Vehicle Routing Problem with Capacity limitation and Time window

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Problem – (Multi) Capacitated Vehicle Routing Problem

- $Node = \{1, 2, 3, 4, 5, 6, 7, 8\}$ → Set of the index of each city.
- $Mode = \{1, 2, 3, 4, 5\}$ → Set of the index of each vehicle.
- x_{ijm} is the states of route (i, j) served by vehicle m, $\forall\, i \neq j$. (Decision Variable – Go or not)
- C_{ijm} is the cost of route (i, j) served by vehicle m, $\forall\, i \neq j$.
- Q_i is the demand volume of city i.
- V_m is the capacity of vehicle m.
- u_{im} is the accumulated delivers at city i served by vehicle m. (Dummy Variable)

$$Minimize\ Z = \sum_{i,j \in Node} \sum_{m \in Mode} C_{ijm} x_{ijm}$$

s.t.

For speeding up

$$(1)\quad \sum_{j \in Node} \sum_{m \in Mode} x_{ijm} \geq 3, \quad \forall\, i = 1$$

$$(2)\quad \sum_{j \in Node} x_{ijm} \leq 1, \quad \forall\, i = 1, m \in Mode$$

$$(3)\quad \sum_{j \in Node} x_{ijm} - \sum_{j \in Node} x_{jim} = 0, \quad \forall\, i = 1, m \in Mode$$

$$(4)\quad \sum_{j \in Node} \sum_{m \in Mode} x_{ijm} = 1, \forall\, i \in Node, m \in Mode, if\ Q_i + Q_j \leq V_m$$

$$(5)\quad \sum_{i \in Node} \sum_{m \in Mode} x_{ijm} = 1, \forall\, j \in Node, m \in Mode\ , if\ Q_i + Q_j \leq V_m$$

$$(6)\quad \sum_{m \in Mode} u_{im} \geq Q_i, \quad \forall\, i \in Node, i \neq 1$$

$$(7)\quad u_{im} \leq V_m, \quad \forall\, i \in Node, i \neq 1, m \in Mode$$

Important !!

$$(8)\quad \boldsymbol{u_{jm} - u_{im} + (V_m - (Q_i + Q_j)) \cdot x_{jim} + V_m \cdot x_{ijm} \leq V_m - Q_i,\quad \forall\, i, j \in Node, i, j \neq 1, m \in Mode}$$

$$(9)\quad u_{jm} + \left(V_m - Q_j\right) \cdot x_{ijm} \leq V_m, \quad \forall\, i = 1, j \in Node, m \in Mode$$

$$(10)\quad \sum_{m \in Mode} u_{jm} - \sum_{i \in Node} \sum_{m \in Mode} x_{ijm} \cdot Q_i \geq Q_j, \quad \forall\, i \neq 1, j \in Node$$

Problem – CVRP (multi capacity) with Time Window

- $Node = \{1, 2, 3, 4, 5, 6, 7, 8\}$ → Set of the index of each city.
- $Mode = \{1, 2, 3, 4, 5\}$ → Set of the index of each vehicle.
- x_{ijm} is the states of route (i, j) served by vehicle m, $\forall\, i \neq j$. (Decision Variable – Go or not)
- C_{ijm} is the cost of route (i, j) served by vehicle m, $\forall\, i \neq j$.
- Q_i is the demand volume of city i.
- V_m is the capacity of vehicle m.
- u_{im} is the accumulated delivers at city i served by vehicle m. (Dummy Variable)
- E_i is the earliest time window of city i.
- L_i is the latest time window of city i.
- S_i is the visit duration at city i.
- t_i is the accumulated delivery time at city i. (Dummy Variable)

- D_{ij} is the distance between city i and j.
- T_{max} is the maximum drive time(99999).
- Trv_{rate} is the travel rate (1.2).

$$\textit{Minimize } Z = \sum_{i,j \in Node} \sum_{m \in Mode} C_{ijm} x_{ijm}$$

s.t.

(1) – **(10)** are as same as the constraints in the previous version (i.e., (Multi) Capacitated Vehicle Routing Problem).

$$(11) \quad t_j \geq t_i + \left((S_i + Trv_{rate} \cdot D_{ij}) \cdot \sum_{m \in Mode} x_{ijm} - L_i \cdot \left(1 - \sum_{m \in Mode} x_{ijm}\right)\right), \quad \forall\, i, j \in Node, \quad j \neq 1$$

$$(12) \quad t_i \geq E_i, \quad \forall\, i \neq 1 \qquad (13) \quad t_i \leq L_i, \quad \forall\, i \neq 1 \qquad (14) \quad t_i + S_i + Trv_{rate} \cdot D_{ij} \cdot \sum_{i \in Node} \sum_{m \in Mode} x_{ijm} \leq T_{max}, \quad \forall\, j = 1$$

Thank you for your listening.