Solutions 6.

Example 1. // First notice that the number 999...999 (n 9's) is obviously divisible by 3. Next, the equality 1000...000=1+999...999, or equivalently $\overline{a000...000} = a \times$ $\times 1000...000 = a \times 999...999 + a$ helps us to rewrite $\overline{a_n a_{n-1}...a_2 a_1 a_0}$ as:

$$\overline{a_n a_{n-1}...a_2 a_1 a_0} = a_n \times 10^n + a_{n-1} \times 10^{n-1} + ... + a_1 \times 10 + a_0 =$$

$$= (a_n \times 999...999 + a_{n-1} \times 999...999 + ... + a_1 \times 9) + (a_n + a_{n-1} + ... + a_1 + a_0)$$

The number in the first bracket is divisible by 3. It means that if the number in the second bracket (which is the sum of the digits of the initial number) is divisible by 3 then the initial number is divisible by 3.

Example 2. Assume that it is possible. Then $n!$ contains at least 4 digits. Therefore n is greater than 3, which means that $n!$ is divisible by 3. But the sum of the digits of our number is $2 + 1 + 5 = 8$. Thus by the divisibility rule for 3 it is not divisible by 3. Here we come to a contradiction.
Answer: No, it's not possible.

Problem 6.1. Repeat the arguments of Example 1 substituting 9 instead of 3.

Problem 6.2. If a number is divisible by 45 then it is divisible by 5 and by 9. Divisibility by 5 implies that the last digit has to be 0. The divisibility by 9 implies that the sum of the digits should be divisible by 9. Combining the arguments we see that the number in question should be 1111111110.
Answer: 1111111110.

Problem 6.3. Denote this number by n and the sum of its digits by m. Then $n = 3 \times m$, so $n \,\vdots\, 3$ and by the divisibility rule, $m \,\vdots\, 3$. The latter means that $n \,\vdots\, 9$, so again by the corresponding divisibility rule $m \,\vdots\, 9$. Thus, we deduce that $n \,\vdots\, 27$.
Answer: Yes, it is.

Problem 6.4. Consider the sum of the digits. It is equal to $2 \times 100 + 1 \times 100 = 300$ and this number is divisible by 3, but not divisible by 9. But if you have $n^2 \,\vdots\, 3$ then $n \,\vdots\, 3$, so n^2 should be divisible by 9 but it's not. This is a contradiction.

Problem 6.5. There are a lot of correct answers. For example, 1379586420. If we cross any 6 digits different from the last 6 digits the last digit of the resulting number will be 0, 2, 4, 6, 8, or 5. By the divisibility rule for 2 or for 5, it will be divisible by 2 or 5, thus it will be composite. And if we cross the last 6 digits the remaining number will be $1379 = 7 \times 197$. So, the result will be a composite number anyway.
Answer: 1379586420.