Universidad Tecnológica de Bolívar
CARTAGENA DE INDIAS

Machine Learning

Bias-Variance

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What is bias?

- Bias is the difference between the average prediction of our model and the correct value which we are trying to predict.
- Model with high bias pays very little attention to the training data and oversimplifies the model. It always leads to high error on training and test data.

What is variance?

- Variance is the variability of model prediction for a given data point or a value which tells us spread of our data.
- Model with high variance pays a lot of attention to training data and does not generalize on the data which it hasn’t seen before.
- As a result, such models perform very well on training data but has high error rates on test data.

Mathematically

- Let the variable we are trying to predict as Y and other covariates as X. We assume there is a relationship between the two such that

  Y=f(X) + e

- Where e is the error term and it’s normally distributed with a mean of 0.
- We will make a model *f^(X)* of *f(X)* using linear regression or any other modeling technique.
- So the expected squared error at a point x is

$$Err(x) = E\left[(Y - \hat{f}(x))^2\right]$$

Mathematically

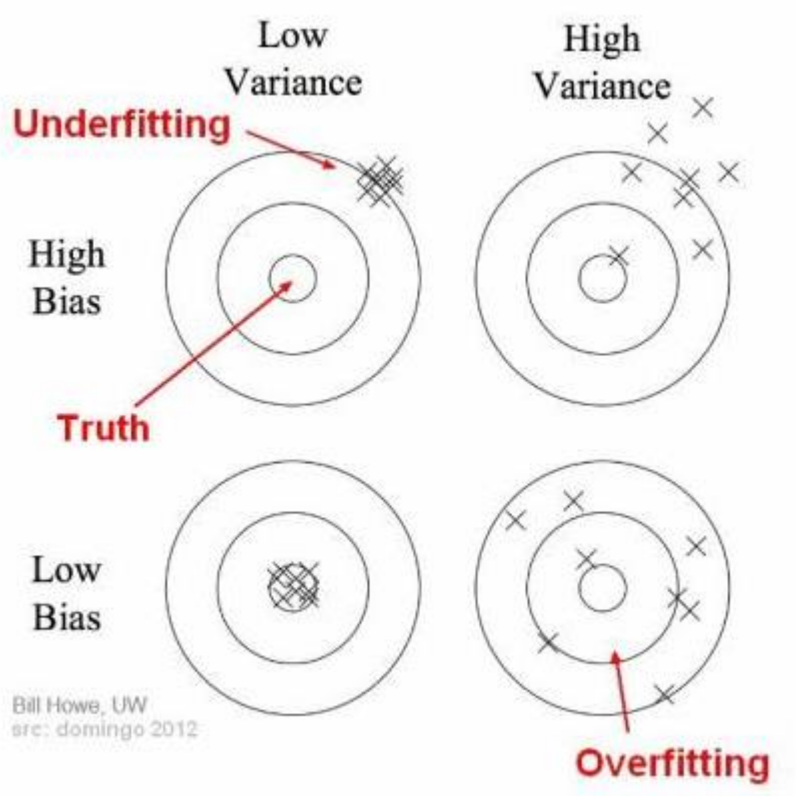
The *Err(x)* can be further decomposed as:

$$Err(x) = \left(E[\hat{f}(x)] - f(x)\right)^2 + E\left[\left(\hat{f}(x) - E[\hat{f}(x)]\right)^2\right] + \sigma_e^2$$

$$Err(x) = \mathrm{Bias}^2 + \mathrm{Variance} + \mathrm{Irreducible\ Error}$$

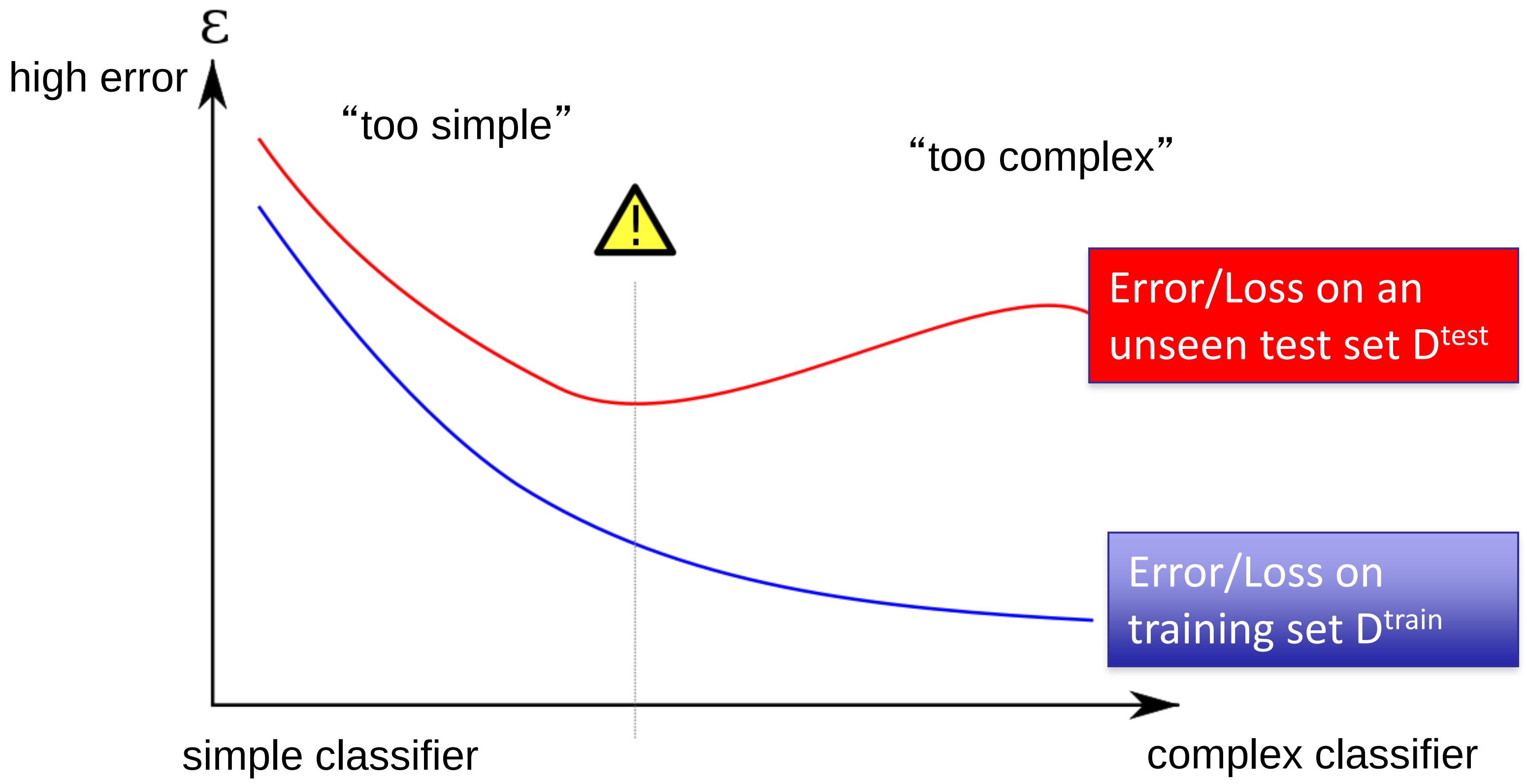
Err(x) is the sum of Bias^2, variance and the irreducible error.

Bias and variance using bulls-eye diagram



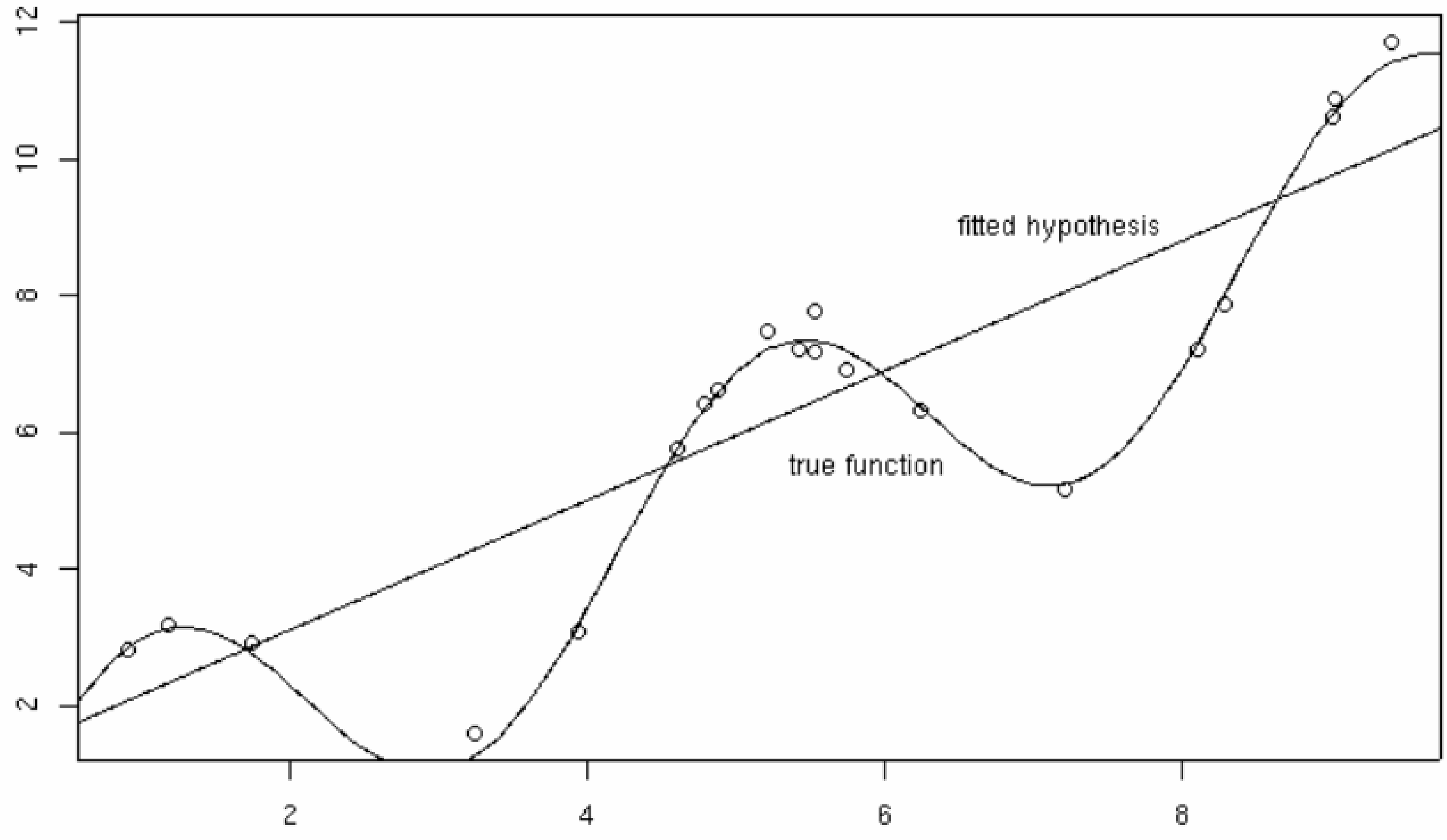
- Center of the target is a model that perfectly predicts correct values.
- As we move away from the bulls-eye our predictions become get worse and worse.
- Repeat our process of model building to get separate hits on the target.

Bias/Variance is a Way to Understand Overfitting and Underfitting



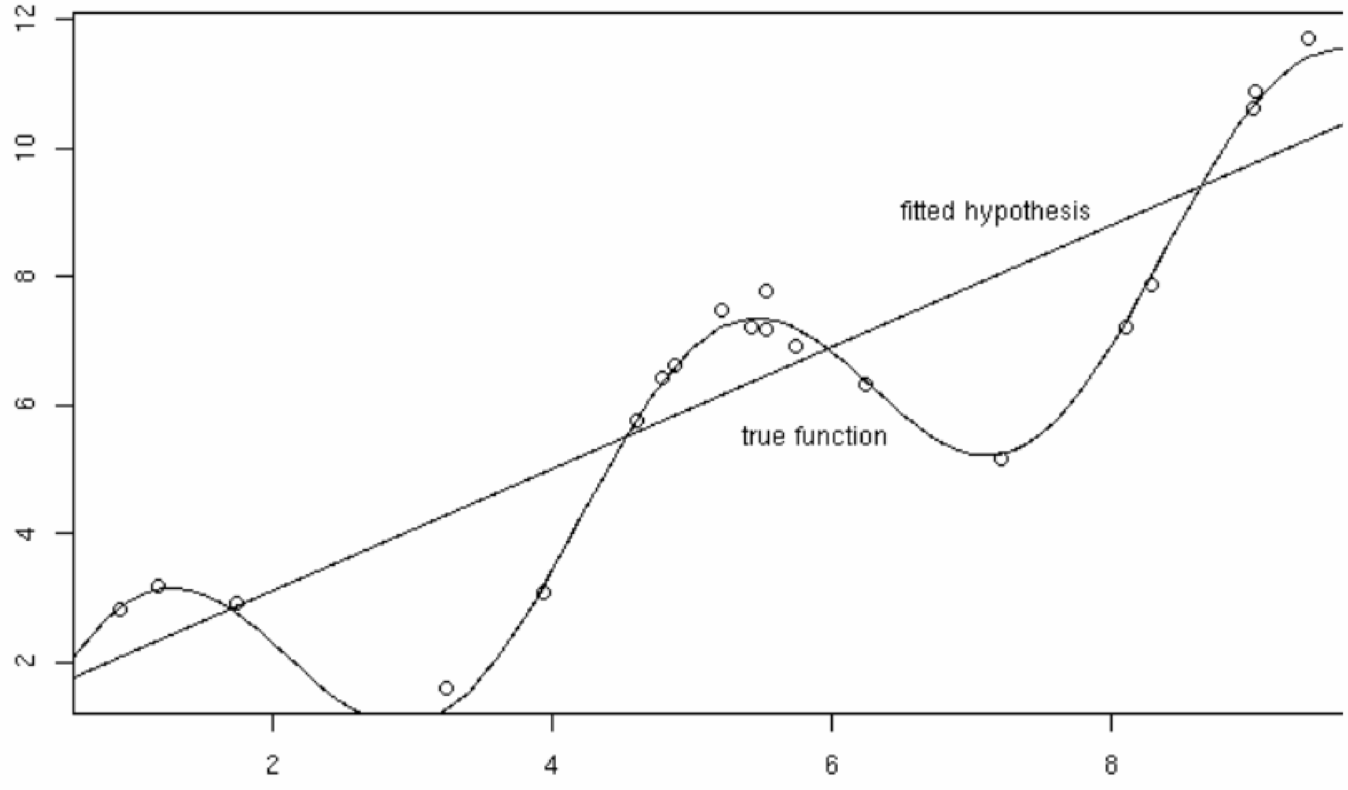
Bias-Variance: An Example

Example



y = x + 2 sin(1.5x) + N(0,0.2)

Example



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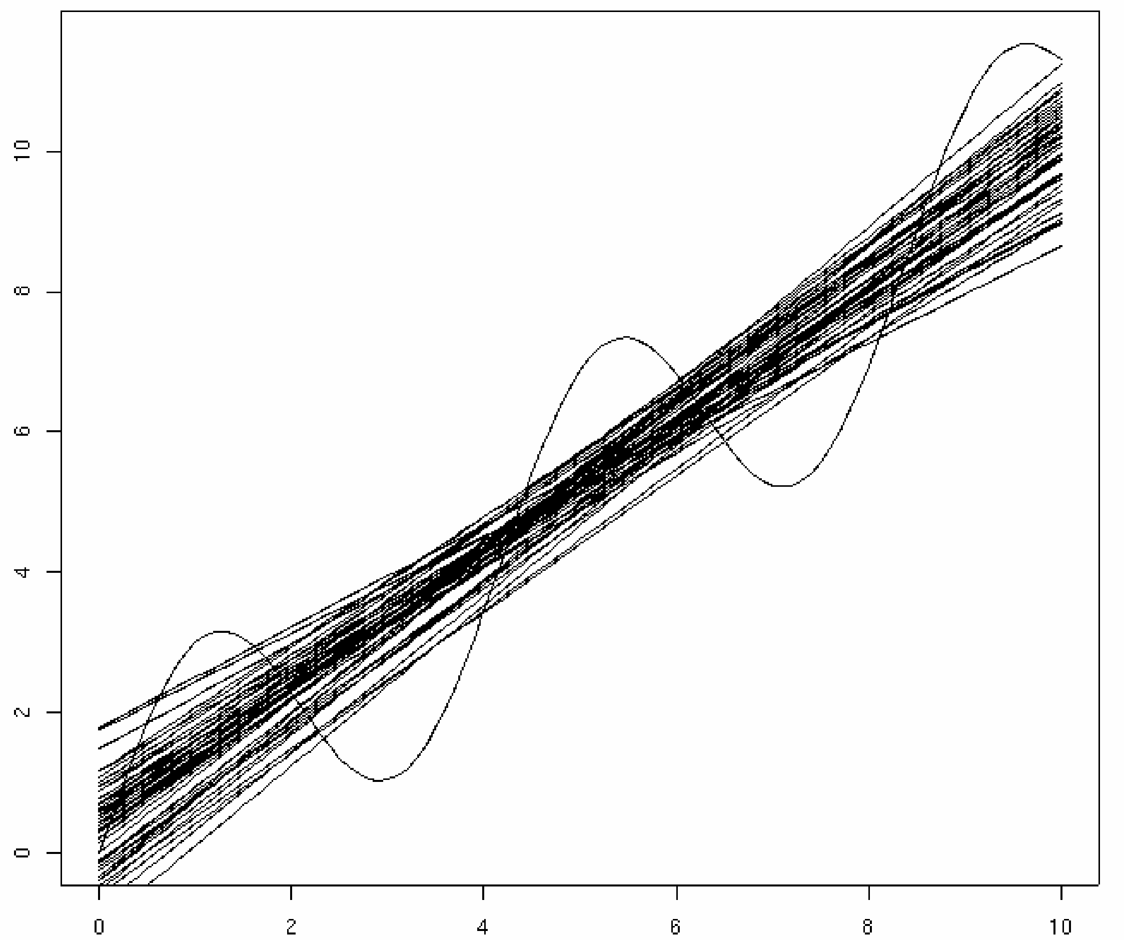
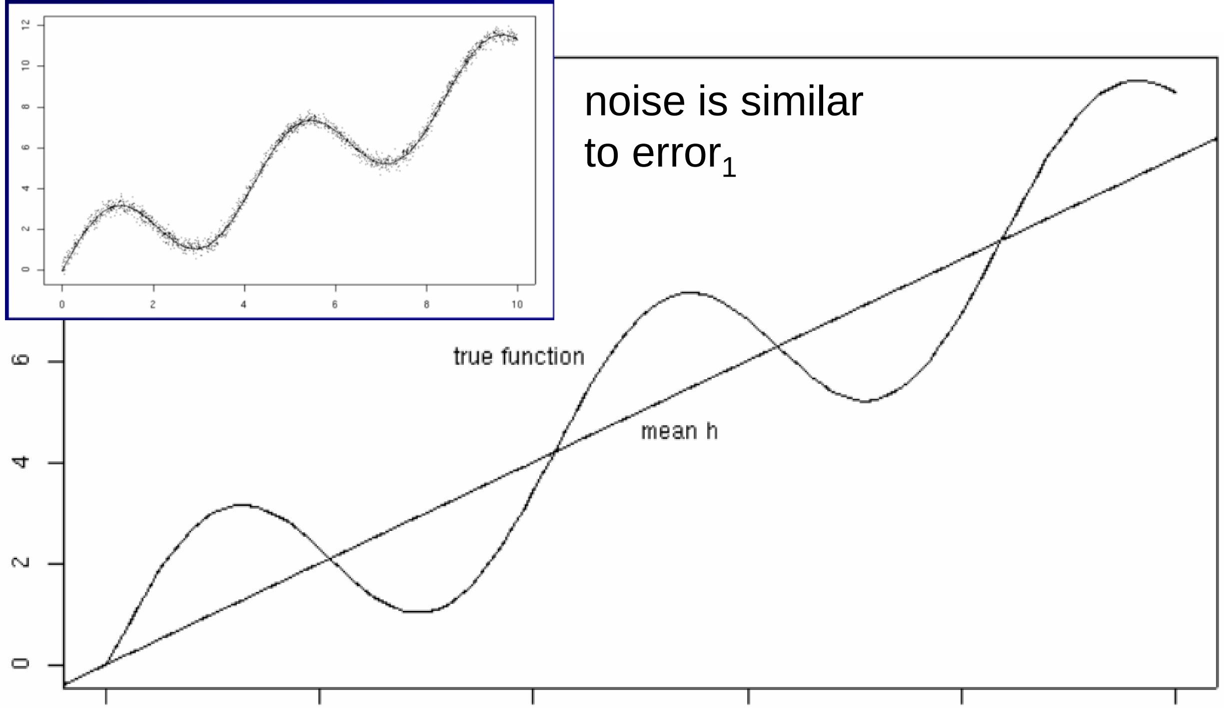
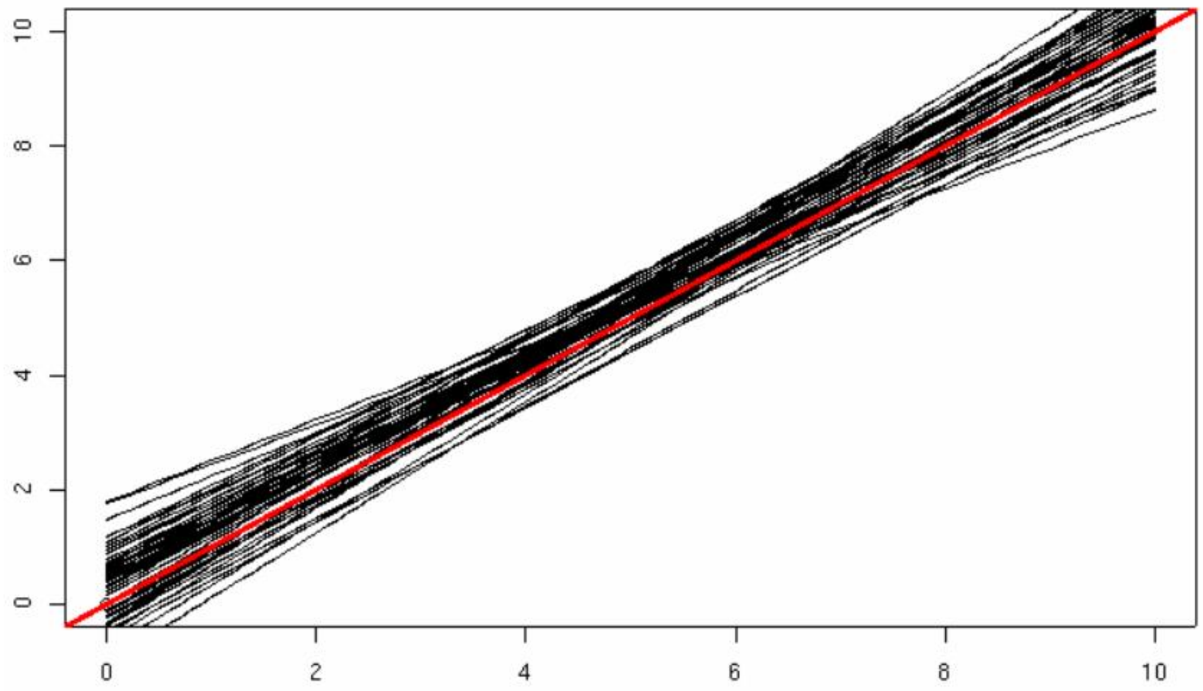


Same experiment, repeated:
with 50 samples of 20 points each



noise is similar to $error_1$



The true function *f* can't be fit perfectly with hypotheses from our class *H* (lines) ➔ $Error_1$

Fix: *more* expressive set of hypotheses *H*

We don't get the best hypothesis from *H* because of noise/small sample size ➔ $Error_2$

Fix: *less* expressive set of hypotheses *H*

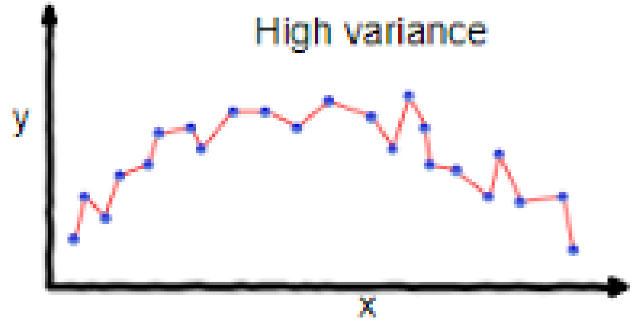
Overfitting

- Overfitting refers to a model that models the training data too well.
- Overfitting happens when a model learns the detail and noise in the training data to the extent that it negatively impacts the performance of the model on new data.
  - It happens when we train our model a lot over noisy dataset.
  - These models have low bias and high variance.
  - These models are very complex like Decision trees which are prone to overfitting.

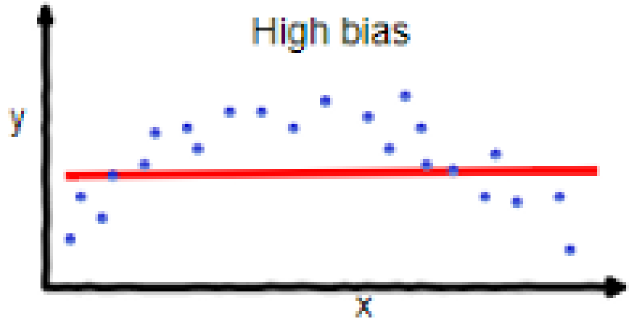
Underfitting

- Underfitting refers to a model that can neither model the training data nor generalize to new data.
- An underfit machine learning model is not a suitable model and will be obvious as it will have poor performance on the training data.
  - These models usually have high bias and low variance.
  - It happens when we have very less amount of data to build an accurate model or when we try to build a linear model with a nonlinear data.
  - Also, these kind of models are very simple to capture the complex patterns in data like Linear and logistic regression.

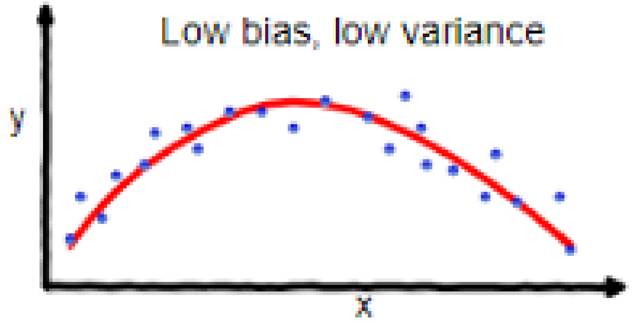
Overfitting vs Underfitting



overfitting



underfitting



Good balance

How to Solve Underfitting and Overfitting

- Cross-validation.
- Regularization.
- Early stopping.
- Pruning.
- Dropout.
- Regularize the weights.