

Solutions 1.

Example 1. Are you a liar?

Example 2. He can not be a liar because in that case his lies eventually turns out to be truth. So A is knight, we believe him and conclude that B is a liar.

Answer: A - knight, B - liar.

Example 3. Assume first is a knight then second and third are liars but then second tells the truth. So first is a liar and there are at least one knight among second and third but second cannot be a liar because in this case third has to be a knight and second tells the truth then. So second is a knight and from his words third has to be a knight.

Answer: First is a liar, second and third are knights.

Problem 1.1. "What time is it?" "What is the parity of the number of times you have heard this question?" etc.

Problem 1.2. a) He cannot be a liar since in this case he will be a knight at the same time. That means that he is a knight and at least one of the others has to be a liar. So we get from 1 to 11 knights. b) Assume he is a liar then his words have to be lies (e.g. if the rest are liars then he is knight) and the only way to verify this is if the rest are liars but, so he has to be knight, and we get a contradiction. Thus he has to be a knight. But then the rest can not be all liars, otherwise he has to be a liar. So there are at least two knights which means that there can be from 0 to 10 liars. Answer: a) From 1 to 11 knights; b) From 0 to 10 liars.

Problem 1.3. Consider two cases. 1) There exists at least one liar then his neighbour also a liar and eventually we conclude that all of them are liars. 2) There are no liars at all. Then all of them are knights and there is no contradiction.

Answer: All of them are liars or all of them are knights at the same time.

Problem 1.4. Assume D is not watching the TV, then by 2) E is watching the TV, so by 5), D is watching the TV. We come to a contradiction and our assumption is incorrect. So D is watching the TV. By 4), C is also watching the TV, then by 3), B is not watching the TV and by 1), A is not watching the TV which means by 5), that E is not watching the TV.

Answer: Only C and D are watching the TV.

Problem 1.5. Every two statements contradict each other. That means that there can be only one correct statement or zero correct statements. In the first case we immediately conclude that there should be 99 false statements, so the 99-th statement is correct. In the second case we get a contradiction since the last statement will be the correct one.

Answer: 99-th statement is correct.

Problem 1.6. You should take any 15 of them for the first pile and turn every coin of this pile upside down. If there were n coins with heads up in the first pile before we turned them then there are exactly $15-n$ coins with heads up in the second pile

and in the first pile after turning them.

Problem 1.7. It is easy to figure out if the dot is in the square with 4 lines; just draw your lines through the edges of the box. But it is also possible to do it with just 3 lines. To do this, draw the first line through one of the diagonals of the square. It divides the square into two triangular halves, and we now know in which half of the square the dot might lie. After this we draw lines through the remaining two sides of this half of the square and figure out if the dot is inside or outside the square. What could be done if the dot lies on the first line?

Why aren't two lines enough? It is because it's impossible to bound a finite region with just two lines. Regardless of how we draw those two lines, the dot might lie within the unbounded angle formed by lines that intersect the square. We have no way of determining if the dot is inside or outside the square.