Tarea #6 Edwin Arbey Valbuena Gamboa 20211005129

$$G(s) = \frac{20(s+2)}{s(s+1)(s+4)} \qquad \begin{cases} OS\% = 9{,}5\% \\ t_s = 0{,}74s \end{cases}$$

$$U(s) \rightarrow \boxed{\frac{1}{s^3+5s^2+4s}} \xrightarrow{X_1(s)} \boxed{0s^2+20s+100} \rightarrow Y(s)$$

$$\frac{X_1(s)}{U(s)} = \frac{1}{s^3+5s^2+4s} \qquad (s^3+5s^2+4s)X_1(s) = U(s) \qquad \dddot{x}_1 + 5\ddot{x}_1 + 4\dot{x}_1 = U$$

$$X_1(s) = x_1 \qquad x_2 = \dot{x}_1 \qquad x_3 = \dot{x}_2 = \ddot{x}_1 \qquad \dot{x}_3 = \ddot{x}_2 = \dddot{x}_1$$

$$\dot{x}_3 + 5x_3 + 4x_2 = U \qquad \dot{x}_3 = -5x_3 - 4x_2 + U$$

$$Y(s) = (b_2 s^2 + b_1 s + b_0)X_1(s) = (0s^2+20s+100)X_1(s) = (20s+100)X_1(s)$$

$$20\dot{x}_1 + 100x_1 = 20x_2 + 100x_1 \qquad y = 20x_2 + 100x_1$$

$$\begin{vmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 6 & -4 & -5 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} + \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix} U \qquad y = \begin{vmatrix} 100 & 20 & 0 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix}$$

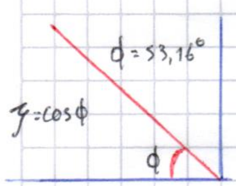
$$\%OS = e^{-(\xi\pi/\sqrt{1-\xi^2})} \qquad 0{,}095 = e^{-(\xi\pi/\sqrt{1-\xi^2})} \qquad \ln(0{,}095) = \ln e^{-(\xi\pi/\sqrt{1-\xi^2})}$$

$$-2{,}3539 = \frac{-\xi\pi}{\sqrt{1-\xi^2}} \longrightarrow \left(-2{,}3539\sqrt{1-\xi^2}\right)^2 = (-\xi\pi)^2$$

$$5{,}5407 - 5{,}5407\xi^2 = \xi^2\pi^2 \qquad 5{,}5407 = \xi^2\pi^2 + 5{,}5407\xi^2$$

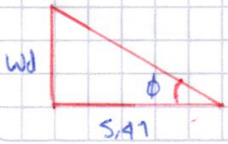
$$5{,}5407 = \xi^2(\pi^2 + 5{,}5407) \qquad \xi^2 = \frac{5{,}5407}{\pi^2+5{,}5407} \longrightarrow \xi = \sqrt{\frac{5{,}5407}{\pi^2+5{,}5407}} \qquad \xi = 0{,}5996$$

$$s = \sigma + j\omega_d \qquad \arccos(0{,}5996) = 53{,}16^\circ$$



$$\sigma = \xi\omega_n \qquad t_s = 0{,}74 \qquad 0{,}74 = \frac{4}{\sigma} \qquad \sigma = \frac{4}{0{,}74} = 5{,}405$$

$$\omega_n = \frac{5{,}405}{0{,}5996} = 9{,}02\ \frac{rad}{s}$$

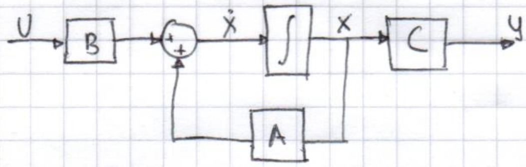


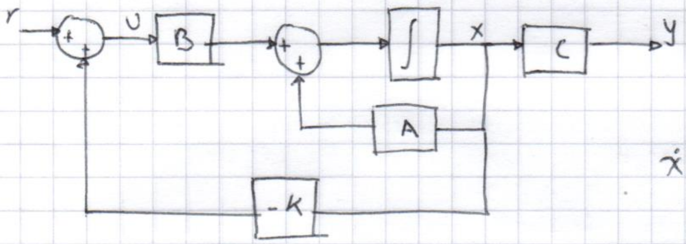
$$\tan\phi = \frac{\omega_d}{5{,}41} \qquad \omega_d = 7{,}21$$

$$\tan(53{,}16)\,5{,}41 = \omega_d$$

$\dot{x} = Ax + BU$

$y = Cx + DU$

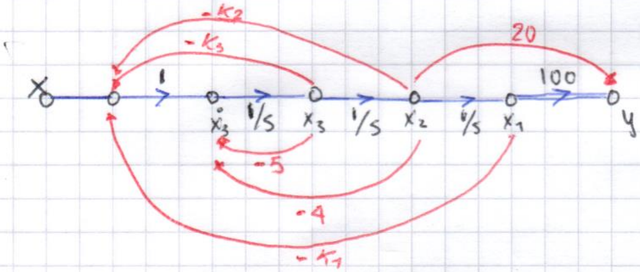




$\dot{x} = Ax + BU$

$\dot{x} = Ax + B(-Kx + r)$

$\dot{x} = -BKx + Br + Ax \rightarrow \dot{x} = (A - BK)x + Br$



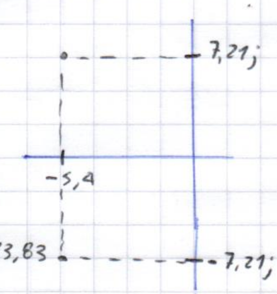
$\dot{x}_3 = -4x_2 - 5x_3 + U$

$= -4x_2 - 5x_3 - k_3x_3 - k_2x_2 - k_1x_1 + r$

$= -k_1x_1 - (4+k_2)x_2 - (5+k_3)x_3 + r$

$$\begin{vmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -k_1 & -(4+k_2) & -(5+k_3) \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} + \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix} r$$

$\det[sI - (A - BK)] = s^3 + (5+k_3)s^2 + (4+k_2)s + k_1 = P_{et}$



$(s + 5,4 + j\,7,2)(s + 5,4 + j\,7,2)$

$(s + 5,1)$

$s^3 + 15,9s^2 + 136,22s + 413,83 = 0$

$s^3 + (5+k_3)s^2 + (4+k_2)s + k_1 = s^3 + 15,9s^2 + 136,22s + 413,83$

$(5+k_3)s^2 = 15,9s^2 \longrightarrow 5 + k_3 = 15,9 \quad k_3 = 10,9$

$(4+k_2)s = 136,22s \longrightarrow 4 + k_2 = 136,22 \quad k_2 = 132,22$

$k_1 = 413,83$

$$\begin{vmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -413,8 & -136,22 & -15,9 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} + \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix} r$$