Probability Rules!

Chapter 13

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Or but NOT disjoint

Your Wallet

- S = {1, 2, 5, 10, 20, 50, 100}
- A = {odd numbered value} = {1, 5}
- B = {bill with a building} = {5, 10, 20, 50, 100}

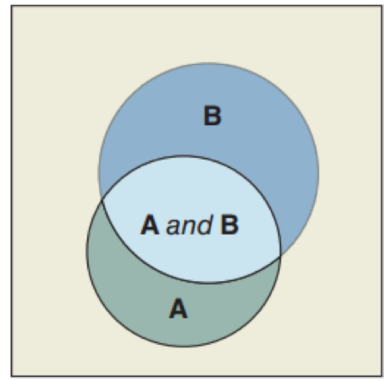
Why is P(A or B) ≠ P(A) + P(B)?

- Answer: A and B are not disjoint.
- The intersection A and B = {$5} is double counted.
- To find P(A or B), subtract P(A and B).

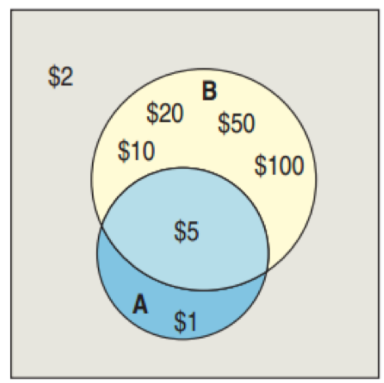
The General Addition Rule

$$P(A \quad or \quad B) = P(A) + P(B) - P(A \quad and \quad B)$$

Add the probabilitiesof the two events and then subtract the probability of their intersection.



P(odd amount or bill with a building) =
P(A) + P(B) – P(A and B} =
P({1, 5}) + P({5, 10, 20, 50, 100}) – P({5})



General Addition Rule Example

Survey

- Are you currently in a relationship?
- Are you involved in sports?

Results

- 33% are in a relationship.
- 25% are involved in sports.
- 11% answered “yes” to both.

Problem

- Find the probability that a randomly selected student is in a relationship or is involved in sports.

General Addition Rule Example

Survey

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- Are you involved in sports?

Results

- 33% are in a relationship.
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Problem

- Find the probability that a randomly selected student is in a relationship or is involved in sports.

33% Relationship, 25% Sport, 11% Both

Events

- R = {in a relationship}
- S = {involved in sports} Calculations
- P(R or S) = P(R) + P(S) – P(R and S) = 0.33 + 0.25 – 0.11 = 0.47

Conclusion

- There is a 47% chance that a randomly selected student is in a relationship or is involved sports.

Using Venn Diagrams

P(not in relationship and no sports)

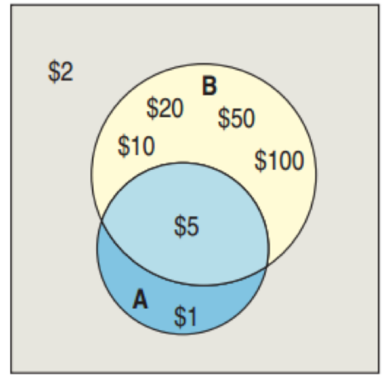
- $P(R^C \quad and \quad S^C)$
- This is the part outside of both circles: 0.53.

P(in a relationship but no sports)

- $P(R \quad and \quad S^C)$
- This is the part in the circle R that is outside S: 0.22.

P(in a relationship or involved in sports but not both)

- $P((R \quad and \quad S^C)$ or $(R^C \quad and \quad S))$
- This is the combination of the circles minus the intersection: 0.22 + 0.14 = 0.36



Contingency Table

A table that displays the results of two categorical questions is called a contingency table.

- P(girl) = 251/478 = 0.525
- P(girl and popular) = 91/478 = 0.190
- P(sports) = 90/478 = 0.188

Sex	Goals: Grades	Popular	Sports	Total
Boy	117	50	60	227
Girl	130	91	30	251
Total	247	141	90	478

Conditional Probability

What if we knew the chosen personwas a girl? Wouldthat change the probability that the girl's goal was sports?

- Yes! We write P(sports I girl)
- Only look at Girl row: P(sports I girl) = 30/251 = 0.120
- Find the probability of selecting a boy given the goal is grades.
- P(boy I grades) = 117/247 = 0.474

		Goals			
		Grades	**Popular**	**Sports**	**Total**
Sex	**Boy**	117	50	60	227
	Girl	130	91	30	251
	Total	247	141	90	478

Conditional Probability Formula

Probability of B given A:

$$P(B|A) = \frac{P(A \quad and \quad B)}{P(A)}$$

		Goals			
		Grades	Popular	Sports	Total
Sex	Boy	117	50	60	227
	Girl	130	91	30	251
	Total	247	141	90	478

$$P(girl \quad | \quad popular) = \frac{P(girl \quad and \quad popular)}{P(popular)}$$

$$P(girl \quad | \quad popular) = \frac{91/478}{141/478}$$

$$P(girl \quad | \quad popular) = \frac{91}{P(141)} = 0.65$$

The General Multiplication Rule

- For A and B independent, we had:
  P(A and B) = P(A) × P(B)

- Rearranging the conditional probability equation, we get the General Multiplication Rule:
  P(A and B) = P(A) × P(B | A)

- Equivalently,
  P(A and B) = P(B) × P(A | B)

Independence

Events A and B are independent if knowing A happened does not change the probability of B. Symbolically:

- A and B are independent $\leftrightarrow$ $P(B \mid A) = P(B)$

Equivalent formulas for independence:

- $P(A \mid B) = P(A)$
- $P(A \text{ and } B) = P(A) \times P(B)$

Grades and Girl Independent?

Determine if the goal of good grades and sex are independent.

- P(grades I girl) = 130/251 ≈ 0.52
- P(grades) = 247/478 ≈ 0.52
- To two decimal places, they are independent.

Are the goal of sports and sex independent?

- P(sports I boy) = 60/227 ≈ 0.26
- P(sports) = 90/478 ≈ 0.19
- No, the goal of sports and sex are dependent.

Sex	Goals: Grades	Popular	Sports	Total
Boy	117	50	60	227
Girl	130	91	30	251
Total	247	141	90	478

Relationships, Sports, and Independence

33% in a relationship, 25% involved in sports, 11% both

Are being in a relationship and being involved in sports independent?

- P(relationship) = 0.33
- P(sports) = 0.25
- P(relationship and sports) = 0.11
- $0.33 \times 0.25 = 0.0825 \neq 0.11$
- No, they are dependent.

Are they disjoint?

- P(relationship and sports) = $0.11 \neq 0$
- No, they are not disjoint.

Independent ≠ Disjoint

Disjoint events cannot be independent.

Consider the events:

- Course grade A
- Course grade B
- Disjoint: You can't get both.
- Not independent: $P(A \mid B) = 0 \neq P(A)$
- A and B are disjoint (also called mutually exclusive) but not independent.

›abilities

Use Text Messaging	Use E-Mail: Yes	No	Total
Yes	0.49		0.62
No			
Total	0.73		1.00

Use Text Messaging	Use E-Mail: Yes	No	Total
Yes	0.49	0.13	0.62
No	0.24	0.14	0.38
Total	0.73	0.27	1.00

73% Use E-mail, 62% Text, 49% Both

1. E-mail and text mutually exclusive?
2. E-mail and text independent?

Plan:

- A = {uses e-mail}
- B = {texts}
- P(A) = 0.73
- P(B) = 0.62
- P(A and B) = 0.49

1. E-mail and text mutually exclusive?

- P(A and B) = 0.49 $\neq$ 0
- Conclusion: E-mail and text are not mutually exclusive.

73% Use E-mail, 62% Text, 49% Both

1. E-mail and text mutually exclusive?
2. E-mail and text independent?

Plan:

- A = {uses e-mail}
- B = {texts}
- P(A) = 0.73
- P(B) = 0.62
- P(A and B) = 0.49

1. E-mail and text independent?

- Make a table

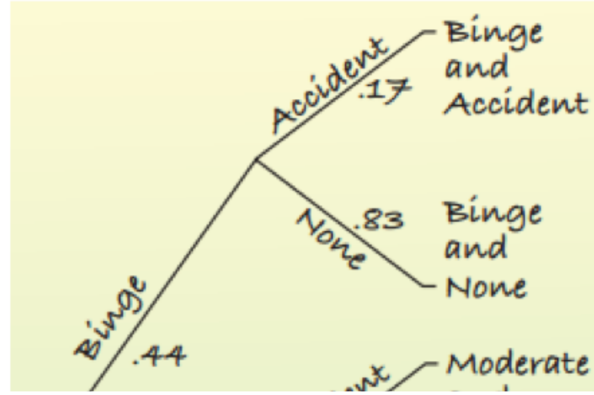
Use Text Messaging	Use E-Mail: Yes	Use E-Mail: No	Total
Yes	0.49	0.13	0.62
No	0.24	0.14	0.38
Total	0.73	0.27	1.00

- P(B I A) = 0.49/0.73 ≈ 0.67
- P(B) = 0.62 ≠ 0.67
- Not independent
- Conclusion: Since the respondents who use

e-mail are more likely to text, they are not independent.

Tree Diagrams

- 44% binge drink, 37% drink moderately, 19% don't drink
- Binge drinkers: 17% in an alcohol related accident
- Non-bingers: 9% in an alcohol-related accident

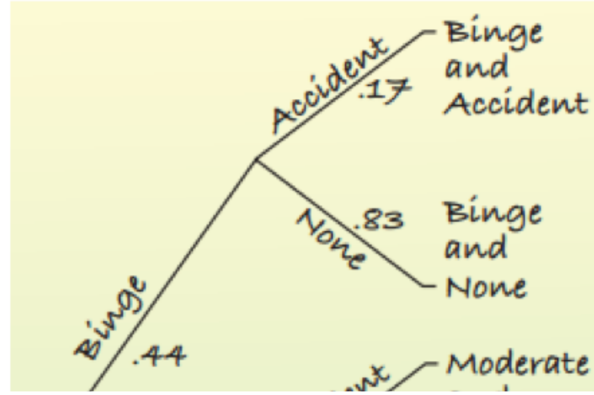


Tree Diagrams

This tree diagram gives the complete information.

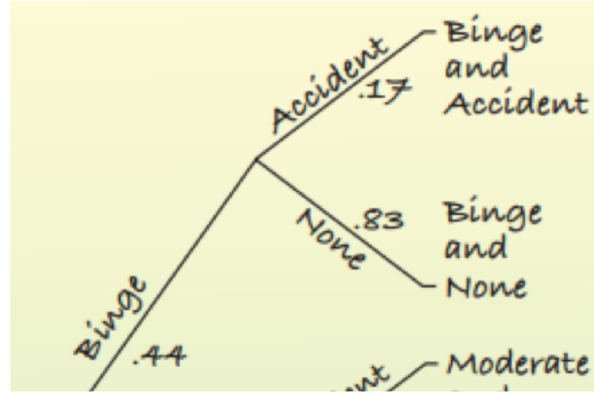
Notice the sums:

- 0.17 + 0.83 = 1
- 0.09 + 0.91 = 1



Probabilities From Trees

- P(moderate and accident) = 0.37 × 0.09 = 0.0333
- P(abstain and accident) = 0.19 × 0 = 0
- P(none) = (0.44 × 0.83) + (0.37 × 0.91) + (0.19 × 1.0) = 0.8919



Tree Diagram Facts

- The sum of the probabilities emanating from any branch is 1.
- The final outcomes are disjoint.
- To find a conditional probability, multiply across.