Homework 1

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1. Transform the following numbers from one base to another

Decimal to binary

- 10 |2   => 1010₂
  - 0 5 |2
  -    ① 2 |2
  -      ⓪ ①

- 1369 |2   = 10101011001₂
  - 1 684 |2
  -    0 342 |2
  -       0 171 |2
  -          1 85 |2
  -             1 42 |2
  -                0 21 |2
  -                   1 10 |2
  -                      0 5 |2
  -                         1 2 |2
  -                            0 1

- 9234876 |2
  - 0 4617438 |2
  -    0 2308719 |2
  -       1 1154359 |2
- 360073 |2   1 577179 |2
  - 1 180036 |2   1 288589 |2
  -    0 90018 |2   1 144294 |2
  -       0 45009 |2   0 72147 |2
  -          1 22504 |2   1 36073

R: 1000110011101001101111100₂

- 2254 |2
  - 0 1127 |2
  -    1 563 |2
  -       1 281 |2
  -          1 140 |2
  -             0 70 |2
  -                0 35 |2
  -                   1 17 |2
  -                      1 8 |2
  -                         0 4 |2
  -                            0 2 |2
  -                               0 1

- 49263749 |2
  - 1 24631874 |2
  -    0 12315937 |2
- 3078984 |2   1 6157968 |2
  - 0 1539492 |2   0 3078984
  -    0 769746 |2
  -       0 384873 |2
- 120057 |2   1 192436 |2
  - 1 6013 |2   0 96218 |2
  -    1 3006 |2   0 48109 |2
  -       0 1503 |2   1 24054 |2
  -          1 751 |2   0 12027
  -             1 375 |2
  -                1 187 |2
  -                   1 93 |2
  -                      1 46 |2
  -                         0 23 |2
  -                            1 11 |2
  -                               1 5 |2
  -                                  1 2 |2
  -                                     0 1

R = 10111011 1110 11010010 0000101₂

Decimal to binary using 2's complement

- $-20 \Rightarrow 20: 0\ 10100 \rightarrow 101011$ 6 bits

$$\begin{array}{r} 101011 \\ +\ 000001 \\ \hline 101100 \end{array} /\!/$$

- $-1025 \Rightarrow 1025: 01000\ 000000\ 1 \rightarrow 10111111110$ 12 bits

$$\begin{array}{r} 10111111110 \\ +\ 00000000001 \\ \hline 10111111111 \end{array} /\!/$$

- $-3925 \Rightarrow 3925: 0111\ 1010101010\ 1 \rightarrow 100001010101010$ 13 bits

$$\begin{array}{r} 100001010101010 \\ +\ 000000000000001 \\ \hline 100001010101011 \end{array} /\!/$$

- $-104596 \Rightarrow 104596: 01100\ 11\ 000\ 100\ 10100 \rightarrow 10011001110110 1011$ 18 bits

$$\begin{array}{r} 100110011101101011 \\ +\ 000000000000000001 \\ \hline 100110011101101100 \end{array} /\!/$$

Unsigned binary to hex

-
1100	1111	0101	0110	0110	1110	1101	1000	0010	1001
C	F	5	6	6	E	D	8	2	9

Short method $/\!/$

Long method: binary to decimal $= 2^0 + 2^3 + 2^5 + 2^{11} + 2^{12} + 2^{14} + 2^{15} + 2^{17} + 2^{18} + 2^{19} + 2^{21} + 2^{22} + 2^{23} + 2^{26} + 2^{22} + 2^{36} + 2^{37}$
$$+ 2^{33} + 2^{34} + 2^{35} + 2^{38} + 2^{39} = 890508335145$$

decimal to hex: $890508335145 \rfloor 16$

$\quad 9\ \ 55656770946 \rfloor 16$

$\quad\quad 2\ \ 3478548184 \rfloor 16$

$13\ 58807\cancel{8} \rfloor 16 \quad\quad\quad 8\ \ 217409261 \rfloor 16$

$\quad 14\ \ 849254 \rfloor 16 \quad\quad\quad 13\ \ 13588678$

$\quad\quad 6\ \ 53078 \rfloor 16$

$\quad\quad 6\ \ 3317 \rfloor 16$

$\quad\quad\quad 5\ \ 207 \rfloor 16$

$\quad\quad\quad 15\ \ 12 \quad = C F 5 6 6 E D 8 2 9 \;/\!/$

-
1000	0111	1000	1110	0011	1000	1110	0011	1111	0011
8	7	8	E	3	8	E	3	F	3

Short method $/\!/$

Long method: binary to decimal $= 2^0 + 2^1 + 2^4 + 2^5 + 2^6 + 2^7 + 2^9 + 2^{13} + 2^{14} + 2^{15} + 2^{19} + 2^{20} + 2^{21} + 2^{25} + 2^{26} + 2^{27} + 2^{31} + 2^{32}$
$$+ 2^{33} + 2^{34} + 2^{39} = 582206678003$$

decimal to hex $\quad 582206678003 \rfloor 16$

$\quad\quad\quad ③\ \ 36387917375 \rfloor 16$

$8883768 \rfloor 16 \quad\quad\quad ⑮\ \ 2274244835 \rfloor 16$

$\quad ⑧\ \ 555235 \rfloor 16 \quad\quad\quad ③\ \ 142140302 \rfloor 16$

$\quad\quad ③\ 34702 \rfloor 16 \quad\quad\quad ⑭\ \ 8883768$

$\quad\quad ④\ \ 2168 \rfloor 16$

$\quad\quad\quad ⑧\ \ 135 \rfloor 16$

$\quad\quad\quad\quad ⑦\ ⑧ \quad = 8 7 8 E 3 8 E 3 F 3 \;/\!/$

• 1010 1101 0101 1100 0110 0101 0100 1010 1010 1010

A   D   5   C   6   5   4   A   A   A   short method

Long method: binary to decimal $= 2^1 + 2^3 + 2^5 + 2^7 + 2^9 + 2^{11} + 2^{14} + 2^{16} + 2^{18} + 2^{21} + 2^{22} + 2^{26} + 2^{27} + 2^{28} + 2^{30} + 2^{32} + 2^{34} + 2^{35}$
$+ 2^{37} + 2^{39} = 744579484330$

decimal to hex: $744579484330 \lfloor 16$

⑩ $46536217770 \lfloor 16$

$710086 \lfloor 16$
⑥ $44380 \lfloor 16$
⑫ $2773 \lfloor 16$
⑤ $173 \lfloor 16$
⑬ ⑩

⑩ $2908513610 \lfloor 16$
⑩ $181782100 \lfloor 16$
④ $11361381 \lfloor 16$
⑤ 710086

= A D 5 C 6 5 4 A A A

          1010
• 1010 0010 1010 ↓1010 1010 1011 1111 1100 0000

A   2   A   A   A   A   B   F   C   0   short method

Long method: binary to decimal $= 2^6 + 2^7 + 2^8 + 2^9 + 2^{10} + 2^{11} + 2^{12} + 2^{13} + 2^{15} + 2^{17} + 2^{19} + 2^{21} + 2^{23} + 2^{25} + 2^{27} + 2^{29} + 2^{31} + 2^{33} + 2^{35} + 2^{37} + 2^{39}$
$= 698648018880$

decimal to hex: $698648018880 \lfloor 16$
⑩ $43679050.1180 \lfloor 16$

$666282 \lfloor 16$
⑩ $41642 \lfloor 16$
⑩ $2602 \lfloor 16$
⑩ $162 \lfloor 16$
② ⑩

⑫ $2729003823 \lfloor 16$
⑮ $176568363 \lfloor 16$
⑪ $10660522 \lfloor 16$
⑩ 666282

= 4 2 A A A A B F C 0

Signed binary to octal

• ①11 111 006 001 111 100 000 001 110 101 011

2's complement: 000 000 111 110 000 011 111 110 001 010 100
         +   1

        000 000 111 110 000 011 111 110 001 010 101

     − 7 6 0 3 7 6 1 2 5   short method

Long method: binary to decimal $= 2^0 + 2^2 + 2^4 + 2^6 + 2^{10} + 2^{11} + 2^{12} + 2^{13} + 2^{14} + 2^{15} + 2^{16} + 2^{22} + 2^{23} + 2^{24} + 2^{25} + 2^{26} = 130153557$

decimal to octal $= 130153557 \lfloor 8$
⑤ $16269194 \lfloor 8$
② $2033649 \lfloor 8$
① $254206 \lfloor 8$
⑥ $31775 \lfloor 8$
⑦ $3971 \lfloor 8$
③ $496 \lfloor 8$
⓪ $62 \lfloor 8$
⑥ ⑦

$= -760376125$

• 010 101 010 101 111 111 111 111 110 000 000
   2   5   2   5   7   7   7   7   6   0   0   short method //

Long method: binary to decimal = $2^7 + 2^9 + 2^{10} + 2^{11} + 2^{12} + 2^{13} + 2^{14} + 2^{15} + 2^{16} + 2^{17} + 2^{18} + 2^{19} + 2^{20} + 2^{21}$
$+ 2^{23} + 2^{25} + 2^{27} + 2^{29} + 2^{31} = 2864709504$

Decimal to hex: $2864709504 | 8$
　　　　　　　　　　② $359088688 | 8$
$10927 | 8$　　　　　　⑥ $44761086 | 8$
　⑦ $1365 | 8$　　　　　　⑥ $5595135 | 8$
　　　⑤ $170 | 8$　　　　　　⑦ $699391 | 8$
　　　　⑤ 21　　　　　　　　　⑦ $87423 | 8$
　　　　　　　　　　　　　　　　　① 10927

$= 25257777600$ //

• ① 110 001 110 000 001 111 111 100 000 101 010

2's complement: 0 001 110 001 111 110 000 000 011 111 010 101
　　　　　+　　　　　　　　　　　　　　　　　　　　　　　　　　1
　　　　　　　0 001 110 001 111 110 000 000 011 111 010 110
　　　　　　ↆ - 1   6   1   7   6   0   0   3   7   2   6   short method //

Long method: binary to decimal = $2^1 + 2^2 + 2^4 + 2^5 + 2^7 + 2^8 + 2^9 + 2^{10} + 2^{14} + 2^{22} + 2^{23} + 2^{27} + 2^{28} + 2^{29} + 2^{30}$
　　　　　　= 1912080342

Decimal to hex = $1912080342 | 8$
　　　　　　　　　　⑥ $239010042 | 8$
$113 | 8$　　　　　　② $29876255 | 8$
　④ $14 | 8$　　　　　　⑦ $3734531 | 8$
　　⑥ ④　　　　　　　③ $466816 | 8$
　　　　　　　　　　　　⓪ $58352 | 8$
$= -16176003726$ //　　　⑥ $7294 | 8$
　　　　　　　　　　　⑤ $911 | 8$
　　　　　　　　　　　⑦ 113

• ① 010 101 010 100 000 101 010 101 011 111 000

2's complement: 0 101 010 101 011 111 010 101 010 100 000 111
　　　　　+　　　　　　　　　　　　　　　　　　　　　　　　　　　1
　　　　　0 101 010 101 011 111 010 101 010 100 001 000
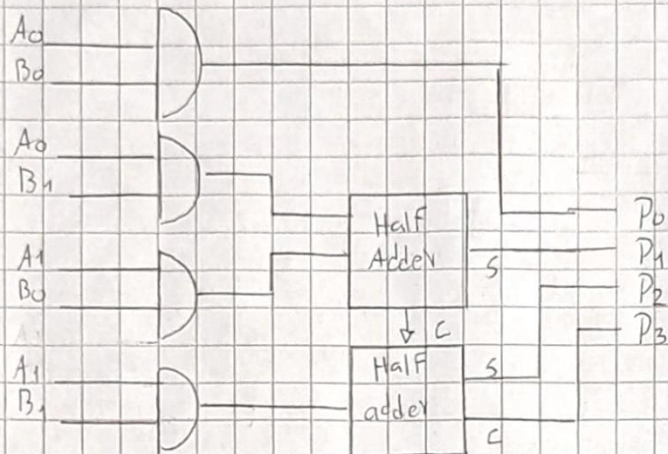　　　　　ↆ - 5   2   5   3   7   2   5   2   4   1   0   short method

Long method: binary to decimal = $2^3 + 2^8 + 2^{10} + 2^{12} + 2^{14} + 2^{16} + 2^{18} + 2^{19} + 2^{21} + 2^{22} + 2^{24} + 2^{26} + 2^{28} + 2^{30} + 2^{31} = 5729244424$

Decimal to hex = $5729244424 | 8$
　　　　　　　　　　⑥ $716155553 | 8$
$218551 | 8$　　　　　① $89514444 | 8$
　⑦ $27311 | 8$　　　　④ $11189930 | 8$
　　③ $3411 | 8$　　　　② $1348741 | 8$
　　　⑤ $42 | 8$　　　　⑤ $174842 | 8$
　　　② ⑤ $= -5253725?410$ //　② 21855

2. Bolean circuits

- Multiplication of two binary numbers of length 2 bits

A_1	A_0	B_1	B_0	P_3	P_2	P_1	P_0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0
0	0	1	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	0	0	0	0	0
0	1	0	1	0	0	0	1
0	1	1	0	0	0	1	0
0	1	1	1	0	0	1	1
1	0	0	0	0	0	0	0
1	0	0	1	0	0	1	0
1	0	1	0	0	1	0	0
1	0	1	1	0	1	1	0
1	1	0	0	0	0	0	0
1	1	0	1	0	0	1	1
1	1	1	0	0	1	1	0
1	1	1	1	1	0	0	1

Referencia:

Electronics Hub (2015), Binary Multiplication methods, electronicshub.org/binary-multiplication/#top

- Two's complement for a binary number of length 3 bits

2 bit of input (A)   1 bit of input (B)   0 bit of input (C)
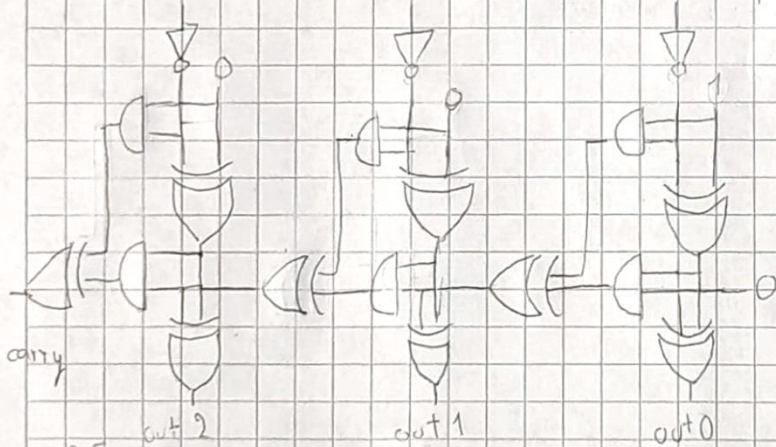
C	B	A	P_2	P_1	P_0
0	0	0	0	0	0
0	0	1	1	1	1
0	1	0	1	1	0
0	1	1	1	0	1
1	0	0	1	0	0
1	0	1	0	1	1
1	1	0	0	1	0
1	1	1	0	0	1

carry

out 2   out 1   out 0

Referencia:

George John (2019), Two's Complement. tutorialspoint.com. cach3.com/twos-complement.html

3. Do the following multiplications in binary

- -5×8 : $5 = 00101 \xrightarrow{2's} 11010 + 1 = 11011$ (md)

  $8 = 01000$ (mr)

MD			MD			MD		
11011			11011			11011		

PD	MR	MX	PD	MR	MX	PD	MR	MX
00000	01000	0	00000	00100	0	00000	00010	0
	CR			CR			CR	
	00101			00100			00011	

MD
11011   10→ PD = PD - MD   11011 → 00100
                                                    1
PD   MR   MX   00101
00000   00001   0   00000
        CR   + 00101
        00010   00101

MD   MD
11011   11011   01→ PD = PD + MD
PD   MR   MX   PD MR   MX
00101   00001   0 →00010   10000   1   00101
        CR   CR   11011
        00010   00001   11101

MD   MD
11011   11011   Resultado 111011000 = -40
PD   MR   MX   PD MR   MY
11101   10000   1 →11110   11000   0   2's → 0000100111
        CR   CR   +   1
        00001   00000   000101000 (40)

○ 11 x (-10) : 11 = 01011   MD
              : 10 = 01010 → 10101 + 1 = 10110 (-10) MR
                        2's

MD   MD
01011   01011   10→ PD = PD - MD : 01011→10100
PD   MR   MX   PD MR   MX   2's
00000   10110   0 →00000   01011   0   ---   00000   10101
        CR   CR   10101
        00101   00100   10101

MD   MD   MD
01011   01011   01011   01→PD=PD+MD
PD   MR   MX   PD MR   MX   PD MR   MX
10101   01011   0 →11010   10101   1 →11101   01010   1   11101
        CR   CR   CR   01011
        00100   00011   00010   100100

MD   MD
01011   01011
PD   MR   MX   PD MR   MX   10→ PD = PD-MD: 00100
01000   01010   1 →00100   00101   0   10101
        CR   CR   11001
        00010   00001

MD   MP
01011   01011
PD   MR   MY   PD MR   MX   Resultado: 111001110 = -110
11001   00101   0 →11100   10010   1
        CR   CR   2's → 000110101
        00001   00000   1
                                    000110110 (110)

• 2×3 ; 2 = 010   MD   10 → PD = PD - MD $2's$
          3 = 011   010   010 → 101
                   PD   MR   Mx   000   1
                   000 011   0   + 110   ‾‾‾‾
                        CR   1 1 0   1 1 0
                        011

        MD   MD   01 → PD = PD + MD
        010   010   010
   PD   MR   Mx   PD   MR   MA   010
   110 011   0   →   111 001   1   → 111 100   1   111
        CR   CR   ‾‾‾‾‾‾
        611   010   001   100 11

        MD   MD
        010   010
   PD   MR   Mx   PD   MR   MA   Resultado : 000 110 = 6 //
   001 100   1   →   000 110   0
        CR   CR
        001   000

• (-4) × (-8) ; 4 = 00100 → $2's$ 11011 + 1 = 11100   (-4) MD
                 8 = 01000 → 10 111 + 1 = 11000 (-8)   MR
                          $2's$

        MD   MD   MD   MD
        11100   11100   11100   11100
   PD   MR   Mx   PD   MR   Mx   PD   MR   Mx   PD   MR   Mx
   00000 11000 0 →   00000 01100 0 →   00000 00110 0 →   00000 00011 0
        CR   CR   CR   CR
        00101   00100   00011   00010

   10 → PD = PD - MD   11100 → $2's$ 00011
                                    1
                                 ‾‾‾‾‾
        00000   00100
        00100   ‾‾‾‾‾‾
      ‾‾‾‾‾‾
        00100

        MD   MD   MD
        11100   11100   11100
   PD   MR   Mx   PD   MR   MA   PD   MR   Mx   Resultado = 0000100000
   00100 00011 0 →   00010 00001 1 →   00001 00000 1
        CR   CR   CR   = 32 //
        00010   00001   00000