06.02.24

15 вар

1.15. $\lim a_n = a$ (указать $N(\varepsilon)$)

$$a_n = \frac{n}{3n-1}; \quad a = \frac{1}{3}$$

По опред последов. $\forall \varepsilon > 0 \ \exists N(\varepsilon) \in \mathbb{N} \mid \forall n \geq N(\varepsilon): |a_n - a| < \varepsilon$

$$\left| \frac{n}{3n-1} - \frac{1}{3} \right| < \varepsilon$$

$$\left| \frac{3n-3n+1}{3(3n-1)} \right| < \varepsilon$$

$$\left(\frac{1}{3(3n-1)} \right) - \frac{1}{3} \left| \left(\frac{1}{3n-1} \right) \right| < \varepsilon$$

$$\left| \frac{1}{3n-1} \right| < \varepsilon$$

$$3n-1 > \frac{1}{\varepsilon}$$

$$3n > \frac{1}{\varepsilon} + 1$$

$$N(\varepsilon) = \left[\frac{1}{3} \left(\frac{1}{\varepsilon} + 1 \right) \right] + 1$$

2.15. $$\lim_{n \to \infty} \frac{8n^3 - 2n}{(n+1)^4 - (n-1)^4} = \left[\frac{\infty}{\infty}\right] = \frac{(8n^3-2n)\cancel{((n+1)^4+(n-1)^4)}}{((n+1)^2-(n-1)^2)((n+1)^2+(n-1)^2)} =$$

$$= \frac{8n^3 - 2n}{(n^2+2n+1-n^2+2n-1)(n^2+2n+1+n^2-2n+1)} = \frac{8n^3 - 2n}{4n(2n^2+2)} =$$

$$= \frac{8n^3 - 2n}{8n^3 + 8n} = \frac{8}{8} = 1$$

3.15 $\lim\limits_{n\to\infty} \frac{\sqrt{4n+1} - \sqrt[3]{27n^3+4}}{\sqrt[4]{n} - \sqrt[3]{n^5+n}} = \left\{\frac{\infty}{\infty}\right\} \Big| : \sqrt[3]{n^5} =$

$$= \frac{\sqrt{\frac{4}{\sqrt[3]{n^7}} + \frac{1}{\sqrt[3]{n^{10}}}} - \sqrt[3]{\frac{27}{n^2} + \frac{6}{n^5}}}{\frac{1}{\sqrt[20]{n^7}} - \sqrt{1 + \frac{1}{n^4}}} = \frac{0-0}{0-1} = 0$$

5.15. $\lim\limits_{n\to\infty} \frac{\sqrt[3]{n^3+5} - \sqrt{3n^4+2}}{1+3+5+\ldots+(2n-1)} = \left\{\frac{\infty}{\infty}\right\} \cdot$

$\cancel{S=an}$ $S_n = \left(\frac{a_1+a_n}{2}\right) n$

$$= \frac{\sqrt[3]{n^3+5} - \sqrt{3n^4+2}}{\frac{(1+2n-1)}{2}\cdot(2n-1)} = \frac{\sqrt[3]{n^3+5} - \sqrt{3n^4+2}}{2n^2-n} =$$

$$= \frac{4n^2-2n}{2} = 2n^2 - n$$

$$= \frac{\frac{\sqrt[3]{n^3+5}}{n^2} - \frac{\sqrt{3n^4+2}}{n^2}}{2 - \frac{1}{n}} = \frac{\sqrt{\frac{1}{n^3} + \frac{5}{n^6}} - \sqrt{3 + \frac{2}{n^4}}}{2 - \frac{1}{n}} = -\frac{\sqrt{3}}{2}$$

6.15 $\lim \left(\frac{3n+1}{3n-1}\right)^{2n+3} = \left(\frac{3n-1+2}{3n-1}\right)^{2n+3} = \left(1 + \frac{2}{3n-1}\right)^{2n+3} =$

$$= \left(\left(1 + \frac{2}{3n-1}\right)^{\frac{3n-1}{2}}\right)^{\left(2n+3\cdot\frac{2}{3n-1}\right)} = e^{\lim\limits_{n\to\infty} \frac{2(2n+3)}{3n-1}} =$$

$$= e^{\frac{4n+6}{3n-1}} = e^{\frac{4}{3}}$$

9.15. $\lim\limits_{x\to-2} \dfrac{x^3+5x^2+8x+4}{x^3+3x^2-4} = \left\{\dfrac{0}{0}\right\} = \Big|\ (x+2)$

$= \lim\limits_{x\to-2} \dfrac{\cancel{(x+2)}(x+1)\cancel{(x+2)}}{\cancel{(x+2)}(x-1)\cancel{(x+2)}} = \dfrac{-2+1}{-2-1} = \dfrac{1}{3}$

10.15. $\lim\limits_{x\to3} \dfrac{\sqrt[3]{9x}-3}{\sqrt{3+x}-\sqrt{2x}} = \left\{\dfrac{0}{0}\right\} = \dfrac{(\sqrt[3]{9x}-3)(\sqrt[3]{9x}^2+3\sqrt[3]{9x}+9)(\sqrt{3+x}+\sqrt{2x})}{(3+x-2x)\cancel{(\sqrt[3]{9x}+3)}(\sqrt[3]{9x}^2+3\sqrt[3]{9x}+9)}$

$= \dfrac{(9x-27)(\overbrace{\sqrt{3+x}}^{\sqrt{6}}+\overbrace{\sqrt{2x}}^{\sqrt{6}})}{(3-2x+x)(\underbrace{\sqrt[3]{(9x)^2}}_{9}+\underbrace{3\sqrt[3]{9x}}_{9}+\underbrace{9}_{9})} = \dfrac{2\sqrt{6}}{27} \lim\limits_{x\to3} \dfrac{9x-27}{3-2x+x} =$

$= \dfrac{9\cancel{(x-3)}}{-\cancel{(x-3)}} = -\dfrac{9\cdot2\sqrt{6}}{27} = -\dfrac{2\sqrt{6}}{3}$

11.15. $\lim\limits_{\alpha\to0} \dfrac{\sin 7x}{x^2+\pi x} = \left[\dfrac{0}{0}\right]$

Замен $\sin\alpha \sim \alpha \quad \alpha\to0$

$\lim\limits_{\alpha\to0} \dfrac{7\cancel{x}}{\cancel{x}(x+\pi)} = \dfrac{7}{x+\pi} = \dfrac{7}{\pi}$

или $\dfrac{7x}{7x}\cdot\overbrace{\dfrac{\sin 7x}{(x+\pi)}}^{=1} = \dfrac{7x}{x+\pi} = \dfrac{7}{\pi}$

12.15 $\lim\limits_{x\to 1} \frac{3^{5x-3}-3^{2x^2}}{tg\,\Pi x} = \left\{\frac{0}{0}\right\} =$

$t = x-1 \quad ; \quad x = t+1$

$x\to 1 \quad t\to 0$

$\lim\limits_{t\to 0} \frac{3^{5(t+1)-3}-3^{2(t+1)^2}}{tg\,\Pi x} =$

$tg\,\alpha \sim \alpha \quad \alpha\to 0$

$= \lim\limits_{t\to 0} \frac{3^{5t+2}-3^{2t^2+4t+2}}{\Pi t} = \frac{9}{\Pi}\lim\limits_{t\to 0} \frac{3^{5t}-3^{2t^2+4t}}{t} =$

$= \frac{9}{\Pi}\lim\limits_{t\to 0}\left(\frac{5(3^{5t}-1)}{5t} - \frac{(2t^2+4t)(3^{2t^2+4t}-1)}{t(2t^2+4t)}\right) =$

$= \frac{9}{\Pi}\lim(5\ln 3 - \ln 3(2t+4)) = \frac{9\ln 3}{\Pi}$

17.15 $\lim\limits_{x\to 0}\left(\frac{x^3+8}{3x^2+10}\right)^{x+2} = \left(\frac{8}{10}\right)^2 = \left(\frac{4}{5}\right)^2 = \frac{16}{25}$

18.15 $\lim\limits_{x\to 3}\left(\frac{3\cdot 9-2x}{3\cdot 3}\right)^{tg\frac{\Pi x}{6}} = \cancel{2x-6x}$

$= 3\cdot\left(1+\frac{2x}{9}\right)^{tg\frac{\Pi x}{6}} = 3\lim\left(\left(1+\frac{2x}{9}\right)^{\frac{9}{2x}}\right)^{\frac{2x}{9}\,tg\frac{\Pi x}{6}} = e$

$= 3\cdot e^{\lim\limits_{x\to 3}\frac{2x}{9}\cdot tg\frac{\Pi x}{6}} = \infty$

$\frac{2}{3}$

$t = x-3$, $x = t+3$, $t \to 0$

$$\lim_{t\to 0}\left(\frac{9-2(t+3)}{3}\right)^{tg\frac{\pi}{6}(t+3)} = \left(\frac{9-2t+6}{3}\right)^{tg\left(\frac{\pi t}{6}+\frac{\pi}{2}\right)} =$$

$$= \left(1-\frac{2t}{3}\right)^{-ctg\frac{\pi}{6}t} = \lim\left(\left(1-\frac{2t}{3}\right)^{-\frac{3}{2t}}\right)^{=e} {}^{-\frac{2t}{3}\left(-\frac{\cos\frac{\pi t}{6}}{\sin\frac{\pi t}{6}}\right)^{=1}} =$$

$$= e^{\frac{2}{3}\lim\limits_{t\to 0}\frac{t}{\sin\frac{\pi t}{6}}} = e^{\frac{0}{0}}$$

$\sin\alpha \sim \alpha \quad \alpha \to 0$

$$= e^{\frac{2}{3}\lim\frac{t}{\frac{\pi t}{6}}} = e^{\frac{4}{\pi}}$$