Elec4621 Lab2, S1 2019

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1. Two first order digital filters are given by their z-transform transfer functions as:

$$H(z) = 1 - az^{-1}$$

and

$$G(z) = \frac{1}{1 - az^{-1}}$$

In each case, the parameter a satisfies $|a| < 1$.

   (a) Write Matlab code to compute the impulse responses $h[n]$ and $g[n]$. Using $a = \frac{1}{2}$ and $a = -\frac{1}{2}$, compare the results obtained and explain their similarities and differences.

   (b) Starting with the sequence $x[n]$, given in question 1 of lab 1, filter first with $H(z)$ to obtain a sequence $y[n]$. Now apply the second filter $G(z)$ to $y[n]$, recovering a new sequence $w[n]$. In doing this, you should use the fact that $w[n] = 0$ for all $n < 0$. Note that you will need to implement $G(z)$ second filter recursively, which may require you to recall some of the things you learned in Elec3004. Ask the lab demonstrators to help you if there is anything you do not understand – we will be reviewing this material also in lectures, but your lab session may come before the revision material is covered. **In any event, you should not be using Matlab built-in functions to do the filtering for you here.** What is the relationship between $x[n]$ and $w[n]$?

   (c) Write the code necessary to calculate and plot the magnitude responses, $\left|\hat{h}(\omega)\right|$ and $|\hat{g}(\omega)|$.

2. A digital oscillator is to be implemented using the following recursive relationship:

$$y[n] = \alpha y[n-1] - \beta y[n-2]$$

(a) Assuming that $y[n] = 0$ for $n < 0$, determine suitable values of $y[0]$, α and β such that the system will oscillate at a frequency of $f_0 = \frac{\omega_0}{2\pi}$ and have the function $\cos(\omega_0 n + \theta_0)$. [Hint: what are the values of the output at $n = 0$, $n = 1$, $n = 2$, ...? Can you then solve for the required quantities?]

(b) Implement the oscillator in Matlab. Verify that it is working as expected for various values of ω_0 and θ_0. For example, plot the output, $y[n]$, for the case, $\omega_0 = \frac{\pi}{6}$ and $\theta_0 = 0$; $\omega_0 = \frac{\pi}{4}$ and $\theta_0 = \frac{\pi}{6}$; and $\omega_0 = \frac{\pi}{3}$ and $\theta_0 = \frac{2\pi}{3}$.

(c) What should θ_0 and consequently $y[0]$, α and β be in order to obtain the output $\sin(\omega_0 n)$?.

(d) Now suppose that the system difference equation is

$$y[n] = x[n] + \alpha y[n-1] - \beta y[n-2]$$

with an input, $x[n]$, having two non-zero sample values, $x[0]$ and $x[-1]$, get the oscillator to produce the signal

$$y[n] = \sin \omega_0 n$$

exactly.

3. A fifth order all-zero filter has the following transfer function,

$$H(z) = \left(\frac{z - r_1}{z}\right)\left(\frac{z^2 - 2r_2 z\cos\theta_2 + r_2^2}{z^2}\right)\left(\frac{z^2 - 2r_3 z\cos\theta_3 + r_3^2}{z^2}\right)$$

where

$$r_1 = e^{-\frac{1}{8}}, \quad r_2 = r_3 = 0.9, \quad \theta_1 = 0.45\pi, \quad \theta_2 = 0.6\pi, \quad \theta_3 = 0.85\pi$$

(a) Find the impulse response of the filter.

Hint: There are a number of ways to do this: One way is to build a vector, $\mathbf{v}$, containing all six complex-valued zeroes, and then use the Matlab function, 'poly', to find the coefficients of the polynomial with these roots – the coefficients of the polynomial are the filter taps. Another way is to proceed from the second order sections of the transfer function and use polynomial multiplication to build 6-th order numerator. Then you will have the filter taps.

(b) Using Matlab code which you write yourself (as you did in Lab 1), find and plot the magnitude and phase responses of the filter.

Hint: You should remember that the frequency response is obtained by substituting $z = e^{j\omega}$. Therefore you can think of this as the evaluation of the transfer function on a fine grid of ω. Thus, you can construct a vector $\mathbf{w}$ of angular frequencies and evaluate the transfer function for this vector, then plot the amplitude and phase of the result.

(c) By selectively taking reciprocals of the filter's zeros (i.e., replacing r_k by $\frac{1}{r_k}$), you can generate seven other FIR filters, all having 7 taps and exactly the same magnitude response as the filter given above. Write Matlab code to walk through these filters, plotting their magnitude and phase responses in turn. What do you notice about the phase responses, in comparison to that found in part (b) above?

Hint: The implementation of this part is quite simple if you use the code of the last part as a function that takes the poles and zeroes as input.

(d) Examine the impulse responses of all 8 filters (the original and the extra 7 found in part (c)). What special characteristic is exhibited by the impulse response of the original filter?

(e) Now assume that the filter coefficients are to be rounded to some number of decimal places, N_{digits}, write code that plots the location of the zeroes and the resulting magnitude and frequency responses for different values of N_{digits}.