

Dependence modelling using copulas

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Subject of analysis

Objective: Modelling dependency structure between stock indexes from three sectors:

- Information technology: Apple (AAPL), Microsoft (MSFT), Nvidia (NVDA),
- Finance: Citigroup Inc (C), Goldman Sachs Group Inc (GS), JPMorgan Chase & Co (JPM),
- Services: The Walt Disney Company (DIS), Netflix (NFLX), Amazon.com (AMZN).

Data: Daily values on close from 2014/08/25 to 2022/08/23.

Metrics: AIC, BIC, Log-likelihood.

Results:

- Marginals – best metrics scores achieved for t-Student distribution.
- Joint distribution – best metrics scores achieved for D-vine copula.

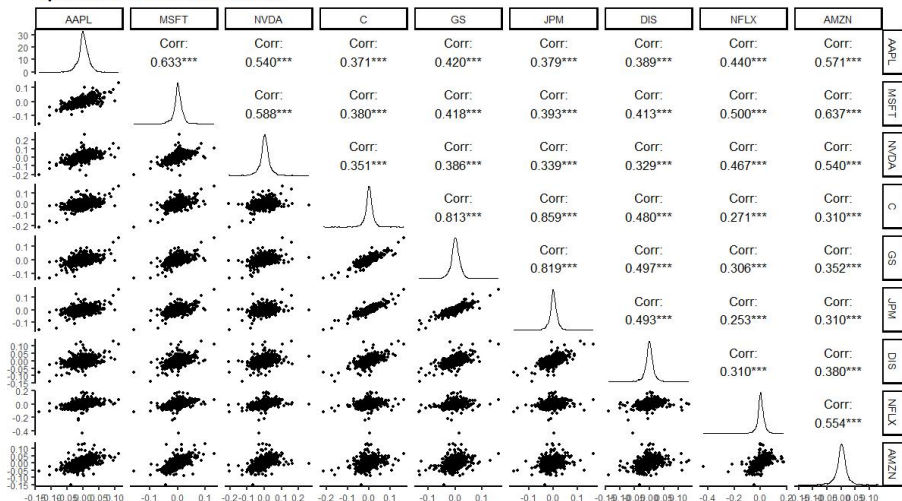
Levels

2014-08-25 / 2022-08-23



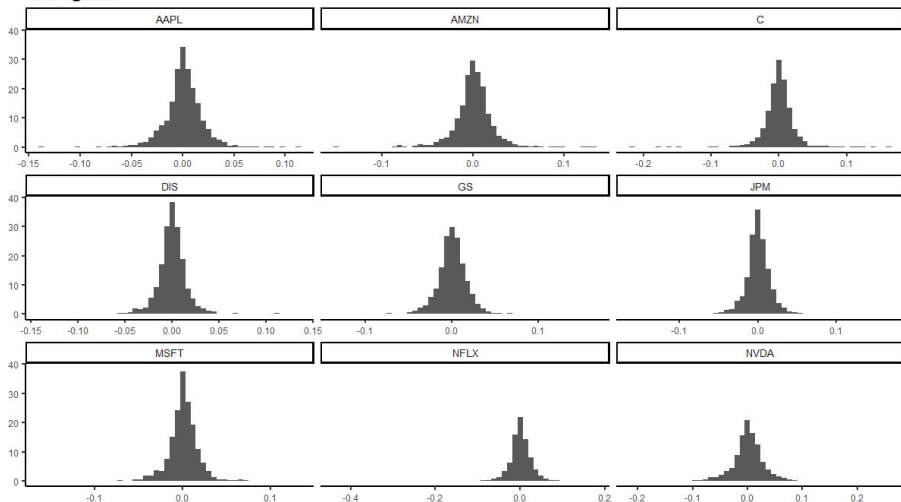
Spearman correlation matrix

Spearman correlation matrix



Histogram of logarithmic increments

Histogram



AIC for fitted marginals

Table: AIC scores for fitted marginals.

Ticker	Gaussian	t-Student	Cauchy	Logistic
AAPL	-10371.47	-10734.44	-10412.40	-10658.04
MSFT	-10633.29	-11042.11	-10809.60	-11003.79
NVDA	-8505.15	-8908.12	-8594.30	-8821.99
C	-9744.16	-10462.98	-10177.07	-10277.89
GS	-10338.31	-10785.15	-10406.27	-10701.03
JPM	-10515.94	-11123.16	-10822.05	-10981.67
DIS	-10691.22	-11326.85	-11118.12	-11198.80
NFLX	-8554.29	-9263.09	-8945.58	-9115.93
AMZN	-9977.81	-10450.31	-10149.33	-10331.41

BIC for fitted marginals

Table: BIC scores for fitted marginals.

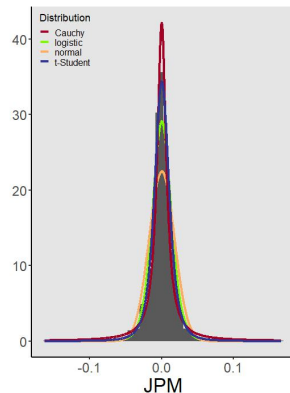
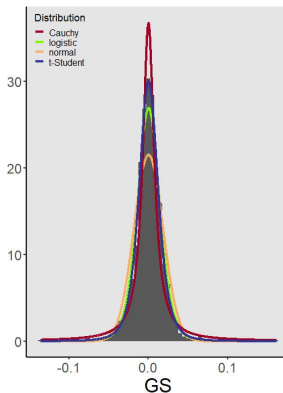
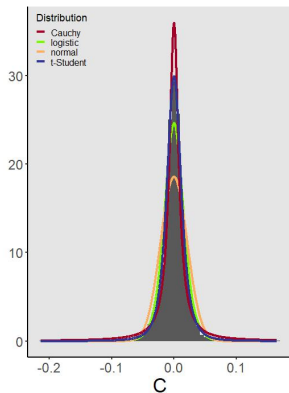
Ticker	Gaussian	t-Student	Cauchy	Logistic
AAPL	-10360.26	-10717.62	-10401.19	-10646.83
MSFT	-10622.08	-11025.29	-10798.39	-10992.57
NVDA	-8493.93	-8891.30	-8583.09	-8810.78
C	-9732.95	-10446.16	-10165.85	-10266.67
GS	-10327.10	-10768.33	-10395.05	-10689.82
JPM	-10504.73	-11106.34	-10810.83	-10970.46
DIS	-10680.00	-11310.03	-11106.91	-11187.58
NFLX	-8543.08	-9246.26	-8934.36	-9104.71
AMZN	-9966.60	-10433.49	-10138.12	-10320.19

Log-likelihood for fitted marginals

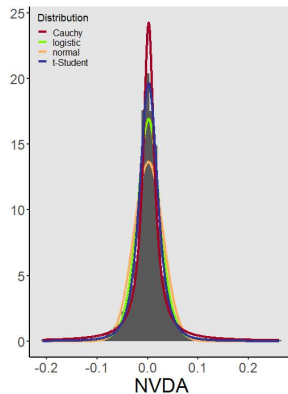
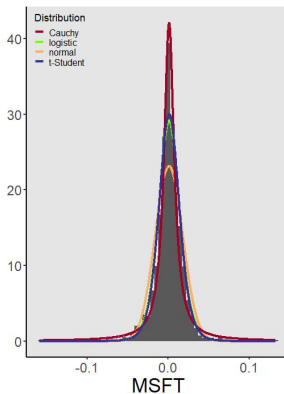
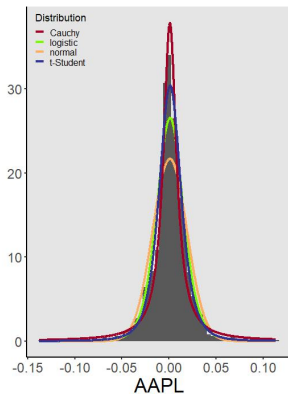
Table: Log-likelihood scores for fitted marginals.

Ticker	Gaussian	t-Student	Cauchy	Logistic
AAPL	5187.74	5370.22	5208.20	5331.02
MSFT	5318.65	5524.06	5406.80	5503.89
NVDA	4254.57	4457.06	4299.15	4413.00
C	4874.08	5234.49	5090.53	5140.94
GS	5171.16	5395.57	5205.13	5352.52
JPM	5259.97	5564.58	5413.02	5492.84
DIS	5347.61	5666.43	5561.06	5601.40
NFLX	4279.15	4634.54	4474.79	4559.96
AMZN	4990.91	5228.15	5076.67	5167.70

Fitted marginals (Finance)



Fitted marginals (IT)



Fitted marginals (Services)

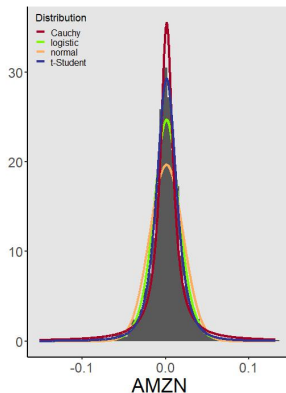
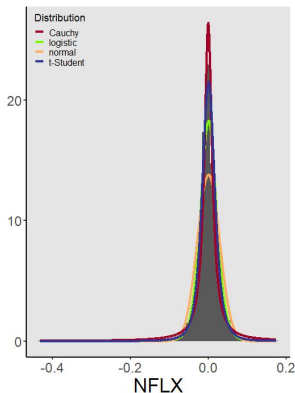
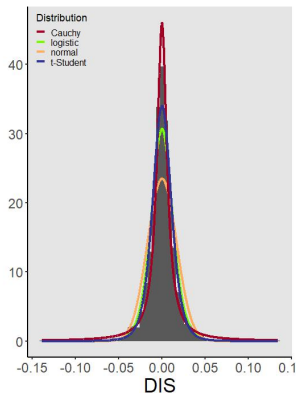


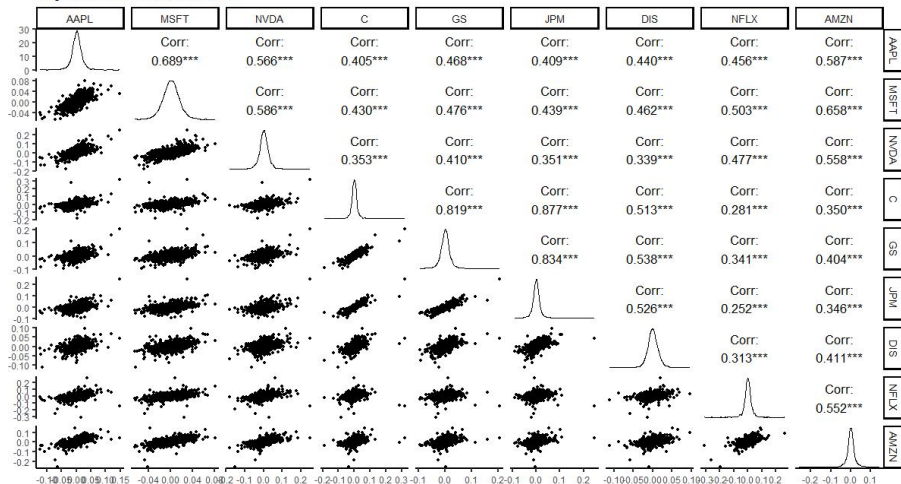
Table: Criterion scores for fitted elliptical & archimedean copulas.

	Gaussian	t-Student	Frank	Clayton	Gumbel
AIC	-11882.96	-12827.35	-6234.22	-6818.93	-6523.37
BIC	-11681.09	-12619.88	-6228.61	-6813.32	-6517.76
Log.likelihood	5977.48	6450.68	3118.11	3410.47	3262.68

Spearman matrix for fitted t-Student copula

Spearman correlation matrix for random sample from calibrated t copula

Marginals come from t-Student distribution



Spearman matrix real data vs data sampled from t-Student

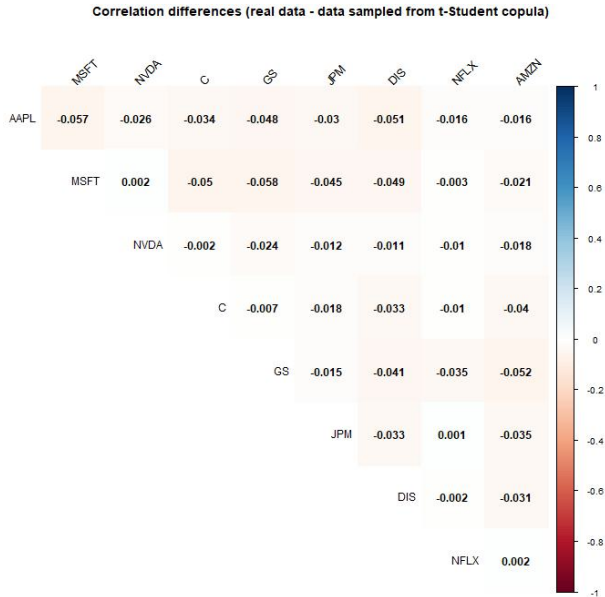


Table: Criterion scores for fitted C-vine, D-vine & R-vine copulas.

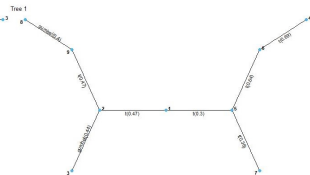
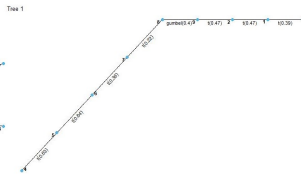
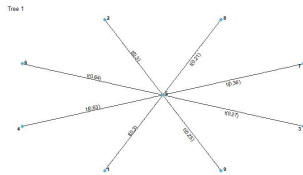
	D-vine	C-vine	R-vine
AIC	-13233.54	-13141.37	-13219.14
BIC	-12925.14	-12810.54	-12899.52
Log.likelihood	6671.77	6629.69	6666.57

Results for different D-vine structures

Table: Criterion scores for different D-vine structures.

Structure	Log-likelihood
5,4,6,1,3,2,7,8,9	6627.32
3,2,1,4,5,6,9,7,8	6601.15
7,8,9,2,1,3,4,6,5	6662.43
5,6,4,7,8,9,3,1,2	6640.17
1,3,2,6,5,4,7,8,9	6632.08
5,6,4,3,1,2,7,9,8	6662.44
3,1,2,7,9,8,5,4,6	6633.64
8,9,7,2,3,1,5,6,4	6599.57
5,6,4,7,8,9,1,2,3	6653.96
3,1,2,9,8,7,6,5,4	6671.77

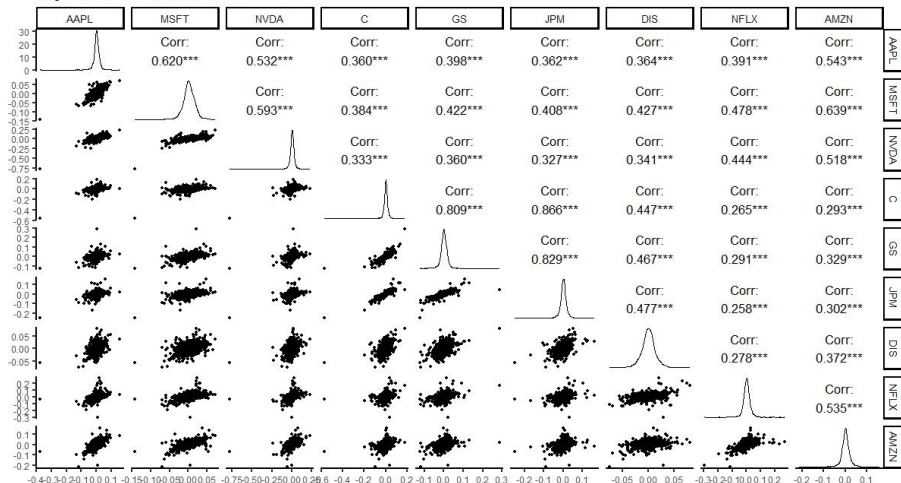
Vines trees (C-vine, D-vine, R-vine)



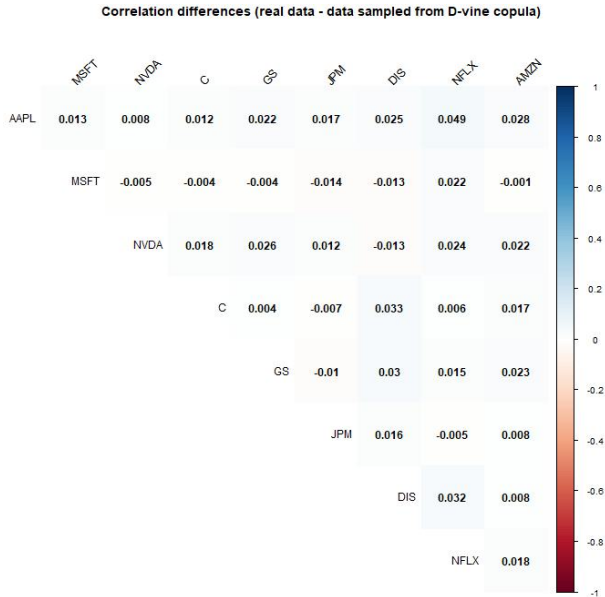
Spearman matrix for fitted D-vine

Spearman correlation matrix for random sample from calibrated dvine copula

Marginals come from t-Student distribution



Spearman matrix real data vs data sampled from D-vine

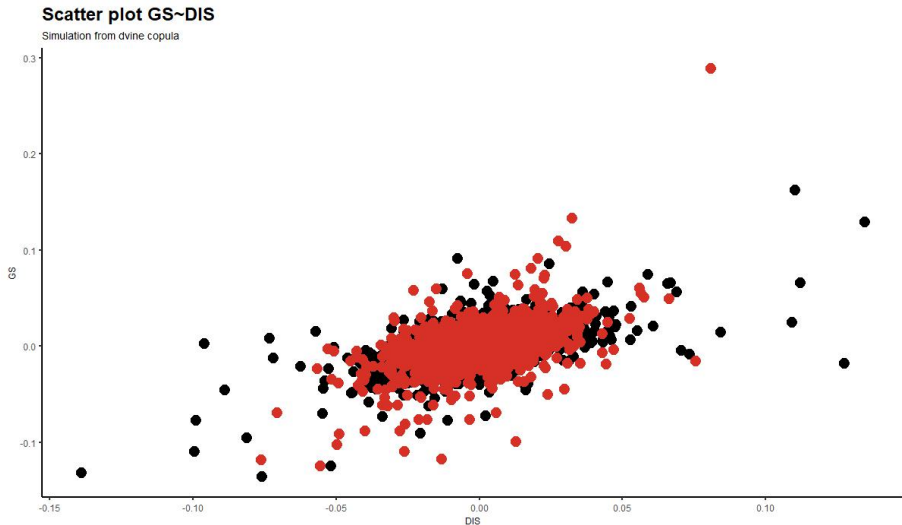


Log-likelihood for different bicopula families

Table: Log-likelihood scores for fitted vine copulas while various bicopula families are in usage.

Bicopula family	C-vine	D-vine	R-vine
all	7845.96	7834.47	7848.46
parametrics_except_tll	6636.08	6674.77	6696.81
nonparametrics_indep_tll	7845.92	7834.45	7848.52
oneparam	6324.18	6356.26	6405.11
twoparam	6636.08	6674.77	6696.80
ellipticals	6602.22	6634.06	6635.83
arch	6215.65	6238.32	6351.13
kendalls_inversion	6629.69	6665.08	6666.57
kernel_transform	7846.26	7834.46	7848.11
epllip_archimed	6629.69	6665.08	6666.57

Scatter plot $GS \sim DIS$ (D-vine)



Scatter plots $GS \sim DIS$

