Adventures in Bayesian Structural Time Series

Part 3: Structural Time Series in Bayesian Context

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Outline

- Without Regression:

Outline

- Without Regression:
  - Prior

Outline

- Without Regression:
  - Prior
  - Posterior

Outline

- Without Regression:
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  - Posterior
- With Regression:

Outline

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Spike and Slab Prior

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San Diego State University

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- **Spike**: prior for probability of regression coefficients being zero
- **Slab**: prior on nonzero regression coefficients and variance

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- Independent Bernoulli prior:

$$\gamma \sim \prod_{i=1}^{K} \pi_k^{\gamma_k} (1 - \pi_k)^{1-\gamma_k}$$

Spike and Slab Prior

Slab

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  - Ω_{γ}^{-1}: submatrix corresponding to $\gamma_k = 1$

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- Ω^{-1}: symmetric matrix
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- Conjugate prior:

$$\beta_\gamma | \sigma_\varepsilon^2, \gamma \sim N(b_\gamma, \sigma_\varepsilon^2 \Omega_\gamma) \qquad \frac{1}{\sigma_\varepsilon^2} | \gamma \sim \text{Gamma}\left(\frac{v}{2}, \frac{ss}{2}\right)$$

Spike and Slab Prior

Full Prior

$$f(\beta, \gamma, \sigma_\varepsilon^2) = f(\beta_\gamma | \gamma, \sigma_\varepsilon^2) f(\sigma_\varepsilon^2 | \gamma) f(\gamma)$$

Specifying Spike and Slab Prior

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  - $\Omega^{-1} = \frac{k}{n} X^T X$