UNIT - 1

Error (E) = Exact Value – Approximation Value

Absolute Error (E_A) = | Exact Value – Approximation Value |

Relative Error (E_R) = $\dfrac{|\text{Exact Value} - \text{Approximation Value}|}{|\text{Exact Value}|}$

Relative Error Percentage (E_R %) = $\dfrac{|\text{Exact Value} - \text{Approximation Value}|}{|\text{Exact Value}|}$ X 100

Bisection Method $x_n = \dfrac{a+b}{2}$

Regula Falsi or False Position Method $x_n = \dfrac{af(b) - bf(a)}{f(b) - f(a)}$

Iterative method $x_n = \phi(x_{n-1}), n \geq 1$

Newton Raphson Method $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

Ramanujan's Method

$$b_1 = 1$$

$$b_2 = a_1 b_1$$

$$b_3 = a_2 b_1 + a_1 b_2$$

$$b_4 = a_1 b_3 + a_2 b_2 + a_3 b_1$$

$$b_5 = a_1 b_4 + a_2 b_3 + a_3 b_2 + a_4 b_1$$

$$b_6 = a_1 b_5 + a_2 b_4 + a_3 b_3 + a_4 b_2 + a_5 b_1$$

Muller' s Method

$$f_0 = f(x_0),\ f_1 = f(x_1),\ f_2 = f(x_2)$$

$$D = (x_2 - x_1)(x_2 - x_0)(x_1 - x_0)$$

$$a = \frac{[(f_2 - f_1)(x_2 - x_0) - (f_2 - f_0)(x_2 - x_1)]}{D}$$

$$b = \frac{[(f_2 - f_1)(x_2 - x_0)^2 - (f_2 - f_0)(x_2 - x_1)^2]}{D}$$

$$c = f_2$$

$$h = \frac{-2c}{b + (signofb)\sqrt{b^2 - 4ac}}$$

$$x_3 = x_2 + h$$

SOLUTION TO SYSTEMS OF NONLINEAR EQUATIONS

$$\left.\begin{aligned} f(x, y) &= 0 \\ \text{and} \quad g(x, y) &= 0 \end{aligned}\right\}$$

Method of Iteration

$$x = F(x, y), \quad y = G(x, y),$$

$$\begin{aligned}
x_1 &= F(x_0, y_0), & y_1 &= G(x_0, y_0) \\
x_2 &= F(x_1, y_1), & y_2 &= G(x_1, y_1) \\
&\vdots & &\vdots \\
x_{n+1} &= F(x_n, y_n), & y_{n+1} &= G(x_n, y_n)
\end{aligned}$$

The process is to be repeated till we obtain the roots to the desired accuracy.

Newton–Raphson Method

$$f(x_0 + h, y_0 + k) = 0, \qquad g(x_0 + h, y_0 + k) = 0$$

$$h\frac{\partial f}{\partial x_0} + k\frac{\partial f}{\partial y_0} = -f_0$$

$$\text{and} \quad h\frac{\partial g}{\partial x_0} + k\frac{\partial g}{\partial y_0} = -g_0$$

$$D = \begin{vmatrix} \dfrac{\partial f}{\partial x_0} & \dfrac{\partial f}{\partial y_0} \\ \dfrac{\partial g}{\partial x_0} & \dfrac{\partial g}{\partial y_0} \end{vmatrix} \neq 0.$$

$$h = \frac{1}{D}\begin{vmatrix} -f_0 & \dfrac{\partial f}{\partial y_0} \\ -g_0 & \dfrac{\partial g}{\partial y_0} \end{vmatrix} \quad \text{and} \quad k = \frac{1}{D}\begin{vmatrix} \dfrac{\partial f}{\partial x_0} & -f_0 \\ \dfrac{\partial g}{\partial x_0} & -g_0 \end{vmatrix}$$

$$x_1 = x_0 + h \quad \text{and} \quad y_1 = y_0 + k$$

The process is to be repeated till we obtain the roots to the desired accuracy.

UNIT - 2

Newton's Forward Interpolation Formula

$$f(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0 \ldots\ldots,\ p = \frac{x-x_0}{h}$$

Newton's Backward Interpolation Formula

$$f(x) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \frac{p(p+1)(p+2)(p+3)}{3!}\nabla^4 y_n \ldots\ldots,\ p = \frac{x-x_n}{h}$$

Gauss forward central difference formula

$$f(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_{-1} + \frac{p(p-1)(p+1)}{3!}\Delta^3 y_{-1} + \frac{p(p-1)(p+1)(p-2)}{4!}\Delta^4 y_{-2} \ldots\ldots,\ p = \frac{x-x_0}{h}$$

Gauss backward central difference formula

$$f(x) = y_0 + p\Delta y_{-1} + \frac{p(p+1)}{2!}\Delta^2 y_{-1} + \frac{p(p-1)(p+1)}{3!}\Delta^3 y_{-2} + \frac{p(p-1)(p+1)(p+2)}{4!}\Delta^4 y_{-2} \ldots\ldots,\ p = \frac{x-x_0}{h}$$

Stirling's Formula

$$y_p = y_0 + p\frac{\Delta y_{-1} + \Delta y_0}{2} + \frac{p^2}{2}\Delta^2 y_{-1} + \frac{p(p^2-1)}{3!}\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2}$$

$$+ \frac{p^2(p^2-1)}{4!}\Delta^4 y_{-2} + \cdots$$

Bessel's Formula

$$y_p = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\frac{\Delta^2 y_{-1} + \Delta^2 y_0}{2} + \frac{p(p-1)(p-1/2)}{3!}\Delta^3 y_{-1}$$

$$+ \frac{(p+1)\,p(p-1)(p-2)}{4!}\frac{\Delta^4 y_{-2} + \Delta^4 y_{-1}}{2} + \cdots$$

Lagrange's Interpolation Formula

$$f(x) = \frac{(x-x_1)(x-x_2)\ldots(x-x_n)}{(x_0-x_1)(x_0-x_2)\ldots(x_0-x_n)}f(x_0) + \frac{(x-x_0)(x-x_2)\ldots(x-x_n)}{(x_1-x_0)(x_1-x_2)\ldots(x_1-x_n)}f(x_1) + \ldots \frac{(x-x_0)(x-x_1)\ldots(x-x_{n-1})}{(x_n-x_1)(x_n-x_2)\ldots(x_n-x_{n-1})}f(x_n)$$

Newton's General Interpolation Formula

$$f(x) = f(x_0) + (x-x_0)f(x_0,x_1) + (x-x_0)(x-x_1)f(x_0,x_1,x_2) + \ldots\ldots\ldots$$

UNIT - 3

Normal equations for fitting a straight line

$$\sum y = ma_0 + a_1 \sum x$$

$$\sum xy = a_0 \sum x + a_1 \sum x^2$$

Normal equations for fitting a parabola

$$\sum y = ma_0 + a_1 \sum x + a_2 \sum x^2$$

$$\sum xy = a_0 \sum x + a_1 \sum x^2 + a_2 \sum x^3$$

$$\sum x^2 y = a_0 \sum x^2 + a_1 \sum x^3 + a_2 \sum x^4$$

Derivatives Using Newton's Forward Difference Interpolation

$$y = f(x_0 + ph) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \ldots\ldots\ldots, \; p = \frac{x - x_0}{h}$$

$$\frac{dy}{dx} = \frac{1}{h}\left[\Delta y_0 + \frac{2p-1}{2}\Delta^2 y_0 + \frac{3p^2 - 6p + 2}{6}\Delta^3 y_0 + \ldots\ldots\right]$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\Delta^2 y_0 + \frac{6p-6}{6}\Delta^3 y_0 + \frac{12p^2 - 36p + 22}{24}\Delta^4 y_0 + \ldots\ldots\right]$$

$$\left(\frac{dy}{dx}\right)_{x=x_0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \ldots\ldots\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_0} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \frac{5}{6}\Delta^5 y_0 \ldots\ldots\right]$$

Derivatives Using Newton's Backward Difference Interpolation

$$y = f(x_n + ph) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \ldots\ldots\ldots, \; p = \frac{x - x_n}{h}$$

$$\frac{dy}{dx} = \frac{1}{h}\left[\nabla y_n + \frac{2p+1}{2}\nabla^2 y_n + \frac{3p^2 + 6p + 2}{6}\nabla^3 y_n + \ldots\ldots\right]$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\nabla^2 y_n + \frac{6p+6}{6}\nabla^3 y_n + \frac{12p^2 + 36p + 22}{24}\nabla^4 y_n + \ldots\ldots\right]$$

$$\left(\frac{dy}{dx}\right)_{x=x_n} = \frac{1}{h}\left[\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \frac{1}{4}\nabla^4 y_n + \ldots\ldots\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_n} = \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \frac{5}{6}\nabla^5 y_n \ldots\ldots\right]$$

Derivatives Using Central Difference Interpolation

$$y = y_0 + p\left(\frac{\Delta y_0 + \Delta y_{-1}}{2}\right) + \frac{p^2}{2}\Delta^2 y_{-1} + \frac{p^3 - p}{12}\left(\Delta^3 y_{-1} + \Delta^3 y_{-2}\right) + \frac{p^4 - p^2}{24}\Delta^4 y_{-2} + ...$$

$$p = \frac{x - x_0}{h}$$

$$\frac{dy}{dx} = \frac{1}{h}\left[\left(\frac{\Delta y_0 + \Delta y_{-1}}{2}\right) + p\Delta^2 y_{-1} + \frac{3p^2 - 1}{12}\left(\Delta^3 y_{-1} + \Delta^3 y_{-2}\right) + \frac{4p^3 - 2p}{24}\Delta^4 y_{-2} + \frac{5p^4 - 15p^2 + 4}{240}\left(\Delta^5 y_{-2} + \Delta^5 y_{-3}\right) + ...\right]$$

$$\frac{d^2 y}{dx^2} = \frac{1}{h^2}\left[\Delta^2 y_{-1} + \frac{6p - 1}{12}\left(\Delta^3 y_{-1} + \Delta^3 y_{-2}\right) + \frac{12p^2 - 2}{24}\Delta^4 y_{-2} + \frac{20p^3 - 30p}{240}\left(\Delta^5 y_{-2} + \Delta^5 y_{-3}\right) + ...\right]$$

$$\left(\frac{dy}{dx}\right)_{x=x_0} = \frac{1}{h}\left[\left(\frac{\Delta y_0 + \Delta y_{-1}}{2}\right) - \frac{1}{12}\left(\Delta^3 y_{-1} + \Delta^3 y_{-2}\right) + \frac{1}{60}\left(\Delta^5 y_{-2} + \Delta^5 y_{-3}\right) + ...\right]$$

$$\left(\frac{d^2 y}{dx^2}\right)_{x=x_0} = \frac{1}{h^2}\left[\Delta^2 y_{-1} - \frac{1}{12}\Delta^4 y_{-2} + \frac{1}{90}\Delta^6 y_{-3} - ....\right]$$

$$\left(\frac{d^3 y}{dx^3}\right)_{x=x_0} = \frac{1}{h^3}\left[\frac{1}{2}(\Delta^3 y_{-1} +^3 y_{-2}) + ...\right]$$

Trapezoidal Rule

$$\int_a^b f(x)dx = \frac{h}{2}[(y_0 + y_n) + 2(y_1 + y_2 + y_3 + ...y_{n-1})]$$

Simpson's 1/3 Rule

$$\int_a^b f(x)dx = \frac{h}{3}[(y_0 + y_n) + 4(y_1 + y_3 + y_5 + ...y_{n-1}) + 2(y_2 + y_4 + y_6 + ...y_{n-2})]$$

Simpson's 3/8 Rule

$$\int_a^b f(x)dx = \frac{3h}{8}[(y_0 + y_n) + 3(y_1 + y_2 + y_4 + ...y_{n-1}) + 2(y_3 + y_6 + y_9 + ...y_{n-3})]$$

Boole's Rule

$$\int_a^b f(x)dx = \frac{2h}{45}[7y_0 + 32y_1 + 12y_2 + 32y_3 + 14y_4 + 32y_5 + 12y_6 + ...]$$

Weddle's Rule

$$\int_a^b f(x)dx = \frac{3h}{10}[y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5 + 2y_6 + ...]$$

UNIT - 4

Taylor's Series

$$y_{n+1} = y(x_0 + (n+1)h) = y_n + hy'_n + \frac{h^2}{2!}y''_n + \frac{h^3}{3!}y'''_n + ...$$

Picard's Method

$$y_n = y_0 + \int_{x_0}^{x} f(x, y_{n-1})dx$$

Euler's Method

$$y_n = y_{n-1} + hf(x_{n-1}, y_{n-1})$$

Modified Euler's Method

$$y_n^{(k)} = y_{n-1} + hf(x_0, y_0)$$

$$y_1^{(k+1)} = y_{n-1} + \frac{h}{2}\left[f(x_{n-1}, y_{n-1}) + f(x_n, y_n^{(k)}) \right], n = 1,2,3,....., k = 0,1,2,.....$$

Runge – Kutta Method

First order	$y_1 = y_0 + hf(x_0, y_0)$
Second order	$y_1 = y_0 + \frac{1}{2}[K_1 + K_2]$ $K_1 = hf(x_0, y_0)$ $K_2 = hf(x_0 + h, y_0 + K_1)$
Third order	$y_1 = y_0 + \frac{1}{6}[K_1 + 4K_2 + K_3]$ $K_1 = hf(x_0, y_0)$ $K_2 = hf(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2})$ $K_3 = hf(x_0 + h, y_0 + 2K_2 - K_1)$
Fourth order	$y_1 = y_0 + \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$ $K_1 = hf(x_0, y_0)$ $K_2 = hf(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2})$ $K_3 = hf(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2})$ $K_4 = hf(x_0 + h, y_0 + K_3)$