Adventures in Bayesian Structural Time Series

Part 2: Structural Time Series

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Outline

- Structural Time Series Models:

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  - Local level model

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  - Local linear trend model

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    - Models with seasonal component

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- Structural Time Series Models:
  - Local level model
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  - Models with regression component

Structural Time Series

- Also called State Space Models

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- Data comes from unobserved variable called the **state space**

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- Data comes from unobserved variable called the **state space**
- We model the state space instead of the observed data directly

Local Level Model

- y_t: observed data

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Local Level Model

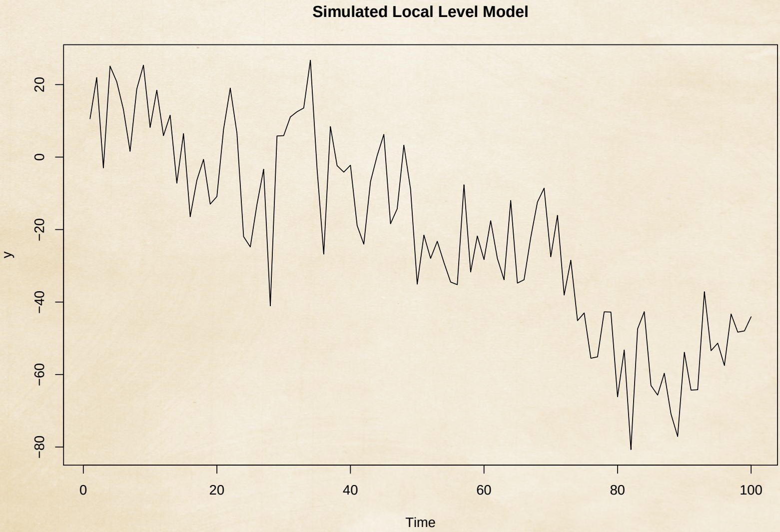
- y_t: observed data
- μ_t: unobserved state

Structural Time Series

Local Level Model

- y_t: observed data
- μ_t: unobserved state
- $$\begin{aligned} y_t &= \mu_t + \varepsilon_t & \varepsilon_t &\sim N(0, \sigma_\varepsilon^2) \\ \mu_{t+1} &= \mu_t + \xi_t & \xi_t &\sim N(0, \sigma_\xi^2) \end{aligned}$$

Local Level Model



Local Linear Trend Model

- y_t, μ_t: same as before

Local Linear Trend Model

- y_t, μ_t: same as before
- ν_t: slope (additional state component)

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Local Linear Trend Model

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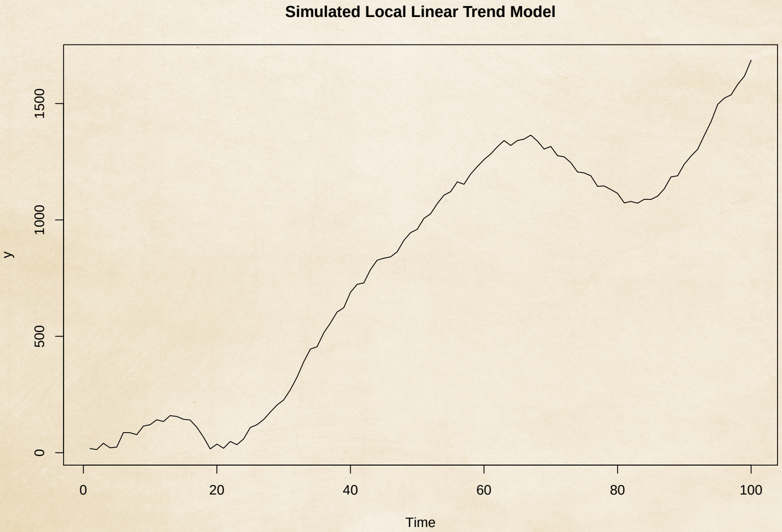
- ν_t: slope (additional state component)
-

$$
\begin{aligned}
y_t &= \mu_t + \varepsilon_t & \varepsilon_t &\sim N(0, \sigma_\varepsilon^2) \\
\mu_{t+1} &= \mu_t + \nu_t + \xi_t & \xi_t &\sim N(0, \sigma_\xi^2) \\
\nu_{t+1} &= \nu_t + \zeta_t & \zeta_t &\sim N(0, \sigma_\zeta^2)
\end{aligned}
$$

Local Linear Trend Model



Local Trend With Seasonality

- μ_t: local linear trend

Local Trend With Seasonality

- μ_t: local linear trend
- τ_t: seasonal component

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Local Trend With Seasonality

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  - S dummy variables (1 for each season)

Local Trend With Seasonality

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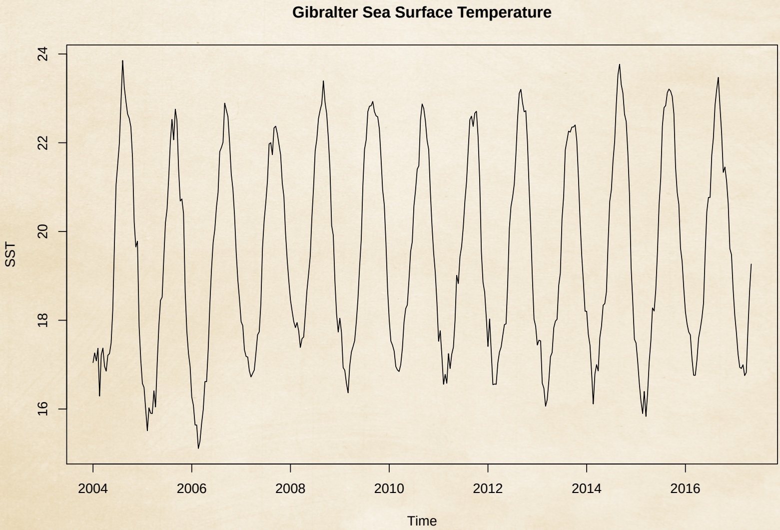
$$y_t = \mu_t + \tau_t + \varepsilon_t \qquad \zeta_t \sim N(0, \sigma_\varepsilon^2)$$

$$\tau_t = \sum_{s=1}^{S-1} \tau_{t-s} + \omega_t \qquad \tau_t \sim N(0, \sigma_\omega^2)$$

Local Trend With Seasonality



Local Trend With Seasonality and Regression

- μ_t: local linear trend

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Local Trend With Seasonality and Regression

- μ_t: local linear trend
- τ_t: seasonal component

Local Trend With Seasonality and Regression

- μ_t: local linear trend
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- $\beta_t^T x_t$: regression component

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Local Trend With Seasonality and Regression

- μ_t: local linear trend
- τ_t: seasonal component
- $\beta_t^T x_t$: regression component
- $$y_t = \mu_t + \tau_t + \beta_t^T x_t + \varepsilon_t \qquad \varepsilon_t \sim N(0, \sigma_\varepsilon^2)$$

- (1): **observation equation**

General Form

- (1): **observation equation**
- (2): **transition equation**

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General Form

- y_t: data

Structural Time Series

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General Form

- y_t: data
- α_t: state variable

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- (1): **observation equation**
- (2): **transition equation**

General Form

- y_t: data
- α_t: state variable
-

$$y_t = Z_t' \alpha_t + \varepsilon_t \qquad \varepsilon_t \sim N(0, H_t) \qquad (1)$$

$$\alpha_{t+1} = T_t \alpha_t + R_t \eta_t \qquad \eta_t \sim N(0, Q_t) \qquad (2)$$