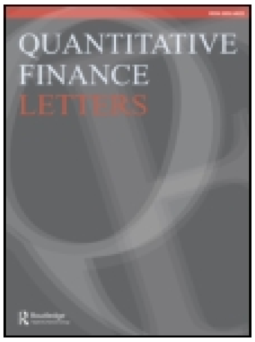

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Asset allocation with risk factors

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In this paper, we propose to build portfolios that offer diversification over so-called 'risk factors' and this within a minimum variance portfolio construction framework. We believe this approach is an important advancement compared with traditional asset allocation as it achieves a higher level of true risk diversification, taking into account the common and unique risk factors that each asset class is exposed to. We apply the methodology to a portfolio invested in European government bonds, corporate bonds, high-yield bonds and equity. The first application consists of an ex post factor risk contribution analysis where we decompose the portfolio risk into the risk associated with the economic activity, inflation, interest rate, exchange rate, credit risk, market risk and idiosyncratic asset class-specific risk factors. In the second application, we construct minimum variance portfolios that satisfy ex ante constraints on the factor risk contributions.

Keywords: Asset allocation; Portfolio risk; Risk factors; Risk contribution

1. Introduction

Risk-based portfolio solutions are portfolio allocation techniques that do not require explicit modelling of expected returns. In asset allocation, the equal risk contribution (ERC) approach is very popular: portfolio weights are dynamically set such that the asset classes contribute equally to the portfolio risk (Boudt *et al*. 2013b). As such, the portfolio loads automatically less on the more risky asset and, by diversifying across asset classes, portfolio drawdowns are reduced.

An important caveat when constructing ERC portfolios is that the power of diversification of the ERC constraint depends on the underlying assets. When those assets are very dependent on underlying common risk factors, the portfolio risk may effectively be very concentrated in a few underlying risk factors. Consider, e.g. an ERC portfolio invested in a bond, equity and convertible bond index. Clearly, the three asset classes have common risk exposures. Depending on the bond/equity exposure of the convertible bond index, an ERC constraint at the asset class level may lead to a concentration of portfolio risk into the underlying bond or equity risk.

We argue that, for asset allocation, not only the risk contribution of the assets, but also the risk contributions of the factors are important elements to consider in the portfolio decision. To identify such sources, we propose the use of a linear factor model that decomposes the risk of an asset class into the exposure to directly observable risk factors explaining the comovement across asset classes and the idiosyncratic asset-specific risk factors that can be identified in the return series of the asset classes.

This parametric set-up corresponds to the related work of Roncalli and Weisang (2012) on risk parity portfolios with risk factors. In order to calculate the factor risk contributions, we then rewrite the portfolio return as an exact linear combination of factors. We follow Zivot (2011) in defining the set of factors as the joint set of risk factors specified by the modeller, on the one hand, and the idiosyncratic asset-specific risk factors corresponding to the unexplained variation in the stock returns, on the other hand. The common risk factors can of course be correlated, while the idiosyncratic risk factors are assumed to be uncorrelated.

Roncalli and Weisang (2012) follow Meucci (2007) in calculating the factor risk contributions in one step. In our view, this comes at the price of tractability. Zivot (2011) proposes a two-step approach of first estimating the exposures by ordinary least squares. Given those exposures, one can then calculate the factor risk contributions in an analogous way as the asset risk contributions are calculated. As such, more tractable analytical expressions are obtained for the factor risk contributions.

Importantly, this two-step framework also allows more flexibility in the choice of estimation methods. In fact, our application is on asset allocation for which macro-economic factors are crucial determinants of the asset returns. Many of these factors can only be observed at a quarterly frequency. Because of the relatively large number of parameters to estimate compared with the number of observations, we recommend not to use the ordinary least squares and sample covariance estimators, but so-called shrinkage estimators in which the estimates are 'de-noised'.

We focus on portfolio standard deviation as a risk measure but, under the assumption of elliptical symmetry or using simulation methods, the approach can be extended to downside risk measures such as value-at-risk and expected shortfall.

In what follows, we first review the methodology of factor risk analysis. We then illustrate the differences between

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asset risk contributions and factor risk contributions for a portfolio invested in EU government bonds, EU corporate bonds, EU high-yield bonds and European equity. We further illustrate the effect on the portfolio weight allocation of constraining the percentage risk contribution of the underlying risk factors. Finally, for all portfolios considered, an out-of-sample return analysis is performed to illustrate the effects of the imposed risk diversification on the portfolio returns.

2. Proposed methodology

Suppose we have N assets with covariance matrix Σ and a portfolio vector $w = (w_1, \ldots, w_N)'$.

The portfolio standard deviation is given by

$$\sigma(w) = \sqrt{w'\Sigma w}.$$

For portfolio risk management purposes, it is important to disentangle the different sources of portfolio risk.

2.1. *Attribution of portfolio volatility to the portfolio assets*

In a first step, we use the Euler decomposition to break down the portfolio volatility into the volatility contributions of each asset and represent these component risk measures as a percentage of total portfolio risk. The percentage volatility risk contribution of the ith asset in the portfolio is given by

$$\%ARC_i = \frac{1}{\sigma(w)}\frac{\partial\sigma(w)}{\partial w_i} = \frac{w_i(\Sigma w)_i}{w'\Sigma w}$$

(see e.g. Boudt *et al.* 2013a).

2.2. *Attribution of portfolio volatility to the underlying risk factors*

The percentage asset contributions to portfolio volatility provide an insight into the distribution of risk across the portfolio assets but, in some regards, it is still superficial as it does not reveal any economic insight in how risk factors drive the portfolio risk. In fact, a common view is to consider that the variation in asset returns is driven by multiple macro-economic factors and idiosyncratic factors that are specific to each asset.

The use of such factor models is now widespread. It has been used by Ross (1976) to derive expected returns under no arbitrage assumptions (the so-called Arbitrage Pricing Theory). Very often, it is used without any further assumptions, as a descriptive tool to inspect the exposures of an investment style.

To introduce the methodology, we first enumerate the assumptions of the linear factor model and the implications it has for the structure of the covariance matrix and the portfolio variance. We then derive the percentage factor risk contributions and discuss its practical implementation.

2.2.1. The linear factor model. Suppose that K observable factors are identified as being influential for the portfolio variability. At a given frequency (e.g. monthly or quarterly), the asset returns $r_t = (r_{1t}, \ldots, r_{Nt})'$ and the factors $f_t = (f_{1t}, \ldots, f_{Kt})'$ are recorded.

The asset returns are assumed to depend linearly on the factors, whereby the variation in the asset returns that is not explained by the factors is assumed to be uncorrelated with each of the factors and also to be uncorrelated across assets. The linear approach leads to the following system of equations:

$$\begin{pmatrix} r_{1t} \\ r_{2t} \\ \vdots \\ r_{Nt} \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{pmatrix} + \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1K} \\ b_{21} & b_{22} & \cdots & b_{2K} \\ & & \ddots & \vdots \\ b_{N1} & b_{N2} & \cdots & b_{NK} \end{pmatrix} \begin{pmatrix} f_{1t} \\ f_{2t} \\ \vdots \\ f_{Kt} \end{pmatrix}$$
$$+ \begin{pmatrix} \sigma_1 & 0 & \cdots & 0 \\ 0 & \sigma_2 & \cdots & 0 \\ & & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_N \end{pmatrix} \begin{pmatrix} e_{1t} \\ e_2 t \\ \vdots \\ e_{Nt} \end{pmatrix}.$$

The covariance matrix of $e_t = (e_{1t}, \ldots, e_{Nt})'$ is the identity matrix and e_t has mean zero. In matrix notation, the system is given by

$$r_t = a + Bf_t + De_t.$$

Let S be the $K \times K$ covariance matrix of the K factors. Because of the assumption that the unexplained asset return variation e_t is uncorrelated with the factors, we can rewrite Σ (the $N \times N$ covariance matrix of the Nasset returns) as

$$\Sigma = BSB' + D.$$

Our interest is in explaining the portfolio return. Premultiplying the asset return by the portfolio weight vector gives us the portfolio return

$$\begin{aligned} w'r_t &= w'a + w'Bf_t + w'De_t, \\ &= \alpha + \beta' f_t + \delta' e_t, \end{aligned}$$

with $\beta = w'B$, the $K \times 1$ row-matrix of exposure of the portfolio return to each of the factors and with $\delta = w'D$, the $K \times 1$ row-matrix of exposure of the portfolio return to each of the asset-specific factors. Following Zivot (2011), we join these two exposures in the vector γ of size $K + N$:

$$w'r_t = \alpha + \begin{pmatrix} \beta \\ \delta \end{pmatrix}' \begin{pmatrix} f_t \\ e_t \end{pmatrix} = \alpha + \gamma' \begin{pmatrix} f_t \\ e_t \end{pmatrix}.$$

The joint covariance matrix of f_t and e_t is

$$\Theta = \begin{pmatrix} S_{11} & S_{12} & \cdots & S_{1K} & 0 & 0 & \cdots & 0 \\ S_{12} & S_{22} & \cdots & S_{2K} & 0 & 0 & \cdots & 0 \\ & & \ddots & & & & \ddots & \\ S_{1K} & S_{2K} & \cdots & S_{KK} & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 0 & 1 & \cdots & 0 \\ & & \ddots & & & & \ddots & \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 1 \end{pmatrix}.$$

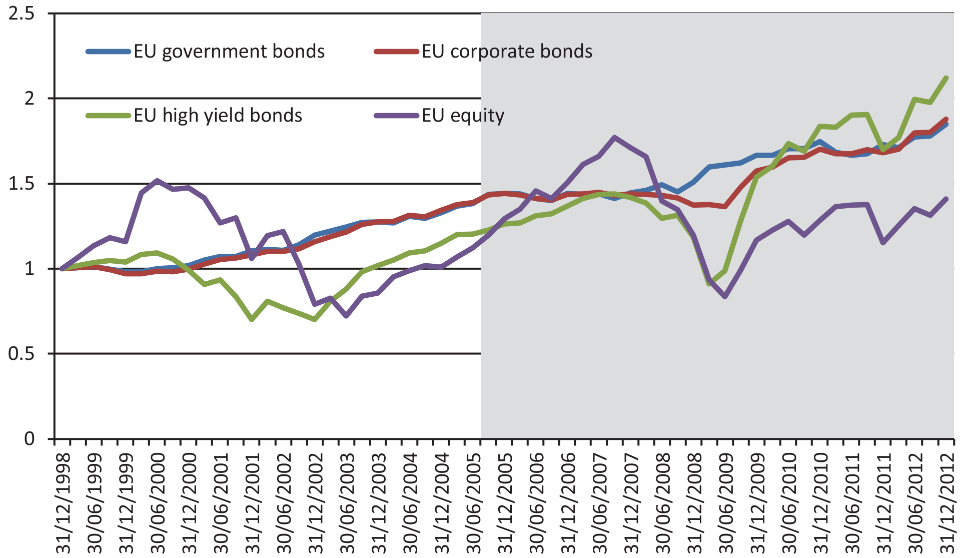

Figure 1. Quarterly cumulative value of the EU government bond, corporate bond, high-yield bond and equity index low risk, value and ERC low-risk value portfolio versus the market portfolio for a global universe over the period December 1998–December 2012. The grey area indicates the out-of-sample evaluation period.

2.2.2. Percentage factor risk contributions under the linear factor model. Under the linear factor model, the portfolio volatility can thus be written as

$$\sigma(\gamma) = \sqrt{\gamma' \Theta \gamma}.$$

In an analogous way to the percentage volatility contribution of the ith asset, the percentage volatility risk contribution of the ith factor is given by

$$\%FRC_i = \frac{1}{\sigma(\gamma)} \frac{\partial \sigma(\gamma)}{\partial \gamma_i} == \frac{\gamma_i(\Theta\gamma)_i}{\gamma' \Theta \gamma}.$$

Because of the one-homogeneity of $\sigma(\gamma)$ (i.e. the property that $\sigma(k\gamma) = k\sigma(\gamma)$), the percentage factor risk contributions add up to one.

2.2.3. Implementation. For the effective calculation of the percentage risk contributions at the level of the individual assets and factors, we need to estimate the covariance matrix of the asset returns (Σ), the covariance matrix of the factors (S) and the factor exposures (γ). The traditional approach is to use the sample covariance estimator and the ordinary least-squares estimate. But, as mentioned in the introduction, in our set-up, we typically have a large number of parameters to estimate and a small number of observations. In such cases, the sample covariance matrix and least-squares estimates are known to be unreliable and shrinkage estimators perform better.

Several shrinkage methods exist. We will use the covariance shrinkage estimator of Ledoit and Wolf (2003) and base all our estimates on the Ledoit–Wolff shrinkage estimate of the covariance matrix of the asset return and factors, jointly. The factor exposures implied by this estimated covariance matrix are obtained following Engle (2012).

3. Applications in risk monitoring and portfolio allocations

The proposed methodology has important applications in monitoring the portfolio risk as well as in the design of optimal portfolios. Next, we illustrate both applications for the universe of European government bonds, corporate bonds, high-yield bonds and equity. The data source is Bloomberg.[†]

As some of the macro-factors that we will consider are only available at a quarterly frequency, we consider quarterly rebalancing of the portfolio.

The cumulative return evolution of each of the asset classes over the period 1999–2012 is shown in figure 1. Besides the differences in volatility and return over the period, the graph clearly shows the diversification potential across the different investment styles.

The shaded area in figure 1 corresponds to the out-of-sample evaluation period used to compare the different portfolio allocation strategies. Panel 1 of table 1 summarises the return performance of the four asset classes over this period. Over this period, the worst performing asset class in all aspects is EU equity with an average annualised return of 1.8%, a volatility of 16.8% and a maximum drawdown of 54%. The corporate and government bond indices perform similarly with returns of 5% and a standard deviation between 4% and 5%. The high-yield bond index has over the period the highest return (9.3%) with an annualised volatility of 15%.

We consider 13 economic factors that we have grouped into 6 categories:

1. Activity: EU GDP growth, industrial production growth and the economic sentiment index as published by the European Commission.

[†] For the asset classes, the index names are Bloomberg/EFFAS Bond Indices (EUGATR), IBOXX € CRP OA TR (EU government bonds), Pan-European High Yield and MSCI EUROPE NR.

Table 1. Monthly returns analysis of single-asset strategies and dynamically rebalanced asset allocation portfolios over the period August 2006–December 2012.

	Annualised return	Annualised standard deviation	Sharpe ratio (RF = 0)	Max drawdown
Panel 1: Single-asset class strategies				
Government bonds	4.8%	4.3%	1.114	−6.7%
Corporate bonds	5.0%	4.7%	1.058	−8.0%
High-yield bonds	9.3%	15.0%	0.618	−37.6%
Equity	1.8%	16.8%	0.106	−54.1%
Panel 2: Standard dynamically rebalanced portfolios				
Equally weighted	5.2%	8.2%	0.638	−23.0%
Equal risk contribution	4.5%	4.7%	0.954	−10.0%
Minimum variance	4.6%	4.1%	1.137	−5.4%
Panel 3: Risk factor constrained minimum variance portfolios				
RFCP1	3.5%	5.9%	0.593	−17.8%
RFCP2	5.1%	4.3%	1.183	−6.4%
RFCP3	4.8%	4.7%	1.040	−8.6%

2. Inflation: consumer prices and commodity prices.
3. Interest rate: real interest rate and slope of the yield curve.
4. Currency: percentage changes real effective exchange rate.
5. Credit risk: EU corporate Baa-AA bonds spread, US corporate Baa-AA bonds spread, TED spread.
6. Market risk: implied volatility S&P 500 (VIX) and DAX.

The first four categories are also considered in the risk factor analysis of Roncalli and Weisang (2012). The data frequency used is quarterly.

3.1. *Application 1: risk monitoring and ex post portfolio risk analysis*

The first application of factor risk budgets is the ex post analysis of the portfolio risk concentration. The risk analysis is done for the following three portfolios:

- The equally weighted portfolio: each of the four assets is attributed a 25% portfolio weight.
- The ERC portfolio: each asset contributes to 25% of the portfolio risk.
- The minimum variance portfolio: portfolio weights are such that the portfolio variance is minimal, under the constraint of full investment and no short sales.

The covariance matrix is estimated on rolling samples of 24 (quarterly) observations. The out-of-sample analysis period corresponds to August 2006–December 2012 and portfolios are rebalanced on a quarterly frequency.

The average portfolio weights over this period are reported in table 2. Note that the ERC and minimum variance portfolios are concentrated in government bonds and have a relatively lower allocation to the more risky high-yield bonds and the equity index.

Panel 2 of table 1 shows the portfolio performance of these three dynamic asset allocation strategies, compared with the single-asset alternatives in Panel 1 of table 1. The differences in portfolio weights translate directly in the portfolio volatility, whereby the (annualised) volatility of the ERC portfolio (4.7%) is in between the volatility of the minimum variance portfolio (4.1%) and the equally weighted portfolio (8.2%). The annualised return of the three portfolios is around 5%.

Table 2. Average quarterly weight allocation of equally weighted, ERC and minimum variance portfolio invested in EU government bonds, corporate bonds, high-yield bonds and equity over the period August 2006–December 2012. Portfolios are rebalanced quarterly.

	Equally weighted	Equal risk contribution	Minimum variance
Government bonds	.25	.50	.52
Corporate bonds	.25	.32	.44
High-yield bonds	.25	.10	.01
Equity	.25	.08	.03

Let us now focus on the key question, namely the attribution of the portfolio volatility to the percentage volatility caused by the assets (yielding the percentage asset risk contributions) and the distribution of the percentage volatility across the different risk factors (yielding the percentage factor risk contributions).

The percentage volatility contributions of each asset are reported in table 3. Comparing the ERC and minimum variance portfolios, we see that increasing the weight of the high-yield bonds from 1% to 10% and of equity from 3% to 8% has a large impact on the percentage volatility contributions of those assets, which increase sharply from 1% and 3% to 25%, respectively.

The percentage volatility contribution of each factor is shown in table 4.

As could be intuitively expected, the volatility of the equally weighted portfolio being 25% invested in all assets is explained by all factors, except currency. The idiosyncratic government and corporate bonds factors have negligible impact, in contrast with the idiosyncratic high-yield bonds and equity factors that explain 13% and 16% of the volatility of the equally weighted portfolio.

The ERC and the minimum variance portfolio have a much lower exposure to the European high-yield bond and equity asset classes. As a consequence, these portfolios have

Table 3. Average quarterly asset-based risk allocation of equally weighted, ERC and minimum variance portfolio invested in EU government bonds, corporate bonds, high-yield bonds and equity over the period August 2006–December 2012. Portfolios are rebalanced quarterly.

	Equally weighted	Equal risk contribution	Minimum variance
Government bonds	−.01	.25	.52
Corporate bonds	.06	.25	.44
High-yield bonds	.42	.25	.01
Equity	.53	.25	.03

Table 4. Average quarterly factor-based risk allocation of equally weighted, ERC and minimum variance portfolio invested in EU government bonds, corporate bonds, high-yield bonds and equity over the period August 2006–December 2012. Portfolios are rebalanced quarterly.

	Equally weighted	Equal risk contribution	Minimum variance
Activity	.20	.19	.11
Inflation	.12	.12	.19
Interest rate	.11	.14	.10
Currency	.00	.00	.00
Credit risk	.16	.09	.07
Market risk	.10	.03	.01
Percentage risk contribution by all macro-financial risk factors	69%	57%	48%
Idiosyncratic government bonds factor	.01	.18	.27
Idiosyncratic corporate bonds factor	.01	.08	.22
Idiosyncratic high-yield bonds factor	.13	.08	.01
Idiosyncratic equity factor	.16	.09	.02
Percentage risk contribution by all idiosyncratic risk factors	31%	43%	52%

a higher exposure to the idiosyncratic government and corporate bond factors. In particular, for the ERC portfolio, 18% and 8% of portfolio volatility are explained by the government and corporate bonds factors, and for the minimum variance portfolios, these factors explain 27% and 22%, respectively.

For all portfolios, the economic activity, inflation and interest rate factors are the three most important macro-economic contributors to the portfolio volatility. Jointly, the macro-economic factors explain 69% of the volatility of the equally weighted portfolio, 57% of the volatility of the ERC portfolio and 48% of the minimum variance portfolio volatility.

3.2. *Application 2: ex ante factor risk constraints in portfolio allocation*

An important shortcoming of the minimum variance portfolio in this application is that only three factors are responsible for 68% of the total portfolio volatility: inflation (19%), idiosyncratic government bonds factor (27%) and the idiosyncratic corporate bonds factor (22%). We now investigate the empirical properties of implementing factor risk budgets that restrict ex ante the risk contributions of the different factors. To illustrate this, we consider the following risk factor constrained minimum variance portfolios:

- [RFCP1] Minimum variance portfolio under the constraint that the maximum percentage factor risk contribution is less than 20%.
- [RFCP2] Minimum variance portfolio under the constraint that the maximum percentage idiosyncratic factor risk contribution is less than 20%.
- [RFCP3] Minimum variance portfolio under the constraint that the maximum asset return percentage contribution is less than 30% and the maximum percentage factor risk contribution is less than 20%.

Tables 5–7 show the corresponding weight and risk allocations, while the out-of-sample return performance is in Panel 3 of table 1.

Let us first consider the RFCP1 portfolio. Remember that the minimum variance portfolio without risk factor constraints is 52% invested in government bonds and 44% in corporate bonds. By imposing that the maximum percentage factor risk contribution should be less than 20%, an important diversification is achieved, at the portfolio weight level (44% is invested in corporate bonds and 33% in government bonds), the asset risk contribution level (the maximum risk contribution drops from 52% to 44%) and the factor risk contribution, where the upper bound of 30% is clearly binding. This increase in diversification is accompanied by a decrease in return (4.6–3.5%), an increase in volatility (4.1–5.9%) and an important increase in the maximum drawdown (−5.4% to −17.8%). Overall, it seems that the maximum 20% constraint on all risk factors is too

Table 5. Average quarterly weight allocation of risk factor-constrained portfolios invested in EU government bonds, corporate bonds, high-yield bonds and equity over the period August 2006–December 2012. Portfolios are rebalanced quarterly.

	RFCP1	RFCP2	RFCP3
Government bonds	.44	.47	.48
Corporate bonds	.33	.44	.35
High-yield bonds	.09	.04	.08
Equity	.14	.05	.09

Table 6. Average asset-based risk allocation of risk factor constrained portfolios invested in EU government bonds, corporate bonds, high-yield bonds and equity over the period August 2006–December 2012. Portfolios are rebalanced quarterly.

	RFCP1	RFCP2	RFCP3
Government bonds	.17	.35	.23
Corporate bonds	.30	.46	.27
High-yield bonds	.14	.10	.23
Equity	.38	.10	.27

Table 7. Average factor-based risk allocation of equally weighted, ERC and minimum variance portfolio invested in EU government bonds, corporate bonds, high-yield bonds and equity over the period August 2006–December 2012. Portfolios are rebalanced quarterly.

	RFCP1	RFCP2	RFCP3
Activity	.16	.15	.19
Inflation	.10	.15	.12
Interest rate	.12	.12	.14
Currency	.00	.00	.00
Credit risk	.12	.09	.09
Market risk	.06	.02	.03
Percentage risk contribution by all macro-financial risk factors	56%	53%	57%
Idiosyncratic government bonds factor	.13	.20	.17
Idiosyncratic corporate bonds factor	.13	.20	.10
Idiosyncratic high-yield bonds factor	.05	.03	.06
Idiosyncratic equity factor	.13	.04	.10
Percentage risk contribution by all idiosyncratic risk factors	44%	47%	43%

restrictive and does not strike a balance between the objectives of high return, low risk and high diversification across both assets and risk factors.

In the RFCP2 portfolio, the maximum 20% percentage factor risk contribution constraint is only imposed on the idiosyncratic factors and the resulting portfolio weight and risk allocation are in between the unconstrained minimum variance portfolio and the RFCP1 portfolio. The maximum 20% bound constraint is binding for the idiosyncratic government and corporate bond factors. The RFCP2 portfolio has a substantially higher Sharpe ratio (1.183) compared with the Sharpe ratio of the minimum variance portfolio (1.137) and a comparable maximum drawdown. The RFCP2 portfolio risk is well diversified across all risk factors, but at the asset class level, the RFCP2 portfolio risk is concentrated in the corporate (46%) and government bonds (35%). Imposing only constraints on the risk contributions of the risk factors, therefore, fails in guaranteeing a sufficient risk diversification across the asset classes.

We therefore consider as a final design the RFCP3 portfolio that has constraints on the risk contribution of the assets and the risk factors. More precisely, the RFCP3 portfolio combines a maximum 20% percentage factor risk contribution constraint on the idiosyncratic factors with a maximum 30% percentage risk contribution on each of the assets. Compared with the RFCP2 portfolio, the RFCP3 portfolio loads significantly less on the corporate bond (portfolio weight reduces from 44% to 35%) and more on high-yield bonds (8% instead of 4%) and equity (9% instead of 5%). Its performance in terms of return and risk is comparable (and even slightly better) to the performance of the ERC portfolio.

4. Conclusions

A mere analysis of the component risk contributions of the portfolio assets is insufficient to uncover the risk factor concentrations of the portfolio. The decision on the risk the portfolio manager is willing to take should be done at different aggregation levels, among which the level of the asset and factor percentage risk contributions.

Based on Meucci (2007) and Zivot (2011), we consider a methodology to do so in a computationally simple and transparent way. The proposed two-step approach combines shrinkage estimation of the exposures and covariance matrices, with the usual percentage risk calculation based on the Euler expansion.

We apply the methodology to the case of a diversified asset allocation portfolio invested in EU government bonds, EU corporate bonds, EU high-yield bonds and European equity. We first illustrate the usefulness of the tool for ex post risk analysis and then analyse the impact of ex-ante risk factor constraints on the portfolio allocation. In our application, it is possible to achieve simultaneously a high diversification at the asset level and risk factor level, while still offering comparable (and even slightly better) performance than the ERC approach that allocates the portfolio risk equally across asset classes.

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