Network theorems

Thevinins theorem

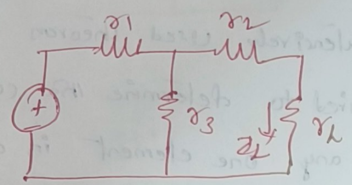
→ This theorem is most extensively used theorem. It is applicable where it is desired to determine the current through on voltage across any one element in a network without going through the method of solving a set of network equations.

Statement

"Any two terminal bilateral linear dc circuits can be replaced by an equivalent circuit consisting of a voltage source and a series resistor."

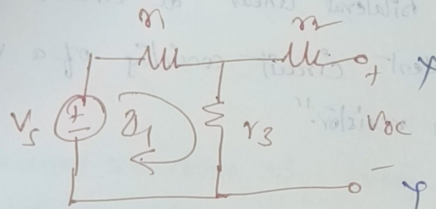
A more general statement of thevinins theorem is that any linear active network consisting of independent and dependent voltage source and current sources and linear bilateral network elements can be replaced by an equivalent circuit consisting of voltage source in series with a resistance the voltage source being the open circuited voltage across the open circuited load terminals and the resistance being the internal resistance of the source network looking through the open circuited load terminals.

Explanation



Let us consider a simple dc circuit as shown in fig. We have to find I_L by thevinin's theorem.

To find V_{oc}



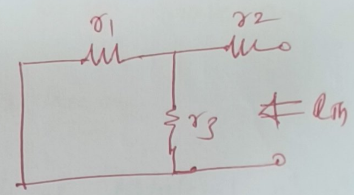
To find V_{oc}, open circuit the load terminals at which we calculate I_L. and calculate open circuit voltage at load terminals which is equal to thevinins voltage V_{th}.

$$V_{oc} = V_s \times \frac{r_3}{r_1 + r_3}$$

As load terminals x are are open circuited the current through r_2 is zero. $\therefore r_2 = 0$.

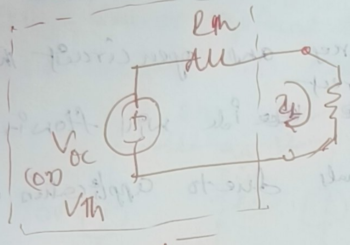
To find R_{th}

To find internal resistance (thevinins resistance (or) equivalent resistance) in series with V_{oc}, the voltage source is removed by short circuit / current source removed by open circuit.



$$R_{Th} = R_2 + \frac{r_1 r_3}{r_1 + r_3}$$

Equivalent Circuit



$$I_L = \frac{V_{oc}}{R_{Th} + R_L}$$

→ Internal resistance of voltage source is zero.

→ Internal resistance of current source is infinity

→ To find R_{th} for the circuits containing dependent sources in addition to (or) in absence of independent sources.

IInd method

Find V_{oc} across the open circuited terminals by conventional network analysis, & short the load terminals and find the short circuit current (I_{sc}) through the shorted terminals.

The internal resistance of the network is obtained as

$$R_{th} = \frac{V_{oc}}{I_{sc}}$$

2nd method

Remove the load terminals resistance and apply a dc driving voltage V_{dc}, at the open circuited load terminals. ~~Kept~~ Keep the other independent source deactivated during this time {short the voltage sources and open circuit the current sources}. A driving ~~voltage~~ current ~~V_{dc}~~ I_{dc} will flow in the circuit from the load terminals due to application of V_{dc}.

$$R_{th} = \frac{V_{dc}}{I_{dc}}$$

Section - I ① - ⓑ State and explain Superposition theorem.

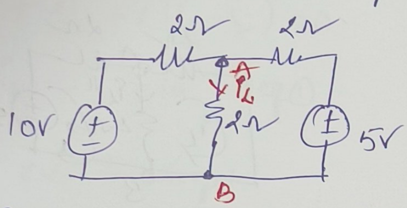
Superposition theorem

6M

"In any linear bilateral circuit the response through an element is equal to the algebraic sum of the responses caused by each source acting at a time while all the other sources are replaced by their internal impedances."

[i.e voltage sources replaced by short circuit current sources replaced by open circuit]

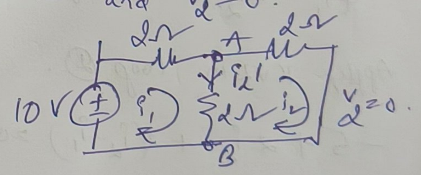
Let us consider an example



To find current i_L at AB Terminals

1. When 10V source is acting alone and $V_2 = 0$.

Apply KVL in loop ①



$$-10 + 2i_1 + 2[i_1 - i_2] = 0.$$

$$4i_1 - 2i_2 = 10 \rightarrow eq ①$$

Apply KVL in loop ②

$$2(i_2 - i_1) + 2i_2 = 0.$$

$$-2i_1 + 4i_2 = 0 \rightarrow eq ②$$

$$\begin{bmatrix} 4 & -2 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \end{bmatrix}$$

$$i_1 = \frac{\Delta_1}{\Delta} \qquad i_2 = \frac{\Delta_2}{\Delta}$$

$\Delta_1 = \begin{vmatrix} 10 & -2 \\ 0 & 4 \end{vmatrix}$ $\Delta = \begin{vmatrix} 4 & -2 \\ -2 & 4 \end{vmatrix}$ $\Delta_2 = \begin{vmatrix} 4 & 10 \\ -2 & 0 \end{vmatrix}$

$\Delta_1 = [40 + 0] = 40$ $\Delta_2 = [0 + 20] = 20.$

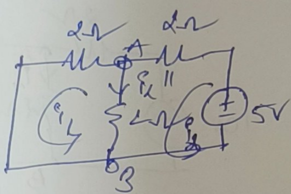
$i_1' = \frac{\Delta_1}{\Delta} = \frac{\cancel{40}^{\,\cancel{20}\,10}}{\cancel{12}_{\,\cancel{6}\,3}}$ $\Delta = [16 - 4] = 12$

$i_2' = \frac{\Delta_2}{\Delta} = \frac{\cancel{20}^{\,10\,5}}{\cancel{12}_{\,\cancel{6}\,3}}$

$i_1' = 3.33A$ $i_2' = \frac{5}{3} = 1.66A$

$i_b' = i_1' - i_2' = 3.33 - 1.66 = 1.67$

When 5V source acting alone



Apply KVL in loop ①.

$2i_1 + 2(i_1 - i_2) = 0.$

$4i_1 - 2i_2 = 0 \rightarrow eq(1)$

Apply KVL in loop ②.

$-5 + 2i_2 + 2(i_2 - i_1) = 0.$

$-2i_1 + 4i_2 = 5 \rightarrow eq(2)$

$$\begin{bmatrix} 4 & -2 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$$

$i_1'' = \frac{\Delta_1}{\Delta}$ $i_2'' = \frac{\Delta_2}{\Delta}$ $\Delta_1 = \begin{vmatrix} 0 & -2 \\ 5 & 4 \end{vmatrix}$ $\Delta_2 = \begin{vmatrix} 4 & 0 \\ -2 & 5 \end{vmatrix}$

$\Delta = \begin{vmatrix} 4 & -2 \\ -2 & 4 \end{vmatrix}$

$\Delta_1 = (0 + 10)$ $\Delta_2 = (20 - 0)$ $\Delta = 16 - 4 = 12$

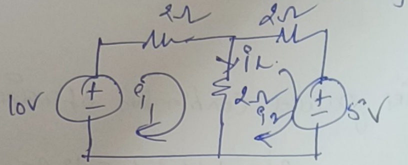
$\Delta_1 = 10$ $\Delta_2 = 20$ $\Delta = 12$

$i_1'' = \frac{\cancel{10}^{\,5}}{\cancel{12}_{\,6}} = 0.833A$ $i_2'' = \frac{\Delta_2}{\Delta} = \frac{\cancel{20}^{\,10\,5}}{\cancel{12}_{\,6\,3}} = 1.66A$

$$i_2'' = i_2'' - i_1'' = 1.66 - 0.833 = 0.834$$

$$i_L = i_L' + i_L'' = 1.667 + 0.834 = \underline{\underline{2.5\,A}}$$

When two sources acting



Apply KVL to loop 1

$$-10 + 2i_1 + 2(i_1 - i_2) = 0.$$

$$4i_1 - 2i_2 = 10 \rightarrow eq(1)$$

Apply KVL in Loop 2

$$2(i_2 - i_1) + 2i_2 + 5 = 0.$$

$$-2i_1 + 4i_2 = -5 \rightarrow eq(2)$$

$$\begin{bmatrix} 4 & -2 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 10 \\ -5 \end{bmatrix}$$

$i_1 = \frac{\Delta_1}{\Delta}$ $\quad i_2 = \frac{\Delta_2}{\Delta}$ $\quad \Delta_1 = \begin{vmatrix} 10 & -2 \\ -5 & 4 \end{vmatrix}$ $\quad \Delta_2 = \begin{vmatrix} 4 & 10 \\ -2 & -5 \end{vmatrix}$

$\Delta_1 = 40 - 10 = 30$ $\quad \Delta = \begin{vmatrix} 4 & -2 \\ -2 & 4 \end{vmatrix}$

$\Delta_2 = -20 + 20 = 0.$

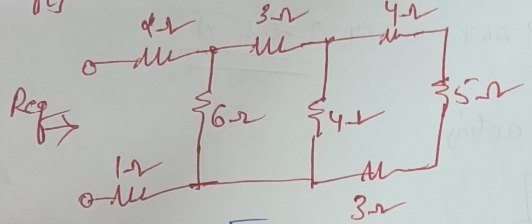
$\Delta = 16 - 4 = 12$

$i_1 = \frac{\Delta_1}{\Delta} = \frac{30}{12}$ $\quad i_2 = \frac{\Delta_2}{\Delta} = \frac{0}{12} = 0\,A$

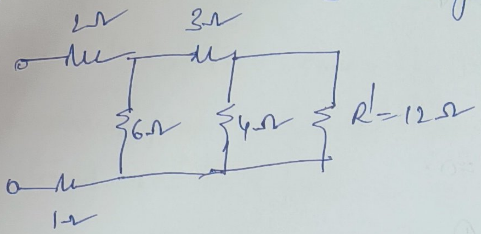
$i_1 = 2.5\,A$

$i_L = 2.5 + 0 = \underline{2.5\,A}$ Hence proved.

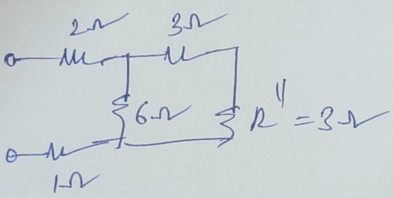
Q. a. Determine Equivalent resistance for the circuit shown in fig. (6M)



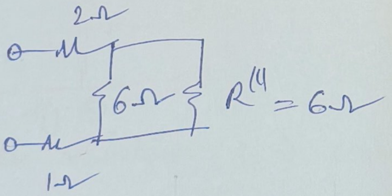
Sol:- Reduce the network using series parallel combination of resistance



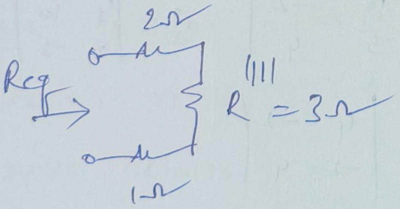
$$R' = 4+5+3 = 12\Omega$$



$$R'' = \frac{12\times 4}{12+4} = \frac{48}{16} = 3\Omega$$



$$R''' = 3+3 = 6\Omega$$



$$R'''' = \frac{6\times 6}{6+6} = \frac{36}{12} = 3\Omega$$

$$Req = 2+3+1 = \underline{6\Omega}$$

⊛ Maximum power Transfer theorem

→ This theorem is used to find the value of load resistance for which there would be maximum amount of power transfer from source to load.

Statement

" A resistance load, being connected to a dc network receives maximum power when the load resistance is equal to the internal resistance (thevinin's equivalent resistance) of the source network as seen from the load terminals."

Explanation

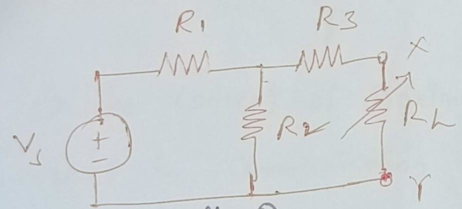


fig. ⓐ

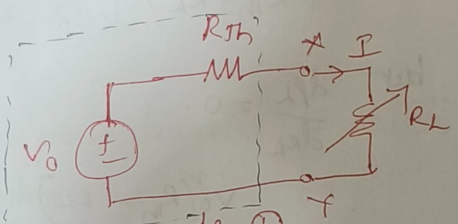


fig ⓑ

A variable resistance R_L is connected to a dc source network as shown in fig. ⓐ while fig ⓑ represents the thevinin voltage V_0 and thevinin's Resistance R_{Th} of the source network.

with reference to fig ⓑ $I = \dfrac{V_0}{R_{Th} + R_L}$

while the power delivered to the resistive load is

$$P_L = I^2 R_L = \left(\frac{V_0}{R_{Th} + R_L}\right)^2 \times R_L$$

P_L can be maximised by varying R_L and hence maximum power can be delivered when $\frac{dP_L}{dR_L} = 0$

However

$$\frac{dP_L}{dR_L} = \frac{1}{\left[(R_{Th} + R_L)^2\right]^2}\left[(R_{Th}+R_L)^2 \frac{d}{dR_L}(V_0^2 R_L) - V_0^2 R_L \frac{d}{dR_L}(R_{Th}+R_L)^2\right]$$

$$\Rightarrow \frac{1}{(R_{Th}+R_L)^4}\left[(R_{Th}+R_L)^2 V_0^2 - V_0^2 R_L \times 2\,[R_{Th}+R_L]\right]$$

$$= \frac{V_0^2 (R_{Th} + R_L - 2R_L)}{(R_{Th}+R_L)^3} = \frac{V_0^2\ R_{Th} - R_L}{(R_{Th}+R_L)^3}$$

but $\frac{dP_L}{dR_L} = 0.$

$$\therefore \frac{V_0^2 (R_{Th} - R_L)}{(R_{Th}+R_L)^3} = 0.$$

Which gives $R_{Th} - R_L = 0$ (or)

$\boxed{R_{Th} = R_L}$ → Condition for maximum power transfer

Hence, it has been proved that power transfer from a dc source network to a resistive network is maximum when the internal resistance of the dc source network is equal to the load resistance.

When $R_{Th} = R_L$ the system being perfectly matched for load and source, the ~~source~~ power transfer becomes maximum and this amount of power (P_{max}) can be obtained as

$$P_{max} = \frac{V_0^2 R_{Th}}{(R_{Th} + R_{Th})^2} = \frac{V_0^2}{4R_{Th}}$$

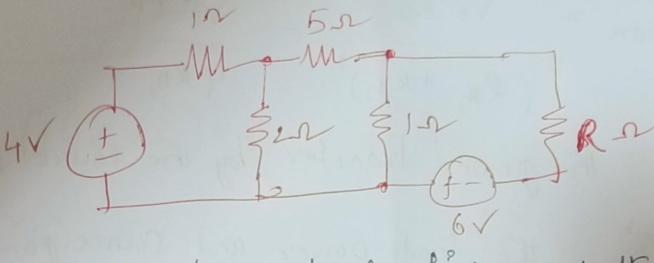
Obviously the power transfer by the source would be also $\frac{V_0^2}{4R_{Th}}$, the load power and source power being same. Total power supplied is $P = 2\frac{V_0^2}{4R_{Th}} = \frac{V_0^2}{2R_{Th}}$

During maximum power transfer the efficiency becomes

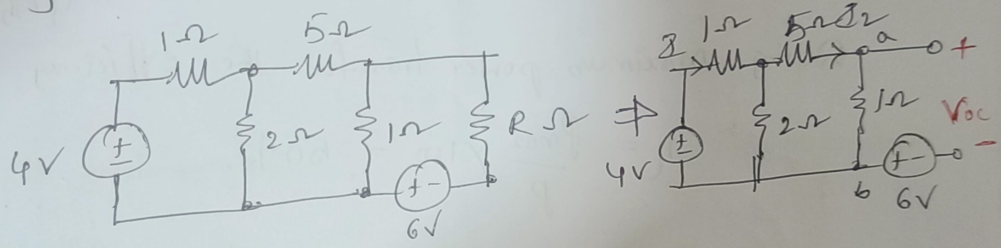
$$\eta = \frac{P_{max}}{P} \times 100 = 50\%.$$

→ The concept of maximum power transfer is mainly used in communication systems.

(*) Find the value of R in the circuit of fig. such that maximum power transfer takes place. What is the amount of this power.



Sol:- Let the R be replaced first and the open circuit voltage be $V_{o.c}$



Here $I = \dfrac{4V}{\left[\left[(5+1)\|2\right]+1\right]} = \dfrac{4}{5/2} = 8/5\ A$

$\therefore \quad I_2 = I \cdot \dfrac{2}{2+5+1} = \dfrac{8}{5} \times \dfrac{1}{4} = \dfrac{2}{5}\ A.$

The drop across ab branch is then

$$V_{ab} = \frac{2}{5} \times 1 = \frac{2}{5}\ V$$

Obviously

$$V_{oc} = V_{ab} + 6 = \frac{2}{5} + 6 = \frac{32}{5}\ V.$$

$$V_{oc} = 6.4\ \text{Volts}$$

When $R_{Th} = R_L$ the system being perfectly matched for load and source, the ~~source~~ power transfer becomes maximum and this amount of power (P_{max}) can be obtained as

$$P_{max} = \frac{V_0^2 R_{Th}}{(R_{Th} + R_{Th})^2} = \frac{V_0^2}{4R_{Th}}$$

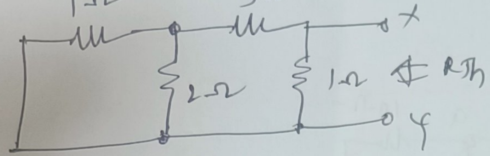
Obviously the power transfer by the source would be also $\frac{V_0^2}{4R_{Th}}$, the load power and source power being same. Total power supplied is $P = 2\frac{V_0^2}{4R_{Th}} = \frac{V_0^2}{2R_{Th}}$

During maximum power transfer the efficiency becomes

$$\eta = \frac{P_{max}}{P} \times 100 = 50\%.$$

→ The concept of maximum power transfer is mainly used in communication systems.

To find the internal resistance of the circuit across x-y with reference to fig (b)



$$R_{Th} = (1 \| 2 + 5) \| 1$$

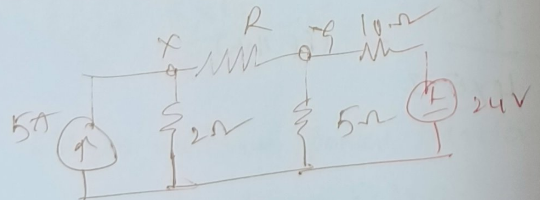
$$= \left[\left(\frac{1\times 2}{1+2}\right) + 5\right] \| 1$$

$$= \frac{\frac{17}{3}\times 1}{\frac{17}{3}+1} = \frac{17}{20} = 0.85\,\Omega$$

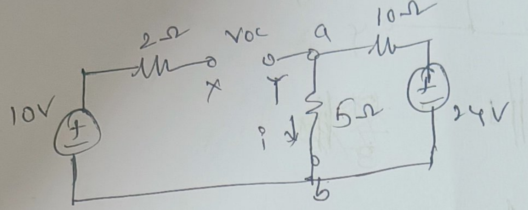
As per maximum power transfer theorem $R_{Th} = R_L = 0.85\,\Omega$

Maximum power $P_{max} = \frac{V_{oc}^2}{4R_{Th}} = \frac{(6.4)^2}{4\times 0.85} = \underline{\underline{12\,W}}$

(*) What should be the value of R such that maximum power transfer takes place from the rest of the network to R in fig. Obtain the amount of this power.



Sol: Let first convert the 'I' source to 'V' source and remove R from X-Y terminal, the voltage of these terminal being $V_{o.c}$.



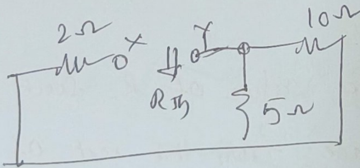
$\therefore V_{a-b}$ = drop across 5Ω $= 1.6 \times 5 = 8V$

Thus in the left loop $-10 + V_{oc} + 8 = 0$.

$V_{oc} = 2$ volts

With reference to R_{Th} (internal resistance looking through x-y) is obtained as

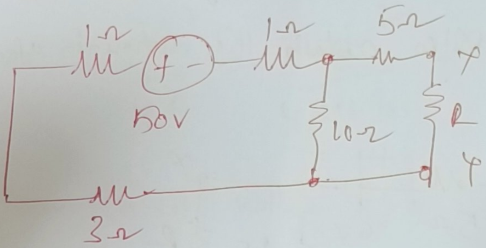
$$R_{Th} = \frac{10 \times 5}{10+5} + 2 = 5.33\,\Omega$$



As per maximum power transfer $R = R_{Th} = 5.33\Omega$

$$P_{max} = \frac{V_{oc}^2}{4R} = \frac{2^2}{4 \times 5.33} = 188\text{ mW}.$$

(*) Assuming Maximum power transfer from the source to R, find the value of this amount of power in the circuit of fig.



Sol:- R is removed by open circuit, with reference to fig.

$$I = \frac{50}{1+1+3+10} = \frac{50}{15} = 3.33\ \Omega$$

$$\therefore\ V_{oc} = V_{10\Omega} = 3.33 \times 10 = 33.33\ \text{volts}$$

It may be noted that b may be positive polarity as the current in 10Ω resistor flows from b to a

$$\therefore\ V_{x-y} = -33.33\ V = V_{oc}$$

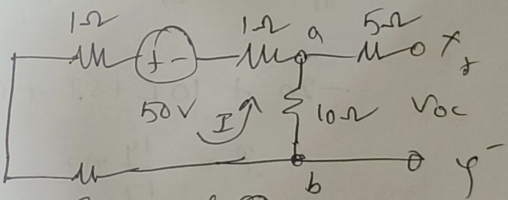


fig (a)

With reference to fig. (b)

$$R_{Th} = \frac{5 \times 10}{5+10} + 5 = 8.33\ \Omega$$

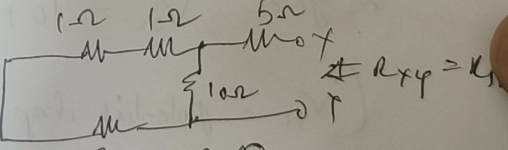


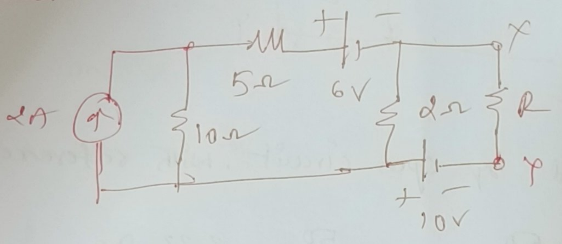
fig (b)

At per maximum power transfer theorem

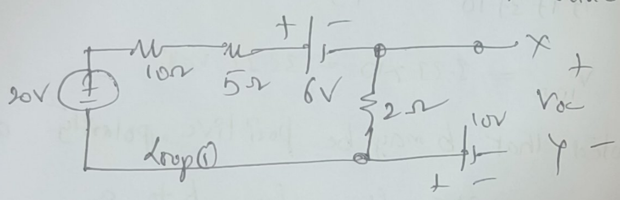
$$R = R_{Th} = 8.33\ \Omega$$

$$P_{max} = \frac{V_{oc}^2}{4R} = \frac{(33.33)^2}{4 \times 8.33} = 33.34 \text{ watts}$$

(7) Find R to have maximum power transfer in the circuit of fig. Also obtain the amount of maximum power.



Sol:- Let us first convert current source to voltage source



The application of mesh analysis in left loop (or loop(1)).

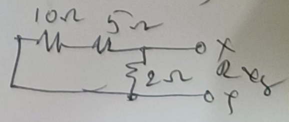
$$-20 + 10I + 5I + 6 + 2I = 0.$$

$$I = \frac{14}{17} A$$

this gives $V_{oc} = 10 + \frac{14}{17} \times 2 = 11.65 V$

(V_{oc} = potential drop across 2Ω + potential of 10V battery)

The internal resistance of the circuit looking from X-Y terminals is $R_{th} = \frac{15 \times 2}{15+2} = \frac{30}{17} = 1.765$



As per theorem maximum power transfer

$$\boxed{R = 1.765 = R_{Th}}$$

$$P_{max} = \frac{V_{Th}^2}{4R} = \frac{11.65^2}{4 \times 1.765} = \underline{\underline{19.22\,W}}$$

(*) The open circuit voltage of a standard car battery is 12.6 V and the short circuit current is approximately 300A. What is the available power from the battery.

Sol:-

$$R_{oc} = \frac{V_{oc}}{I_{sc}} = \frac{12.6}{300} = 0.042\,\Omega$$

$$P_{ave} = \frac{V_{Th}^2}{4R_{Th}} = \frac{V_{oc}^2}{4R_o} = \frac{(12.6)^2}{4 \times 0.042} = 945\,W$$

(*) A stereo sound system is operated from a battery which is made from 8 dry cells connected in series. Each cell has an emf of 1.5V and an internal resistance of 0.75 Ω. How much is the available power from this battery.

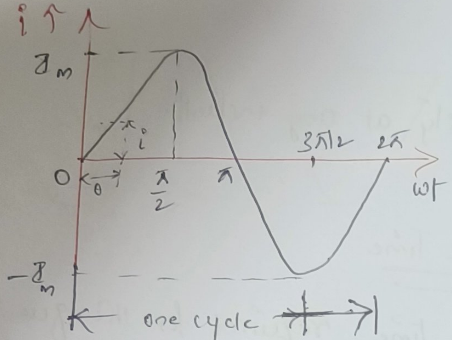
Sol:- $V_{oc} = 8 \times 1.5 = 12\,V$

output impedance $R_o = 8 \times 0.75 = 6\,\Omega$

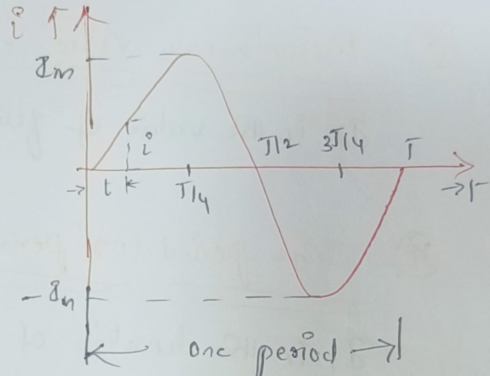
$$P_{max} = \frac{V_{oc}^2}{4R_o} = \frac{(12)^2}{4 \times 6} = \underline{\underline{6\,W}}$$

Alternating voltage and currents and their Mathematical Relation and graphical representation

→ In ac power like voltages and currents vary with time Sinusoidally. Such a variation is shown graphically in fig-



Current i (vs) angle ωt

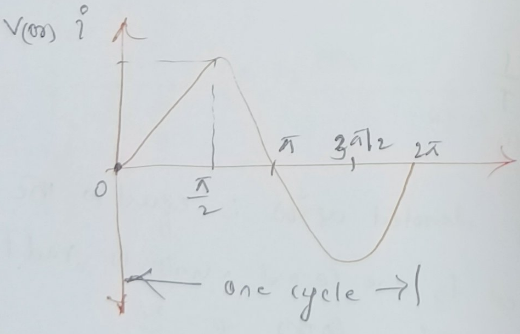


current i (vs) t

Mathematically $i = I_m \sin \omega t$

Terminology

Cycle : One complete set of positive and negative Values of the function is Called a cycle.



Maximum value (or) peak value

It is the maximum value (I_m) (or) positive (or) negative of the quantity. It is also sometimes called amplitude of the sinusoid.

Instantaneous value :

It is the value of quantity at any instant.

Time period (or) periodic Time

It is the duration of the time required for the quantity to complete one cycle. It is denoted as T.

Frequency

It is the no.of cycles that occur in one second. It is denoted as f. The unit of frequency is hertz (Hz) which is same as cycles/second. Obviously frequency is reciprocal of time period.

$$f = \frac{1}{T}$$

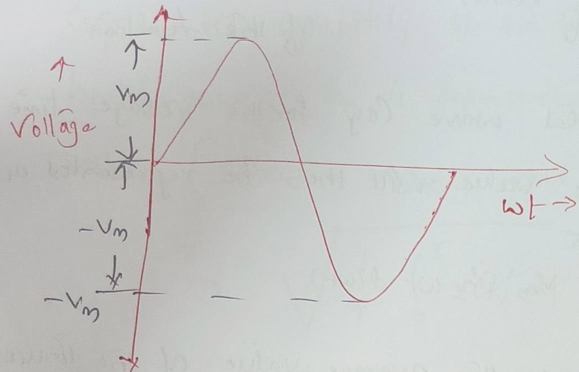
Angular frequency

Angular frequency denoted as 'ω' is equal to the number of radians covered in one second. unit is rad/sec.

$$\omega = 2\pi f \quad (\text{or}) \quad \omega = \frac{2\pi}{T}$$

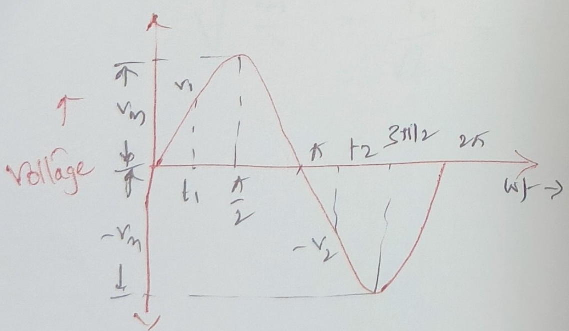
Peak value (Amplitude)

It is the Maximum value of the wave during either positive (or) negative half cycle. V_m is the peak value of sinusoidal voltage wave.



Instantaneous value :

It is the value of sinewave at any instant of the cycle. this value is different points along the wave form. In fig. V_1 is the instantaneous value of time t_1 and $-V_2$ is the instantaneous value of time t_2



Rms value (Root Mean square value)

As the effective value of a sine wave is zero for the entire cycle hence inorder to get the effective value, we compute the capability of the sine wave interms of its heating power. this is represented by the root mean square value [effective value] and canbe represented by

$$x_{rms} = \sqrt{\frac{1}{T}\int_0^T x(t)^2\,dt}$$

for a periodic function, T being the time period.

Hence for a sinusoidal voltage waves.

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} (V_m \sin\omega t)^2\, d(\omega t)}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V_m^2 \left[\frac{1-\cos 2\omega t}{2}\right] d(\omega t)}$$

$$= \sqrt{\frac{V_m^2}{2}} = \frac{V_m}{\sqrt{2}} = 0.707\, V_m$$

(*) A voltage wave is represented by $V = 200 \sin 314t$

Find (a) Maximum value (b) Rms value (c) Average value (d) Frequency (e) Time period (f) Instantaneous value after 0.05 sec.

Sol:- Given $V = 200 \sin 314t$

(a) from above peak value (or) maximum value is 200V (V_m)

(b) Rms value is

$$V = \frac{V_m}{\sqrt{2}} = 141.4\,V$$

(c) average value is

$$V_{avg} = \frac{2V_m}{\pi} = 0.637\,V_m = 127.4\,V$$

(d) Frequency $(f) = \frac{\omega}{2\pi} = \frac{314}{2\pi} = 50\,Hz$

(e) Time period $T = 1/f = 0.02\,sec.$

(f) Instantaneous value of the given wave after 0.05 sec

$V = 200 \sin(314 \times 0.05) = \cancel{0.41V} = \underline{54.12\,V}$

* Form factor

The ratio of effective value to the average value is known as the form factor of waveform.

$$\text{Form factor} = \frac{\text{Rms value}}{\text{Average value}}$$

* peak factor (or) Crest factor (or) amplitude factor

The ratio of peak value to rms value is known as peak factor.

$$\text{peak factor} = \frac{\text{peak (or) maximum value}}{\text{Rms value.}}$$

Average Value

In general the average value of a periodic function $x(t)$ can be represented as

$$x_{avg} = \frac{1}{T}\int_0^T x(t)\,dt$$

where T is the time period of the function.

In case of a sinusoidal wave say for the voltage time wave, the average value will then be represented as

$$V_{avg} = \frac{1}{\pi}\int_0^{\pi} V_m \sin \omega t \; d(\omega t)$$

It may be noted that the average value of the sinusoidal voltage wave is computed for $\frac{1}{2}$ cycle as the average value of the sine wave would be zero for a complete cycle.

$$V_{avg} = \frac{1}{\pi}\left[-V_m \cos \omega t\right]_0^{\pi}$$

$$= \frac{2V_m}{\pi} = 0.637\, V_m$$