Dependence modelling using copulas

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Subject of analysis

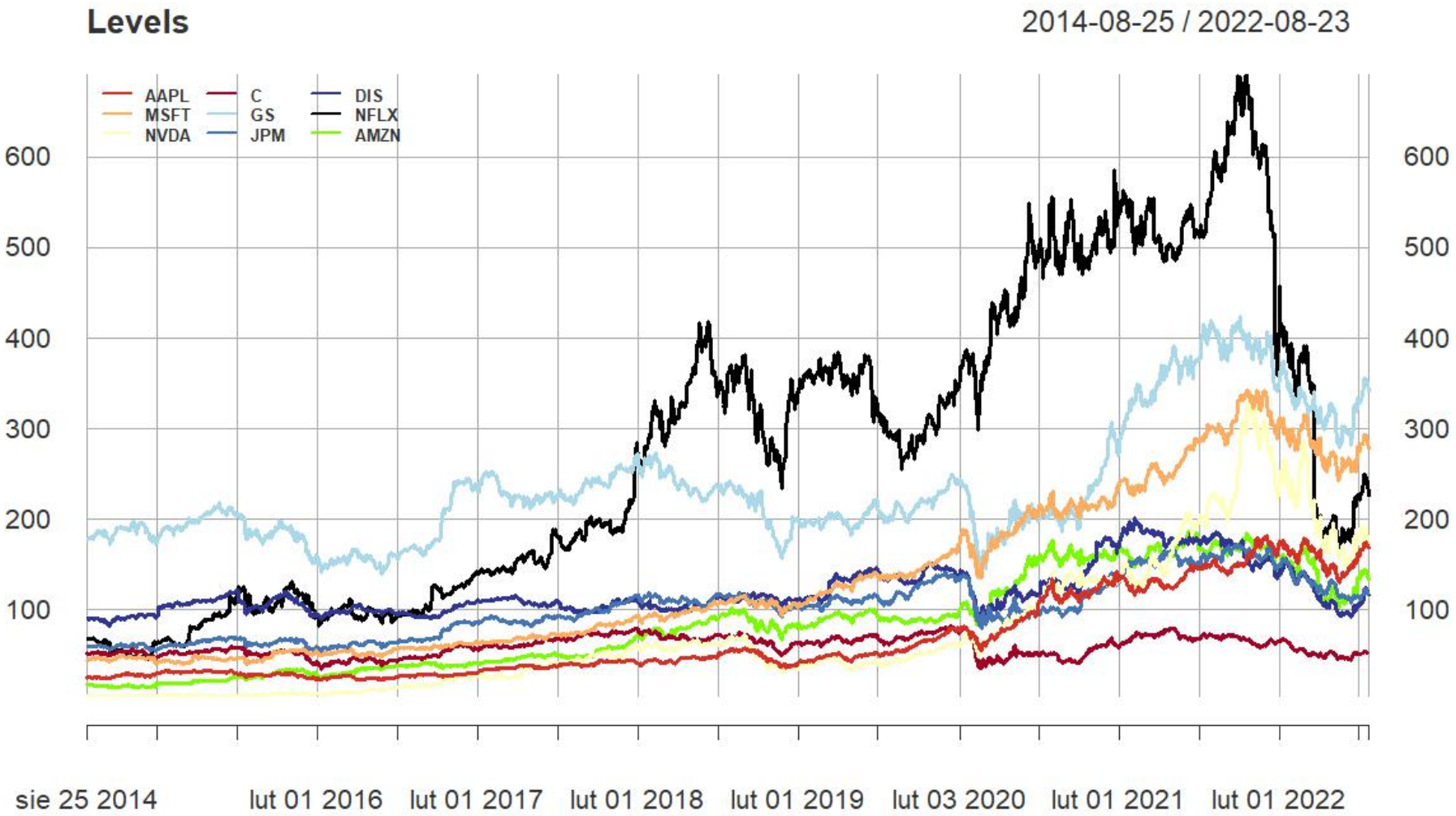
Objective: Modelling dependency structure between stock indexes from three sectors:

- Information technology: Apple (AAPL), Microsoft (MSFT), Nvidia (NVDA),
- Finance: Citigroup Inc (C), Goldman Sachs Group Inc (GS), JPMorgan Chase & Co (JPM),
- Services: The Walt Disney Company (DIS), Netflix (NFLX), Amazon.com (AMZN).

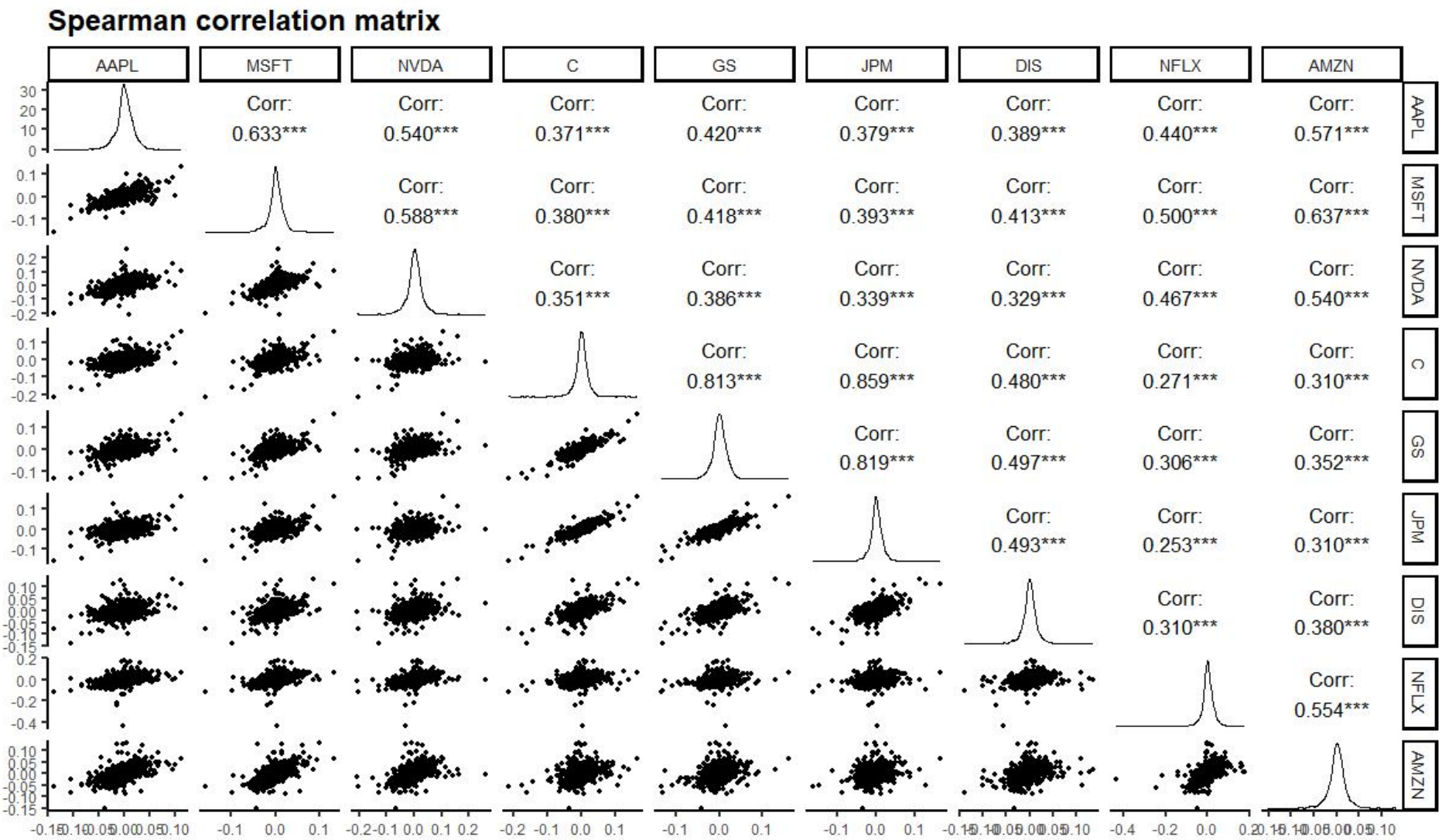
Data: Daily values on close from 2014/08/25 to 2022/08/23.
Metrics: AIC, BIC, Log-likelihood.
Conclusion:

- Marginals – best metrics scores achived for t-Student distribution.
- Joint distribution – best metrics scores achived for D-vine copula.

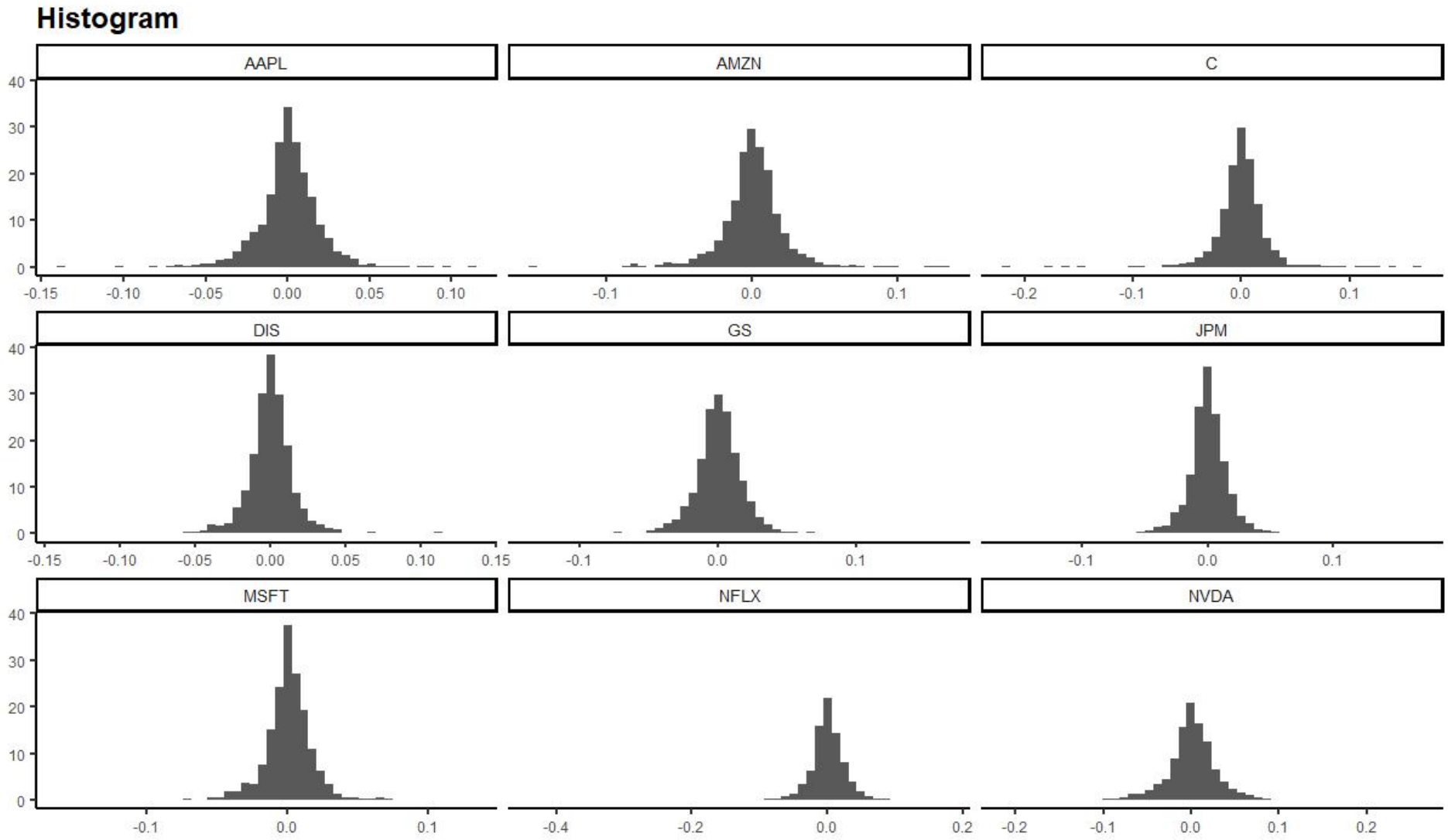
Indexes


Levels
2014-08-25 / 2022-08-23
AAPL
MSFT
NVDA
C
GS
JPM
DIS
NFLX
AMZN
600
500
400
300
200
100
sie 25 2014
lut 01 2016
lut 01 2017
lut 01 2018
lut 01 2019
lut 03 2020
lut 01 2021
lut 01 2022

Spearman correlation matrix



Histogram of logarithmic increments



AIC for fitted marginals

Table: AIC scores for fitted marginals.

Ticker	Gaussian	t-Student	Cauchy	Logistic
AAPL	-10371.47	-10734.44	-10412.40	-10658.04
MSFT	-10633.29	-11042.11	-10809.60	-11003.79
NVDA	-8505.15	-8908.12	-8594.30	-8821.99
C	-9744.16	-10462.98	-10177.07	-10277.89
GS	-10338.31	-10785.15	-10406.27	-10701.03
JPM	-10515.94	-11123.16	-10822.05	-10981.67
DIS	-10691.22	-11326.85	-11118.12	-11198.80
NFLX	-8554.29	-9263.09	-8945.58	-9115.93
AMZN	-9977.81	-10450.31	-10149.33	-10331.41

BIC for fitted marginals

Table: BIC scores for fitted marginals.

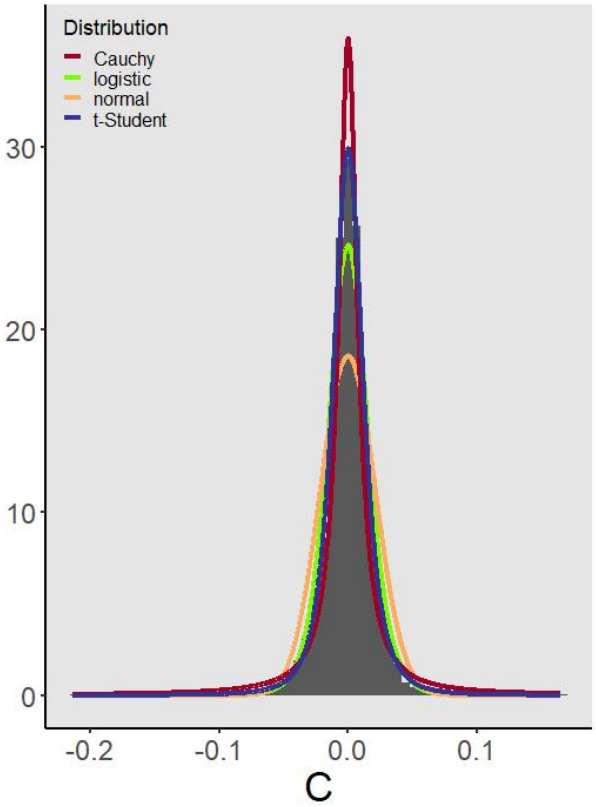
Ticker	Gaussian	t-Student	Cauchy	Logistic
AAPL	-10360.26	-10717.62	-10401.19	-10646.83
MSFT	-10622.08	-11025.29	-10798.39	-10992.57
NVDA	-8493.93	-8891.30	-8583.09	-8810.78
C	-9732.95	-10446.16	-10165.85	-10266.67
GS	-10327.10	-10768.33	-10395.05	-10689.82
JPM	-10504.73	-11106.34	-10810.83	-10970.46
DIS	-10680.00	-11310.03	-11106.91	-11187.58
NFLX	-8543.08	-9246.26	-8934.36	-9104.71
AMZN	-9966.60	-10433.49	-10138.12	-10320.19

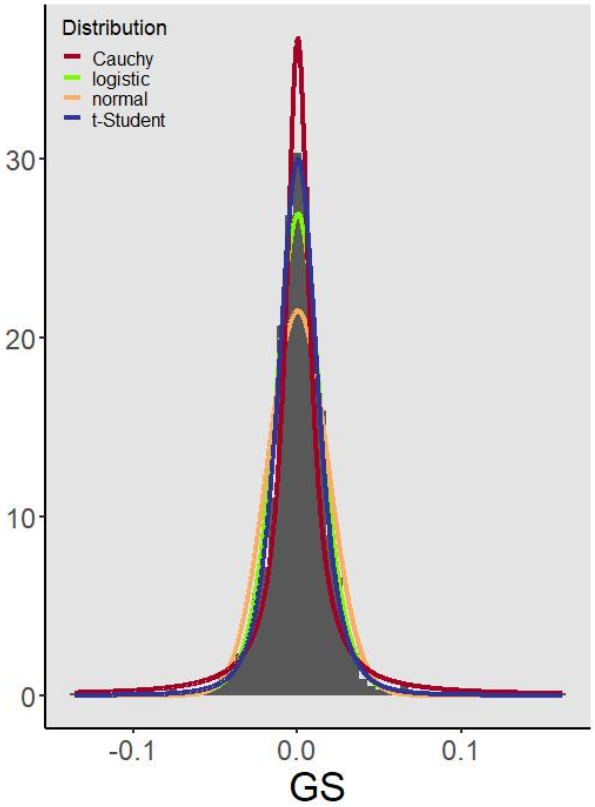
Log-likelihood for fitted marginals

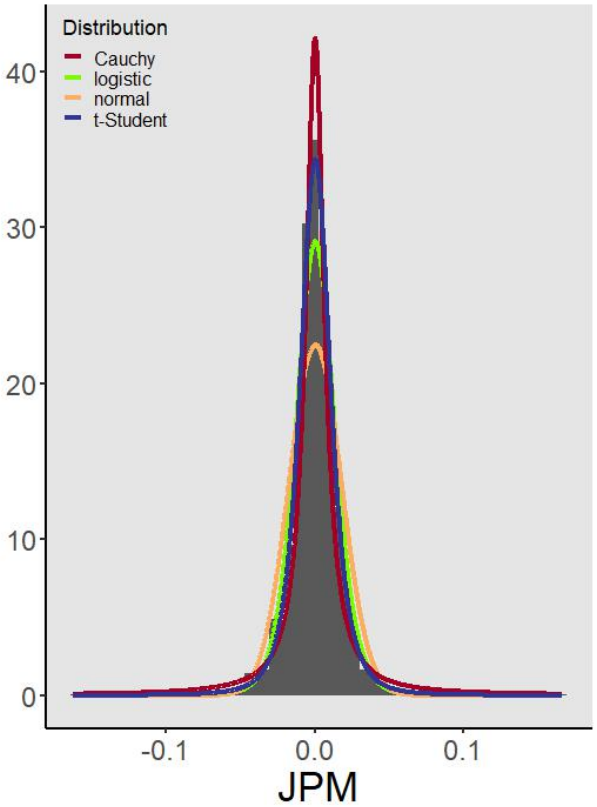
Table: Log-likelihood scores for fitted marginals.

Ticker	Gaussian	t-Student	Cauchy	Logistic
AAPL	5187.74	5370.22	5208.20	5331.02
MSFT	5318.65	5524.06	5406.80	5503.89
NVDA	4254.57	4457.06	4299.15	4413.00
C	4874.08	5234.49	5090.53	5140.94
GS	5171.16	5395.57	5205.13	5352.52
JPM	5259.97	5564.58	5413.02	5492.84
DIS	5347.61	5666.43	5561.06	5601.40
NFLX	4279.15	4634.54	4474.79	4559.96
AMZN	4990.91	5228.15	5076.67	5167.70

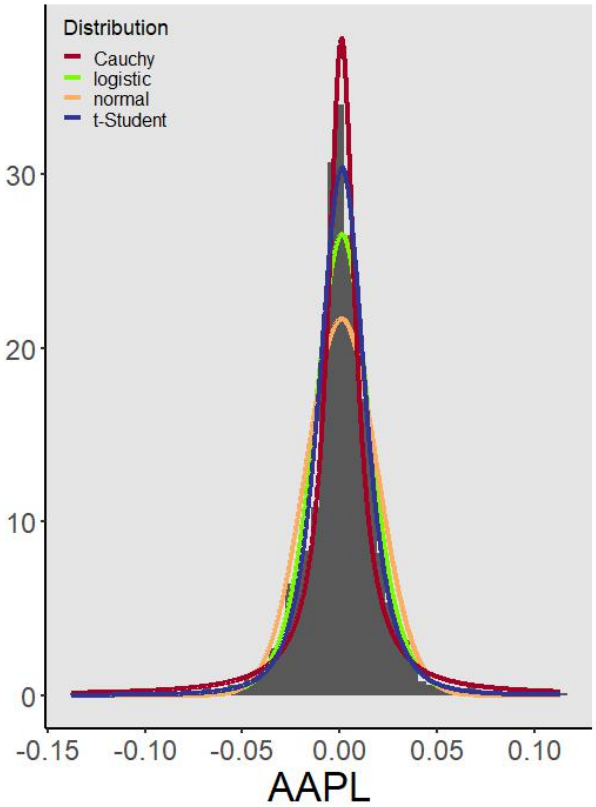
Fitted marginals (Finance)

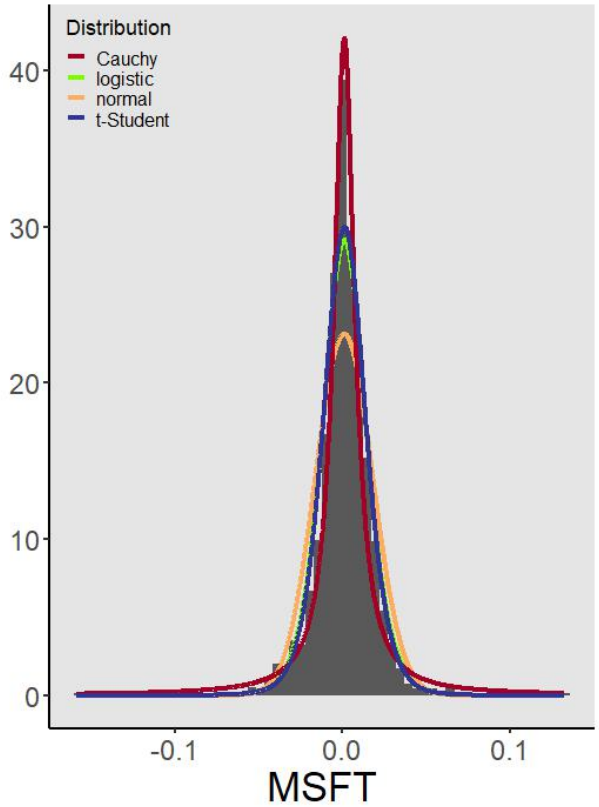


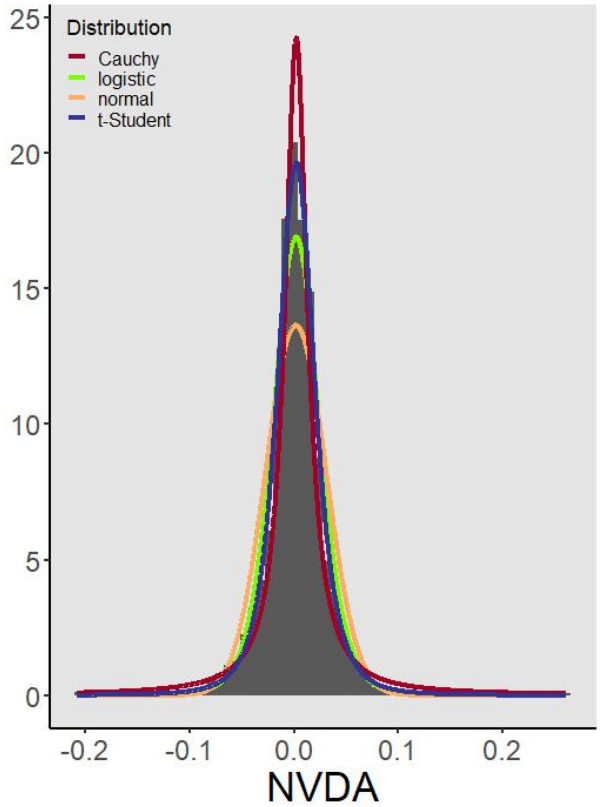




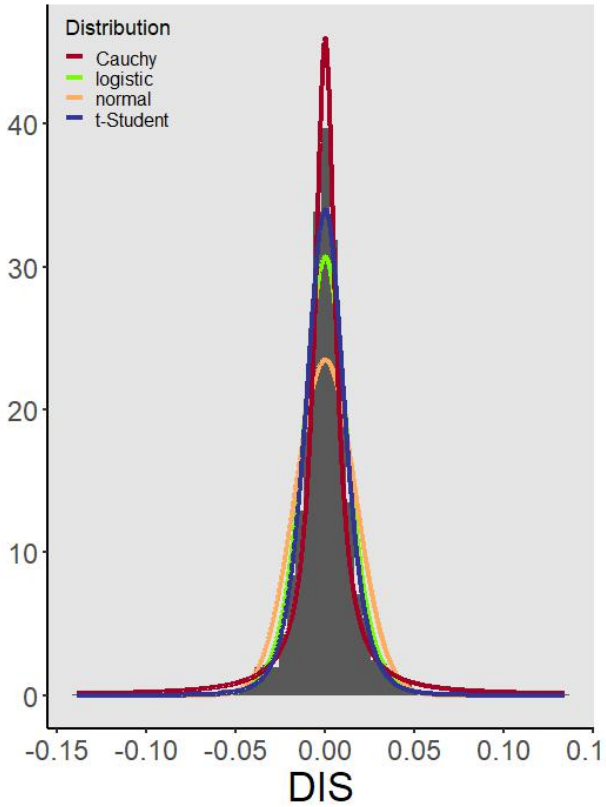
Fitted marginals (IT)

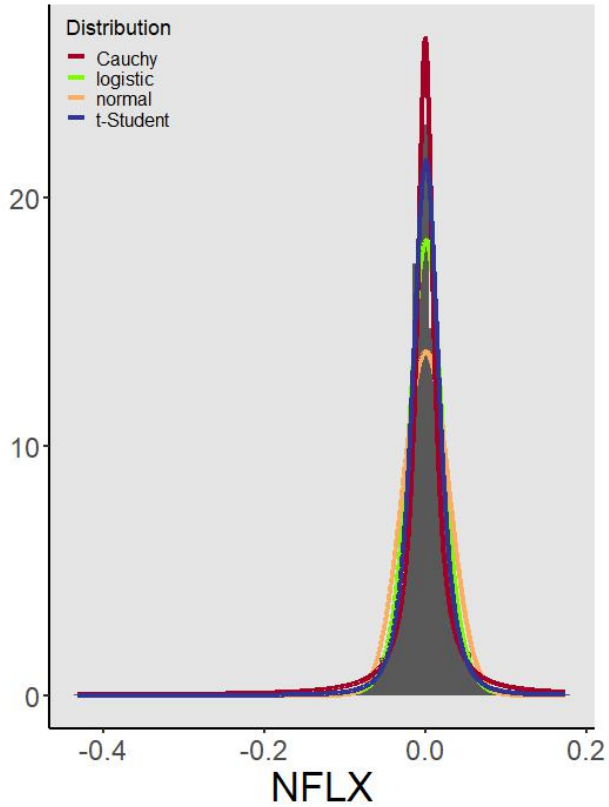


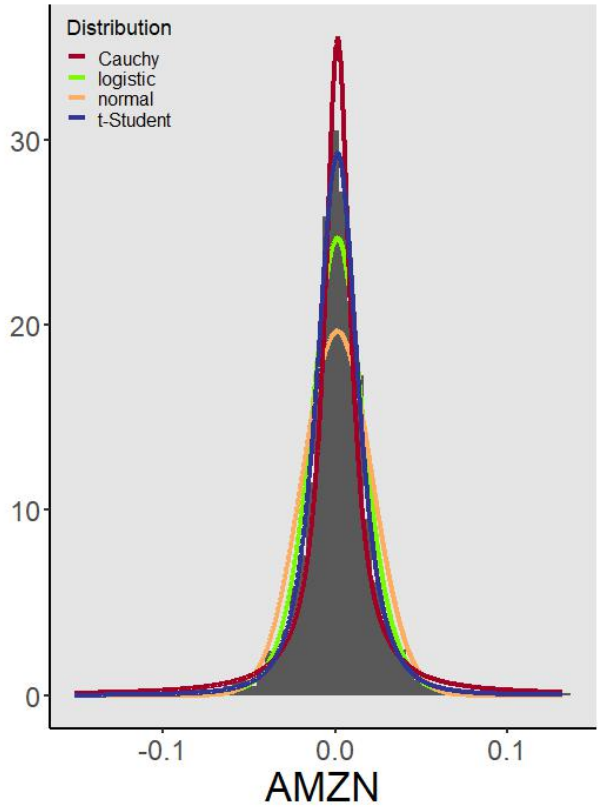




Fitted marginals (Services)





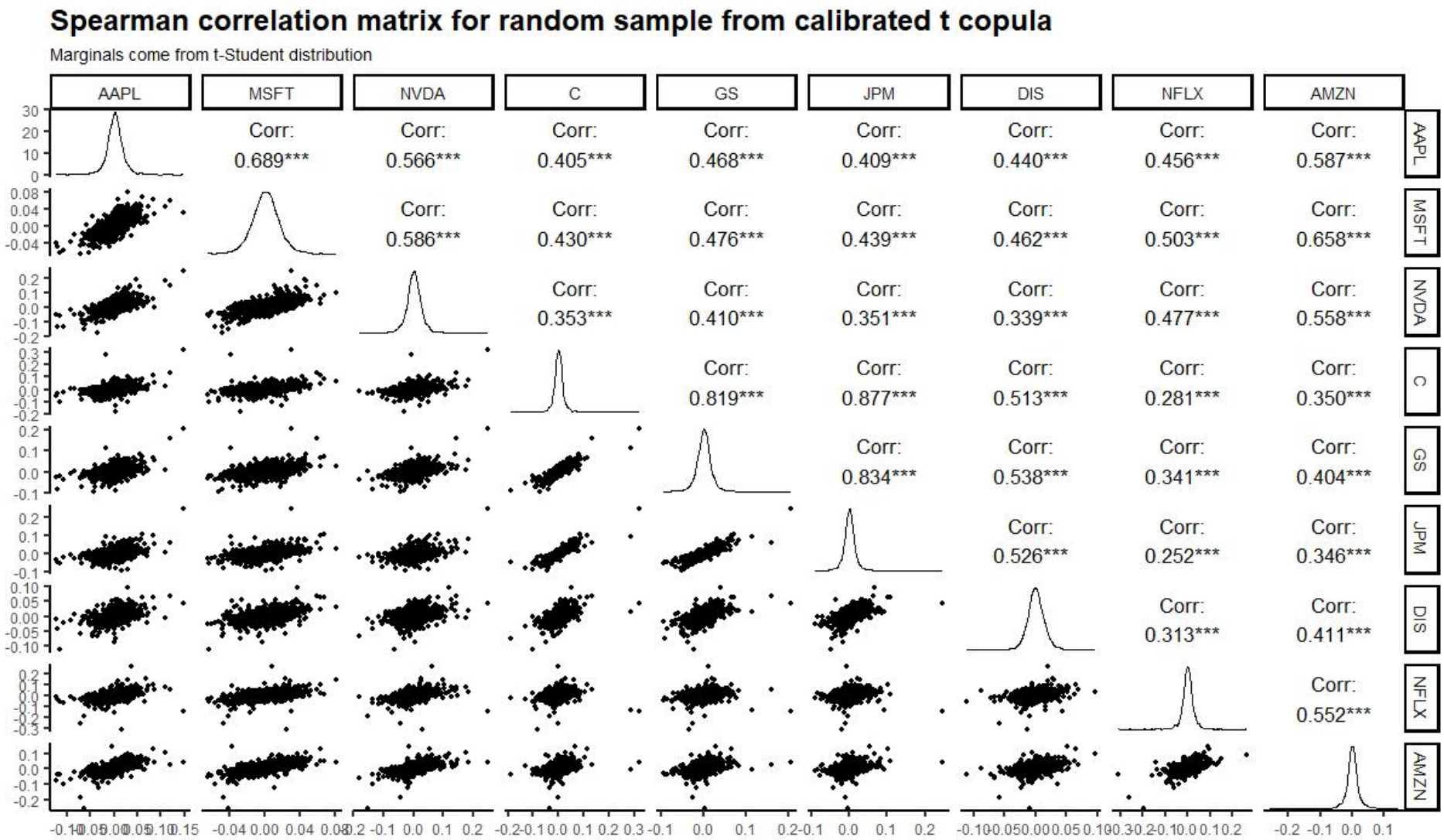


Elliptical & Archimedean copulas

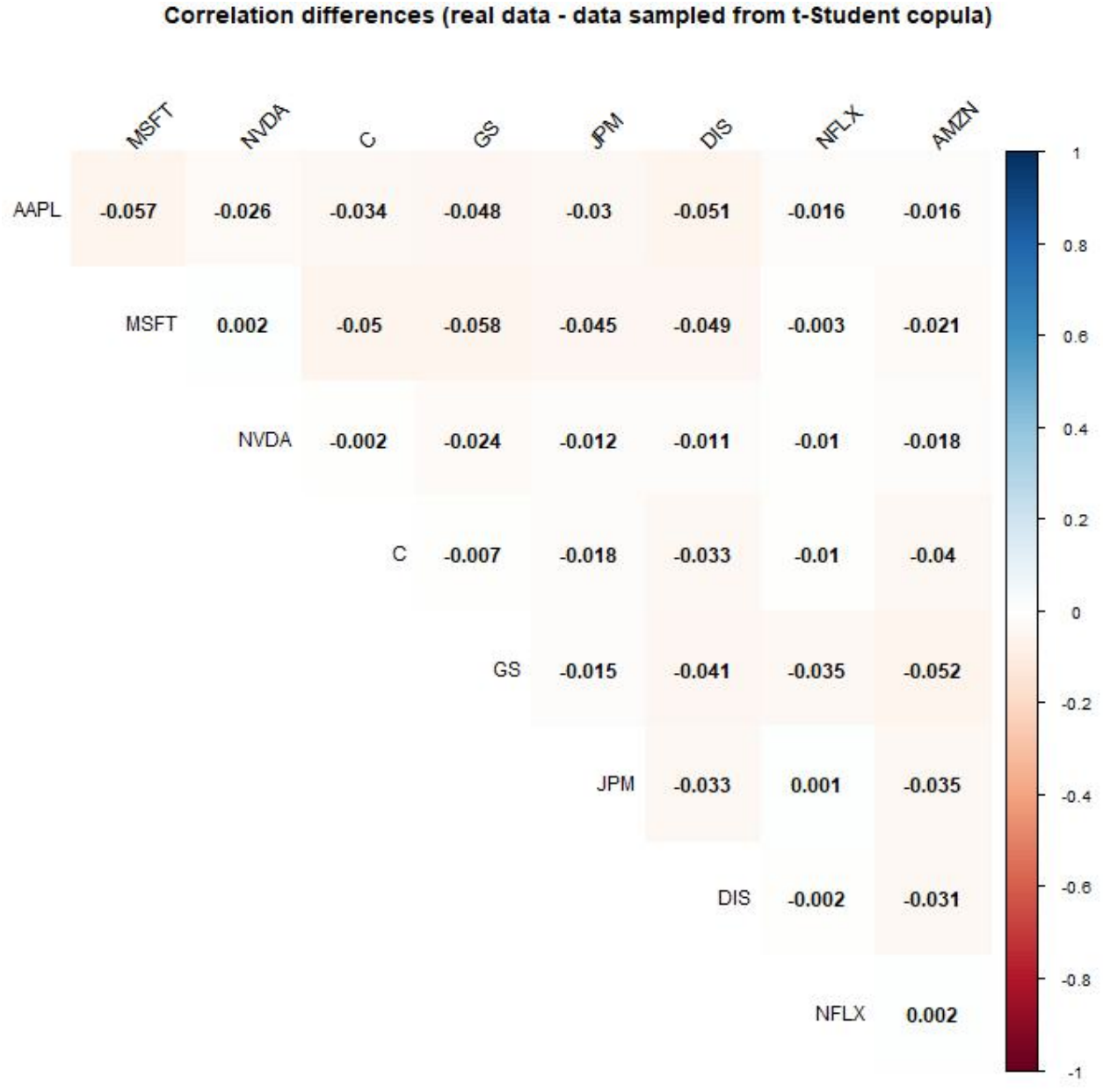
Table: Criterion scores for fitted elliptical & archimedean copulas.

	Gaussian	t-Student	Frank	Clayton	Gumbel
AIC	-11882.96	-12827.35	-6234.22	-6818.93	-6523.37
BIC	-11681.09	-12619.88	-6228.61	-6813.32	-6517.76
Log.likelihood	5977.48	6450.68	3118.11	3410.47	3262.68

Spearman matrix for fitted t-Student copula



Spearman matrix real data vs data sampled from t-Student



Vines copulas

Table: Criterion scores for fitted C-vine, D-vine & R-vine copulas.

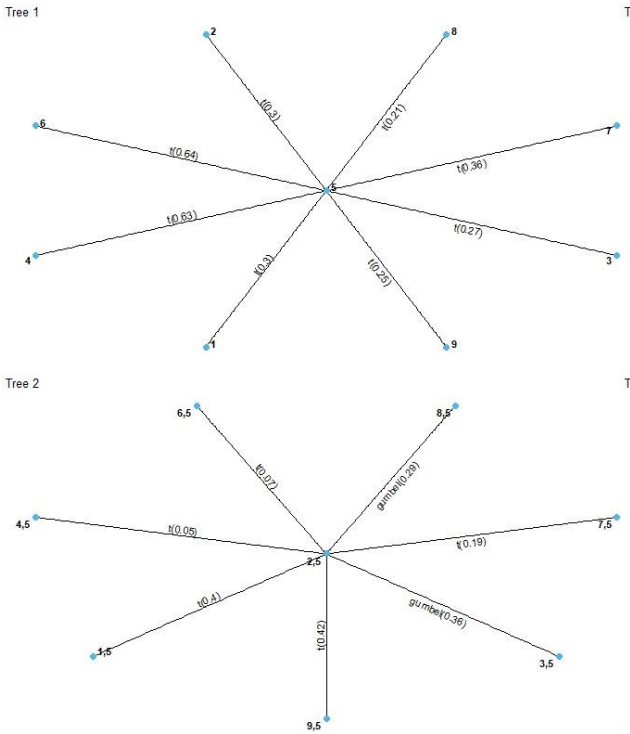
	D-vine	C-vine	R-vine
AIC	-13233.54	-13141.37	-13219.14
BIC	-12925.14	-12810.54	-12899.52
Log.likelihood	6671.77	6629.69	6666.57

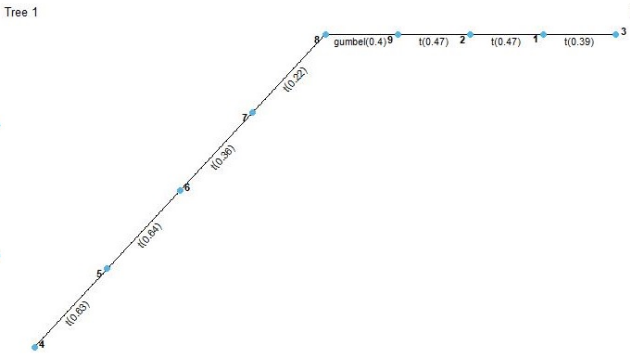
Results for different D-vine structures

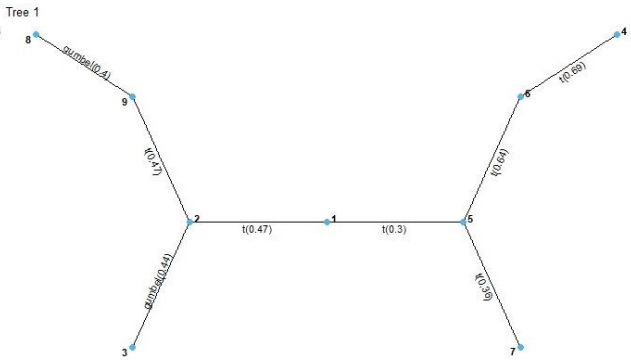
Table: Criterion scores for different D-vine structures.

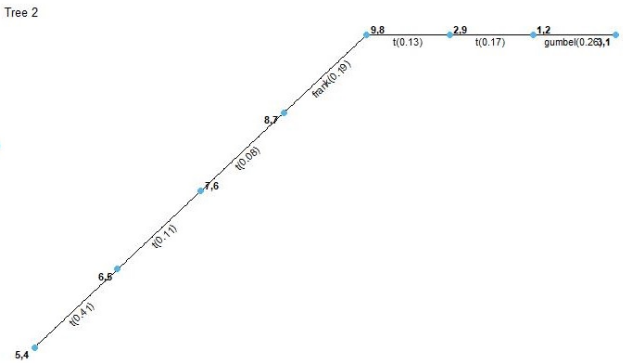
Structure	Log-likelihood
5,4,6,1,3,2,7,8,9	6627.32
3,2,1,4,5,6,9,7,8	6601.15
7,8,9,2,1,3,4,6,5	6662.43
5,6,4,7,8,9,3,1,2	6640.17
1,3,2,6,5,4,7,8,9	6632.08
5,6,4,3,1,2,7,9,8	6662.44
3,1,2,7,9,8,5,4,6	6633.64
8,9,7,2,3,1,5,6,4	6599.57
5,6,4,7,8,9,1,2,3	6653.96
3,1,2,9,8,7,6,5,4	6671.77

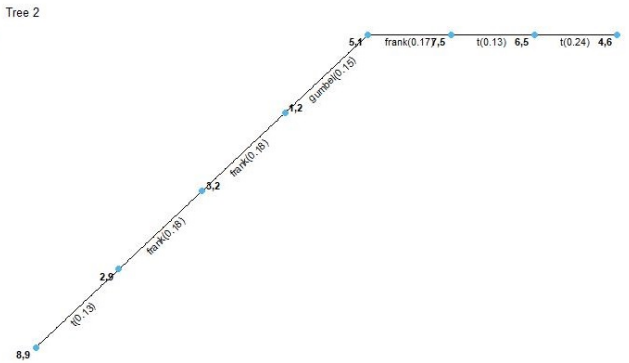
Vines trees (C-vine, D-vine, R-vine)



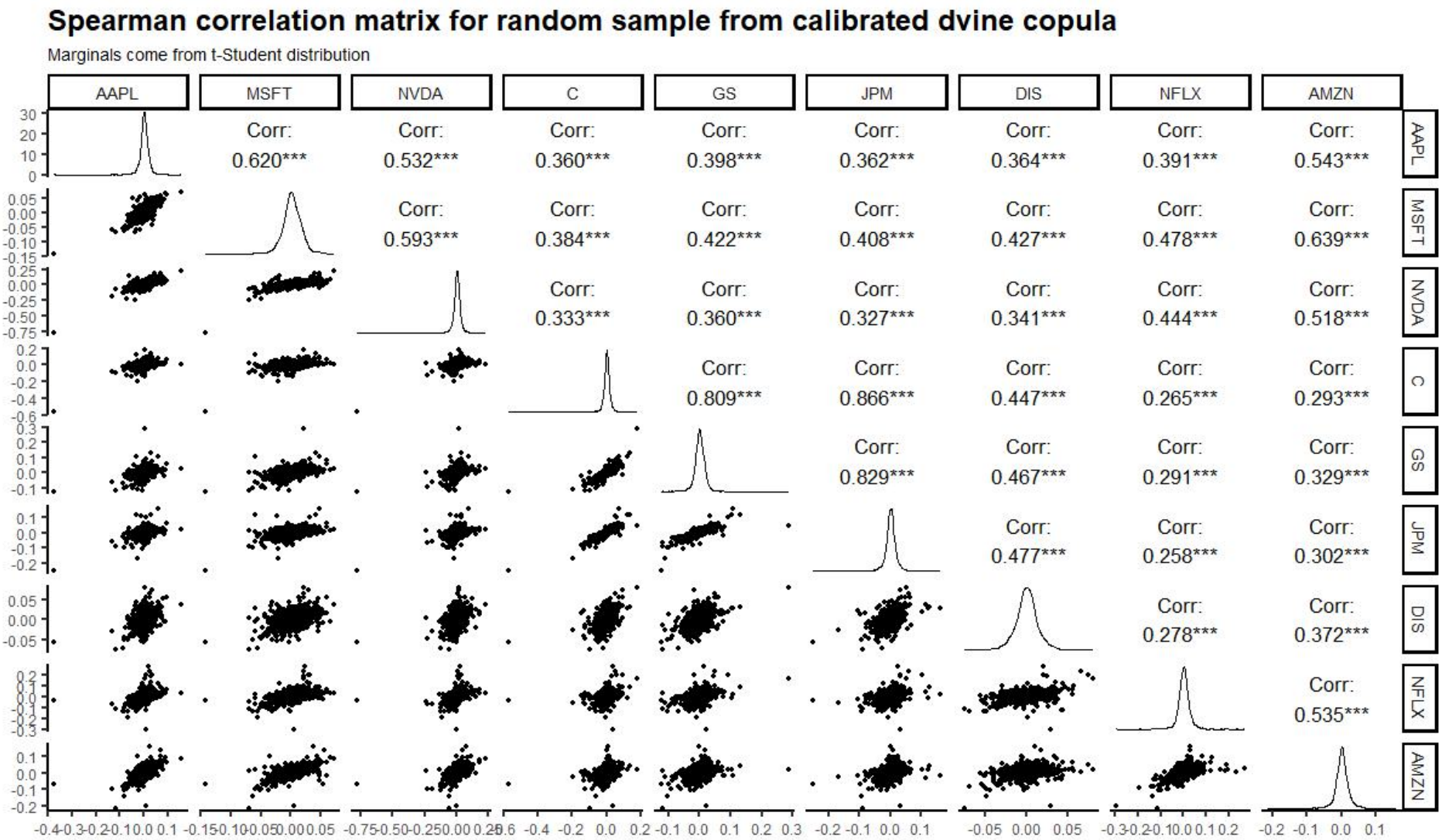




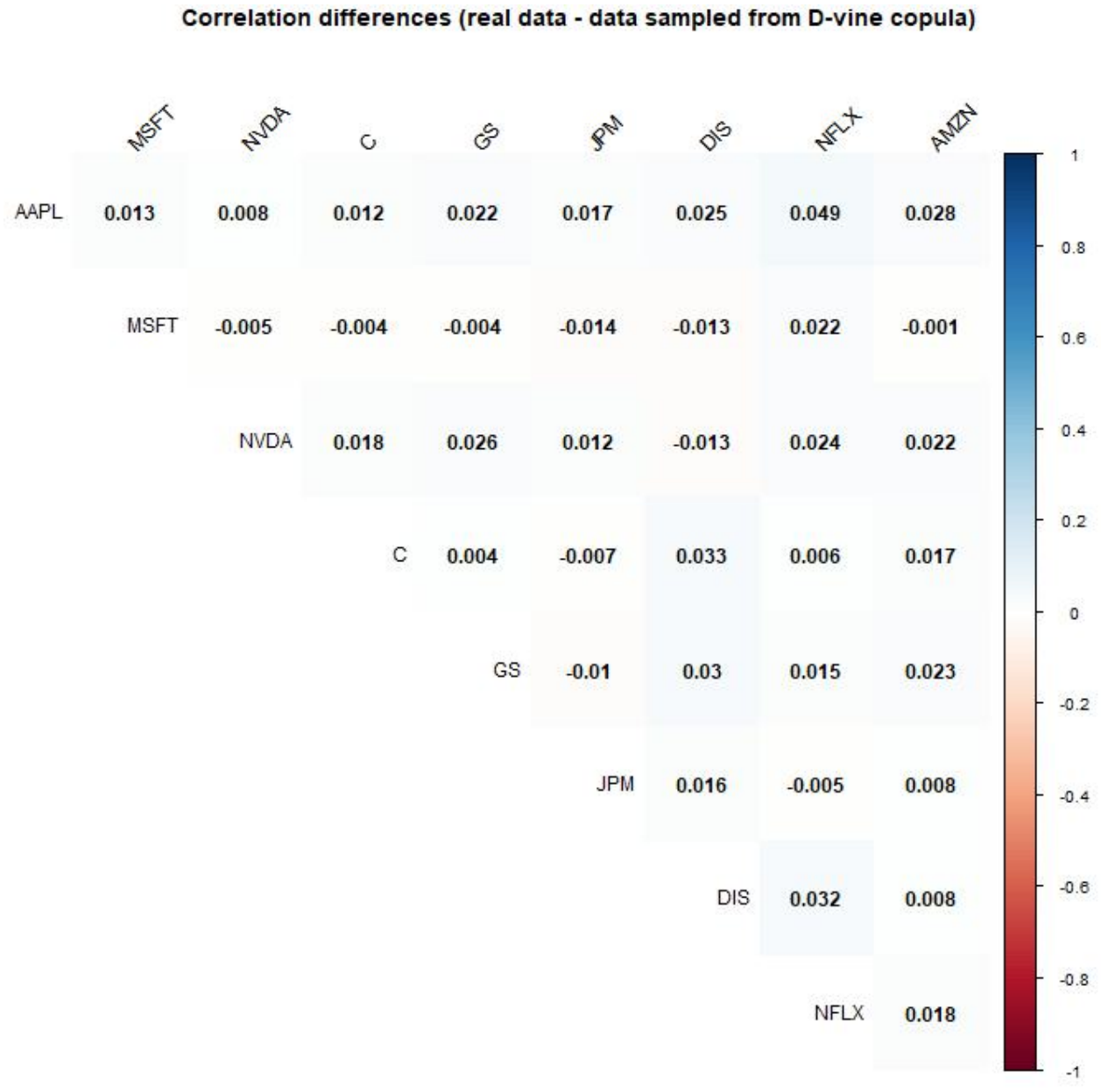




Spearman matrix for fitted D-vine



Spearman matrix real data vs data sampled from D-vine

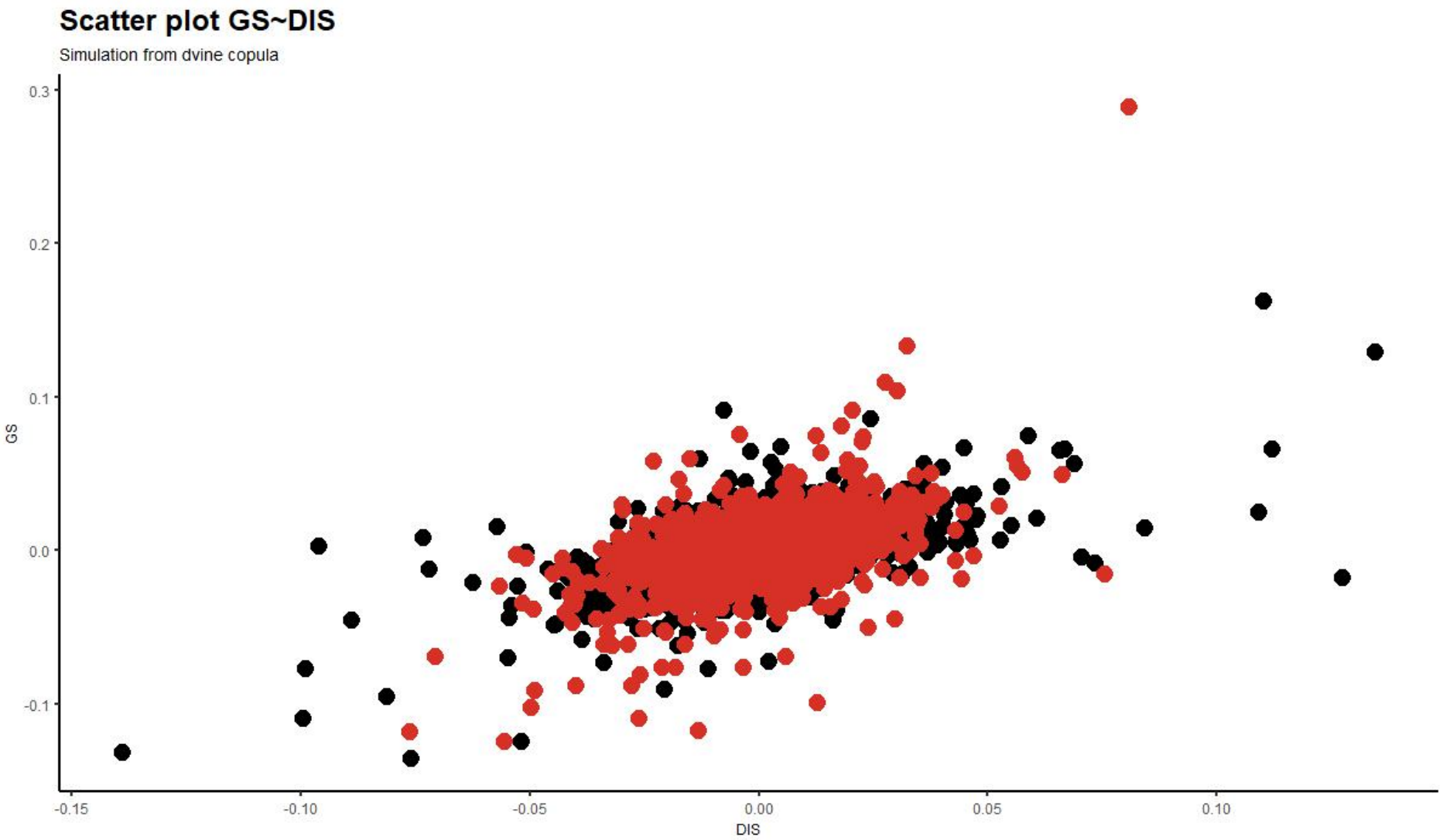


Log-likelihood for different bicopula families

Table: Log-likelihood scores for fitted vine copulas while variour bicopula families are in usage.

Bicopula family	C-vine	D-vine	R-vine
all	7845.96	7834.47	7848.46
parametrics_except_tll	6636.08	6674.77	6696.81
nonparametrics_indep_tll	7845.92	7834.45	7848.52
oneparam	6324.18	6356.26	6405.11
twoparam	6636.08	6674.77	6696.80
ellipticals	6602.22	6634.06	6635.83
arch	6215.65	6238.32	6351.13
kendalls_inversion	6629.69	6665.08	6666.57
kernel_transform	7846.26	7834.46	7848.11
epllip_archimed	6629.69	6665.08	6666.57

Scatter plot $GS \sim DIS$ (D-vine)



Scatter plots $GS \sim DIS$