

Solutions 3.

Example 1. Suppose it can be covered. Each domino covers 2 boxes of the checkerboard. Therefore, in total only the even number of boxes will be covered, but we should have 25 covered boxes. This is a contradiction.

Answer: It is impossible.

Example 2. At each move, a knight jumps from a square of one colour to a square of the opposite colour. Since the knight must make 63 moves, the last (odd) move must bring him to a square of the opposite colour from the square on which he started. However, squares *a1* and *h8* are of the same colour.

Answer: No, he cannot.

Example 3. Suppose it is possible. Name the vertices of the Knave's trajectory as A_1, A_2, \dots, A_9 . Then King's hit crosses all the edges $A_1A_2, A_2A_3, \dots, A_8A_9, A_9A_1$. It means that A_1, A_3, A_5, A_7, A_9 lie on one side of the line corresponding to King's hit and A_2, A_4, A_6, A_8, A_1 lie on another. But the line shouldn't pass through the vertices, so A_1 cannot lie on both sides simultaneously.

Answer: No, he cannot.

Problem 3.1. All the values of new coins are odd numbers. So any 11 coins will give us an odd total amount. Therefore, 100 cannot be obtained.

Answer: No, it won't be possible.

Problem 3.2. Suppose that the first gear rotates clockwise. Then the second gear must rotate counter-clockwise, the third clockwise again, the fourth counter-clockwise, and so on. It is clear that the "odd" gears must rotate clockwise, while the "even" gears must rotate counter-clockwise. But then the first and the eleventh gears must rotate in the same direction. This is a contradiction.

Answer: No.

Problem 3.3. At each move, he goes to a box of the opposite colour. Since he started and finished at the same box, he had changed the colour of the box an even number of times. Thus, he made an even number of moves.

Problem 3.4. (a) Reduce 4 fractions on the left-hand side to the same denominator $abcd$ and sum them up:

$$\frac{bcd + acd + abd + abc}{abcd} = 1.$$

The numerator of the fraction on the left-hand side is even as a sum of 4 odd numbers. The denominator is odd. Thus, it is impossible for this quotient to be

equal to 1.

(b) Consider a well-known equality: $\frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 1$. Multiply both sides by $\frac{1}{2}$ and add $\frac{1}{2}$. You obtain $a = 2, b = 4, c = 6, d = 12$.

Answer: (a) Yes.

Problem 3.5. Since every number from 1 to 15, in particular 1, appears only once in each row and column, there will be 15 boxes with number 1 written in. Assume that 1 is not written in any box of the main diagonal. We know that the boxes symmetrical to the main diagonal contain equal numbers. Thus, we can divide the boxes containing 1 into pairs. Consequently, the total number of boxes with 1 should be even. it cannot be But 15 is not even. This is a contradiction. Therefore, some box of the main diagonal contains 1. Similar arguments work for 2,...,15.

Problem 3.6. When does the product of integers equal 1? Only when all of them are 1 or -1. Assume that the sum of 26 numbers which are equal to 1 or -1 is zero. Then there should be 13 numbers equal to 1 and 13 numbers equal to -1. But in this case the product is -1. This is a contradiction. Thus, the sum have to be nonzero.

Problem 3.7. (First solution): Denote by x (hours) the time the bus spent on travelling between observers A and B, and by t (hours) the common time interval. From the formulation of the problem we deduce that the motorcycle spent $x - t$ hours on travelling between A and B, and the truck spent $x + t$ hours. Now we can express the speed of the motorcycle and the truck in terms of x and t : $\frac{1}{x-t} = 60$ and $\frac{1}{x+t} = 30$. Solving this system of equations we obtain $x = \frac{1}{40}$. Therefore, the bus speed is $\frac{1}{\frac{1}{40}} = 40$ mph.

(Second solution): Let t be the common time interval between the moments the vehicles pass the observers. The motorcycle reaches the first observer t hours after the truck and reaches the second observer t hours before the truck. Therefore the motorcycle covers the distance between the two observers $2t$ hours faster than the truck does. The motorcycle is travelling twice as fast as the truck so it covers the distance between the observers in half the time. Therefore the truck must take $4t$ hours to cover the distance between the observers while the motorcycle takes $2t$ hours.

Now the bus passes the first observer t hour ahead of the truck and it passes the second observer $2t$ hours ahead of the truck. The truck takes $4t$ hours to go between the observers so the bus takes $3t$ hours to go the same distance. Consequently, the bus is going $\frac{4}{3}$ the speed of the truck or 40 mph.

Answer: 40 mph.