FM408, Financial Engineering **Jean-Pierre Zigrand**
The London School of Economics **Michaelmas 2014/15**

Take-Home Exam

Administrative Items

This is your take-home exam that will count towards 20% of your final grade. Marks add up to 100. You can work in groups of up to **four** people (that's 4 or less). Each student needs to submit his or her own copy of the assignment. You need to specify group membership on your submissions. All members of the same group will receive the same grade. The deadline for this assignment and the precise submission guidelines are available from the department.

Statement of your problem

You are the Chief Risk Officer of a single-strategy equity-focused bank called Bank of Risky and Uncontrollable Proprietary Trading, BANK RUPT for short.

Your traders have decided to specialise on volatility and variance trading because that sounds sexy and may well attract investors. Unfortunately, they have not benefitted from a solid LSE finance degree, which is how you as the CRO come in.

Assume that the true (non-dividend paying) stock price process under $\mathbb{P}$ follows the process

$$dS_t = \mu_t S_t dt + \sigma_t S_t dW_t$$

with $S_0 = 100$. Furthermore, assume that stochastic volatility is governed by the Heston process. Denoting the variance by $\nu := (\sigma)^2$, under $\mathbb{Q}$

$$d\nu_t = -\lambda(\nu_t - \bar{\nu})dt + \eta\sqrt{\nu_t}dW_t^V$$

where W^V is a second Brownian motion, correlated with W at a constant correlation coefficient ρ.

Assume also that the riskless short rate is zero, $r = 0$. The trading horizon is always a year. Let Δt be the discrete trading horizon, so that each Δt periods the trader adjusts his positions. Say $\Delta t \in \mathbf{\Delta} := \{1/2500, 1/250\}$ initially, roughly hourly and daily. Those two periods represent two distinct trading strategies and the trader allocated half his capital into each.

1. [15 marks] One of your traders overheard some LSE FM408 students excitedly discussing a concept they called "the bowl." First your trader thought the students were betting on the Superbowl, but then he got the gist of it. He now tries to implement it naively as follows. He goes long the call with maturity $T = 1$ and delta hedges the stock price exposure.

Even though the true world is Heston, the trader is blissfully ignorant and assumes the world is Black-Scholes-Merton. He decided to rebalance his positions each $\Delta t \in \boldsymbol{\Delta}$ and readjust to the then current BS delta $\frac{\partial C_{BS}}{\partial S_t}(S_t, t; \sigma_t^{IV})$ at the moment of rebalancing.

The trader gets quoted the implied vol corresponding to the Heston price, given that the Heston model is the true model. He buys the quoted call option and does the long vol trade.

What is the actual (not the approximate one as described on the slides on page 249) P&L path of the trader over the year for a large number of paths, some of which do display realised variance on average above the implied one and some below? Summarise the T information as a density.

2. [10 marks] Now repeat the exercise in the previous subquestion, but assume that the trader rehedges each period at the BS delta but evaluated at the realised variance - $(S_{t+\Delta t} - S_t)^2$ - of the previous period. Compare results.

3. [15 marks] The trader having bombed his third annual bonus on a negative P&L (comment carefully why this may be the case), you decide to have a word with him and explain the general bowl to him that is valid also in SV environments (see notes pp 416). Your message got a bit garbled, however, and the trader still does the bowl with one call and he follows his first strategy still rebalancing each $\Delta t \in \boldsymbol{\Delta}$ but resetting it now at the initial delta of the BS call pricing function $\frac{\partial C_{BS}}{\partial S_t}(S_t, t; \sigma_0^{IV})$.

   Compute his actual P&L at the end of the year for a large number of different paths and comment.

4. [10 marks] Compare the P&L profiles of the trader having used the three rebalancing strategies of subquestions 1, 2 and 3, namely using as volatility parameter in the BS delta 1) the then current implied vol of the call, 2) the (square root of the) realised variance and 3) the initial implied vol.

5. [10 marks] Having bombed his fourth consecutive annual bonus (why may this have been the case?), he decided that neither the call not the combination of an ATM call and an ATM put is the right strategy to follow. A bloke at the pub mentioned a thing called VIX to him, and the trader duly googled it up and found a rather incomprehensible formula involving a bunch of options, in fact $\frac{1}{K^2}$ of each each OTM option with strike price K. Given the current stock price of $S_0 = 100$, the trader buys $\frac{1}{K^2}$ OTM for strikes $K \in \{90, 95, 100, 105, 110\}$ and follows the same strategy as the one he followed in this past year.

6. [20 marks] Having bombed his fifth consecutive annual bonus (why may this have been the case?), he now bribed you, being an ex-LSE finance student, to sell him a copy of the FM408 notes (of course you refused to relinquish them) and carefully read the lecture note on volatility derivatives. His new strategy now is to rebalance discretely as before at $\Delta t \in \boldsymbol{\Delta}$ but to use minus a number of log contracts and at each such moment to use the delta of the BS log contract evaluated at the initial BS implied vol of the log contract.

Compute the final P&L after the year for a large number of different paths. Compare to the expression in the notes on page 418 and 419.

7. [20 marks] The trader now got a bit more ambitious and wondered why he should always reset his delta hedges pretending that the derivative value is the BS value at the initial σ_0^{IV}. In particular, the trader not only has a view on the future path of realised volatility, but at the same time he has a view about what the implied volatility ought to be, namely the constant $\sigma_h \neq \sigma_0^{IV}$.

   The trader guesses that if this is his opinion, he may as well rehedge his delta at each rehedging period (which is continuously here) to the new BS delta, but pretending the implied vol is σ_h throughout, rather than σ_0^{IV} as before.

   Repeating the proof on pages 417-419, what is the final P&L of this trader at time T? Interpret carefully.

Notes on Implementation

Assume throughout that the Heston model is the "true model." Use the estimated parameters chosen to fit the SPX (S&P 500) options as of the close on September 15, 2005, as reported in Gatheral's textbook on pages 36-42, as well as S_0 the short rate r and the dividend yield δ:

ν	0.0174
$\bar{\nu}$	0.0354
ν_0	$\bar{\nu}$
η	0.3877
$\rho_{S,\sigma}$	-.7165
λ	1.3253
r	0
δ	0
S_0	100

To save on time and power, perhaps you may wish to store the Monte Carlo paths and reuse them.

Good luck!