Adventures in Bayesian Structural Time Series

Part 2: Structural Time Series

Andrew Bates, Josh Gloyd, Tyler Tucker

Outline

- Structural Time Series Models:

Outline

- Structural Time Series Models:
  - Local level model

Outline

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- Structural Time Series Models:
  - Local level model
  - Local linear trend model

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  - Local level model
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  - Models with a seasonal component

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- Structural Time Series Models:
  - Local level model
  - Local linear trend model
  - Models with a seasonal component
  - Models with a regression component

Structural Time Series

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- Also called State Space Models

Structural Time Series

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- Data comes from unobserved variable called the **state space**

Structural Time Series

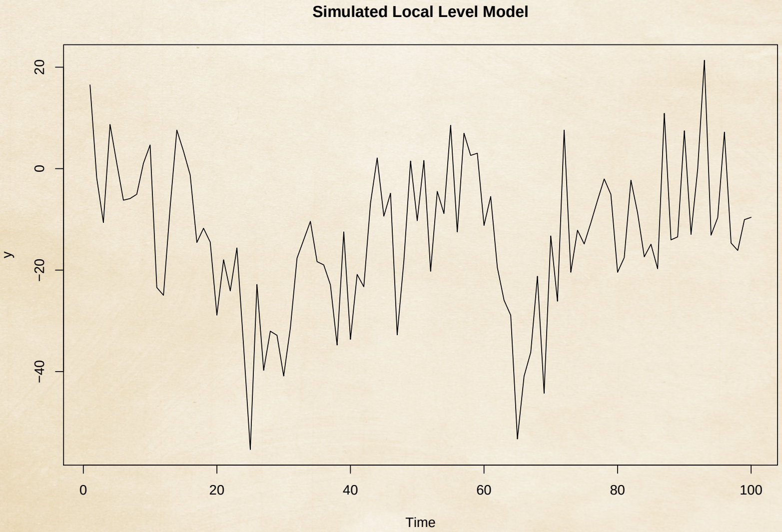
- Also called State Space Models
- Data comes from unobserved variable called the **state space**
- We model the state space instead of the observed data directly

Local Level Model

- y_t: observed data
- μ_t: unobserved state
-

$$\begin{aligned} y_t &= \mu_t + \varepsilon_t & \qquad \varepsilon_t &\sim N(0, \sigma_\varepsilon^2) \\ \mu_{t+1} &= \mu_t + \xi_t & \qquad \xi_t &\sim N(0, \sigma_\xi^2) \end{aligned}$$

Local Level Model



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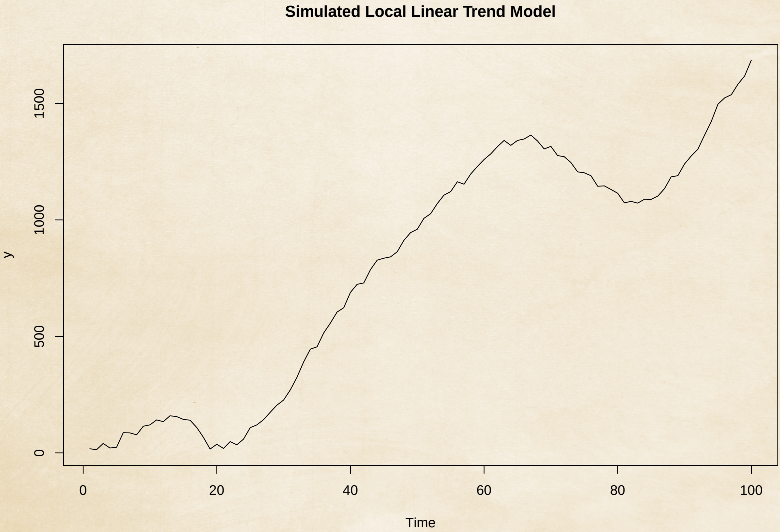
Local Linear Trend Model

- y_t, μ_t: same as before
- ν_t: slope (additional state component)
-

$$
\begin{aligned}
y_t &= \mu_t + \varepsilon_t & \varepsilon_t &\sim N(0, \sigma_\varepsilon^2) \\
\mu_{t+1} &= \mu_t + \nu_t + \xi_t & \xi_t &\sim N(0, \sigma_\xi^2) \\
\nu_{t+1} &= \nu_t + \zeta_t & \zeta_t &\sim N(0, \sigma_\zeta^2)
\end{aligned}
$$

Local Linear Trend Model



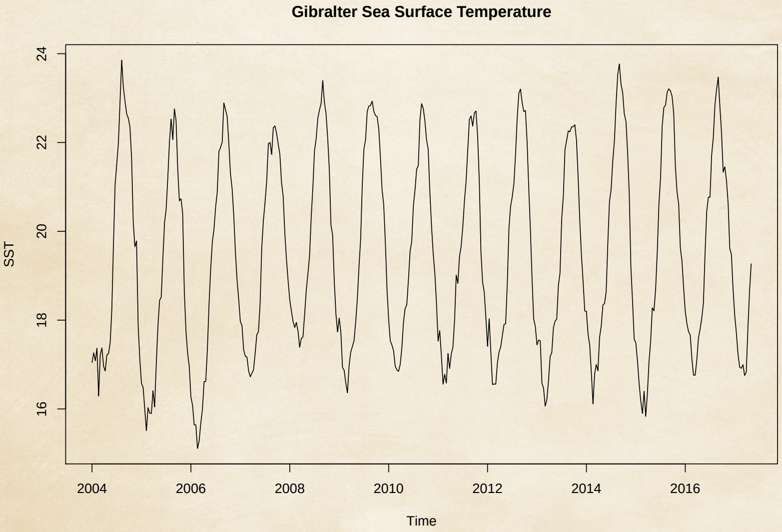
Local Trend With Seasonality

- μ_t: local linear trend
- τ_t: seasonal component
    - S dummy variables (1 for each season)
-

$$y_t = \mu_t + \tau_t + \varepsilon_t \qquad \zeta_t \sim N(0, \sigma_\varepsilon^2)$$

$$\tau_t = -\sum_{s=1}^{S-1} \tau_{t-s} + \omega_t \qquad \tau_t \sim N(0, \sigma_\omega^2)$$

Local Trend With Seasonality



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Local Trend With Seasonality and Regression

- μ_t: local linear trend
- τ_t: seasonal component
- $\beta^T x_t$: regression component
- $$y_t = \mu_t + \tau_t + \beta^T x_t + \varepsilon_t \qquad \varepsilon_t \sim N(0, \sigma_\varepsilon^2)$$

Structural Time Series

- (1): **observation equation**
- (2): **transition equation**

General Form

- y_t: data
- α_t: state variable
-

$$
\begin{aligned}
y_t &= Z_t'\alpha_t + \varepsilon_t & \varepsilon_t &\sim N(0, H_t) & (1)\\
\alpha_{t+1} &= T_t\alpha_t + R_t\eta_t & \eta_t &\sim N(0, Q_t) & (2)
\end{aligned}
$$