

$$\frac{\partial X^2}{\partial \theta} = \sum \frac{\partial}{\partial \theta} [(y_i - (a_i x_i + a_0))^2]$$

$$\sum (y_i - (a_i x_i + a_0)) = 0$$

$$\sum_{i=1}^N y_i - \sum_{i=1}^N a_i x_i + \sum_{i=1}^N a_0 = 0$$

$$\bar{y} - a_i \bar{x} - a_0 = 0$$

$$a_0 = \bar{y} - a_i \bar{x}$$

$$\frac{\partial X^2}{\partial a_i} = \sum_{i=1}^N \frac{\partial}{\partial a_i} [y_i - (a_i x_i + a_0)]^2 = 0$$

$$\textcircled{1} \sum_{i=1}^N x_i y_i - a_i \sum_{i=1}^N x_i^2 + a_0 \sum_{i=1}^N x_i = 0$$

$$\textcircled{2} -2 \sum_{i=1}^N x_i y_i + a_i \sum_{i=1}^N x_i^2 \left[\frac{1}{N} \sum_{i=1}^N y_i - \frac{a_i}{N} \sum_{i=1}^N x_i \right] \sum_{i=1}^N x_i = 0$$

$$\textcircled{1} = \textcircled{2}$$

Por tanto:

$$a_i = \frac{2 \sum_{i=1}^N x_i y_i - \frac{1}{N} \sum_{i=1}^N x_i \sum_{i=1}^N y_i}{\sum_{i=1}^N x_i^2 - \frac{1}{N} \left(\sum_{i=1}^N x_i \right)^2}$$

$$\bullet \frac{\partial X^2}{\partial a_0} = \sum_i \left[\frac{\partial}{\partial a_0} (y_i - (a_0 + a_1 x_i + a_2 x_i^2))^2 \right]$$

$$\sum_{i=1}^N [a_0 + a_1 x_i + a_2 x_i^2 = y_i]$$

$$\bullet \frac{\partial X^2}{\partial a_1} = \sum_i \left[\frac{\partial}{\partial a_1} (y_i - (a_0 + a_1 x_i + a_2 x_i^2))^2 \right]$$

$$= \sum_{i=1}^N [a_0 x_i + a_1 x_i^2 + a_2 x_i^3 = x_i y_i]$$

$$\bullet \frac{\partial X^2}{\partial a_2} = \sum_i \left[\frac{\partial}{\partial a_2} (y_i - (a_0 + a_1 x_i + a_2 x_i^2))^2 \right]$$

$$= \sum_{i=1}^N [a_0 x_i^2 + a_1 x_i^3 + a_2 x_i^4 = x_i^2 y_i]$$