

# Two Quick Proofs as an Expression of Affection for Mathematics

Edwin Trejo Balderas

August 2022

## 1 Introduction

This document aims to offer a glimpse into the independent math work I do in my free time as, unfortunately, I do not believe the activities section on the WISE application offers me enough space to do that. I also hope it serves as sufficient proof of my deep love for mathematics and interest in MIT's course 18. In this document, I prove that every positive even power of 2 is one more than a multiple of 3 and every positive odd power of 2 is one less than a multiple of 3 using mathematical induction. I came up with the propositions presented by accident, simply wondering about the powers of 2 and the numbers near them, which is how many of my propositions originate. This creativity that math allows is what I mean when I say it is an art.

## 2 Observations and Propositions

The first few powers of 2 (starting at  $n = 1$ ) are shown below:

$$a_n = 2^n = 2, 4, 8, 16, 32, 64, \dots$$

Notice how adding 1 to each of 2, 8, and 32 gives a multiple of 3:

$$2 + 1 = 3 = 3(1)$$

$$8 + 1 = 9 = 3(3)$$

$$32 + 1 = 33 = 3(11)$$

Also notice how 2, 8, and 32 are odd powers of 2:

$$2 = 2^1, 8 = 2^3, 32 = 2^5$$

From this, I propose this holds for all odd powers of 2:

**Proposition 1.**

$$2^{2n-1} + 1 = 3(x) \text{ such that } x \text{ is an integer for all } n = 1, 2, 3, \dots$$

Alternatively, we notice how subtracting 1 from each of 4, 16, and 64 gives a multiple of 3:

$$4 - 1 = 3 = 3(1)$$

$$16 - 1 = 15 = 3(5)$$

$$64 - 1 = 63 = 3(21)$$

Also notice how 4, 16, and 64 are even powers of 2:

$$4 = 2^2, 16 = 2^4, 64 = 2^6$$

From this, I propose this holds for all even powers of 2:

**Proposition 2.**

$$2^{2n} - 1 = 3(x) \text{ such that } x \text{ is an integer for all } n = 1, 2, 3, \dots$$

### 3 Proof of Proposition 1

To prove the following:

$$2^{2n-1} + 1 = 3(x) \text{ such that } x \text{ is an integer for all } n = 1, 2, 3, \dots$$

We first must show it is true for the case where  $n = 1$ , indeed:

$$2^{2(1)-1} + 1 = 2 + 1 = 3$$

Now, assuming  $n = k$  for all  $k = 1, 2, 3, \dots$ , we have:

$$2^{2k-1} + 1 = 3(x) \text{ such that } x \text{ is an integer for all } k = 1, 2, 3, \dots$$

It follows that this is also true for all  $n = k + 1$ :

$$2^{2(k+1)-1} + 1 = 3(x)$$

$$2^{2(k+1)-1} + 1 = 2^{2k-1+2} + 1 = 2^2 2^{2k-1} + 1 = 4(2^{2k-1}) + 1 = 3(2^{2k-1}) + 2^{2k-1} + 1$$

From our assumption, we know that  $2^{2k-1} + 1$  is in fact a multiple of 3, so we can replace it with  $3m$  where  $m$  is some integer:

$$3(2^{2k-1}) + 3m = 3(2^{2k-1} + m)$$

Since both factors of 3 are integers, we have shown that the above expression is indeed a multiple of 3, and have proven proposition 1

### 4 Proof of Proposition 2

The proof of the second proposition, shown below, follows a very similar progression as the first.

$$2^{2n} - 1 = 3(x) \text{ such that } x \text{ is an integer for all } n = 1, 2, 3, \dots$$

For  $n = 1$ , indeed:

$$2^2 - 1 = 4 - 1 = 3$$

Assuming  $n = k$  for all  $k = 1, 2, 3, \dots$ , we have:

$$2^{2k} - 1 = 3(x) \text{ such that } x \text{ is an integer for all } k = 1, 2, 3, \dots$$

It follows that this is also true for all  $n = k + 1$ :

$$2^{2(k+1)} - 1 = 3(x)$$

$$2^{2(k+1)} - 1 = 2^2 2^{2k} - 1 = 4(2^{2k}) - 1 = 3(2^{2k}) + 2^{2k} - 1 = 3(2^{2k}) + 3m = 3(2^{2k} + m)$$

Thus proving proposition 2.

### 5 Bonus Proof

Since all even powers of 2,  $2^{2n}$ , can be simplified to powers of 4,  $4^n$ , we can see that, using the proof of proposition 2, the following holds:

$$4^n - 1 = 3(x) \text{ such that } x \text{ is an integer for all } n = 1, 2, 3, \dots$$

### 6 Extensions

Currently, I am working towards generalizing these two theorems to the rest of the integers, such as even/odd powers of 3 being 1 greater/less than multiples of 4, or rather even/odd powers of  $n$  being 1 greater/less than multiples of  $n + 1$ . I am also working to see if this holds in reverse, where all even/odd powers of  $n$  could be 1 greater/less than multiples of  $n - 1$ . Thank you so much for reading! :)