San Diego State University

Adventures in Bayesian Structural Time Series

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Outline

- Structural Time Series Models:

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    - Local level model

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  - Local level model
  - Local linear trend model

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  - Models with seasonal component

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  - Prior
  - Posterior

Structural Time Series

- Also called State Space Models

Local Level Model

- y_t: observed data
- μ_t: latent state

$$
\begin{aligned}
y_t &= \mu_t + \varepsilon_t & \varepsilon_t &\sim N(0, \sigma_\varepsilon^2) \\
\mu_{t+1} &= \mu_t + \xi_t & \xi_t &\sim N(0, \sigma_\xi^2)
\end{aligned}
$$

Structural Time Series

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- Also called State Space Models
- Data comes from unobserved variable called the **state space**

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Structural Time Series

- Also called State Space Models
- Data comes from unobserved variable called the **state space**
- We model the state space instead of the observed data directly

Local Level Model

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- μ_t: latent state

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\mu_{t+1} &= \mu_t + \xi_t & \xi_t &\sim N(0, \sigma_\xi^2)
\end{aligned}
$$

Local Level Model

- will have plot here

Local Linear Trend Model

- y_t, μ_t: same as before
- ν_t: slope (additional state component)

$$
\begin{aligned}
y_t &= \mu_t + \varepsilon_t & \varepsilon_t &\sim N(0, \sigma_\varepsilon^2) \\
\mu_{t+1} &= \mu_t + \nu_t + \xi_t & \xi_t &\sim N(0, \sigma_\xi^2) \\
\nu_{t+1} &= \nu_t + \zeta_t & \zeta_t &\sim N(0, \sigma_\zeta^2)
\end{aligned}
$$

Local Linear Trend Model

- will have picture here

Structural Time Series

Basic Structural Model

- Local Trend Model with seasonal component
- μ_t: local trend
- τ_t: seasonal component

$$y_t = \mu_t + \tau_t + \zeta_t \qquad \zeta_t \sim N(0, \sigma_\zeta^2)$$