



Adventures in Bayesian Structural Time Series

Part 2: Structural Time Series

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⊠ Structural Time Series Models:



⊠ Structural Time Series Models:

- ⊠ Local level model



⊠ Structural Time Series Models:

- ⊠ Local level model
- ⊠ Local linear trend model



- ⊗ Structural Time Series Models:
 - ⊗ Local level model
 - ⊗ Local linear trend model
 - ⊗ Models with seasonal component



⊠ Structural Time Series Models:

- ⊠ Local level model
- ⊠ Local linear trend model
- ⊠ Models with seasonal component
- ⊠ Models with regression component



⬠ Also called State Space Models



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- ⊠ Data comes from unobserved variable called the **state space**



- ⊠ Also called State Space Models
- ⊠ Data comes from unobserved variable called the **state space**
- ⊠ We model the state space instead of the observed data directly



Local Level Model

⬠ y_t : observed data



Local Level Model

- ⊠ y_t : observed data
- ⊠ μ_t : unobserved state



Local Level Model

⬢ y_t : observed data

⬢ μ_t : unobserved state



$$y_t = \mu_t + \varepsilon_t$$

$$\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$$

$$\mu_{t+1} = \mu_t + \xi_t$$

$$\xi_t \sim N(0, \sigma_\xi^2)$$



Simulated Local Level Model





Local Linear Trend Model

⊠ y_t, μ_t : same as before



Local Linear Trend Model

- ⊠ y_t, μ_t : same as before
- ⊠ ν_t : slope (additional state component)



Local Linear Trend Model

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$$y_t = \mu_t + \varepsilon_t$$

$$\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$$

$$\mu_{t+1} = \mu_t + \nu_t + \xi_t$$

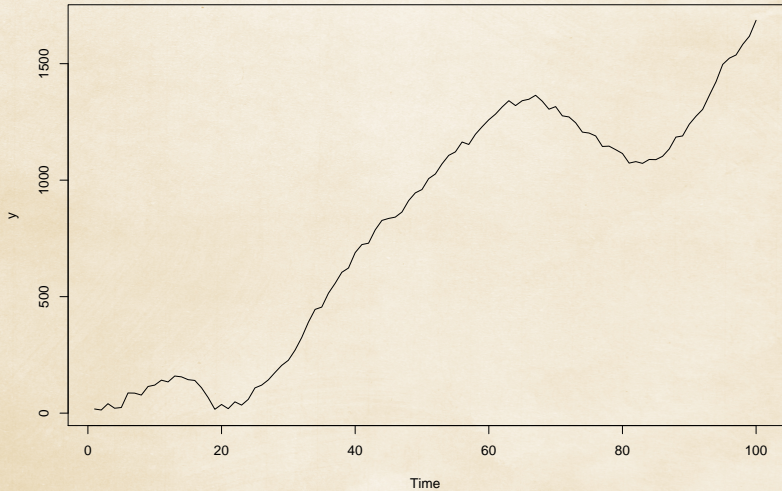
$$\xi_t \sim N(0, \sigma_\xi^2)$$

$$\nu_{t+1} = \nu_t + \zeta_t$$

$$\zeta_t \sim N(0, \sigma_\zeta^2)$$



Simulated Local Linear Trend Model





Local Trend With Seasonality

⊠ μ_t : local linear trend



Local Trend With Seasonality

- ⊠ μ_t : local linear trend
- ⊠ τ_t : seasonal component



Local Trend With Seasonality

- ⊠ μ_t : local linear trend
- ⊠ τ_t : seasonal component
 - ⊠ S dummy variables (1 for each season)



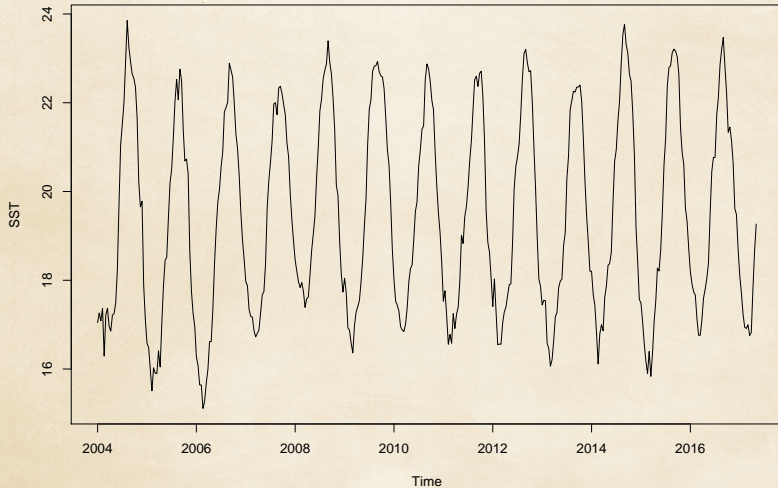
Local Trend With Seasonality

- ⊠ μ_t : local linear trend
- ⊠ τ_t : seasonal component
 - ⊠ S dummy variables (1 for each season)
- ⊠

$$y_t = \mu_t + \tau_t + \varepsilon_t \qquad \zeta_t \sim N(0, \sigma_\varepsilon^2)$$
$$\tau_t = \sum_{s=1}^{S-1} \tau_{t-s} + \omega_t \qquad \tau_t \sim N(0, \sigma_\omega^2)$$



Gibraltar Sea Surface Temperature





Local Trend With Seasonality and Regression

⊗ μ_t : local linear trend



Local Trend With Seasonality and Regression

- ⊠ μ_t : local linear trend
- ⊠ τ_t : seasonal component



Local Trend With Seasonality and Regression

- ⊠ μ_t : local linear trend
- ⊠ τ_t : seasonal component
- ⊠ $\beta_t^T x_t$: regression component



Local Trend With Seasonality and Regression

- ⊠ μ_t : local linear trend
- ⊠ τ_t : seasonal component
- ⊠ $\beta_t^T x_t$: regression component
- ⊠

$$y_t = \mu_t + \tau_t + \beta_t^T x_t + \varepsilon_t \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2)$$



⊠ (1): **observation equation**

General Form



- ⊠ (1): **observation equation**
- ⊠ (2): **transition equation**

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General Form

- ⊠ y_t : data



- ⬢ (1): **observation equation**
- ⬢ (2): **transition equation**

General Form

- ⬢ y_t : data
- ⬢ α_t : state variable



- ⊠ (1): **observation equation**
- ⊠ (2): **transition equation**

General Form

- ⊠ y_t : data
- ⊠ α_t : state variable
- ⊠

$$y_t = Z_t' \alpha_t + \varepsilon_t \quad \varepsilon_t \sim N(0, H_t) \quad (1)$$

$$\alpha_{t+1} = T_t \alpha_t + R_t \eta_t \quad \eta_t \sim N(0, Q_t) \quad (2)$$