Bayesian Data Analysis Session 1

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Overview

Session 1: Edwin

What is Bayesian statistics? Theory and simple examples.

Session 2: Rick

Building hierarchical models with Stan.

Introduction

Intuition of Bayesian Statistics

Statistics describes the world in probality distributions.

Collect data to learn about the distributions: $\hat{\theta}$

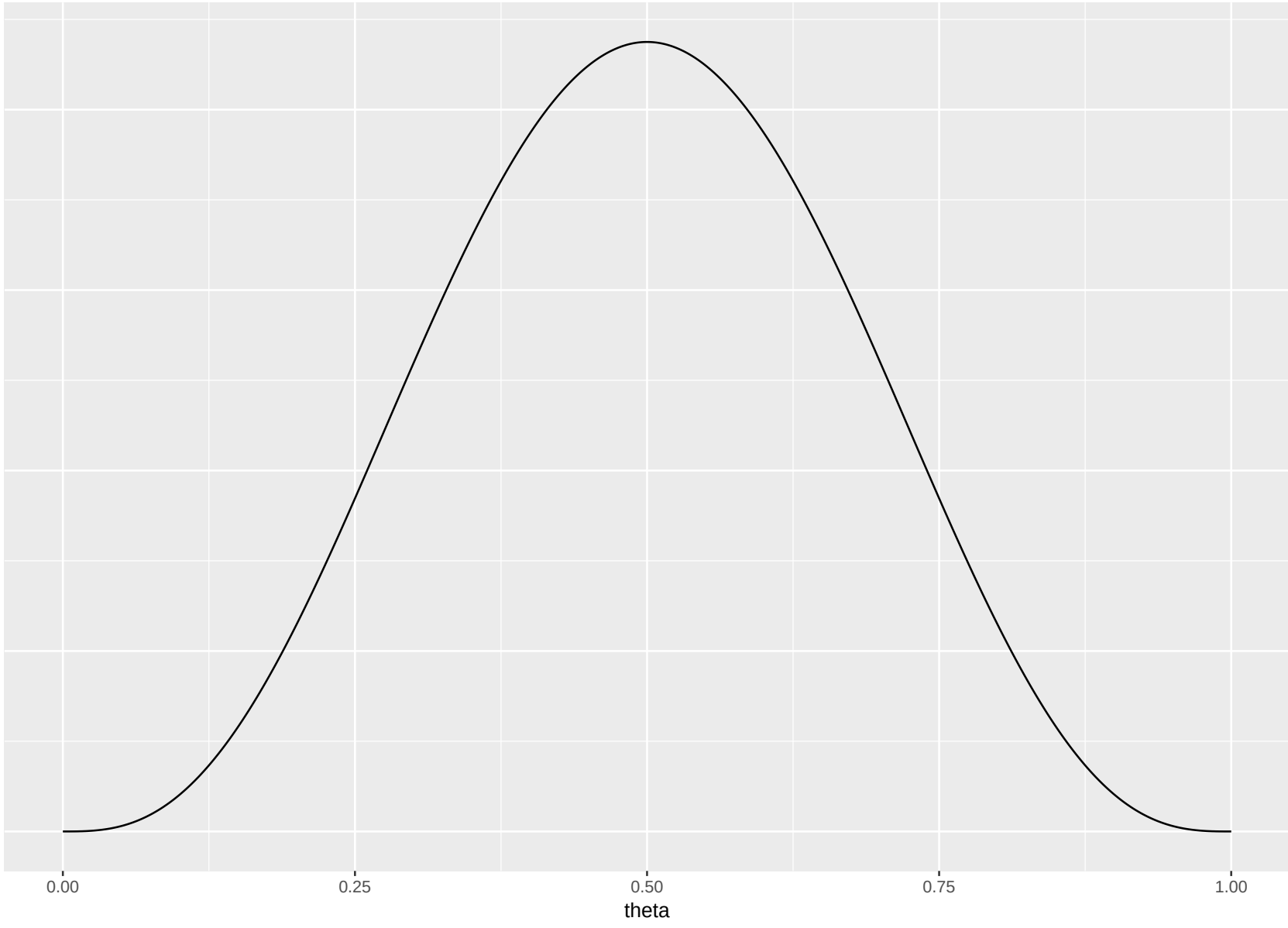
Reverse engineering of parameters producing the data.

How do we deal with uncertainty?

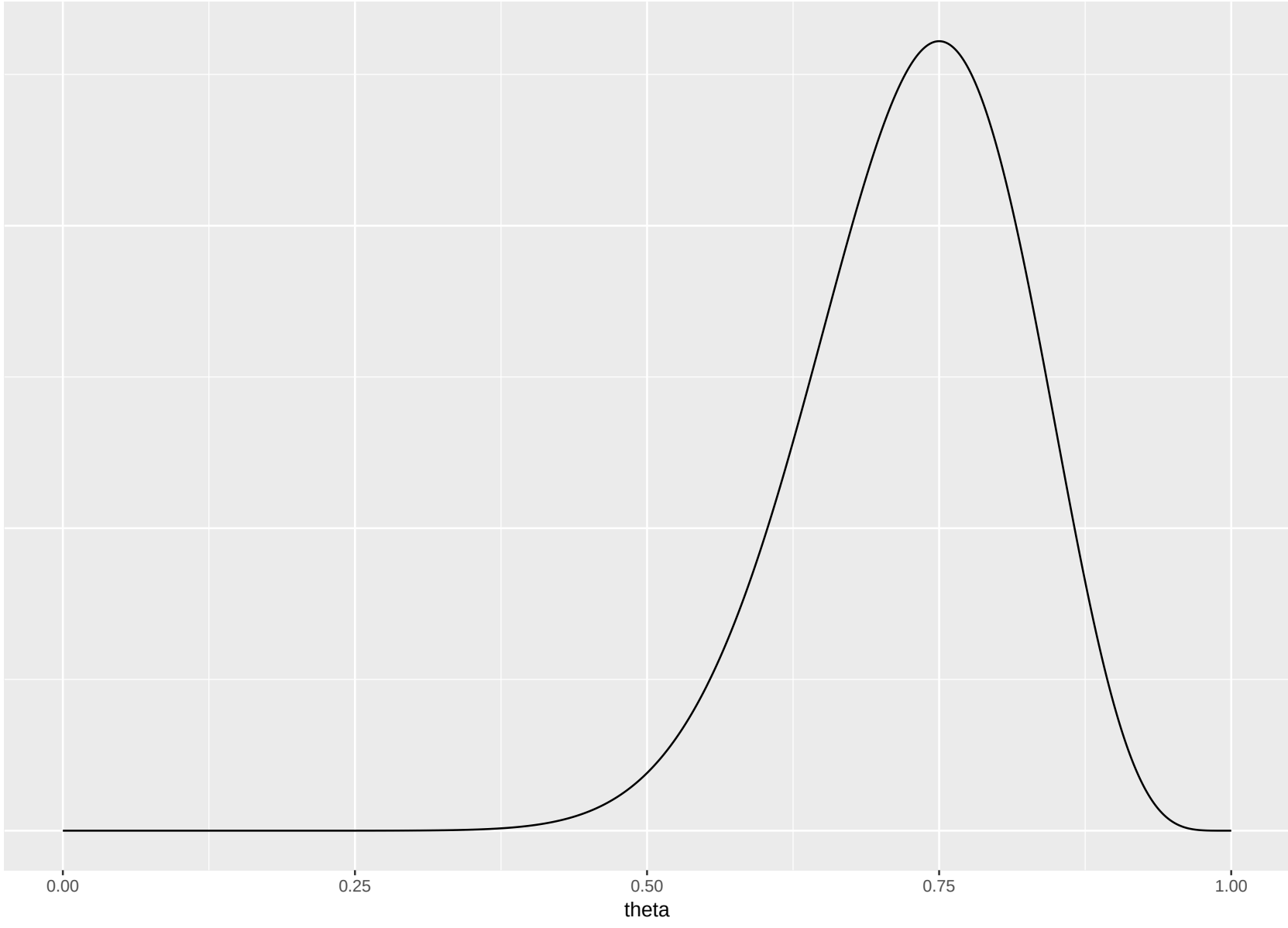
A Bayesian:

- Sets a probabilty distribution on all θ.
- Updates his beliefs with data.

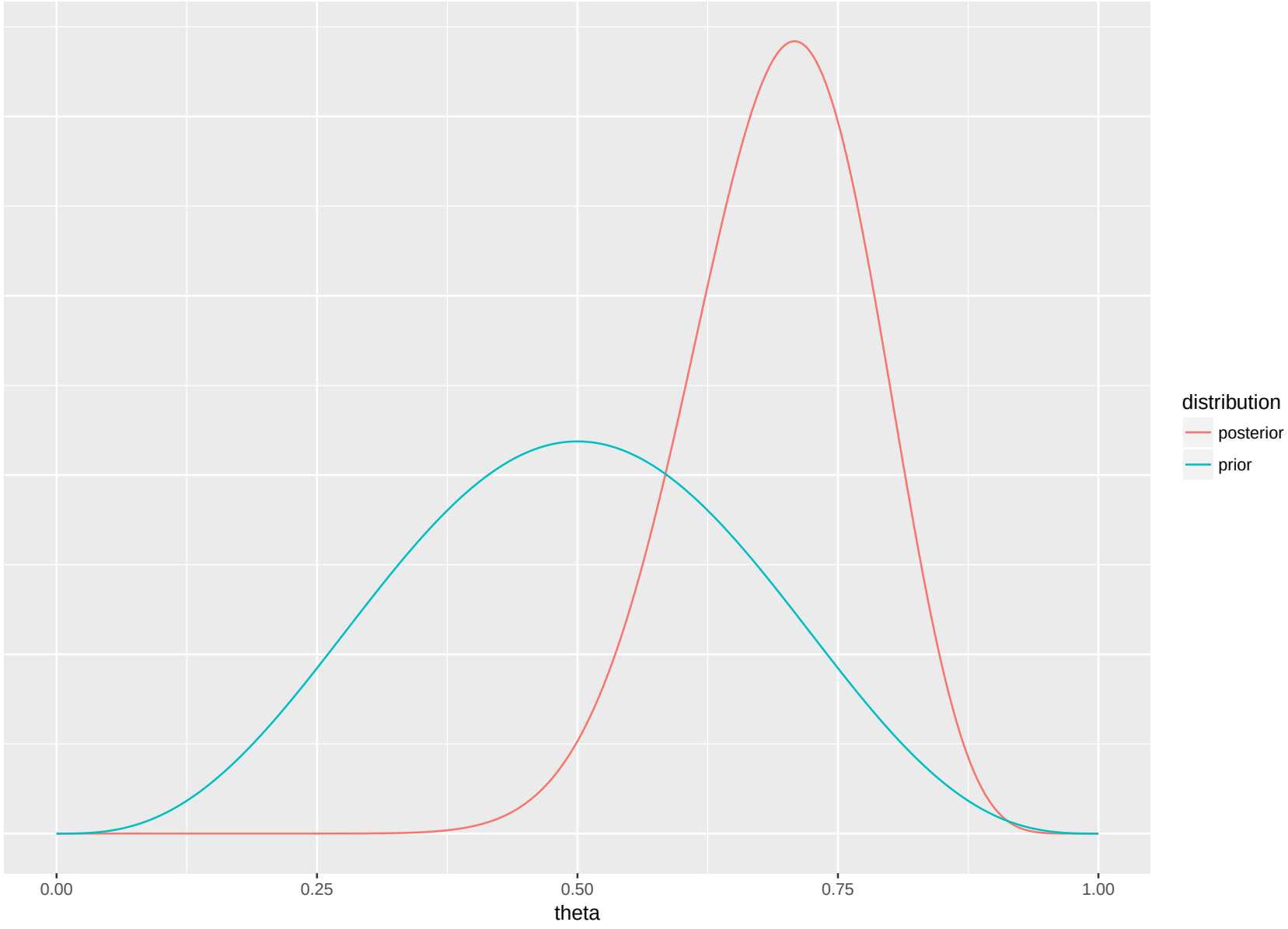
Set a prior: $P(\theta)$



Get the likelihood function: $P(D|\theta)$



Update prior to posterior with likelihood: $P(\theta|D)$



Likelihood not in this plot, on different scale (why?).

Bayesian data analysis

The essence of BDA is **credibility (re)allocation**.

We have an a priori idea about θ:

- expert opinion
- previous research
- educated guess

Data provides evidence of the parameter value.

The posterior is a compromise between prior and likelihood. It reflects the current knowledge.

Bayesian vs frequentist

- Frequentist only consider the likelihood.
- Frequentits have an objective view of probability. For Bayesians it is a subjective best guess.
- Frequentists: data random, parameters fixed. Bayesians: data fixed, parameters random.

Why do we want BDA in the first place?

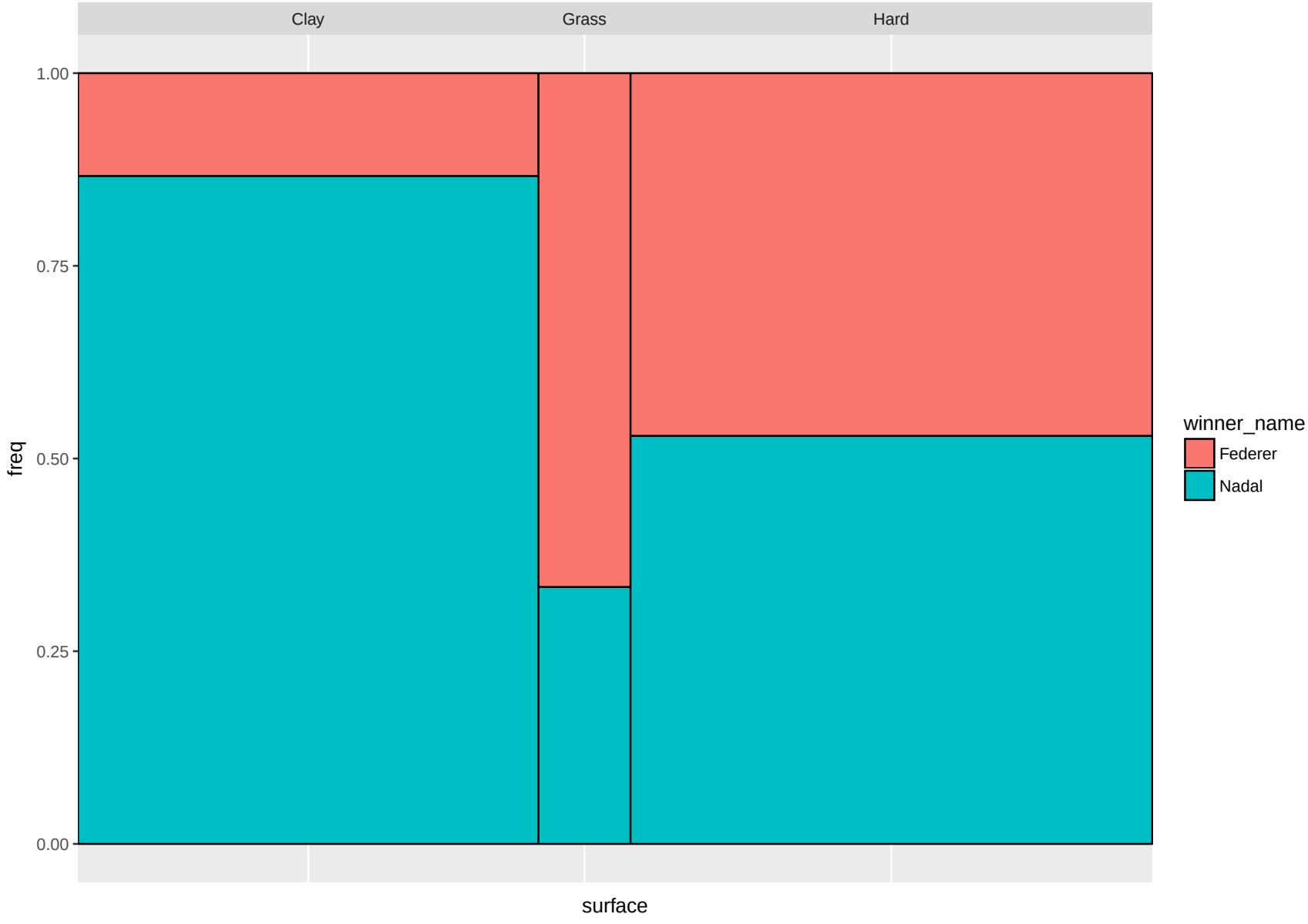
- Elegant and intuitive paradigm.
- Incorparation of previous knowledge and allowing for updating.
- Describe complex relationships without huge amounts of data.

Probability

Considered known

- sample space Ω.
- probability functions.
- discrete and continuous random variables.
- expected value and variance of random variables.
- from probability distribution to likelihood.

Probability of multiple events



Joints, marginals and conditionals

(We assume the probabilities here as given, not as estimated.)

The joint is the probability two events coincide. $P(A = a \cap B = b)$ or for brevity $P(A \cap B)$

winner_name	Clay	Grass	Hard
Federer	0.057	0.057	0.229
Nadal	0.371	0.029	0.257

Joints, marginals and conditionals

The marginals are the univariate distributions of A and B. Sum over the other (or integrate them out when continuous).

Clay	0.428
Grass	0.086
Hard	0.486

Federer	0.343
Nadal	0.657