Problem Sheet 3.

Parity is a mathematical term that describes the property of an integer's inclusion in one of two categories: **even** or **odd**. An even number is said to have even parity, and an odd number, odd parity.

Keep in mind:
(a) A sum of odd number of odd numbers is odd.
(b) A sum of even number of odd numbers is even.
(c) A sum of any number of even numbers is even.

Example 1. Can a 5×5 square checkerboard be covered by 1×2 dominoes?

Example 2. Can a knight start at square a1 of a chessboard, and go to square h8, visiting each of the remaining squares exactly once on the way?

Example 3. The King and Knave of Hearts were playing a game of croquet. The Knave of Hearts went first and made a sensational hit that created a closed trajectory of 9 line segments. It is now the King's turn and he is worried that he cannot possibly match the same sensational hit of the Knave's move. Can he be lucky enough to cross all the 9 segments (of the hit created by the Knave) with one straight hit not passing through the vertices?

A timeless children's book, Alice's Adventures in Wonderland is 150 years old in 2015 and the story of this beguiling tale begins with its creator, Charles Ludwidge Dodgson. Charles was a natural storyteller who regularly invented new stories to entertain his friends. He knew that for the best stories to work, the child must be at the centre of the narrative so that their imagination could be led to marvellous and wonderful places.

Inspired by real events and a real child, Alice's Adventures in Wonderland was dreamt up on a summer's day in Oxford. The river outing with the Liddell family on 4 July 1862, from Folly Bridge to Godstow, is now famous for the Alice story to which it led. Charles L. Dodgson travelled downriver with the Dean of Christ Church, Oxford (the college at which Charles was a lecturer in Mathematics), and his family. Along the way he told the family a story about a bored little girl called Alice who goes looking for an adventure. The family loved it and at the end of that day, the daughter, Alice Liddell, asked for the story to be written down. Charles Dodgson agreed and began writing the manuscript the next day. It eventually took him two and a half years to complete. (taken from http://aliceinwonderland150.com/)

Problem 3.1. The Queen has introduced a new currency in the world of Wonderland. This currency consists of three golden coins with values 3, 5 and 15. Will it be possible for Alice to change an old note with value 100 using 11 new coins?

Problem 3.2. One sunny day Alice met the White Rabbit. The Rabbit told her that he owns a pocket watch which has 11 gears arranged in a chain loop. The rabbit asked Alice if it was possible for all the gears to rotate simultaneously. What is your opinion on this matter? Can all the gears rotate simultaneously?

Problem 3.3. After the Mad Tea-Party, the Hatter was so excited that he decided to cool down by going on a short walk across the chessboard. He started at position a1, then walked around in steps taking each step as if he was a knight, and eventually returned back to a1. Show that he made an even number of steps.

Problem 3.4. (a) Is it possible that odd a, b, c, d satisfy $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} = 1$?
(b) Can you find different natural numbers a, b, c and d satisfying $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} = 1$?

Problem 3.5. The Cheshire Cat wrote one of the numbers $1, 2, \ldots, 15$ into each box of a 15×15 square table in such a way, that boxes which are symmetric to the main diagonal contain equal numbers. Every row and column consists of 15 different numbers. Show that no two numbers along the main diagonal are the same.