Demostrar que:

$$E = \int_a^b \varepsilon(x)\,dx = \int_a^b \frac{f'''(\xi)}{4!}(x-a)(x-b)\left(x-\left(\frac{a+b}{2}\right)\right)dx = 0$$

$$E = \int_a^b \frac{f'''(\xi)}{4!}\left(x^3 + \frac{-3ax^2 - 3bx^2 + 4abx + b^2x + a^2x - a^2b - ab^2}{2}\right)dx$$

$$E = \frac{f'''(\xi)}{4}\left[\frac{x^4}{4} - \frac{ax^3}{2} - \frac{bx^3}{2} + abx^2 + \frac{b^2x^2}{4} + \frac{a^2x^2}{4} - \frac{a^2bx}{2} - \frac{ab^2x}{2}\right]\Bigg|_a^b$$

$$E = \frac{f'''(\xi)}{4}\left[\left(\frac{b^4}{4} - \frac{ab^3}{2} - \frac{b^4}{2} + ab^3 + \frac{b^4}{4} + \frac{a^2b^2}{4} - \frac{a^2b^2}{2} - \frac{ab^3}{2}\right) - \right.$$

$$\left.\left(\frac{a^4}{4} - \frac{a^4}{2} - \frac{ba^3}{2} + a^3b + \frac{b^2a^2}{4} + \frac{a^4}{4} - \frac{a^3b}{2} - \frac{a^2b^2}{2}\right)\right]$$

$$E = \frac{f'''(\xi)}{4}\left[\left(-\frac{1}{4}a^2b^2\right) - \left(-\frac{1}{4}a^2b^2\right)\right]$$

$$E = \frac{f'''(\xi)}{4}[0] = 0$$