QF $\int_S f \, \mathrm{d}\mu.$ QUANTUM FORMALISM quantumformalism.com

Homework 4

Directions: Answer the following questions. You are encouraged to work together, join the discussion sessions, use discord, and ask me questions!

1. Let $\mathcal{A} \subset P(X)$ be an algebra, $\mathcal{A}_\sigma$ the collection of countable unions of sets in $\mathcal{A}$, and $\mathcal{A}_{\sigma\delta}$ the collection of countable intersections of sets in $\mathcal{A}_\sigma$. Let μ_σ be a premeasure on $\mathcal{A}$ and μ^* the induced outer measure. Show that:

   a) For any $E \subset X$ and $\epsilon > 0$ there exists $A \in \mathcal{A}_\sigma$ with $E \subset A$ and $\mu^*(A) \leq \mu^*(E) + \epsilon$.

   b) If $\mu^*(E) < \infty$, then E is μ^*-measurable if and only if there exists $B \in \mathcal{A}_{\sigma\delta}$ with $E \subset B$ and $\mu^*(B \backslash E) = 0$.

2. Let μ^* be an outer measure on X induced by a premeasure μ_0 where $\mu_0(X) < \infty$. If $E \subset X$, define the inner measure of E to be $\mu_*(E) = \mu_0(X) - \mu^*(E^c)$. Then E is μ^*-measurable if and only if $\mu^*(E) = \mu_*(E)$.

3. Come up with an example of a set X, an algebra $\mathcal{A}$, outer measure μ^* and a set in $E \subset X$ such that $\mu_*(E) \neq \mu^*(E)$.
