Solutions 5.

Example 1. No, it is not true. For example, $4|12$ and $6|12$ but $24 \nmid 12$ (does not divide).
Answer: No.
Example 2. If a number is divisible by 5 and by 7 then its prime decomposition contains 5 and 7. This means that it can be written as $5 \times 7 \times ... = 35 \times ...$, so it is divisible by 35.
Answer: Yes.
Example 3. The prime decomposition of A ($p_1^{\alpha_1} \times p_2^{\alpha_2} \times ... \times p_k^{\alpha_k}$) does not contain 3. Then the prime decomposition of $2A$ (which is just 2 multiplied by the prime decomposition of A) does not contain 3 either.
Answer: It is not possible.
Problem 5.1. Here is the list: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97. This number can be divided by $10 = 2 \times 5$ and the quotient can be divided by 3. We will have $2910 = 2 \times 3 \times 5 \times 97$. The last number is prime (by our list), so it is the prime decomposition.
Answer: $2910 = 2 \times 3 \times 5 \times 97$.
Problem 5.2. Since this number is even its prime decomposition contains 2. Then the prime decomposition of $3A$ contains 3 and 2, thus together they will give the factor 6, so $3A$ is divisible by 6.
Answer: Yes, it is divisible.
Problem 5.3. If we know $a = c \times b + d$ $(d < b)$ then multiplying by 3 we arrive at $3a = c \times 3b + 3d$. Here $3d < 3b$, so this representation of $3a$ satisfies the one we get from the modulo operation. That means that the remainder is 3 times bigger and the quotient stays the same.
Answer: The remainder is $3d$, the quotient is c.
Problem 5.4. (a) Since every third natural number is divisible by 3 and every second natural number is divisible by 2, any three consecutive numbers will contain one which is divisible by 3 and one which is divisible by 2. It may be the same number. Anyhow, the prime decomposition of their product will contain 2 and 3 meaning that it is divisible by 6.
(b) Similarly to the arguments in (a) we deduce that any four consecutive numbers will contain two even numbers and one of those is divisible by 4. And also there will be one number which is divisible by 3. Some of them may coincide, but still the

prime decomposition will contain the factor 2 three times $2^3 = 2 \times 4$ and the factor 3 at least one time. So, it will contain the factor $2 \times 3 \times 4 = 24$.
Answer: (b) Yes it is.
Problem 5.5. Consider the prime decomposition of 2016 - $2016 = 2^5 \times 3^2 \times 7$. It follows that n should be at least 7 to give the factor 7, but if $n = 7$ there will be only four 2's in the decomposition of 7!. And the next value $n = 8$ works out: $(6 \times 3) \vdots 3^2$, $(2 \times 4 \times 6 \times 8) \vdots 2^5$.
Answer: n=8.
Problem 5.6. This number may be equal to $2017!+1$. Then $n+1$, $n+2$, ..., $n+2016$ are equal to $2017! + 2$, $2017! + 3$, ..., $2017! + 2017$ respectively. These numbers are divisible by 2, 3, ... 2017 respectively by the definition of $2017! = 1 \times 2 \times ... \times 2017$ being the product of all natural numbers from 1 to 2017.
Remark: Note that considering $n = 2016!$ is more complicated, since it is not an easy question whether $2016! + 1$ is prime or composite.
Answer: $2017! + 1$.
Problem 5.7. Assume that 2^{2016} has n digits and 5^{2016} has m digits. We know that 10^a is written with $a + 1$ digits (a 0's and one 1) and is the smallest number if that kind. So the following inequalities hold:

$$10^{n-1} < 2^{2016} < 10^n \qquad and \qquad 10^{m-1} < 5^{2016} < 10^m$$

Multiplying them arrives us at:

$$10^{n+m-2} < 2^{2016} \times 5^{2016} = 10^{2016} < 10^{n+m}$$

It follows that $n+m-2 < 2016 < n+m$ which is equivalent to $2016 < n+m < 2018$. Thus, we obtain $n + m = 2017$ which is the total number of digits written on the page.
Answer: 2017 digits.