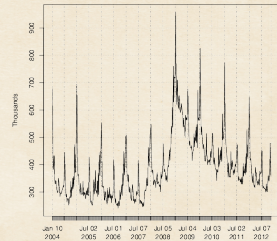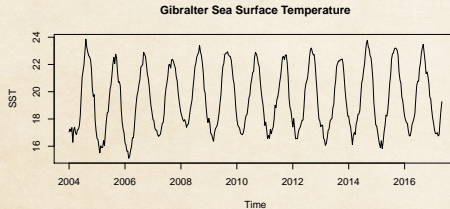Adventures in Bayesian Structural Time Series

Andrew Bates, Josh Gloyd, Tyler Tucker

What are Time Series?

Gibralter Sea Surface Temperature

- Predicting the Present with Bayesian Structural Time Series by Steven L. Scott and Hal Varian (Google)

- Predicting the Present with Bayesian Structural Time Series by Steven L. Scott and Hal Varian (Google)

- Implementation

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- Predicting the Present with Bayesian Structural Time Series by Steven L. Scott and Hal Varian (Google)

- Implementation
  - R: bsts

❀ Predicting the Present with Bayesian Structural Time Series
by Steven L. Scott and Hal Varian (Google)

❀ Implementation

   ❀ R: bsts

      ❀ or Causal Impact

⚙ Predicting the Present with Bayesian Structural Time Series by Steven L. Scott and Hal Varian (Google)

⚙ Implementation

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        ⚙ or Causal Impact

    ⚙ Python: Causal Impact

What are BSTS?

⬡ Predicting the Present with Bayesian Structural Time Series by Steven L. Scott and Hal Varian (Google)

⬡ Implementation
  ⬡ R: bsts
     ⬡ or Causal Impact
  ⬡ Python: Causal Impact

⬡ Comprised of 3 components:

- Predicting the Present with Bayesian Structural Time Series by Steven L. Scott and Hal Varian (Google)

- Implementation
    - R: bsts
        - or Causal Impact
    - Python: Causal Impact

- Comprised of 3 components:
    - Structural Time Series model (a.k.a. state space model)

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- Implementation
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- Comprised of 3 components:
  - Structural Time Series model (a.k.a. state space model)
  - Spike and Slab regression

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- Predicting the Present with Bayesian Structural Time Series by Steven L. Scott and Hal Varian (Google)

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- Comprised of 3 components:
  - Structural Time Series model (a.k.a. state space model)
  - Spike and Slab regression
  - Bayesian model averaging

⬡ Data from unobserved **state space** plus noise
⬡ Model the latent state space instead of the data directly

Local Level Model

⬡ y_t: data
⬡ μ_t: latent state

$$y_t = \mu_t + \varepsilon_t \qquad \varepsilon_t \sim N(0, \sigma_\varepsilon^2)$$
$$\mu_{t+1} = \mu_t + \xi_t \qquad \xi_t \sim N(0, \sigma_\xi^2)$$

Analogous to the inercept in linear regression but allowing for
the intercept to vary over time

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Local Linear Trend Model

- ✸ y_t, μ_t: same as before
- ✸ ν_t: slope (additional state component)

$$y_t = \mu_t + \varepsilon_t \qquad\qquad \varepsilon_t \sim N(0, \sigma_\varepsilon^2)$$
$$\mu_{t+1} = \mu_t + \nu_t + \xi_t \qquad\qquad \xi_t \sim N(0, \sigma_\xi^2)$$
$$\nu_{t+1} = \nu_t + \zeta_t \qquad\qquad \zeta_t \sim N(0, \sigma_\zeta^2)$$

Structural Time Series

General Form

- ✠ y_t: data
- ✠ α_t: state component

$$y_t = Z_t'\alpha_t + \varepsilon_t \qquad \varepsilon_t \sim N(0, H_t) \qquad (1)$$
$$\alpha_{t+1} = T_t\alpha_t + R_t\eta_t \qquad \eta_t \sim N(0, Q_t) \qquad (2)$$

- ✠ (1): **observation equation**
- ✠ (2): **transition equation**

Bayesian Context

- ❂ Spike and slab regression
  - ❂ Used when regression components are included
  - ❂ Variable selection technique
  - ❂ Prior on regression coefficients

- ❂ Bayesian Model Averaging
  - ❂ Consequence of spike and slab prior
  - ❂ Different βs included in each draw of posterior (i.e. different model on each draw)

- ❂ Prior Elicitation and Posterior Sampling
  - ❂ Inclusion probabilities for regression coefficients
  - ❂ Or: expected model size, expected R^2, weight given to R^2
  - ❂ Gibbs sampler (stochastic search variable selection) to draw from posterior
  - ❂ For details see paper by Scott and Varian