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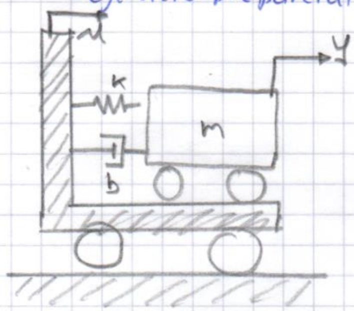
Ejercicio preparcial

1)



$$\ddot{y} + \frac{b}{m}\dot{y} + \frac{k}{m}y = \frac{b}{m}\dot{u} + \frac{k}{m}u$$

$$\ddot{y} + a_1\dot{y} + a_2 y = b_0\ddot{u} + b_1\dot{u} + b_2 u$$

$$a_1 = \frac{b}{m} \quad a_2 = \frac{k}{m} \quad b_0 = 0 \quad b_1 = \frac{b}{m} \quad b_2 = \frac{k}{m}$$

$$\beta_0 = b_0 = 0 \qquad \beta_1 = b_1 - a_1\beta_0 = \frac{b}{m}$$

$$\beta_2 = b_2 - a_1\beta_1 - a_2\beta_0 = \frac{k}{m} - \left(\frac{b}{m}\right)^2$$

$$x_1 = y - \beta_0 u = y$$

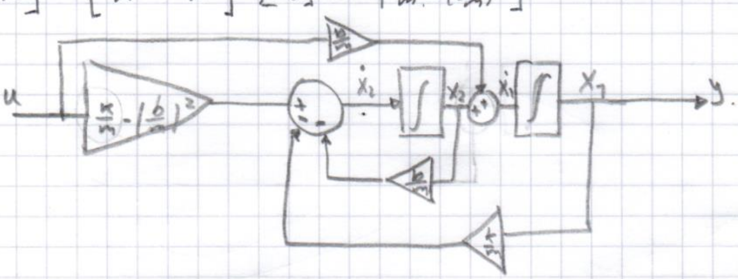
$$x_2 = \dot{x}_1 - \beta_1 u = \dot{x}_1 - \frac{b}{m}u$$

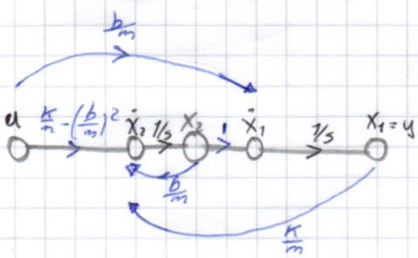
$$y = x_1$$

$$\dot{x}_1 = x_2 + \beta_1 u = x_2 + \frac{b}{m}u$$

$$\dot{x}_2 = -a_2 x_1 - a_1 x_2 + \beta_2 u = -\frac{k}{m}x_1 - \frac{b}{m}x_1 + \left[\frac{k}{m} - \left(\frac{b}{m}\right)^2\right]u$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{b}{m} \\ \frac{k}{m} - \left(\frac{b}{m}\right)^2 \end{bmatrix} u \qquad y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$





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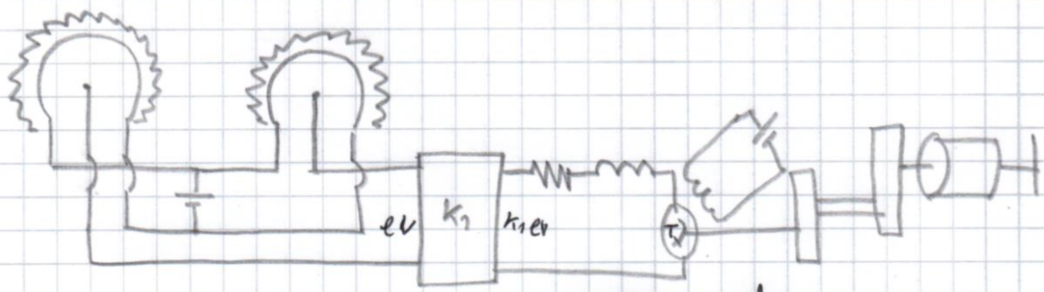


$$L_a \frac{di_a}{dt} + R_a i_a + k_3 \frac{d\theta}{dt} = k_1 e_v \qquad J_0 \frac{d^2\theta}{dt^2} - b_0 \frac{d\theta}{dt} = T = k_2 i_a$$

$$\frac{\Theta(s)}{E_v(s)} = \frac{k_1 k_2}{s(L_a s + R_a)(J_0 s + b_0) + k_2 k_3 s} = \frac{k_0 k_1 k_{2} n}{s(L_a s + R_a)(J_0 s + b_0) + k_3 k_2}$$

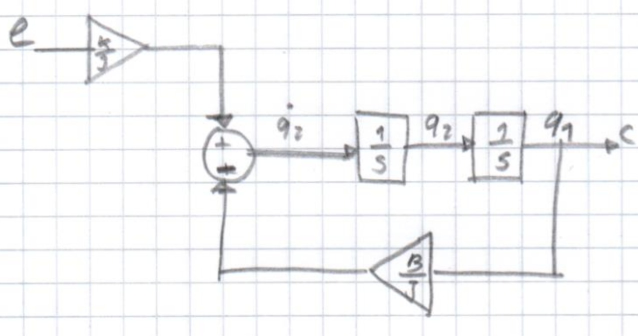
$$G(s) = \frac{k_0 k_1 k_2 n / R_a}{J_0 s^2 + \left(b_0 + \frac{k_2 k_3}{R_a}\right) s} \qquad J = \frac{J_0}{n^2} \qquad K = \frac{k_0 k_1 k_2}{n R_a}$$

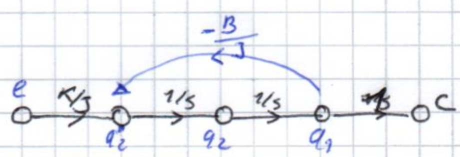
$$B = \frac{[b_0 + (k_2 k_3 / R_a)]}{n^2} \qquad K_m = \frac{K}{B} \qquad T_m = \frac{J}{B} = \frac{R_a J_0}{R_a b_0 + k_2 k_3}$$

$$G(s) = \frac{K}{Js^2 + Bs} = \frac{K_m}{s(T_m s + 1)} \qquad eK = J\ddot{c} + B\dot{c} \qquad \ddot{c} = \frac{eK}{J} - \frac{B\dot{c}}{J}$$

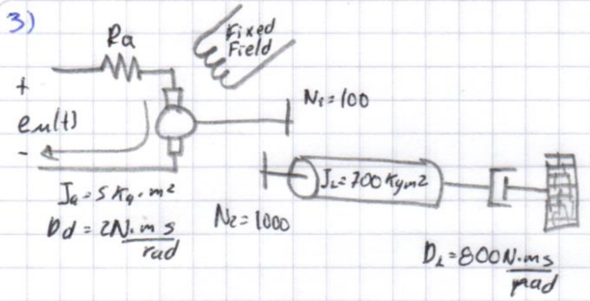
$$q_1 = c \qquad q_2 = \dot{q}_1 = \dot{c} \qquad \dot{q}_2 = \ddot{q}_1 = \ddot{c}$$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{B}{J} & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{K}{J} \end{bmatrix} e \qquad c = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} + [0] e$$





3)



$$E_{a(s)} \rightarrow \boxed{\frac{0{,}0417}{s(s+1{,}667)}} \rightarrow \theta_{L(s)}$$

$$J_m = J_a + J_L\left(\frac{N_1}{N_2}\right)^2 = 5 + 700\left(\frac{1}{10}\right)^2 = 12$$

$$D_m = D_a + D_L\left(\frac{N_1}{N_2}\right)^2 = 2 + 800\left(\frac{1}{10}\right)^2 = 10$$

$T_{stall} = 500$ $\quad \omega_{no\,Load} = 50$ $\quad e_a = 100$

$$\frac{K_t}{R_a} = \frac{T_{stall}}{e_a} = \frac{500}{100} = 5 \qquad K_b = \frac{e_a}{\omega_{noload}} = \frac{100}{50} = 2$$

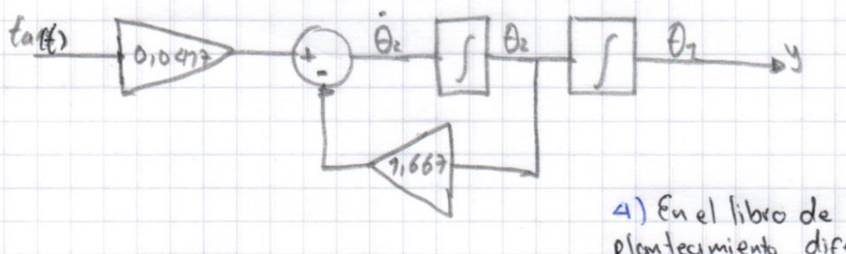
$$\frac{\theta_{m(s)}}{E_{a(s)}} = \frac{5/12}{s\left(s + \frac{1}{12}[10 + (5)(2)]\right)} = \frac{0{,}417}{s(s+1{,}667)} \quad \text{por} \quad \frac{N_1}{N_2} = \frac{1}{10} \longrightarrow \frac{0{,}0417}{s(s+1{,}667)}$$

$$\frac{0{,}0417}{s^2 + 1{,}667 s} = \frac{\theta_{m(s)}}{E_{a(s)}} \longrightarrow 0{,}0417\, E_{a(s)} = s^2\,\theta_{m(s)} + 1{,}667\, s\, \theta_{m(s)}$$

$$\theta_{m(s)} = \theta_1 \qquad \dot{\theta}_1 = \theta_2 \qquad \dot{\theta}_2 = \ddot{\theta}_1$$

$$0{,}0417\, E_{a(s)} = \dot{\theta}_2 + 1{,}667\, \theta_2$$

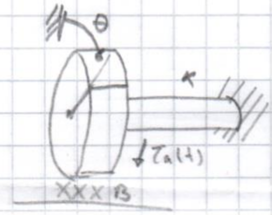
$$\begin{vmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{vmatrix} = \begin{vmatrix} 0 & 1 \\ 0 & -1{,}667 \end{vmatrix} \begin{vmatrix} \theta_1 \\ \theta_2 \end{vmatrix} + \begin{bmatrix} 0 \\ 0{,}0417 \end{bmatrix} E_{a(t)} \qquad y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{vmatrix} \theta_1 \\ \theta_2 \end{vmatrix}$$

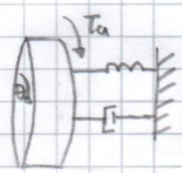


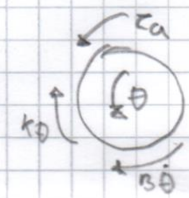
4) En el libro de Ogata se hace un planteamiento diferente ya que se plantea el ejercicio usando muchas constantes, sin embargo haciendo cambios a estas constantes y se forza una reducción, de allí en adelante el procedimiento de resolución es el mismo.

Examen #2 Sistemas Dinamicos

1)







$$I_o \ddot{\theta} + B\dot{\theta} + k\theta = \tau_a \qquad q_1 = \theta \qquad q_2 = \dot{q}_1 = \dot{\theta}_1 \qquad \dot{q}_2 = \ddot{q}_1 = \ddot{\theta}$$

$$I_o \dot{q}_2 + Bq_2 + kq_1 = \tau_a$$

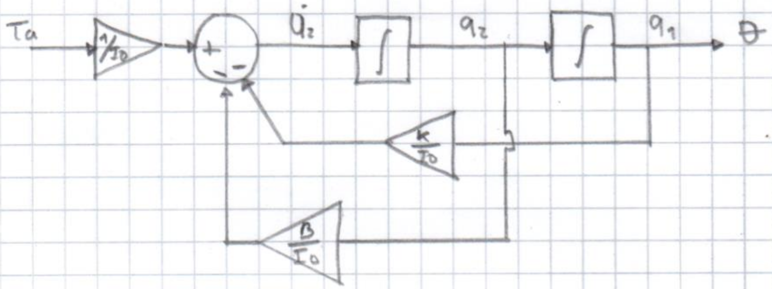
$$I_o \dot{q}_2 = \tau_a - Bq_2 - kq_1$$

$$\dot{q}_2 = \frac{\tau_a}{I_o} - \frac{B}{I_o} q_2 - \frac{k}{I_o} q_1$$

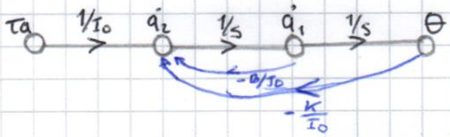
$$\begin{vmatrix} \dot{q}_1 \\ \dot{q}_2 \end{vmatrix} = \begin{vmatrix} 0 & 1 \\ -\frac{k}{I_o} & -\frac{B}{I_o} \end{vmatrix} \begin{vmatrix} q_1 \\ q_2 \end{vmatrix} + \begin{vmatrix} 0 \\ 1/I_o \end{vmatrix} \tau_a(t)$$

$$\theta = [1 \quad 0] \begin{vmatrix} q_1 \\ q_2 \end{vmatrix}$$

b)



c)



función de transferencia

$$I_o \ddot{\theta} + B\dot{\theta} + k\theta = \tau_a$$

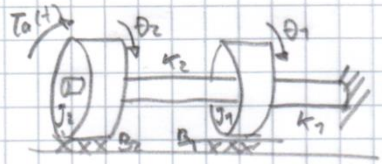
$$I_o s^2 \theta(s) + Bs\,\theta(s) + k\,\theta(s) = \tau_a(s)$$

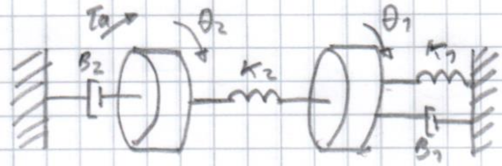
$$\theta(s)\{I_o s^2 + Bs + k\} = \tau_a(s)$$

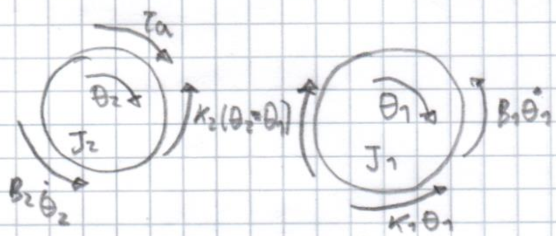
$$G(s) = \frac{\theta(s)}{\tau_a(s)} = \frac{1}{J_o s^2 + Bs + k}$$

2) $\theta_2 > \theta_1$

a. función de transferencia entre θ_2 y τ_a







$$\tau_a - J_2\ddot{\theta}_2 - k_2(\theta_2-\theta_1) - B_2\dot{\theta}_2 = 0$$

$$\tau_a = J_2\ddot{\theta}_2 + k_2(\theta_2-\theta_1) + B_2\dot{\theta}_2$$

$$\tau_a = J_2\ddot{\theta}_2 + k_2\theta_2 - k_2\theta_1 + B_2\dot{\theta}_2$$

$$k_2(\theta_2-\theta_1) - J_1\ddot{\theta}_1 - k_1\theta_1 - B_1\dot{\theta}_1 = 0$$

$$T(s) = J_2 s^2\theta_2(s) + k_2\theta_2(s) - k_2\theta_1(s) + B_2 s\,\theta_2(s)$$

$$k_2\theta_2 - k_2\theta_1 - J_1\ddot{\theta}_1 - k_1\theta_1 - B_1\dot{\theta}_1 = 0$$

$$T(s) = \theta_2(s)\left(J_2 s^2 + k_2 + B_2 s\right) + \theta_1(s)(-k_2)$$

$$k_2\theta_2(s) - k_2\theta_1(s) - J_1 s^2\theta_1(s) - k_1\theta_1(s) - B_1 s\,\theta_1(s) = 0$$

$$\theta_1(s)\left(-B_1 s - k_1 - k_2 - J_1 s^2\right) + k_2\theta_2(s) = 0$$

$$\begin{bmatrix} -k_2 & J_2 s^2 + k_2 + B_2 s \\ -B_1 s - k_1 - k_2 - J_1 s^2 & k_2 \end{bmatrix}\begin{bmatrix}\theta_1(s) \\ \theta_2(s)\end{bmatrix} = \begin{bmatrix} Ts \\ 0 \end{bmatrix}$$

$$\frac{\theta_2(s)}{T(s)} = \frac{+J_2 s^2 + k_2 + B_2 s}{-k_2^2\,(J_2 s^2 + k_2 + B_2 s)(B s + k_1 + k_2 + J_1 s^2)}$$

b) Espacio de estados

$\theta_1 = q_1$ $\qquad q_2 = \dot{\theta}_1$ $\qquad \dot{q}_2 = \ddot{\theta}_1$

$\theta_2 = q_3$ $\qquad q_4 = \dot{\theta}_2$ $\qquad \dot{q}_4 = \ddot{\theta}_2$

$$\tau_a = J_2\ddot{\theta}_2 + k_2(\theta_2-\theta_1) + B_2\dot{\theta}_2$$

$$\ddot{\theta}_2 = \frac{\tau_a}{J_2} - \frac{k_2}{J_2}\theta_2 + \frac{k_2}{J_2}\theta_1 - \frac{B_2}{J_2}\dot{\theta}_2 \longrightarrow \dot{q}_4 = \frac{\tau_a}{J_2} - \frac{k_2}{J_2}q_3 + \frac{k_2}{J_2}q_1 - \frac{B_2}{J_2}q_4$$

$$k_2\theta_2 - k_2\theta_1 - J_1\ddot{\theta}_1 - k_1\theta_1 - B_1\dot{\theta}_1 = 0$$

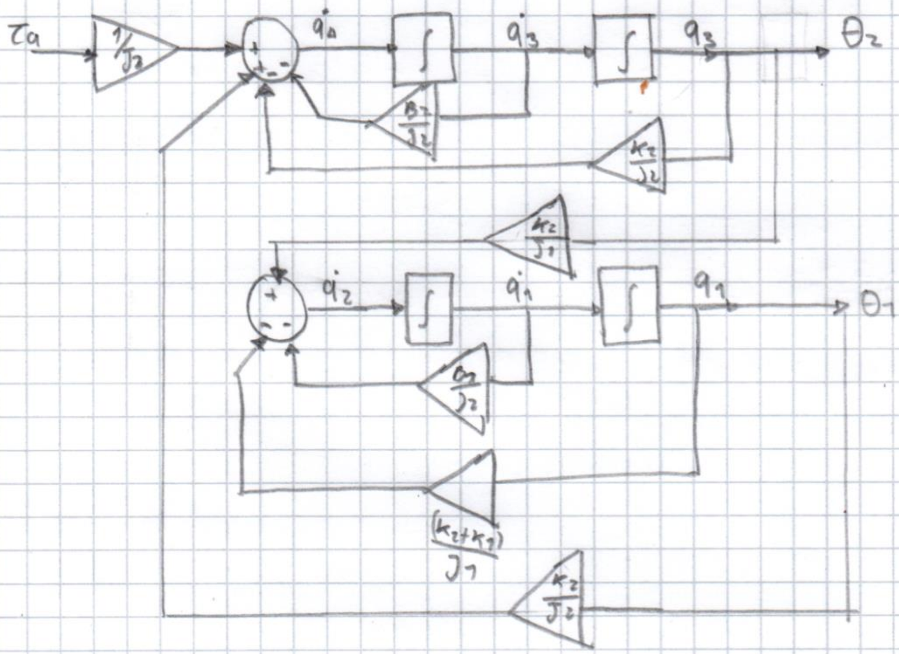
$$\ddot{\theta}_1 = \frac{k_2}{J_1}\theta_2 - \frac{k_2}{J_1}\theta_1 - \frac{k_1}{J_1}\theta_1 - \frac{B_1}{J_1}\dot{\theta}_1$$

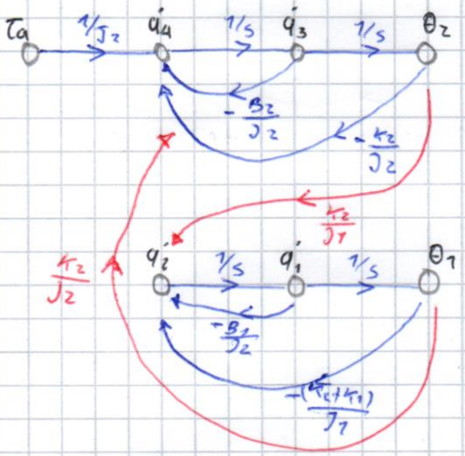
$$\dot{q}_2 = \frac{k_2}{J_1}q_3 - \frac{(k_2+k_1)}{J_1}q_1 - \frac{B_1}{J_1}q_2$$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{(K_2+K_1)}{J_1} & -\frac{B_1}{J_1} & \frac{K_2}{J_1} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{K_2}{J_2} & 0 & -\frac{K_2}{J_2} & -\frac{B_2}{J_2} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{J_2} \end{bmatrix} \tau_a$$

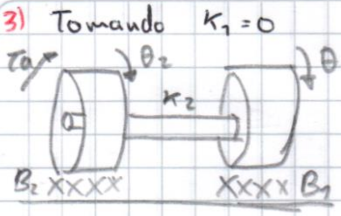
$\theta_2 = q_3$

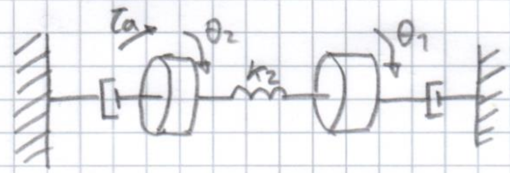
$$y = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$

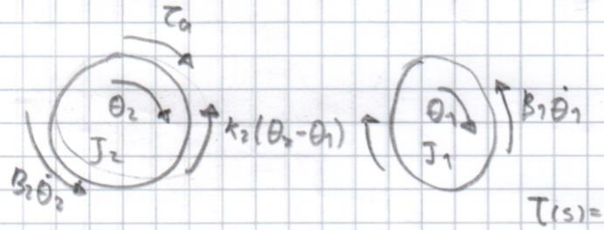




3) Tomando $k_1 = 0$







$$\tau_a = J_2\ddot\theta_2 + k_2\theta_2 - k_2\theta_1 + B_2\dot\theta_2$$

$$k_2\theta_2 - k_2\theta_1 - J_1\ddot\theta_1 - B_1\dot\theta_1 = 0$$

$$T(s) = \Theta_2(s)\left(J_2 s^2 + k_2 + B_2 s\right) + \Theta_1(s)(-k_2)$$

$$\Theta_1(s)\left(-B_1 s - k_2 - J_1 s^2\right) + k_2\Theta_2(s) = 0$$

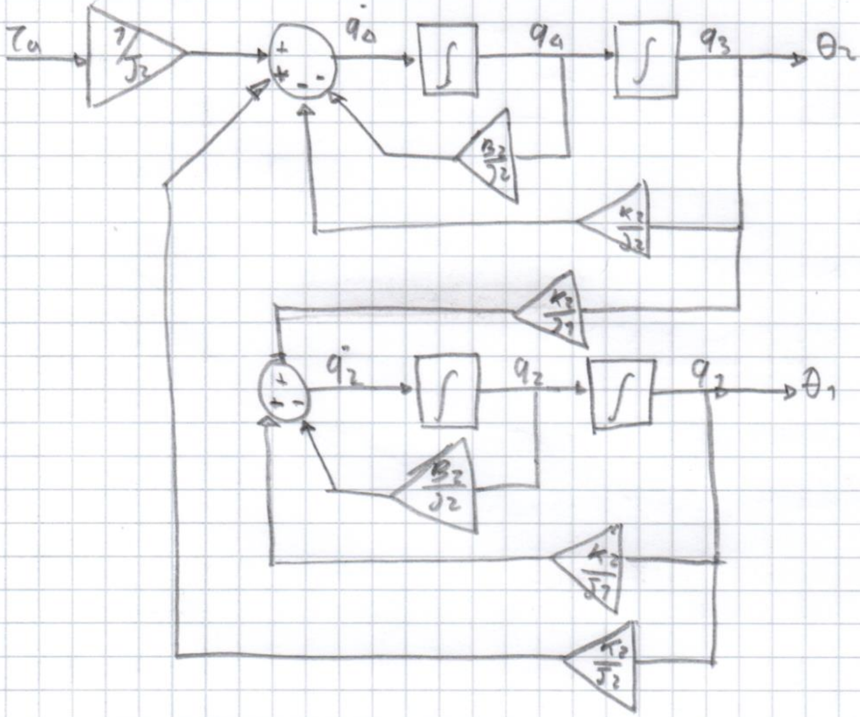
$$\begin{bmatrix} -k_2 & J_2 s^2 + k_2 + B_2 s \\ -B_1 s - k_2 - J_1 s^2 & k_2 \end{bmatrix}\begin{bmatrix} \Theta_1(s) \\ \Theta_2(s) \end{bmatrix} = \begin{bmatrix} T(s) \\ 0 \end{bmatrix}$$

$$\frac{\Theta_2(s)}{T(s)} = \frac{J_2 s^2 + k_2 + B_2 s}{-k_2^2 \quad (J_2 s^2 + k_2 + B_2 s)(B_1 s + k_2 + J_1 s^2)}$$

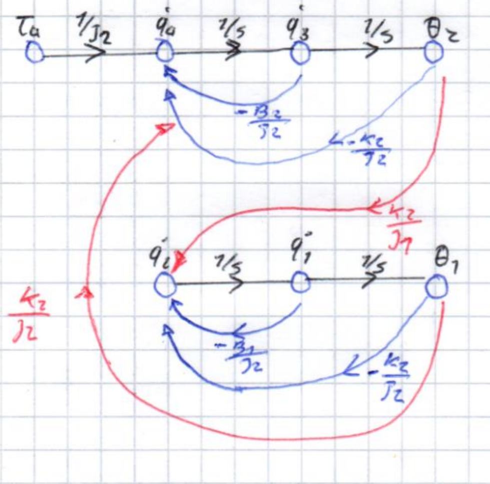
Espacio de estados $\quad \theta_1 = q_1 \quad q_2 = \dot\theta_1 \quad \dot q_2 = \ddot\theta_1 \quad \theta_2 = a_3 \quad q_3 = \dot\theta_2 \quad \dot q_3 = \ddot\theta_2$

$$\dot q_4 = \frac{\tau_a}{J_2} - \frac{k_2}{J_2}q_3 + \frac{k_2}{J_2}q_1 - \frac{B_2}{J_2}q_4 \qquad \dot q_2 = \frac{k_2}{J_1}q_3 - \frac{k_2}{J_1}q_1 - \frac{B_1}{J_1}q_2$$

$$\begin{vmatrix} \dot q_1 \\ \dot q_2 \\ \dot q_3 \\ \dot q_4 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_2}{J_1} & -\frac{B_1}{J_1} & \frac{k_2}{J_1} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k_2}{J_2} & 0 & -\frac{k_2}{J_2} & -\frac{B_2}{J_2} \end{vmatrix}\begin{vmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{vmatrix} + \begin{vmatrix} 0 \\ 0 \\ 0 \\ 1/J_2 \end{vmatrix}\tau_a \qquad y = [0 \quad 0 \quad 1 \quad 0]\begin{vmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{vmatrix}$$

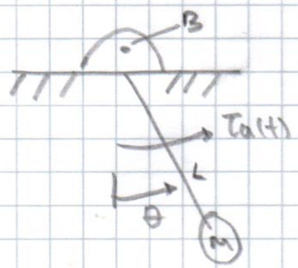


τa
1/J2
q̇4
q4
q3
θ2
B2/J2
K2/J2
K2/J1
q̇2
q2
q1
θ1
B2/J2
K2/J1
K2/J2

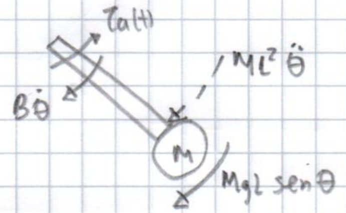


τa
1/J2
q̇4
1/s
q̇3
1/s
θ2
-B2/J2
-K2/J2
K2/J1
q̇2
1/s
q̇1
1/s
θ1
-B1/J2
-K2/J2
K2/J2

4)



$I = ml^2 \qquad Ta(t) - mgl\,sen\,\theta$



$$ML^2\ddot{\theta} + B\dot{\theta} + MgL\,sen\,\theta = Ta(t)$$

$$\dot{\theta} = \omega \qquad \ddot{\theta} = \dot{\omega}$$

$$\dot{\omega} = \frac{1}{ML^2}\left[-MgL\ sen\,\theta - B\omega + Ta(t)\right] \qquad \theta = 0{,}5\ rad$$

$$ML^2\ddot{\theta} + B\dot{\theta} + MgL\theta = Ta(t)$$

$$\dot{\theta} = \omega \qquad \theta = q_1 \quad q_2 = \dot{q}_1 = \dot{\theta} = \omega \quad \dot{q}_2 = \ddot{\theta} = \dot{\omega}$$

$$\dot{\omega} = \frac{1}{ML^2}\left[-MgL\theta - B\omega + Ta(t)\right] \longrightarrow \dot{q}_2 = \frac{1}{ML^2}\left[-MgLq_1 - Bq_2 + Ta(t)\right]$$

$$q_2 = -\frac{g}{L}q_1 - \frac{B}{ML^2}q_2 + \frac{Ta(t)}{ML^2}$$

$$\begin{bmatrix}\dot{q}_1 \\ \dot{q}_2\end{bmatrix} = \begin{bmatrix}0 & 1 \\ -\frac{g}{L} & -\frac{B}{ML^2}\end{bmatrix}\begin{bmatrix}q_1 \\ q_2\end{bmatrix} + \begin{bmatrix}0 \\ 1/ML^2\end{bmatrix}Ta(t) \qquad \theta = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}q_1 \\ q_2\end{bmatrix}$$