Adventures in Bayesian Structural Time Series

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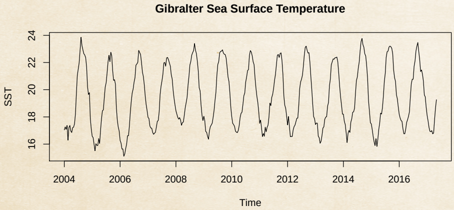
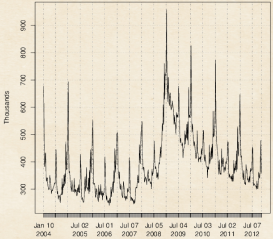
What are Time Series?





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  by Steven L. Scott and Hal Varian (Google)

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  - Spike and Slab regression
  - Bayesian model averaging

Structural Time Series

- Data from unobserved **state space** plus noise
- Model the latent state space instead of the data directly

Local Level Model

- y_t: data
- μ_t: latent state

$$
\begin{aligned}
y_t &= \mu_t + \varepsilon_t & \varepsilon_t &\sim N(0, \sigma^2_\varepsilon) \\
\mu_{t+1} &= \mu_t + \xi_t & \xi_t &\sim N(0, \sigma^2_\xi)
\end{aligned}
$$

Analogous to the inercept in linear regression but allowing for the intercept to vary over time

Local Linear Trend Model

- y_t, μ_t: same as before
- ν_t: slope (additional state component)

$$
\begin{aligned}
y_t &= \mu_t + \varepsilon_t & \varepsilon_t &\sim N(0, \sigma_\varepsilon^2) \\
\mu_{t+1} &= \mu_t + \nu_t + \xi_t & \xi_t &\sim N(0, \sigma_\xi^2) \\
\nu_{t+1} &= \nu_t + \zeta_t & \zeta_t &\sim N(0, \sigma_\zeta^2)
\end{aligned}
$$

Structural Time Series

General Form

- y_t: data
- α_t: state component

$$
\begin{aligned}
y_t &= Z_t' \alpha_t + \varepsilon_t & \varepsilon_t &\sim N(0, H_t) & (1)\\
\alpha_{t+1} &= T_t \alpha_t + R_t \eta_t & \eta_t &\sim N(0, Q_t) & (2)
\end{aligned}
$$

- (1): **observation equation**
- (2): **transition equation**

Bayesian Context

- Spike and slab regression
  - Used when regression components are included
  - Variable selection technique
  - Prior on regression coefficients
- Bayesian Model Averaging
  - Consequence of spike and slab prior
  - Different βs included in each draw of posterior (i.e. different model on each draw)
- Prior Elicitation and Posterior Sampling
  - Inclusion probabilities for regression coefficients
  - Or: expected model size, expected R^2, weight given to R^2
  - Gibbs sampler (stochastic search variable selection) to draw from posterior
  - For details see paper by Scott and Varian

Joint Prior

- $\gamma_k = 1$ if $\beta_k \neq 0$
- $\gamma_k = 0$ if $\beta_k = 0$

$$p(\beta, \gamma, \sigma_\epsilon^2) = p(\beta_\gamma | \gamma, \sigma_\epsilon^2) p(\sigma_\epsilon^2 | \gamma) p(\gamma)$$

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"Spike" Prior

- π_k: probability a particular β_k is included in the model

$$\gamma \sim \prod_{k=1}^{K} \pi_k^{\gamma_k}(1 - \pi_k)^{1-\gamma_k}$$

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"Slab" Prior

- b_γ: b is a vector of prior means
- Ω_γ^{-1}: denotes the rows and columns of symmetric matrix Ω^{-1} corresponding to $\gamma_k = 1$
- ss: prior sum of squares
- ν: prior sample size

$$\beta_\gamma | \sigma_\epsilon^2, \gamma \sim N\Big(b_\gamma, \sigma_\epsilon^2 (\Omega_\gamma^{-1})^{-1}\Big) \qquad \frac{1}{\sigma_\epsilon^2} | \gamma \sim Ga\Big(\frac{\nu}{2}, \frac{ss}{2}\Big)$$