Solutions 2.

Example 1. In this example the pine trees are the "pigeons" and the needles are the "pigeon holes" (numbered from 0 to 600000). Since we have at least 600002 pigeons and 600001 pigeon holes, then, by the Pigeon Hole Principle, there will be a hole with 2 pigeons in it. That is, there will be two trees with the same number of needles on them.
Example 2. Divide a square 4×4 into 16 equal squares 1×1. As $16 > 15$, it is impossible to place 15 trees in such a way that each small square has a tree inside it.
Problem 2.1. Assume the contrary, i.e. that every pigeon hole must contain no more than k pigeons. It means that altogether N pigeon holes must contain no more than Nk pigeons. But $Nk < Nk + 1$. Thus we come to a contradiction.
Problem 2.2. Divide the garden into 25 equal square parts 2×2. Our "pigeons"are the apple trees (=51) and our "pigeon holes"are the small squares (=25). Since $51 = 25 \times 2 + 1$, we can find a pigeon hole with $2 + 1 = 3$ pigeons by the General Pigeon Hole Principle. This is exactly what we need to prove.
Problem 2.3. Assume the contrary. Divide the chess board into 16 equal squares 2×2. Our "Pigeons"are the kings, our "pigeon holes"are the squares 2×2. By the Pigeon Hole Principle, we can find a square with 2 kings in it. But the kings, if placed within the same square 2×2, will check each other. Thus, we arrive at a contradiction.
Problem 2.4. Suppose it is possible to form such 9 units from 44 knights. Let us arrange them in the ascending order. A unit have at least one knight. Therefore, the smallest unit should have at least one knight, the next unit should have at least two knights,..., the largest unit should have at least nine knights. Thus, in total, we have at least $1 + 2 + ... + 9 = 45$ knights. It means that Sir Lancelot have to participate in this mission.
Problem 2.5. How many friends can a single knight have? From 0 to 10. But there can't be a knight with zero friends and a knight with 10 friends at the same time. If two people are friends, we call it a friendship. Let's count the number of friendships. We have 10 possible friendship options for a particular person corresponding to him having 0,...,9 friends or to 1,...,10 friends. Such friendships are the "pigeon holes", and our 11 knights are the "pigeons" in this context. It follows from the Pigeon Hole Principle that there will be two knights in the same pigeon hole. Thus, two knights

will have the same number of friends.
Problem 2.6. Denote by N the number of names with more than one representative. N can't be greater or equal to 5. Otherwise, it is possible to take 2 representatives from 5 of these "pigeon holes" and form a unit of 10 knights from which we can't choose 3 with the same name. Assume it was a joke. That means that every "pigeon hole" consists of less or equal to 14 number of "pigeons". We have $N \leq 4$, so in the corresponding "pigeon holes" there will be no more than $4 \times 14 = 56$ knights in total. Thus at least $60 - 56 = 4$ "pigeon holes" will have only one "pigeon" inside. Therefore, we can form a unit with these 4 "pigeons" and 3 pairs of "pigeons" from "pigeon holes" with more than one "pigeon" if $N \geq 3$. And if $N \leq 2$ we will have at least $60 - 2 \times 14 = 32$ "pigeon holes" with only one pigeon. So we can form a unit of 10 pigeons from the later "pigeon holes". Existence of such unit in both cases contradicts with the task. So our assumption was incorrect.
Problem 2.7. (a) No, not necessarily. Let the train move at a constant speed of 90 mph for the first half hour, at 110 mph the next half hour, at 90 the next, and so on. Clearly, the train covered the first 100 miles in an hour. Now think of any one-hour time interval. During that time, the train goes 90 mph for 30 minutes of that hour and 110 mph for the remaining 30 minutes. One of these 30-minute times will most likely consist of some number of minutes at the start of the hour and the remainder of the 30 minutes at the end of the hour. The train goes 100 miles in that hour and, since the train never stops, the 100 miles are covered in exactly one hour.
(b) No. Average speed is the ratio of total distance to total time. Thus the problem can be reformulated as follows: does the train necessarily have to have moved through 550 miles over the five and a half hours? The answer is no. In moving according to the scheme outlined in part (a), the train will travel through 545 miles. Check yourself.