IB Math AA SL Investigation: *Volumes of Cones*

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The following gives the page(s) on which questions are answered:

1. Quesiton 1

   (a) Using data from the table, the system of equations is:

$$\begin{aligned} y(x) &= a_0 + a_1 x + a_2 x^2 + a_3 x^3 \\ y(-1) &= a_0 + a_1(-1) + a_2(-1)^2 + a_3(-1)^3 = 4 \\ y(0) &= a_0 + a_1(0) + a_2(0)^2 + a_3(0)^3 = 2 \\ y(1) &= a_0 + a_1(1) + a_2(1)^2 + a_3(1)^3 = 4 \\ y(2) &= a_0 + a_1(2) + a_2(2)^2 + a_3(2)^3 = 4 \end{aligned}$$

   Simplifying gives:

$$\begin{aligned} a_0 - a_1 + a_2 - a_3 &= 4 \\ a_0 + 0a_1 + 0a_2 + 0a_3 &= 2 \\ a_0 + a_1 + a_2 + a_3 &= 4 \\ a_0 + 2a_1 + 4a_2 + 8a_3 &= 4 \end{aligned}$$

   Constructing an augmented matrix from this system gives:

$$\begin{bmatrix} 1 & -1 & 1 & -1 & 4 \\ 1 & 0 & 0 & 0 & 2 \\ 1 & 1 & 1 & 1 & 4 \\ 1 & 2 & 4 & 8 & 4 \end{bmatrix}$$

   Where the right-most column is the column containing constant terms

   (b) Applying: row operations to the matrix from (a) gives:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 2 \\ 1 & 1 & 1 & 1 & 4 \\ 1 & -1 & 1 & -1 & 4 \\ 1 & 2 & 4 & 8 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 1 & 1 & 2 \\ 0 & -1 & 1 & -1 & 2 \\ 0 & 2 & 4 & 8 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 1 & 1 & 2 \\ 0 & 0 & 2 & 0 & 4 \\ 0 & 0 & 2 & 6 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 1 & 1 & 2 \\ 0 & 0 & 2 & 0 & 4 \\ 0 & 0 & 0 & 6 & -6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 1 & 1 & 2 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

   Which is in RREF.

   (c) The RREF matrix from part (b), gives the values of the coefficients of the polynomial to be:

$$a_0 = 2, a_1 = 1, a_2 = 2, a_3 = -1$$

   Which gives:

$$y(x) = 2 + x + 2x^2 - x^3$$

2. Question 2

   (a) Filling in the places for variables with 0 coefficients in the system gives:

$$\begin{aligned} 2x_1 + x_2 + x_3 + 0x_4 + 7x_5 &= 20 \\ x_1 + 0x_2 + x_3 + x_4 + 3x_5 &= 9 \\ 0x_1 + x_2 + x_3 + 0x_4 + 5x_5 &= 10 \end{aligned}$$

   Converting this system to an augmented matrix gives:

$$\begin{bmatrix} 2 & 1 & 1 & 0 & 7 & 20 \\ 1 & 0 & 1 & 0 & 3 & 9 \\ 0 & 1 & 1 & 0 & 5 & 10 \end{bmatrix}$$

   (b) Applying row operations to the matrix in (a) gives:

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 3 & 9 \\ 0 & 1 & 1 & 0 & 5 & 10 \\ 2 & 1 & 1 & 0 & 7 & 20 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 3 & 9 \\ 0 & 1 & 1 & 0 & 5 & 10 \\ 0 & 1 & -1 & 0 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 3 & 9 \\ 0 & 1 & 1 & 0 & 5 & 10 \\ 0 & 0 & -2 & 0 & -4 & -8 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 3 & 9 \\ 0 & 1 & 1 & 0 & 5 & 10 \\ 0 & 0 & 1 & 0 & 2 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 5 \\ 0 & 1 & 0 & 0 & 3 & 6 \\ 0 & 0 & 1 & 0 & 2 & 4 \end{bmatrix}$$

   Which is in RREF.

   (c) Translating the augmented matrix in RREF from (b) back into a system of equations gives:

$$\begin{aligned} x_1 + x_5 &= 5 \\ x_2 + 3x_5 &= 6 \\ x_3 + 2x_5 &= 4 \end{aligned}$$

   Solving for our basic variables, x_1, x_2, and x_3, gives:

$$\begin{aligned} x_1 &= 5 - x_5 \\ x_2 &= 6 - 3x_5 \\ x_3 &= 4 - 2x_5 \end{aligned}$$

   Thus, our solution set becomes:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 5 - x_5 \\ 6 - 3x_5 \\ 4 - 2x_5 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ 4 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ x_4 \\ 0 \end{bmatrix} + \begin{bmatrix} -x_5 \\ -3x_5 \\ -2x_5 \\ 0 \\ x_5 \end{bmatrix} =$$

$$\begin{bmatrix} 5 \\ 6 \\ 4 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -1 \\ -3 \\ -2 \\ 0 \\ 0 \end{bmatrix}$$

3. Question 3

   (a) Name: Edwin Trejo Balderas; Facilitator: LaRosa Johnson; School: Douglas County High School