's charge

28.10 Fluorescence and Phosphorescence

- **Fluorescence -** The absorption of UV photons and release of visible photons by a material
- **Fluorescent Light bulbs -** Use this phenomenon, they excite a gas that emits UV photons on a coating then then emits visible light
- **Phosphorescence -** The phenomenon of atoms being excited and releasing their energy much later due to the jump being forbidden

28.11 Lasers

- **Laser -** A device that can produce a narrow beam of monochromatic, coherent light, meaning at any point, all the waves are in phase
- **Stimulated Emission -** The action of a photon pushing an excited atom to release energy at the same wavelength as the original photon, thus two photons of the same wavelength exist
- To obtain coherent light, the atoms must be excited so that more excited ones exist than ground state ones, called an *inverted population* and the excited state must be metastable or exist excited longer than usual

Creating an Inverted Population

- **Optical Pumping -** Method of creating an inverted population that uses strong flashes of light to put chromium atoms in a metastable state, once these atoms drop down the photons they emit continue to stimulate emission
- **PN Junction Laser -** Uses the inverted population of electrons between the conduction and valence band to stimulate emission

Applications

- DVDs and CDs use lasers to be read by having the laser reflect off of pits and valleys on their surface
- Lasers are also used in medicine as they can destroy a local area

28.12 Holography

- **Hologram -** A 3-d image produced using lasers
- This is achieved by splitting lasers on a half-silvered mirror and having half of it reflect off an object and the other half continue onto a film
- The light reflected off the object reaches the film and produces the image
- **Volume Holograms -** Do not require a laser, but can be viewed with white light on a thick emulsion that shows where destructive interference occured when making the hologram

29 Molecules and Solids

Quantum Mechanics has had a great impact on our lives. As we have come to understand more about the topic, we have been able to use it to our advantage to develop and improve the study of semiconductors.

29.1 Bonding in Molecules

- **Chemical Bond** - The attachment of a group of multiple atoms and are held together as a single unit, two main types

Covalent Bonds

- The *sharing* of electrons between two atoms
- Covalent bonds are a result of constructive interference of electron wave functions in the space between two atoms
- **Bond Energy** - The energy required to break a bond

Ionic Bonds

- Unequal sharing of electrons
- For example, in sodium chloride (NaCl) sodium's outer electron spends most of its time around the chlorine atom
- In an ionic bond, atoms become ions, that is, they become electrically charged by gaining or losing electrons
- Ionic bonds are caused by the fact that the atoms that gain electrons exert a stronger force on the electrons than the atoms that lose electrons

Partial Ionic Character of Covalent Bonds

- **Pure Covalent Bond** - A covalent bond in which electrons are shared equally across atoms
- Many covalent bonds are not purely covalent and share electrons somewhere between a pure covalent and ionic bond, which is called *partial ionic character*
- Molecules with partial ionic character are polar, that is they have parts with a net positive and net negative charge

29.2 Potential-Energy Diagrams for Molecules

- **Potential Energy Diagram** - A plot of the potential energy versus separation distance
- For two point charges, the potential energy, PE, is given by

$$PE = k\frac{q_1 q_2}{r}$$

  Where k is a constant $k = 9.0 * 10^9 N * m^2/C^2 q_1$ and q_2 are the strengths of the two charges and r is the separation distance

- As PE increases, KE increases
- For two polar molecules, the PE diagram is negative and increasing in magnitude as r decreases since the closer the atoms get, the stronger the electric force get; After a certain point, the diagram curves up sharply as the nuclei of the atoms/molecules repel each other
- There is an optimal separation of atoms that causes potential energy to be at its lowest, called the *Binding Energy*, which tells how much energy must be put in to make the atoms separate to infinity
- Some bonds have Pe diagrams that are positive at larger distances and require more energy to get over the initial hump and attract atoms, called the *activation energy*
- Other bonds require energy input to form, and release energy when they are broken

29.3 Weak (van der Waals) Bonds

- **Strong Bonds** - Bonds that hold atoms together to form molecules
- **Weak Bonds** - An attachment between molecules due to electrostatic attraction, usually cause by attraction between dipoles
- **Electric Dipole** - A pair of point charges with equal magnitude and opposite sign
- Dipole-Induced Dipole bonds are caused by a dipole molecule inducing a dipole in a non-polar molecule
- **Hydrogen Bond** - The strongest of the weak (Van der Waals) bonds, occurs in a dipole-dipole bond when one of the dipoles is hydrogen as it has some covalent nature and can form longer lasting bonds

Protein Synthesis

- Weak bonds are crucial to protein synthesis
- Proteins are made of chains of molecules called *amino acids*, the instructions for which are provided by an organism's *genetic code*
-

29.4 Molecular Spectra

- When atoms bond, the probability distributions of their electrons intersect and their energy levels change
- **Band Spectra -** THe overlapped transitions from one energy level to the next in a molecule, these are difficult to discern

Rotational Energy Levels in Molecules

- The kinetic energy of a diatomic molecule rotating about its center is
$$E_{rot} = \frac{1}{2}I\omega^2$$
Where $I\omega$ is
$$I\omega = \sqrt{l(l+1)}$$
Where h is plank's constant and l is an integer
- Rotational energy is quantized
- Transitions between rotational energy levels are subject to the selection rule

Vibrational Energy Levels in Molecules

- The vibrational energy of a diatomic molecule is given by
$$E_{vib} = (v + \frac{1}{2})hf$$
Where v is an integer called the *vibrational quantum number*
- The lowest energy state is given by $v = 0$ which means there is still some energy, called the *zero-point energy*
- Transitions between vibrational energy levels are subject to the selection rule, so a change in vibrational energy can only be $\Delta E_{vib} = hf$

29.5 Bonding in Solids

- **Solid-STate Physics -** The study of matter in solid form
- **Amorphous -** Describes matter where particles are not in an arranged pattern
- **Crystalline -** Describes matter where particles are aranged in a pattern, called a *lattice*
- In ionic bonding, one atoms does not "belong" to a specific bond, rather the atom is shared by the particles around it
- **Metallic Bond -** The idea that in a metal structure, electrons flow freely around atoms instead of belonging to a specific atom
- A metallic structure is held together by the electrostatic attraction between the lattice of metal ions and sea of free roaming electrons
- The free electrons are what give metals conductivity; they carry vibrational energy throughout the structure to the atoms and conduct heat, and if they all move in the same direction they can carry electrical potential
- There also exist weak bonds in substances such as noble gases that make their particles hold on very weakly

29.6 Free-Electron Theory of Metals; Fermi Energy

- The free electron theory views electrons in a metal structure as always moving no matter what and obey the exclusion principle in that they can only move at explicit energy states
- **Fermi-Dirac Statistics -** Another quantum statistic obeyed by electrons, states that no two electrons can have the same set of quantum numbers
- **Fermi Level -** The highest energy level that can be filled by two electrons in an atom
- **Fermi Energy -** The energy state at the fermi level
- In a metal, the average energy in its electrons changes very little when temperature changes, note that this is different from an ideal gas
- Thinking of electrons in a metal lattice as a gas provides a good explanation for the conductive properties of metals

29.7 Band Theory of Solids

- As two atoms get closer, their energy electron shell states split into 2; when 6 atoms do the same, their energy levels split into 6
- When many atoms come together to form a solid, their electron states split into a band, energy levels so close together they seem continuous
- This explains why the spectrum of a heated solid appears continuous
- Good conductors have a highest energy band that is only partially filled
- A good insulator has a full highest energy level, so that their valence band and conduction bands are far apart
- Semiconductors have their valence and conduction bands close together

29.8 Semiconductors and Doping

- **Doping** - The process of introducing an impurity into the crystal structure of silicon to change its conductive properties
- Silicon has 4 valence electrons, and each atom is bonded to 4 others, thus its valence shell is full and there are no empty spots for electrons to move freely, so pure silicon is not a conductor
- Adding elements with different amounts of valence electrons is called *doping* and can improve the conductivity of Silicon
- Adding an element from group 5 of the periodic table introduces excess electrons and creates an *n-type* Semiconductor
- Adding an element from group 3 of the periodic table introduces an empty space in the silicon structure where electrons can move around in and creates a *p-type* Semiconductor
- Note that n-type and p-type semiconductors have no net charge
- Band theory states that doping has additional energy states between the valence and conduction bands, and thus makes silicon electrons require less energy to jump between bands

29.9 Semiconductor Diodes, LEDS, OLEDs

- **pn Junction Diode** - Forms when an n- and p-type semiconductor meet, some electrons from the n-type flow over and fill in holes in the p-type, giving the n- a positive charge and the p- a negative charge, forming a potential difference
- The junction area where all holes are filled is called the *depletion layer* because all extra electrons and holes are depleted
- If the positive terminal of a battery is connected to the p-side and the negative terminal to the n-side of a pn junction diode, the external voltage will oppose the intrinsic potential difference, and the diode is said to be *forward biased*
- A great enough voltage will overcome the natural potential difference and allow current to flow
- When a diode is reversed biased, the positive holes and extra electrons are pulled on by the battery's terminals in opposite directions and do not get close enough to allow a current to flow
- **Breakdown** - Occurs when too high of a voltage is applied to a reverse biased diode, its atoms ionize and allow electrons to flow in the direction of the batteries terminals **Zener Diode** - A designed to regulate voltage supply as long as voltage is maintained across their breakdown point
- Diodes are called *non-linear devices* because voltage an current across them is not proportional

Rectifiers

- Diodes can serve as *rectifiers* - devices that convert AC to DC
- **Half-Wave Rectification** - Occurs when a diode is place on an AC circuit, the diode blocks current flow on the backwards cycle of the AC cycle
- **Full-Wave Rectification** - Uses 2 or 4 diodes to make the entire wave of an AC current flow forwards, these circuits sometimes use an RC circuit to smooth out the wave flow

Photovoltaic Cells

- **Photovoltaic Cells** - Heavily doped pn junctions that convert sunlight to electricity
- Photons are absorbed and excite an electron to a higher energy level, leaving a hole, the produced electron and hole then allow current to flow

LED

- **Light Emitting Diode -** A type of diode where electrons emit a photon when flowing across the pn junction
- Light emission is achieved with compound semiconductors which usually involve a group III and group V element bonded together
-

Pulse Oximeter

- **Pulse Oximeter -** A device that uses 2 LEDs to measure the % of oxygen saturation in blood
- The diodes release light which is detected by a photodiode, the ratio of absorbed light (red/IR) is used to calculate oxygen saturation

pn Diode Lasers

- **Diode Lasers -** use a pn-junction in forward bias
- When one electron drops into a lower energy state, it releases a photon that pushes other electrons to drop in state and release photons as well

OLED

- Organic LEDs use organic compounds to perform the same basic functions as a conventional LED
- They consist of an emissive layer and conductive layer placed between two electrodes
- OLEDs can be smaller and more power efficient than conventional LEDs
- How they work: At a high enough voltage, electrons move into a higher state in the emissive layer and holes form in the conductive layer, these electrons and holes meet at the junction and combine to emit a photon

29.10 Transistors: Bipolar and MOSFETs

- **Bipolar Junction Transistor -** Consist of a crystal of one type of doped semiconductor between two of the other type, called npn or pnp
- Each semiconductor is given the name *collector, emitter, and base*
- **Amplifier -** One application of an npn transistor, a battery maintains a DC voltage across the collector and emitter and a voltage is aplied to the base called the *base bias voltage*
- If the BBV is positive, the electrons in the emitter are attracted into the base, then flow into the collector, thus a large current flows through the collector and emitter but only a small current flows through the base
- **Current Gain -** Defined as
$$\frac{output\ ac\ current}{input\ ac\ current}$$
- **Voltage Gain -** Defined as
$$\frac{output\ ac\ voltage}{input\ ac\ voltage}$$
- **MOSFET -** A type of transistor commonly used in digital circuits as a switch, the emitter is called the source, the collector is called the drain, and the base is called the gate

29.11 Integrated Circuits, 22-nm Technology

- Circuits can be made by inserting tiny impurities in a silicon wafer to produce components such as diodes, transistors, and resistors (called integrated circuits)
- The amount of transistors that can fit on a given surface area of silicon has been doubling every 2 to 3 years
- **Technology Generation -** Refers to the minimum width of a conducting line in a circuit, but the gate of a MOSFET can be smaller

30 Nuclear Physics and Radioactivity

Rutherford's experiments hinted at the center of atoms being a tiny, massive, positively charged nucleus. Later, quantum, theory was developed and more was being learned about the nucleus, this chapter focuses on nuclear physics

30.1 Structure and Properties of the Nucleus

- **Proton** - Has a mass of
$$m_p = 1.67262 * 10^{-27} kg$$
- **Neutron** - Has a mass of
$$m_p = 1.67493 * 10^{-27} kg$$
- Protons and neutrons are referred to as *nucleons*
- **Nuclides** - Used to refer to different nuclei
- **Atomic Number** - The number of protons in a nucleus, denoted by Z
- **Atomic Mass Number** - The number of protons plus the number of neutrons in an atom, denoted by A
- Nuclides can be defined by
$$^A_Z X$$
Where X is the chemical symbol for the element
- **Isotope** - Nuclei that share the same atomic number but different mass numbers
- **Natural Abundance** - The percentage of a naturally occurring isotope of a certain element
- Nuclei have a roughly spherical shape, the radius of which can be approximated by
$$r \approx (1.2 * 10^{-15} m)(A^{\frac{1}{3}})$$
- **Unified Atomic Mass Units** - A unit of mass used to specify the masses of nuclei, defined as
$$1u = 1.66054 * 10^{-27} kg$$
- Converting this to energy gives,
$$1u = 931.5 MeV/c^2$$
- **Nuclear Spin** - A quantum number of nuclei that can be either an integer or a half integer

30.2 Binding Energy and Nuclear Forces

Binding Energy

- The mass of a nucleus is always less than the sum of the masses of its individual nucleons
- **Total Binding Energy** - The difference in mass between a nuclei's true mass and the sum of its nucleons, represents the energy that must be put into a nucleus to separate it into its components
- Binding energy is something a nucleus lacks
- **Binding Energy per Nucleon** - Defined as the total binding energy divided by A

Nuclear Forces

- **Strong Nuclear Force** - The force that holds nucleons together an opposes the electrostatic repulsion between protons
- The strong nuclear force is a *short range* force, it acts over a very short distance
- **Weak Nuclear Force** - A weaker type of nuclear force that is only known about because of its presence in certain types of nuclear decay
- The four fundamental forces are the strong and weak nuclear forces, the electromagnetic force, and gravity

30.3 Radioactivity

- **Radioactivity** - The phenomenon of certain materials producing radiation that are not x-rays or weaker
- Radioactivity is the result of the *decay* of an unstable nucleus
- **Radioisotope** - A radioactive isotope of a certain element
- The tree types of radiation are alpha, beta, and gamma, each have a different charge

30.4 Alpha Decay

- An alpha particle is defined as (^4_2He) or a helium nucleus
- When alpha (α decay occurs, a new element is formed and its nucleus is referred to as the daughter nucleus while the original is called the parent
- **Transmutation -** The conversion from one element to another
- Alpha decay occurs when the strong nuclear force is no longer to hold large nuclei together due to electrostatic repulsion
- **Disintegration Energy -** Total energy released when a alpha decay occurs
- Alpha particles are held very strongly together, so their mass is less than the sum of their nucleons, so it is easier for them to separate from a parent nucleus
- **Smoke Detectors -** Devices that use alpha decay from Americium to ionize nitrogen and oxygen in the surrounding air, allowing current to flow, when smoke is present the smoke particles absorb the radiation instead and stop the current flow, setting off the alarm

30.5 Beta Decay

β^- **Decay**

- A β^- particle is an electron
- During beta decay, a neutrino is also released, which is a particle with a very small mass and no charge
- Because the electron is emitted from the nucleus itself, its charge is +1e more than it was previously
- What really happens is one of the neutrons within the nucleus turns into a proton and releases an electron, thus
$$n = p + e + neutrino$$
- The kinetic energy of a beta particle can range between 0 and 156keV
- The neutrino was discovered in an effort to determine why the kinetic energy was not the same for all beta particles
- The weak nuclear force is only present during beta decay, as the neutrino only interacts with matter through the weak nuclear force

β^+ **Decay**

- Some isotopes have too many protons and too few neutrons, these decay by emitting a positron instead of an electron
- A positron has the same mass an electron but has a positive charge, it is also called the antiparticle to the electron

Electron Capture

- **Electron Capture -** Occurs when a nucleus absorbs one of its orbiting electrons
- A proton in the nucleus then becomes a neutron and the nucleus moves down an atomic number

30.6 Gamma Decay

- **Gamma Ray -** An extremely high-energy photon, come from decaying nuclei
- Nuclei can have an excited state just like an atom, when they jump from a higher to lower energy state, they release a photon called a γ ray
- The energy states of nuclei vary on the order of keV to MeV, thus the photons they emit can have energies of a few MeV
- There is no change in an element as a result of gamma decay
- Nuclei are denoted with an asterisk to represent an excited state
- Sometimes, excited nuclei can remain that way for a long time, they are said to be in a *metastable* state and are called isomers
- **Internal Conversion -** The process through which an excited nucleus returns to its ground state not by releasing a gamma ray, but by releasing an orbital with the same kinetic energy a gamma ray would

30.7 Conservation of Nucleon Number and Other Conservative Laws

- In all three types of decay, conversion laws hold
- **The Law of Conversion of Nucleon Number -** States that the total number of nucleons, remains constant in any process

30.8 Half-Life and Rate of Decay

- A sample of a radioactive isotope does not release all of its radiation at the same time, but the nuclei do so one by one randomly
- The number of decays that occur over a time interval is proportional to the time interval and the number of radioactive nuclei present, represented by

  $$\Delta N = -\lambda N \Delta T$$

  or

  $$\frac{\Delta N}{\Delta t} = -\lambda N$$

  Where ΔN is the number of decays, Δt is the time interval, the - means N is decreasing, N is the number of radioactive nuclei in the sample, and λ is the *decay constant*
- **Decay Constant** - A value that varies for different isotopes

Exponential Decay

- **Radioactive Decay Law** - Solving for N gives

  $$N = N_0 e^{-kt}$$

  Where N_0 is the original amount of radioactive nuclei, e is the natural exponential, k is a constant, and t is time passed thus the number of radioactive nuclei decreases exponentially with time.
- The decay rate of a substance is also called its *activity*
- **Radioactive Decay Law** - Used to calculate decay rate of radioactivity, given by

  $$R = |\frac{\Delta N}{\Delta t}| = R_0 e^{-kt}$$

Half-Life

- **Half-Life** - Defined as the amount time it takes half the original amount of parent isotope to decay

30.9 Calculations Involving Decay Rates and Half-Life

- Problems involving half-life can be easily reasoned through using dimensional analysis
- Another important prerequisite to approaching such problems is knowledge of logarithms and what they can physically represent

30.10 Decay Series

- **Decay Series** - A chain of successive radioactive decays where daughter isotopes lead to other daughter isotopes
- Decay series cause us to observe isotopes with short half-lives occur naturally as they are replenished by the decay of isotopes higher in the series

30.11 Radioactive Dating

- **Radioactive Dating** - A technique used to determine the age of a sample by comparing the presence of radioactive isotopes before and after
- THe age of living matter can be determined by measuring the presence of carbon-14 since its presence in the atmosphere is relatively constant
- A similar technique is used to measure the age of rocks by measuring the presence of uranium-238

30.12 Stability and Tunneling

- A question that arises is why don't all radioactive nuclei immediately decay
- This is because decay particles must surpass the *coulomb barrier*, which it does so via *quantum mechanical tunneling*
- This occurs when, for a brief period of time, the particle violates the conservation of energy and is able to surpass the barrier, giving it enough time to "tunnel" through the barrier

30.13 Detection of Particles

Counters

- **Geiger Counter -** A device consisting of a metal tube filled with a gas and a wire kept at a high positive voltage
- When a charged particle enters the tube, it ionizes a few gas particles whose electrons move to the wire and ionize other gas particles, repeating the process
- The avalanche of electrons then creates a voltage pulse that the counter receives
- **Scintillation Counter -** Uses a scintillator or phosphor whose particles are easily excited and emit light when they return to ground state, they detect the light released
- **Photomultiplier Tube -** Converts the light emitted by a scintillator to an electric signal
- **Semiconductor Detector -** Uses a reverse-biased pn junction diode to detect a charged particle when it excites electrons into the conduction band

Visualization

- **Silicon Wafer Semiconductors -** Used to visualize charged particles, have pixels etched onto their surfaces that each give particle position information
- **Cloud Chamber -** Works bu super cooling a gas, tiny droplets form around charged particles and the reflection of light is used to detect the movement of particles
- **Bubble Chamber -** Similar to a cloud chamber, super heats a liquid and bubble of vapor form around charged particles
- **Multi wire Chamber -** Consists of many wires in a gas, some of which are charged. Charged particles excite molecules in the gas and ionize them, causing them to be attracted by the charged wires and sending a pulse once they come in contact

31 Nuclear Energy; Effects and Uses of Radiation

This chapter focuses on applications and effects of nuclear radiation.

31.1 Nuclear Reactions and the Transmutation of Elements

- **Transmutation** - Changing from one element to another, also occurs during nuclear reactions
- **Nuclear Reaction** - Occurs when a nucleus is struck by another nucleus or a simpler particle
- In a nuclear reaction, charge and nucleon number are maintained
- **Reaction Energy** - Used to calculate the energy used/expelled when particle a hits nucleus X and turns into nucleus Y and particle b, defined as

  $$Q = (M_a + M_X - M_b - M_Y)c^2$$

  This value is negative when energy is used and positive when energy is released

Neutron Physics

- Enrico Fermi discovered that hitting nuclei with neutrons had a much higher chance of causing a reaction
- The first transuranic (elements with atomic number greater than 92) elements were discovered by bombarding uranium nuclei with neutrons, creating heavier nuclei of elemtns 93, 94, ...

Collision Cross Section

- **Collision Cross Section** - Used to predict the probability of certain nuclear reactions happening
- Imagine a large area made of individual objects, each object has a cross sectional area, σ, and is being hit by smaller objects, the area has a total cross section of

  $$A' = nAl\sigma$$

  Where the quantity nAl is the total number of object that comprise the area
- σ can be determined by

  $$\sigma = \frac{R}{R_0 nl}$$

31.2 Nuclear Fission: Nuclear Reactors

Nuclear Fission and Chain Reactions

- Fission is the process through which a large nucleus splits into two smaller nuclei roughly half the size of the original
- **Liquid Drop Model** - A uranium-235 nucleus absorbs a neutron and becomes a *compound nucleus*, uranium-236
- The extra energy puts the nucleus in an excited state and elongates slightly, weakening the weak nuclear force between the two ends and the nucleus splits in two
- **Fission Fragments** - The resulting nuclei from a fission reaction
- A chain reaction could occur as uranium fission releases some neutrons too, which could be used to cause fission in neighboring uranium molecules
- Nuclear reactors take advantage of the chain reaction and use the collective energy of each fission event to produce heat

Nuclear Reactors

- **Moderator** - A substance used to slow down neutrons as they usually travel too fast to be absorbed by uranium atoms
- **Enrichment** - A process used to increase the percentage of uranium-235 in a sample of uranium in order to increase the likelihood of a nuclear reaction
- **Critical Mass** - The minimum mass of uranium required to achieve a self-sustaining reaction
- **Neutron Multiplication Factor** - The average number of neutrons produces by a reaction,represented by f, a self-sustaining reaction must have a NMF of $f \geq 1$

Atom Bomb

- Nuclear Bombs use a similar principle, but do not allow the uranium to reach its critical mass until it is time to detonate, after which it releases its energy quickly in an unhindered burst
- Atom bombs produce radioactive fragments called *fallout*

31.3 Nuclear Fusion

Nuclear Fusion; Stars

- **Nuclear Fusion** - Bringing two nuclei together to build a larger nuclei
- Fusion produces energy because the binding energy per nucleon is less for lighter nuclei than for heavier nuclei
- The proton-proton chain where hydrogen nuclei fuse into helium nuclei in the sun goes as follows

$$^1_1H \rightarrow^4_2 He + 2e^+ + 2v + 2\gamma$$

Possible FUsion Reactors

- One of the primary reactions that humans could use in nuclear fusion reactors to produce energy is

$$^2_1H +^3_1 H \rightarrow^4_2 He + n$$

- Using a fission bomb to detonate a fusion bomb is easy but harnessing usable energy from fusion has proven difficult as the extreme conditions necessary to do so are not readily available on Earth
- **Magnetic Confinement** - One such method developed to contain the high temperature plasma necessary for fusion, uses strong magnetic fields to keep the plasma suspended in air and rotate it to increase its kinetic energy
- **Lawson Criterion** - A value that the product of ion density ($ions/m^3$ and confinement time, τ, must surpass in order to produce ignition of a fusion reaction, defined as $3 * 10^20 s/m^3$

31.4 Passage of Radiation Through Matter: Biological Damage

- **Ionizing Radiation** - Any type of radiation that can ionize an atom of any material
- Radiation damage in organisms is caused by ionization in cells
- The radiation disrupts regular processes in the cell by knocking out electrons that hold together proteins or altering genetic code
- If enough radiation is received, it could kill too many cells or cause cancer in the organism

31.5 Measurement of Radiation-Dosimetry

- **Dosimetry** - The study of measuring dosage of radiation
- **Source Activity** - The rate of nuclear decays per second from a source, unit is the *curie*, defined as

$$1Ci = 3.70 * 10^{10} decays/s$$

- SI unit for source activity is the becquerel, defined as

$$1Bq = 1decay/s$$

- **Absorbed Dose** - Another method of measuring exposure to radiation, this measures the effect the radiation has on the absorbing material
- Absorbed dose is measured in *rads*, defined as the amount of radiation that deposits $0.01J/kg$ of energy per units mass, another unit is the gray, defined as

$$1Gy = 1J/kg = 100rad$$

- **Relative Biological Effectiveness** - The number of rads or X-ray or γ radiation that produces the same biological damage as 1 rad of the given radiation
- **Effective Dose** - Defined as the product of doese in rads and RBE, units is rem (rad equivalent man)

$$rem = rad * RBE$$

- Another unit of effective dose is the sievert, defined as

$$1Sv = 100rem$$

Human Exposure to Radiation

- We are regularly exposed to radiation from sources such as cosmic rays and rocks
- **Natural Radioactive Background** - A source of radiation that averages 0.30rem per year per person in the U.S.
- People who work around radiation should only receive a maximum of 20mSv of radiation per year, their dosage is measured by a dosimeter they carry around
- **Radiation Sickness** - A disease caused by large doses of radiation, symptoms include loss of body hair and nausea

31.6 Radiation Therapy

- Radiation can be used to treat cancer
- **Radiation Therapy -** A cancer treatment that works by focusing large amount of radiation on cancer cells in order to destroy them
- Protons release most of their energy at the end of their path, thus they can be set to destroy cells at a chosen depth by changing the amount of energy behind the proton
- Another technique used is placing a radioactive source in the tumor which will eventually kill most of the cells

31.7 Tracers in Research and Medicine

- **Tracer -** A compound that incorporates a radioactive isotope and can be traced within the body
- Tracers can be used to track how food is digested in the body or where certain compounds are diverted to
- **Auto radiography -** Radioactive isotopes are tracked on film
- **Gamma Camera -** Simultaneously record radioactivity at many points which can be displayed on a screen for ease of visualization

31.8 Emission Tomography: PET and SPECT

- **Single Photon Emission Computed Tomography (SPECT) -** The process of using gamma cameras to detect radioactive intensity at many points
- **Positron Emission Tomography -** Uses positron emitters in a compound that accumulates in a single area to further observe it
- Both SPECT and PET give images tha show biochemistry and metablism

31.9 Nuclear Magnetic Resonance (NMR) and Magnetic Resonance Imagine (MRI)

Nuclear Magnetic Resonance (NMR)

- This technique places the sample to be studied in a static magnetic field, photons are applied to the sample, if the frequency of the photons correspond to a difference in energy levels then nuclei in the sample will be excited
- For hydrogen nuclei, this frequency is 42.58MHz for a magnetic field of 1.0T
- Neighboring nuclei and electrons will affect the frequency at which a nucleus becomes excited

Magnetic Resonance Imaging

- This is the use of NMR for medical imaging
- Hydrogen is often used for MRI as it is the most abundant element in the body and produces the strongest NMR signals

32 Elementary Particles

After the discovery of protons and neutrons, more particles began being discovered, each with unique properties. Fundamental particles are defined as particles so simple they have no internal structure, the fundamental constituents of matter are quarks and leptons. The theory that describes out present view is the standard model.

32.1 High-Energy Particles and Accelerators

- **High-Energy Accelerator** - Devices used to accelerate particles to high energies for experimental purposes

Wavelength and Resolution

- De Broglie's wavelength formula,
$$\lambda = \frac{h}{p}$$
Says that as momentum increases, the wavelength of a particle decreases
- Thus, using a higher energy accelerator can lead to more detail as the shorter wavelength allows more to be seen

Cyclotron

- A device that uses a magnetic field to maintain charged ions in a nearly circular path, still used for treating cancer today
- The frequency, f, of the applied voltage must be the same as the circulating ions, the force, F, on each ion is given by
$$F = qvB$$
thus
$$qvB = \frac{mv^2}{r}$$
- **Synchrocyclotron** - A device used in a cyclotron that reduces frequency in time to correspond to the increase in γm

Synchrotron

- A device that accelerates particles by increasing the magnetic field, B, particles move in a circle
- The largest accelerators in the world are synchrotrons and use rows of magnets in order to carry particles
- **Synchrotron Radiation** - Radiation emitted by the EM fields generated by a synchrotron

Linear Accelerator

- **Linear Accelerator** - Particles are accelerated along a straight path passing through tubular conductors, voltage within the tubes alternates to attract particles between the gaps and allow them to flow more

Colliding Beams

- **Colliding Beams** - Two beams of particles are accelerated and made to collide head on, accomplished in a single accelerator through the use of storage rings

32.2 Beginning of Elementary Particle Physics - Particle Exchange

- A force can be thought of as a field OR an exchange of particles between two objects
- For the electromagnetic force, photons are exchanged which gives rise to the force between two objects
- The photon is emitted by one charged particle and absorbed by another, so it cannot be seen and is referred to as a *virtual* photon, and is said to carry the EM force
- Another particle serves a similar purpose but for the strong nuclear force, called the *meson*
- Mesons carry the strong nuclear force between protons and neutrons
- The particles that carry the weak force are referre to as $W^+,\ W^-,\ Z^0$
- The particle that carries the gravitational force is called the graviton, but it has not yet been detected

32.3 Particles and Antiparticles

- Antiparticles are very similar to regular particles but have some properties reversed
- **Antiprotons** - Have the same mass as protons but has a negative charge
- All particles have an antiparticle
- **Antimatter** - Refers to matter composed of antiparticles

Negative Sea of Electrons

- It was discovered that that equation

  $$E = \pm\sqrt{p^2c^2 + m^2c^4}$$

  Has negative solutions
- This implies there are negative energy states, which were thought to be filled with negative of electrons
- If a photon hit one of these negative electrons, it would go up an energy level and leave a hole, a positron
- The uncertainty principle allows a particle to jump briefly to a normal energy level, thus creating a particle-antiparticle pair

32.4 Particle Interactions and Conservation Laws

- **Baryon Number** - A generalization of nucleon number, nucleons have a baryon number of 1 and antinucleons have a baryon number of -1
- Some reactions do not occur due to the law of conservation of baryon number
- Sometimes, beta decay emits a muon instead of an electron, which is more massive and has its own neutrino
- There also exists the conservation of electron lepton number, where electrons and their neutrinos are assigned $L_e = +1$ and their antiparticles $L_e = -1$
- **Muon Lepton Number** - Muons and muon neutrinos have a muon lepton number of $L_\mu = +1$ and their antiparticles have a number of $L_\mu = -1$
- **Tau Lepton Number** - Tau leptons and tau neutrinos have a tau lepton number of $L_\tau = +1$ and their antiparticles have a number of $L_\tau = -1$
- All the lepton numbers must be conserved in a reaction

32.5 Neutrinos

- **Solar Neutrino Problem** - The sun emits electron neutrinos but when measurements began in the 1960s, rates of neutrinos were much lower than expected
- It was proposed and proved that electron neutrinos can turn into mu or tau neutrinos under certain circumstances, thus lepton numbers are not perfectly conserved but their sum is
- It has also been speculated that not all neutrinos are massless
- Neutrinos may be *majorana* particles, or be there own antiparticles

32.6 Particle Classification

- The fundamental particles include the gauge bosons (photons, gluons, W and Z particles), and leptons
- Another particle category is the hadrons which are composite (made of quarks) particles and interact via the strong nuclear force

32.7 Particle Stability and Resonances

- The lifetime of an unstable particle depends on which force is most active in causing its decay
- Particles that decay via the strong nuclear force typically have very short lifetimes
- A *resonance* occurs when two particles interact significantly at a certain energy level

32.8 Strangeness? Charm? Towards a New Model

- New strange particles were discovered that did not allow some reactions to occur, thus the strange quantum number was invented and its conservation law was introduced
- Strangeness is a partially conserved quantity, it is conserved by the strong force but not the weak

32.9 Quarks

- Hadrons are made of smaller, point-like particles called quarks
- The 6 quarks are up, down, top, bottom, charm, and strange
- All quarks have a spin of 1/2 and a charge of $+2/3e$ or $-1/3e$
- Current models suggest it is impossible for quarls to exist freely and must always be bound together

32.10 The Standard Model: QCD and Electroweak Theory

- Quarks also have a color property, and can be classified as red, green, or blue, note that they are not actually colored particles
- Particles that obey the exclusion principle are called *fermions*
- Each quark is assumed to carry a color charge and the strong force between them is the color force
- There have been efforts to find a unified basis for all four fundamental forces
- The standard model is a combination of the electroweak theory (which combines the electromagnetic and weak nuclear forces) and the strong interaction, all that is missing is gravity

32.11 Grand United Theories

- **Grand Unified Theory -** A theory that incorporates multiple forces
- One theory attempts to unite the EM and strong and weak nuclear forces into one by stating their particles can switch at any time but only at an energy level of around $10^{16} GeV$, at this level, distinguishing between these would become almost impossible
- **Symmetry Breaking -** The phenomenon that occurs when different types of particles are closer than their unification distance, the force between elementary particles at a distance of $10^{-31} metersisthoughttobethesameforc$

Proton Decay

- One testable prediction is that protons decay (and would violate the conservation of baryon number) if two quarks within it came within $10-31m$ of each other

GUT and Cosmology

- Another theory suggests that at the beginning of the universe, it was so hot that particles reached energis on the unification scale, which would have caused baryon number to not be conserved
- This could have led to the imbalance of matter and antimatter we see today

32.12 Strings and Supersymmetry

- **String Theory -** An attempt to unify all 4 forces, it suggests fundamental particles are actually strings that vibrate at a specific standing wave pattern
- Applying another idea, supersymmetry, to string theory gives superstring theory
- **Supersymmetry -** A theory that suggests interactions that would change fermions to bosons and vice versa exist
- Some think current theories are approximations of a more fundamental M-Theory

33 Astrophysics and Cosmology

Astrophysics is defined as the use of physics techniques to study celestial bodies. Cosmology is the study of the universe as a whole.

33.1 Stars and Galaxies

- Objects in space are so far apart their distances are given in light-years, defined as
$$1ly = 9.46 * 10^{15} km$$
- **Nebula** - A large cloud of dust or gas in space
- Galaxies tend to group themselves into galaxy clusters, held to gether by gravitational attraction
- Clusters then group themselves into superclusters
- **Active Galactic Nuclei** - The very bright points in the center of galaxies, thought to come from the matter falling into the center of the black hole

33.2 Stellar Evolution: Birth and Death of Stars, Nucleosynthesis

- **Luminosity** - Total power radiated in watts
- **Apparent Brightness** - The power crossing unit area at the Earth perpendicular to the path of the light
- Luminosity is spread over a sphere's surface area with its radius being the distance, d, from which it is measured, thus apparent brightness is
$$b = \frac{L}{4\pi d^2}$$
- The more massive a star is, the greater its luminosity

H-R Diagram

- The color of a star is related to its luminosity and thus its color is also related to its mass
- An H-R diagram graphs the temperature and luminosity of stars in an attempt to identify relationships

Stellar Evolution; Nucleosynthesis

- **Stellar Evolution** - The life-cycle that stars go through
- Stars are born from large clouds of gas and dust, usually nebulae and start nuclear fusion in their cores once they are massive enough
- Once the star has fused much of its hydrogen, it shrinks slightly and releases less energy as heavier nuclei are fused
- After this, the star moves to the red giant stage where it grows in volume by a factor of 100 or more
- Then, if the star has too low of a mass, it expands more then loses its outer layers, revealing its core, becoming a white dwarf
- If the star has a high enough mass, the electrons and protons within it can undergo inverse beta decay, becoming neutrons which make the star much more dense
- Eventually, a neutron star's core will collapse and undergo supernova, forming virtually all the elements of the periodic table

33.3 Distance Measurements

Parallax

- **Parallax** - The apparent motion of a star against the background of more distance stars

Parsec

- Defined in terms of seconds of arc, one parsec is
$$1pc = 3.26ly$$

Distant Stars and Galaxies

- Further than about 100 light-years, parallax angles are too small to measure
- One technique used to measure far object is comparing the apparent brightness of two stars or two galaxies and using an inverse square law to solve for the unknown distance
- Another technique uses an H-R diagram by determining a star's surface temperature an using the veritcal axis of the diagram to determine distance

Distance via SNIa, Redshift

- The largest distances are estimated using Type Ia supernovae, which have similar origins and similar luminosities

33.4 General Relativity: Gravity and the Curvature of Space

- Einstein proved that it is impossible for an observer to tell if an inertial reference frame is stationary or moving in a straight line
- **Principle of Equivalence** - No experiment can be performed that could distinguish between a uniform gravitational field and an equivalent uniform acceleration
- Although they are equivalent, inertial and gravitational mass are in fact different
- Light is affected by gravity
- **Gravitational Lensing** - Nearby galaxies acting as a magnifying glass that bends light from distant objects towards us
- Einstein's general relativity states that gravitational forces bend space itself, which explains the bending of light
- The curvature of space can be measured by summing the angles of a triangle, which may not add up to 180 degrees if space is curved

Curvature of the Universe

- If the universe has a positive curvature, it would be closed and have a finite volume
- If the universe has a curvature of 0 or less, it would be open and could be infinite
- Current suggestions say the universe is so flat that we cannot tell if its curvature is slightly positive or slightly negative

Black Holes

- Einstein's GR states that massive objects curve the space around them
- Black Holes curve space the most, to become one, an object mass must be squeezed into a radius threshold called the Schwarzschild radius, defined as

$$R = \frac{2GM}{c^2}$$

  Where G is the gravitational constant, M is its mass, and c is the speed of light

- This radius also gives the event horizon of a black hole, which is the surface from which no emitted signals can be detected from

33.5 The Expanding Universe: Red shift and Hubble's Law

- The doppler effect for light is given by

$$\lambda_{obs} = \lambda_{rest}\sqrt{\frac{1+v/c}{1-v/c}}$$

- When light moves away from us, it appears longer and thus appears more red, called *redshift*
- The distance a galaxy moves away from us is proportional to its distance, given by

$$v = H_0 d$$

  where H_0 is the hubble parameter, defined as

$$H_0 = 21km/s/Mly$$

- Another way redshift occurs is through *gravitational redshift*

Expansion and the Cosmological Principle

- It has been established that the universe looks the same from different points
- Every galaxy in the universe is racing away from each other at the Hubble Parameter
- **Cosmological Principle** - The assumption that the universe is isotropic (looks the same from all directions) and homogeneous (looks the same from different locations)
- **Steady State Model** - Assumes the universe is infinitely old and, on average, looks the same now as it always has
- The steady state model suggests no large scale changes have occurred in the universe and that uniformity requires mass to be created at a very slow rate

33.6 The Big Bang and the Cosmic Microwave Background

- **Cosmic Microwave Background** - Evidence towards the big bang, this is radiation that comes from the universe as a whole, measured at $\lambda = 7.35cm$
- **Anisotrophy** - Having different properties in different directions, this is a property of the universe that has been confirmed
- Once the universe cooled to about 3000K, particles could form atoms and free electrons became rare, which allowed photons to travel freely, leading to the CMB

Looking Back toward the Big Bang

- **Lookback Time** - The amount of time it takes light from an event to reach an observer
- **Surface of last scattering** - The edge of the observable universe, we cannot see past this

The Observable Universe

- Many theories suggest that the entire universe is larger than the observable universe

33.7 The Standard Cosmological Model: Early History of the Universe

- **The Standard Cosmological Model** - The current most popular theory for the history and current state of the universe
- It is believed that after an imbalance of matter and antimatter annihilated most of the antimatter, the universe entered the lepton era where electrons and positrons were formed and destroyed each other, releasing photons
- Later, the radiation era began, there were mainly photons and neutrinos but the neutrinos only interacted via the weak force and thus the primary interaction was radiation
- The universe then became matter dominated with eh formation of the first atoms and molecules

33.8 Inflation: Explaining Flatness, Uniformity, and Structure

Flatness

- One theory that explains the flatness of the universe is that the universe has a curvature of 0 and as the observable universe expands, we see more of the flatness

CMB Uniformity

- The CMB is almost entirely uniform, which can be attributed to the fact that in the beginning, the universe was in thermal equilibrium

Galaxy Seeds, Fluctuations

- Forces can undergo quantum fluctuations that are so small they are not detectable unless magnified
- It is believed that inflation magnified the tiny fluctuations of the early universe and created the density irregularities we see today

33.9 Dark Matter and Dark Energy

- **Big Crunch** - The end of the universe if it has a positive curvature, it would be finite and gravity will eventually overtake to bring everything back together

Critical Density

- **Critical Density** - The average mass per volume density threshold that, if surpassed, would give the universe positive curvature, defined as roughly $10^{-26} kg/m^3$
- We currently believe the universe is expanding at an accelerating rate

Dark Matter

- Current experiments have many convinced that the universe is flat and that the critical density is exactly the true density, but normal matter only accounts for about 5% of the critical density's requirements
- Dark matter is believed to be the other 95%, and observations of stars or galaxies in motion suggest they have considerably more mass than previously thought
- Dark matter, if it exists, is likely made of weakly interacting massive particles, which makes it very difficult to detect

Dark Energy - Cosmic Acceleration

- The expansion of the universe is accelerating
- **Dark Energy** - An unknown force that has been suggested to be responsible for this acceleration
- Current estimations on the composition of the universe put the planets and the stars to be about 0.5% of all the mass-energy available

33.10 Large-Scale Structure of the Universe

- Simulations with cold dark energy and dark matter have fit current observations of the universe quite well
- Observations of the CMB have been increasing in precision and soon may be able to detect the effects of gravity waves and provide info for both cosmic inflation and particle physics

33.11 Finally...

- **Anthropic Principle -** States that if the universe were even a little different than it is, we would not be here