Linear Algebra: Final Project

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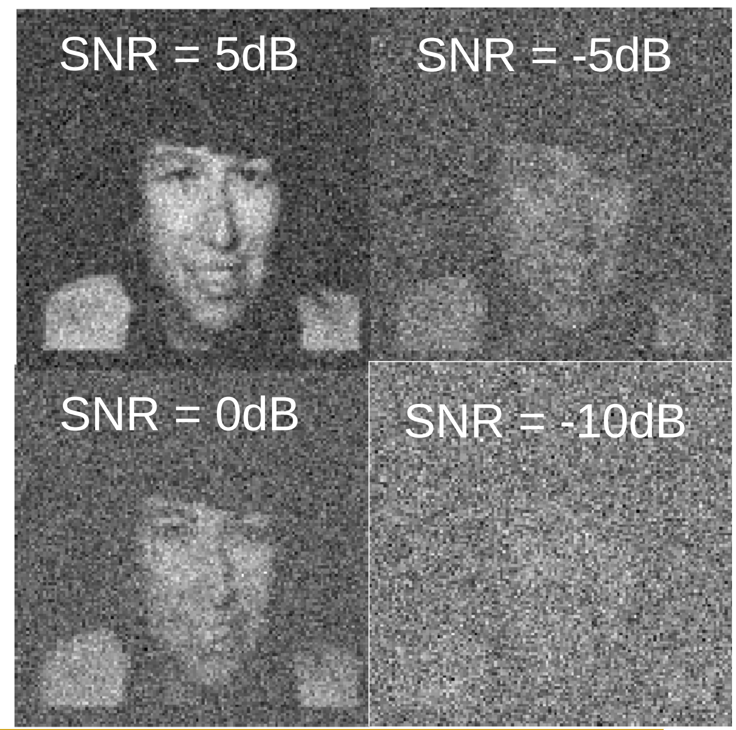
Final project

- In this project, you are asked
  - Program different parts of the “eigenface” facial identification
  - Show your results quantitatively by plotting the so-called miss detection rate vs. signal-to-noise ratio (SNR).
  - Answer some additional questions

Eigenface – face detection

Training set

Testing set



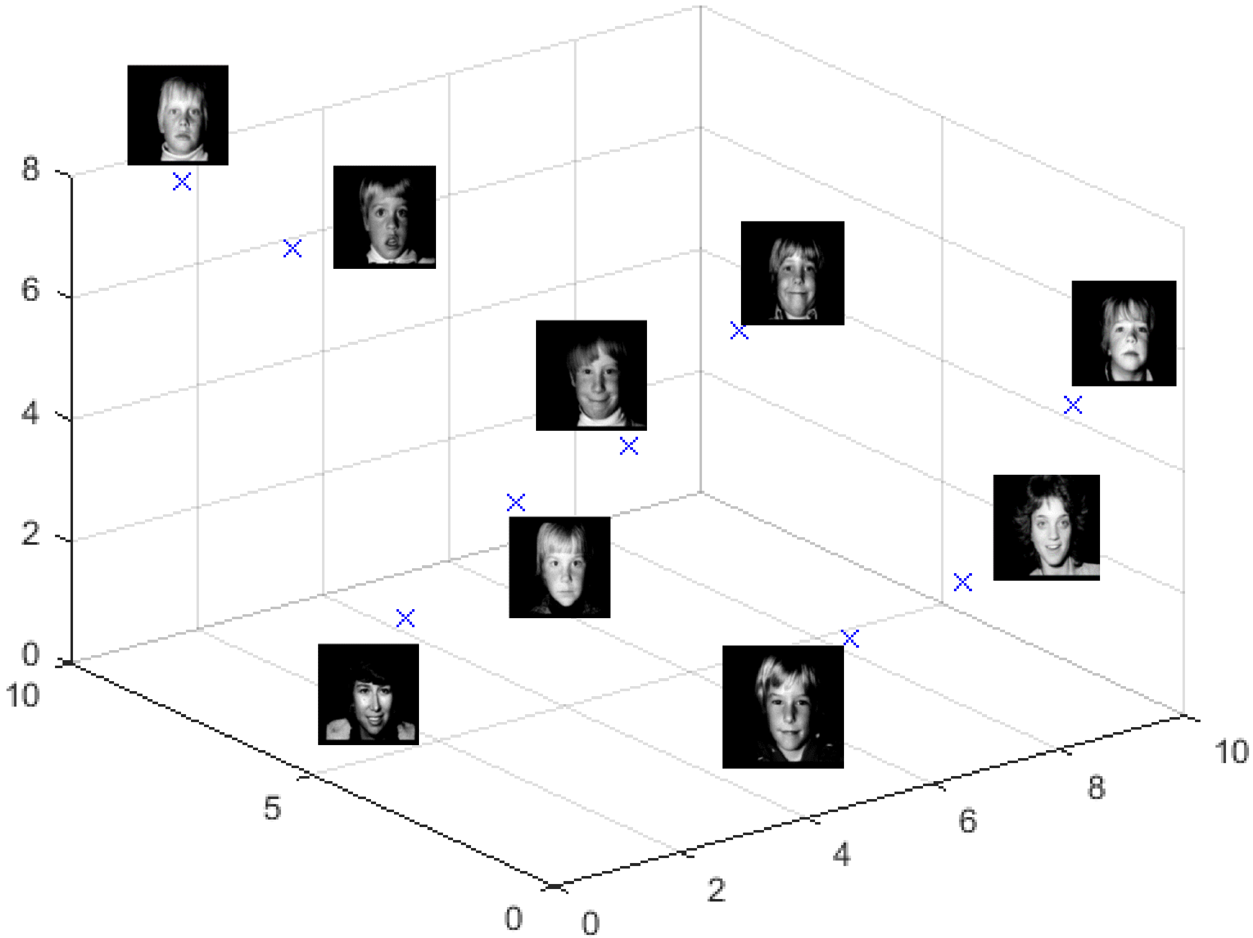
Facial Identification (and Recognition) using Eigenface

- Uses the idea of low rank approximation
- Pro
  - Easy to implement
- Cons
  - Complete retraining is necessary when new faces become available
  - Not robust again occlusion, shadow, …
  - Use of global information (correlation matrix of all data) fails to capture local information of images
  - In terms of identification, detector is not optimal

Eigenface Training Steps

- Treat each new (face) image as a point in Hilbert space H
  - $N \times N$ pixel image ➔ N^2 point in H
  - M training images, usually $M<N^2$
    - It is usually the dimension of the data which causes problem in such problems, not the number of data
      - Curse of dimensionality
      - Big data is a misnomer
- Training phase
  - Calculate average of face data and subtract from each training data
    - Let $\mathbf{f}_1, \mathbf{f}_2, \ldots, \mathbf{f}_M \in \mathbb{R}^{N\wedge 2}$ be the training images, and $\boldsymbol{\gamma} \in \mathbb{R}^{N\wedge 2}$ denotes their mean
    - $\mathbf{g}_i = \mathbf{f}_i - \boldsymbol{\gamma}, \forall i = 1, \ldots M$
  - Calculate corresponding correlation matrix
    - $\mathbf{C} = \mathbf{G}\mathbf{G}^T \in \mathbb{R}^{N\wedge 2 \times N\wedge 2} = \mathbf{U}_C \boldsymbol{\Lambda}_C^{1/2} \boldsymbol{\Lambda}_C^{T/2} \mathbf{U}_C^T, \qquad \mathbf{U}_C, \boldsymbol{\Lambda}_C \in \mathbb{R}^{N\wedge 2 \times N\wedge 2}$
    - $\mathbf{G} = [\mathbf{g}_1 \ \ \mathbf{g}_2 \ \ \ldots \ \mathbf{g}_M] \in \mathbb{R}^{N\wedge 2 \times M}$
      - Note that $\mathbf{C}$ will usually be rank deficient as $N^2 >> M$
      - $\mathbf{U}_C(:,1:M)$ is a basis for $\mathbf{g}_i$, for all i

Data representation



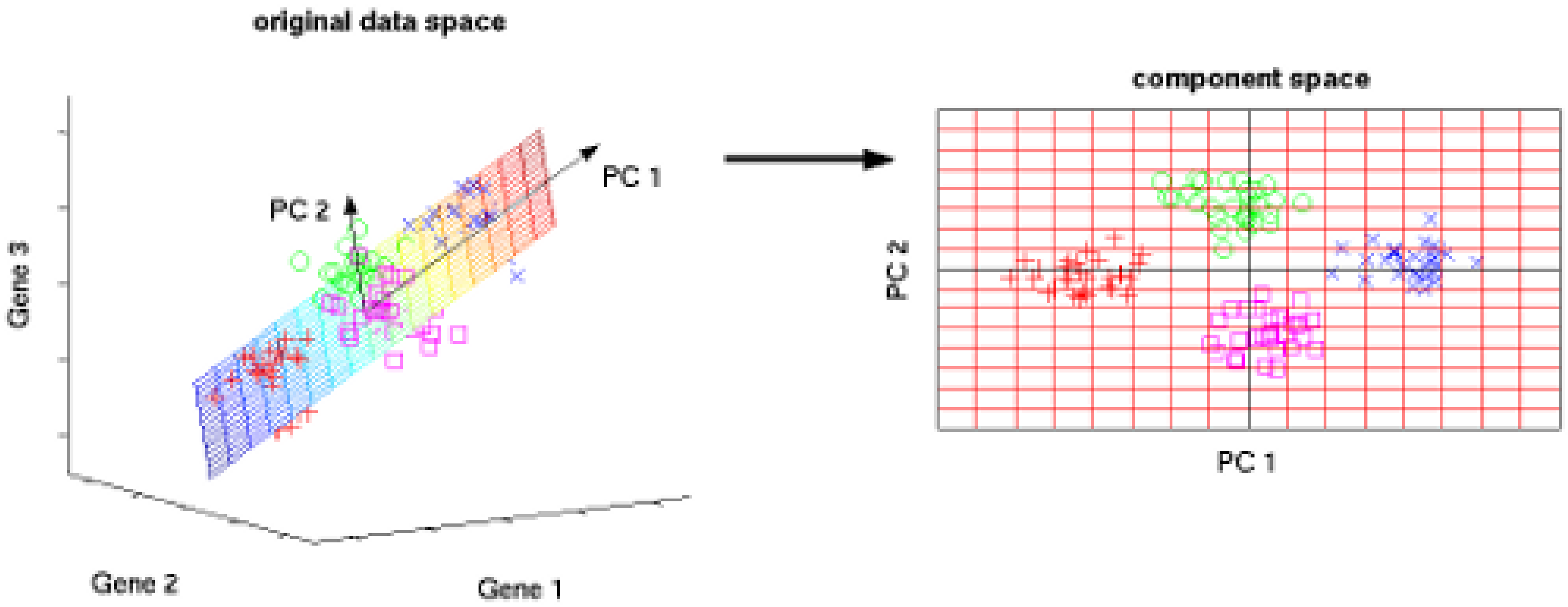
Eigenface Training Steps

- Only need to use M eigenvectors to represent all training data
  - Calculating the $N \times N$ $\mathbf{U}_C$ matrix is computationally inefficient and not necessary
- Consider that if $\mathbf{R}=\mathbf{G}^T\mathbf{G} \in \mathbb{R}^{M\times M}$ and symmetric
  - $\mathbf{R}=\mathbf{U}_R \mathbf{\Lambda}_R^{1/2} \mathbf{\Lambda}_R^{T/2} \mathbf{U}_R^T \in \mathbb{C}^{N\times M}$, then $\mathbf{U}_R \in \mathbb{R}^{M\times M}$, $\mathbf{\Lambda}_R \in \mathbb{R}^{M\times M}$
  - In matrix vector form: $\mathbf{R}\mathbf{u}_{R,i}=\mathbf{G}^T\mathbf{G}\mathbf{u}_{R,i}=\lambda_{R,i}\mathbf{u}_{R,i}$
  - Notice that $\mathbf{G}\mathbf{R}\mathbf{u}_{R,i}=\mathbf{G}\mathbf{G}^T\mathbf{G}\mathbf{u}_{R,i}=\mathbf{C}\mathbf{G}\mathbf{u}_{R,i}=\lambda_{R,i}\mathbf{G}\mathbf{u}_{R,i} \Leftrightarrow \mathbf{C}\mathbf{v}_i=\lambda_{R,i}\mathbf{v}_i$, for $i = 1, 2, \ldots, M$
    - So $\mathbf{v}_i=\mathbf{G}\mathbf{u}_{R,i} \in \mathbb{R}^{N^{\wedge}2\times M}$ and it equals first M eigenvectors of $\mathbf{C}$
  - Calculation of $\mathbf{v}_i$ only needs computing M $\mathbf{u}_{R,i}$ vectors, and multiplying them with $\mathbf{G}$
  - Note that $\mathcal{V}(\mathbf{G}) = \text{span}(\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_M)$
  - "face" space = eigenspace of $\mathbf{C}$ = $\text{span}(\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_M)$

Idea of data projection

3D space

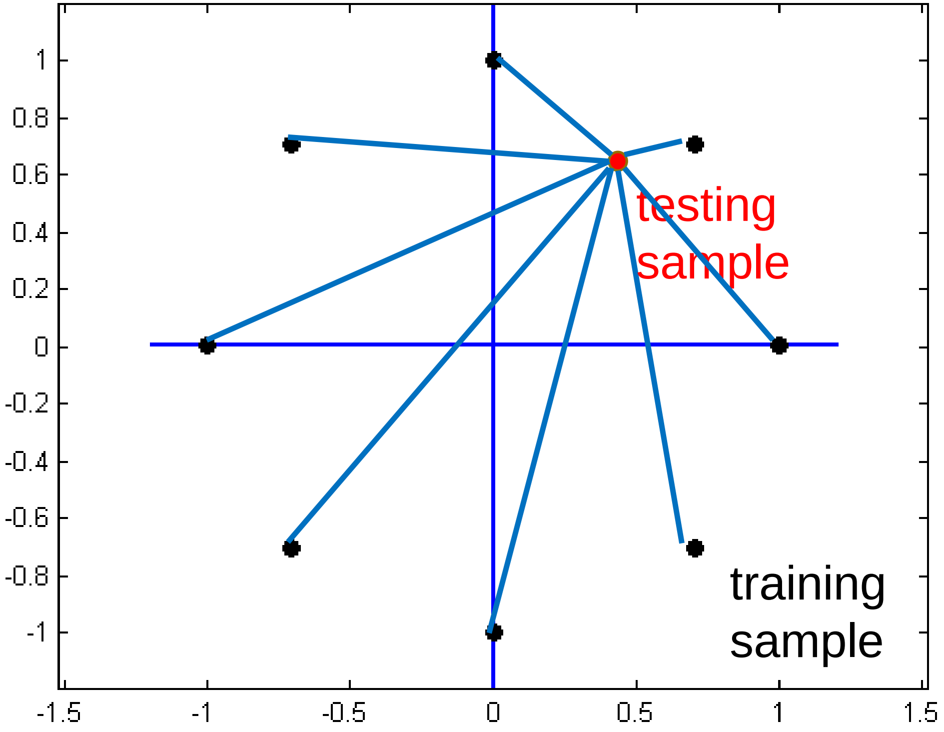
2D space



Eigenface Testing – Facial Identification

- Note that not all M $\mathbf{v}_i$'s are needed
  - Only $M' < M$ $\mathbf{v}_i$'s are retained
- Transform training images as $\mathbf{v}_i^T\mathbf{g}_i = g_{T,i}$, for $i = 1,2, \ldots, M'$
- Form the coefficient vector $\mathbf{g}_T = [g_{T,1}, g_{T,2}, \ldots, g_{T,M'}]^T$ using all images
  - For centroid (mean) of images of same person k, called $\mathbf{g}_{T,k}$
- Suppose a new face point $\mathbf{h}$ appears (already zero-mean)
- Classifying a new face by first computing its coefficient $\mathbf{v}_i^T\mathbf{h} = \mathbf{h}_{T,i}$, for $i = 1, 2, \ldots, M'$
- Form coefficient vector $\mathbf{h}_T = [h_{T,1}, h_{T,2}, \ldots, h_{T,M'}]^T$
- Decide $\mathbf{h}_T$ is person $\mathbf{g}_{T,k,opt}$ if $\mathbf{g}_{T,k,opt} = \operatorname{argmin}_{\mathbf{g}_{T,k}} \| \mathbf{h}_T - \mathbf{g}_{T,k} \|_2^2$
  - This is not optimal detector

Principle of min norm detector

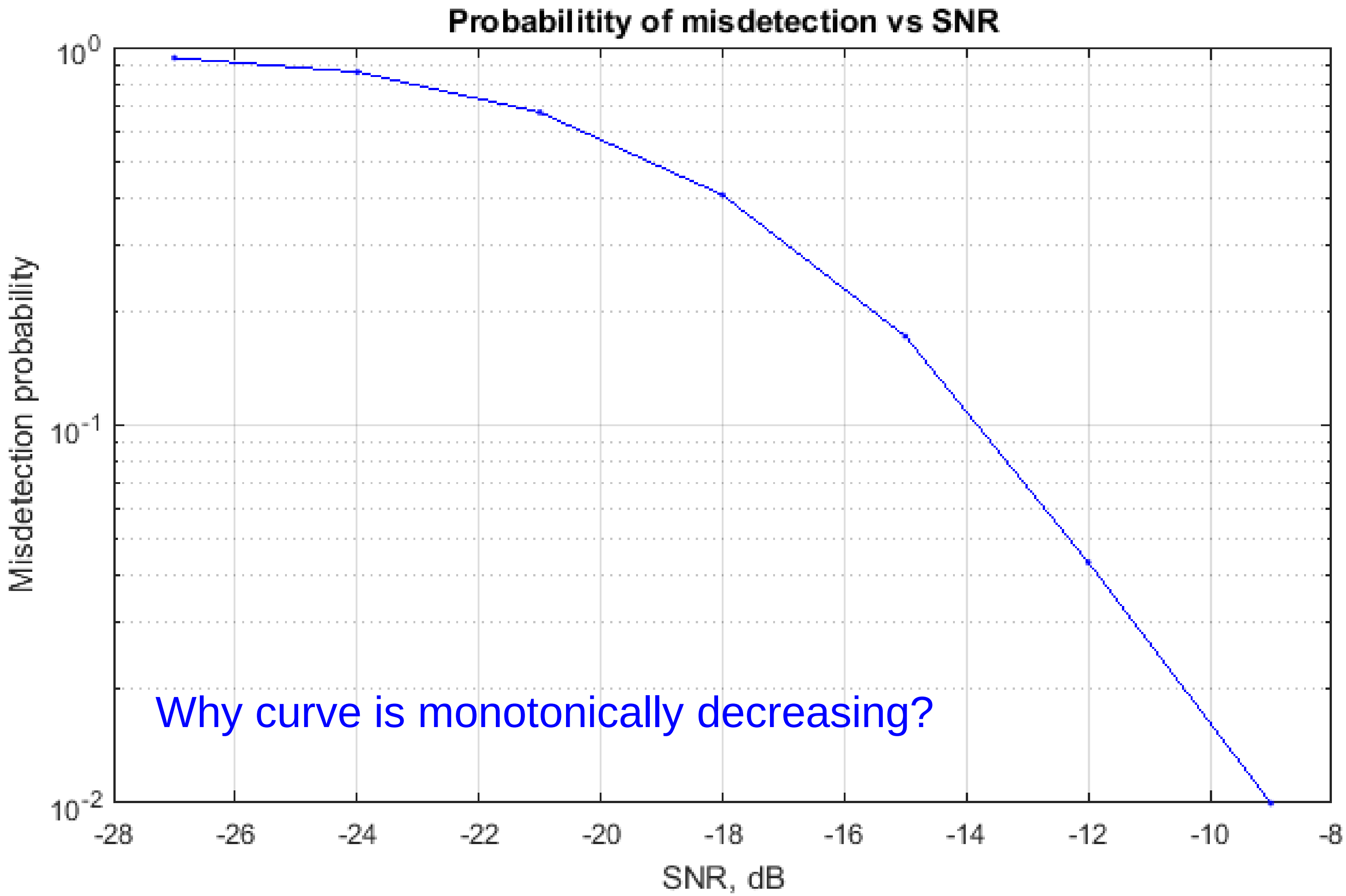


1. Compute distances to all training data points

2. Pick the smallest

$$\mathbf{g}_{T,k,opt} = \operatorname{argmin}_{\mathbf{g}_{T,k}} \| \mathbf{h}_T - \mathbf{g}_{T,k} \|_2^2$$

Probability of wrong detection vs SNR



Assignment

- Complete functions of
  - Image I/O (10%)
  - Efficient computation of singular vectors(15%)
  - Min norm identification(15%)
- Obtain smooth and correct miss detection probability vs. SNR curve using all of the eigenvectors from the covariance matrix (20%)
- Implement algorithm using
  - 10% of the total number of eigenvectors (10%)
  - 1% of the total number of eigenvectors (10%)
- Answer question: how results for reduced number of eigenvectors are different from taking full set?(10%)
- Plot miss probability error vs. SNR curve for all three cases(full set, 10% and 1% of the total number of eigenvectors).(10%)

Good luck!