



Adventures in Bayesian Structural Time Series

Part 2: Structural Time Series

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⊠ Structural Time Series Models:



⊠ Structural Time Series Models:

- ⊠ Local level model



⊠ Structural Time Series Models:

- ⊠ Local level model
- ⊠ Local linear trend model



⊠ Structural Time Series Models:

- ⊠ Local level model
- ⊠ Local linear trend model
- ⊠ Models with a seasonal component



⊠ Structural Time Series Models:

- ⊠ Local level model
- ⊠ Local linear trend model
- ⊠ Models with a seasonal component
- ⊠ Models with a regression component



⬠ Also called State Space Models



- ⊠ Also called State Space Models
- ⊠ Data comes from unobserved variable called the **state space**



- ⊠ Also called State Space Models
- ⊠ Data comes from unobserved variable called the **state space**
- ⊠ We model the state space instead of the observed data directly



Local Level Model

⬠ y_t : observed data

⬠ μ_t : unobserved state



$$y_t = \mu_t + \varepsilon_t$$

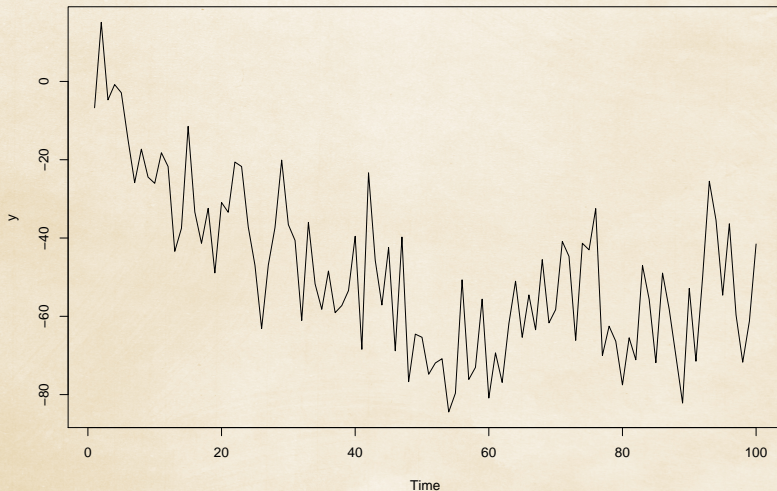
$$\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$$

$$\mu_{t+1} = \mu_t + \xi_t$$

$$\xi_t \sim N(0, \sigma_\xi^2)$$



Simulated Local Level Model





Local Linear Trend Model

⊠ y_t, μ_t : same as before

⊠ ν_t : slope (additional state component)



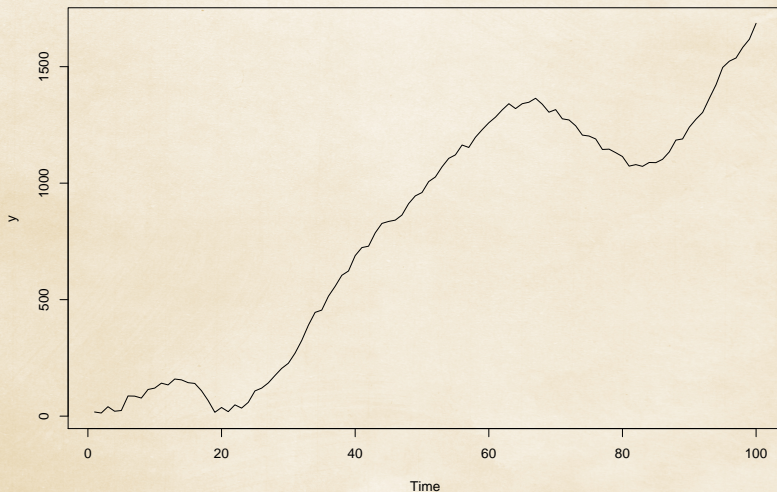
$$y_t = \mu_t + \varepsilon_t \qquad \varepsilon_t \sim N(0, \sigma_\varepsilon^2)$$

$$\mu_{t+1} = \mu_t + \nu_t + \xi_t \qquad \xi_t \sim N(0, \sigma_\xi^2)$$

$$\nu_{t+1} = \nu_t + \zeta_t \qquad \zeta_t \sim N(0, \sigma_\zeta^2)$$



Simulated Local Linear Trend Model





Local Trend With Seasonality

⬠ μ_t : local linear trend

⬠ τ_t : seasonal component

⬠ S dummy variables (1 for each season)



$$y_t = \mu_t + \tau_t + \varepsilon_t$$

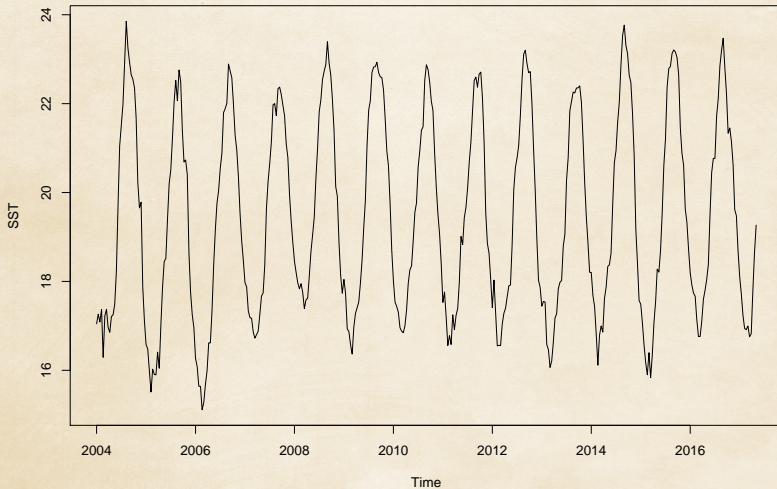
$$\zeta_t \sim N(0, \sigma_\varepsilon^2)$$

$$\tau_t = - \sum_{s=1}^{S-1} \tau_{t-s} + \omega_t$$

$$\tau_t \sim N(0, \sigma_\omega^2)$$



Gibraltar Sea Surface Temperature





Local Trend With Seasonality and Regression

- ⊠ μ_t : local linear trend
- ⊠ τ_t : seasonal component
- ⊠ $\beta_t^T x_t$: regression component
- ⊠

$$y_t = \mu_t + \tau_t + \beta_t^T x_t + \varepsilon_t \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2)$$



Structural Time Series

- ⊠ (1): **observation equation**
- ⊠ (2): **transition equation**

General Form

- ⊠ y_t : data
- ⊠ α_t : state variable
- ⊠

$$y_t = Z_t' \alpha_t + \varepsilon_t \quad \varepsilon_t \sim N(0, H_t) \quad (1)$$

$$\alpha_{t+1} = T_t \alpha_t + R_t \eta_t \quad \eta_t \sim N(0, Q_t) \quad (2)$$