Topology & Differential Geometry Crash Course: Lecture 09

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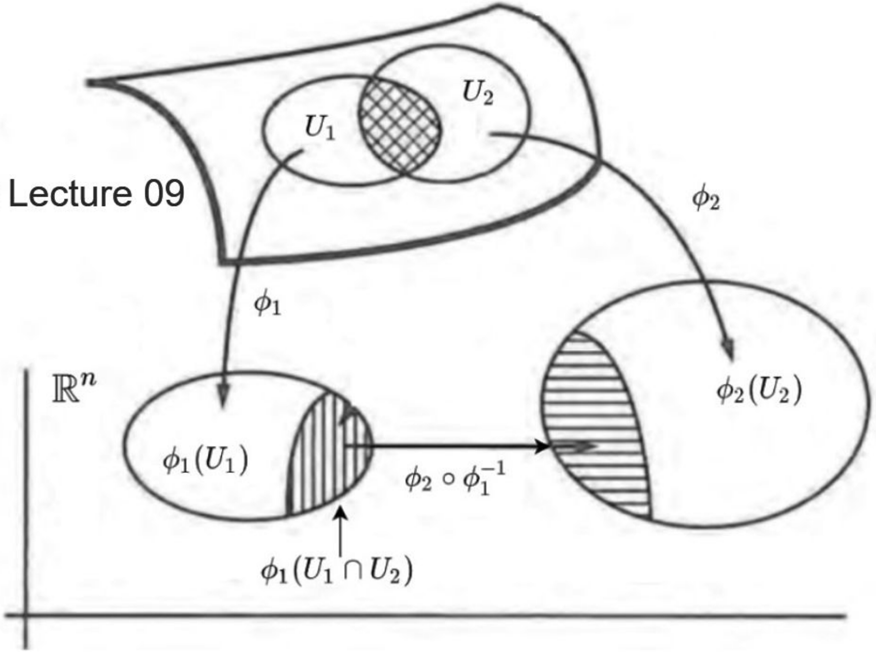


Bambordé Baldé | Co-Founder at Zaiku Group | Twitter: @zaikubalde • **zaikugroup.com** • November 12, 2021

Session Agenda

1. Diffeomorphisms PART II
2. Diffeomorphism Invariance of Dimension
3. Diffeomorphisms Challenge
4. The Diffeomorphism Group
5. Diffeomorphism Group Challenge
6. The Notion of Exotic Smooth Structures
7. Exotic Smooth Structures & Classification
8. **Study Materials**
9. **What next?**

1. Smooth Maps at a Point on a Manifold
2. Smooth Maps on Manifolds
3. Smooth Maps Challenge

Pre-lecture blah blah blah

PART A + **PART B**

Lecture 9 Coverage

General Coverage

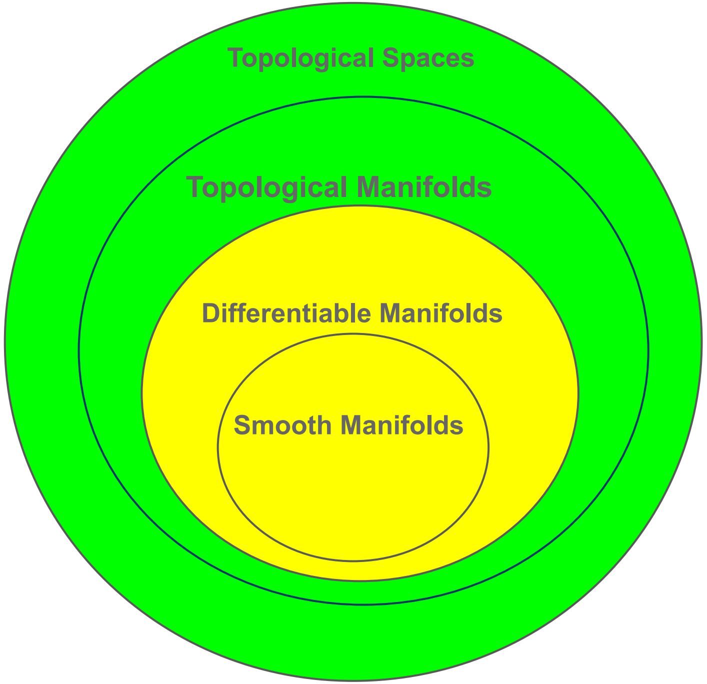
Including a brief intro to Fiber Bundles?

Point-set Topology 101
2/2

Metric Topology 101
3/3

Manifolds 101
4/4

#n is the number of live lectures | Completed | We're here!



Topology Crash Course

Completed | We’re here!

Crash Course Motivation

Lie Groups, Lie Algebras & Representations

Module II (Jan or Feb 2022 start date?)

Lie group

From Wikipedia, the free encyclopedia

Not to be confused with Group of Lie type.

In mathematics, a **Lie group** (pronounced /liː/ "Lee") is a group that is also a differentiable manifold. A manifold is a space that locally resembles Euclidean space, whereas groups define the abstract, generic concept of multiplication and the taking of inverses (division). Combining these two ideas, one obtains a continuous group where points can be multiplied together, and their inverse can be taken. If, in addition, the multiplication and taking of inverses are defined to be smooth (differentiable), one obtains a Lie group.

PART A

Smooth Maps at a Point on a Manifold

Definition (1.0)

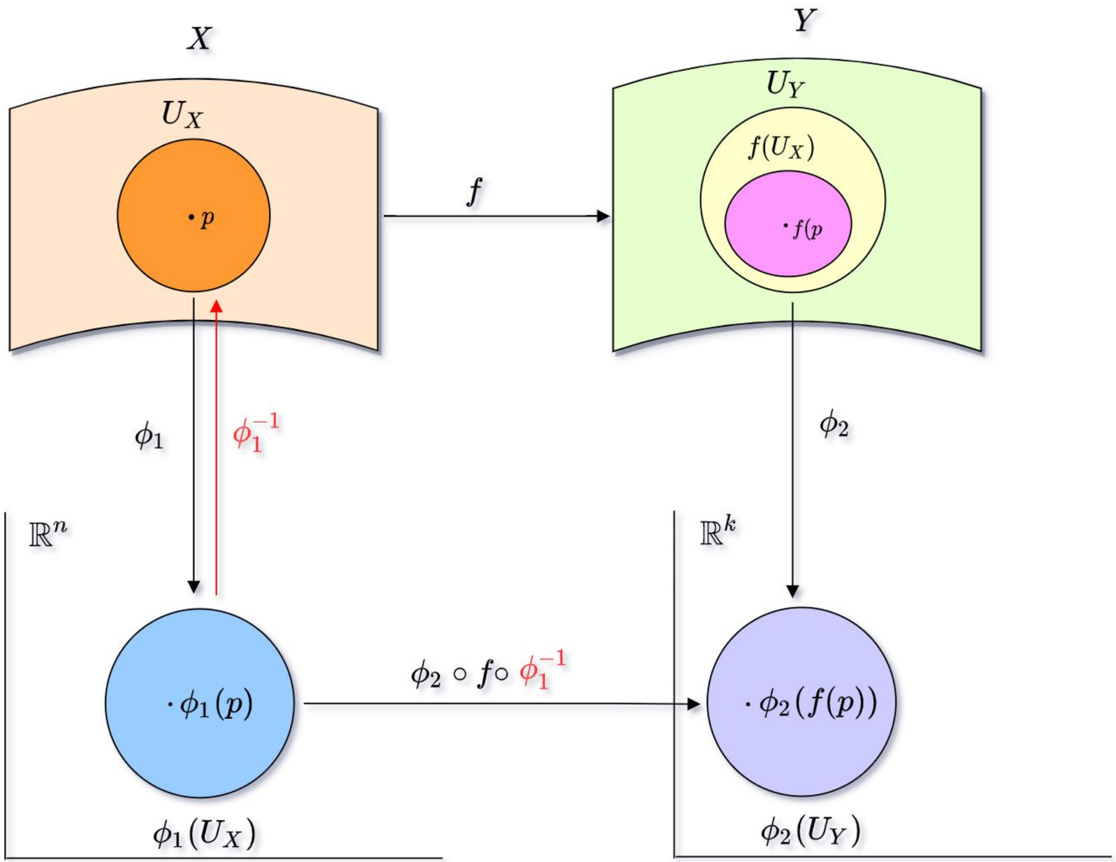
Let X be an $n-$ dimensional smooth manifold and Y be a $k-$ dimensional smooth manifold, with respective smooth atlases $\mathcal{A}_X$ and $\mathcal{A}_Y$. Then a map $f : X \longrightarrow Y$ is smooth at a point $p \in X$ if there are two smooth charts $(U_X, \phi_1) \in \mathcal{A}_X$ and $(U_Y, \phi_2) \in \mathcal{A}_Y$ such that the following three conditions are satisfied:

1. $p \in U_X$ i.e. (U_X, ϕ_1) is a smooth chart around p.
2. $f(U_X) \subseteq U_Y$ i.e. $f(p) \in U_Y$ and so (U_Y, ϕ_2) is a smooth chart around $f(p)$.
3. The composition map $\phi_2 \circ f \circ \phi_1^{-1}$ is a smooth map.

- Here is a general diagrammatic picture of $\phi_2 \circ f \circ \phi_1^{-1}$:

$$\begin{tikzcd}
U_X \subseteq X \arrow[rr, "f"] \arrow[d, shift right, "\phi_1"'] & & U_Y \subseteq Y \arrow[d, "\phi_2"] \\
\phi_1(U_X) \subseteq \mathbb{R}^n \arrow[u, shift right, "\phi_1^{-1}"'] \arrow[rr, "\phi_2 \circ f \circ \phi_1^{-1}"] & & \phi_2(U_Y) \subseteq \mathbb{R}^k
\end{tikzcd}$$

- A more complete picture of $\phi_2 \circ f \circ \phi_1^{-1}$:



Smooth Maps on Manifolds

Definition (1.1)

Let X be an $n-$ dimensional smooth manifold and Y be a $k-$ dimensional smooth manifold, with respective smooth atlases $\mathcal{A}_X$ and $\mathcal{A}_Y$. Then a map $f : X \longrightarrow Y$ is smooth if it is smooth at all $p \in X$.

- Hence, the map $\phi_2 \circ f \circ \phi_1^{-1} : \phi(U_1) \subseteq \mathbb{R}^n \longrightarrow \phi(U_2) \subseteq \mathbb{R}^k$ is essentially a clever trick to enable us to:
  1. Define the notion of differentiability directly at manifold level by using charts.
  2. Make a smooth coordinate change from manifold X to manifold Y!
- Indeed, as you may guess $\phi_2 \circ f \circ \phi_1^{-1}$ is called the coordinate representation of f.
- Obviously, the definition above is so general that the map f can be a map from the manifold X to itself!

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Smooth Maps Challenge I

Convention: When X is an $n-$ dimensional smooth manifold and $Y = \mathbb{R}$. Then the set of all smooth maps $f : X \longrightarrow \mathbb{R}$ is denoted $C^\infty(X)$.

- Let X be the unit sphere $S^2 = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 = 1\}$ and $Y = \mathbb{R}$. Is the map $f : S^2 \longrightarrow \mathbb{R}$ defined for all points $p = (x, y, z) \in S^2$ as $f(p) = z$ smooth i.e. $f \in C^\infty(S^2)$?
- You are encourage to construct the following:
  1. At least four smooth maps $f_1, f_2, f_3, f_4 \in C^\infty(\mathbb{R}^4)$.
  2. At least three smooth maps $f_1, f_2, f_3 \in C^\infty(\mathbb{R}^3)$.
  3. At least two smooth maps $f_1, f_2 \in C^\infty(\mathbb{R}^2)$.
  4. At least two smooth maps $f_1, f_2 \in C^\infty(S^2)$.
- Let $f_1 : X \longrightarrow Y$ and $f_2 : Y \longrightarrow Z$ be smooth maps. Prove that the composition $f_2 \circ f_1 : X \longrightarrow Z$ is also smooth.

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Smooth Maps Challenge II

- Is it true that for any $n-$ dimensional smooth manifold X, the following holds:
  1. The point-wise scalar multiplication defined as $\alpha \cdot f = \alpha f(p)$ is smooth for all $p \in X$, $\alpha \in \mathbb{R}$ and $f \in C^\infty(X)$.
  2. The point-wise addition defined as $(f_1 \oplus f_2)(p) = f_1(p) + f_2(p)$ is smooth for all $p \in X$ and $f_1, f_2 \in C^\infty(X)$.
- With the point-wise operations above, does $C^\infty(X)$ form a real vector space structure? If yes, what is the dimension of $C^\infty(X)$?
- For the super curious, let us define the point-wise multiplication as $(f_1 \otimes f_2)(p) = f_1(p)f_2(p)$ for all $p \in X$ and $f_1, f_2 \in C^\infty(X)$. Does this together with the point-wise addition make $C^\infty(X)$ into a ring with identity? If yes, what is:
  1. What is the ring identity of $C^\infty(X)$?
  2. What is the characteristics of the ring $C^\infty(X)$?
  3. Is $C^\infty(X)$ an abelian or nonabelian ring?

PART B

Diffeomorphisms (PART II)

Definition (1.2)

Let X and Y be smooth manifolds. A map $f : X \longrightarrow Y$ is called a 'diffeomorphism' if the following holds:

1. f is a smooth map.
2. f is invertible and the inverse map f^{-1} is also smooth.

- We say X and Y are diffeormorphic and write $X \simeq_{diff} Y$ if there is at least a diffeomorphism between the two.
- As you can guess, diffeomorphisms are the structure preserving maps for smooth manifolds.
- Obviously, diffeomorphisms are homeomorphisms right? But the other way around is not true! An easy example is the $f : \mathbb{R} \longrightarrow \mathbb{R}$ defined as $f(x) = x^3$ for all $x \in \mathbb{R}$. This map is a homeomorphism but not a diffeomorphism because its inverse is not smooth.

- It is also possible for two smooth manifolds to be homeomorphic at topological level but fail to be diffeomorphic!

Diffeomorphism Dimension Invariance

Theorem (1.0)

Let X be an $n-$ dimensional smooth manifold and Y be an $K-$ dimensional smooth manifold. Then X is diffeomorphic to Y iff $n = k$.

Proof : Textbook or try prove it yourself?!

- Hence, the dimension of smooth manifold is an invariant!

- Even more fascinating, given an $n-$ dimensional topological manifold X, it may be possible to equip X with two smooth structures $\mathcal{A}_1$ and $\mathcal{A}_2$ (or more structures) such that the resulting smooth manifolds $(X, \mathcal{A}_1)$ and $(X, \mathcal{A}_2)$ are not diffeomorphic!

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Diffeomorphisms Challenge

- As homework challenge, you are encourage to prove the following:
  1. If the maps $f_1 : X \longrightarrow Y$ and $f_2 : Y \longrightarrow Z$ are diffeomorphisms between smooth manifolds. Then the composition $f_2 \circ f_1 : X \longrightarrow Z$ is also a diffeomorphism i.e. if $X \simeq_{diff} Y$ and $Y \simeq_{diff} Z$ then $X \simeq_{diff} Z$.
  2. If X is a smooth manifold then the identity map $id_X : X \longrightarrow X$ is a diffeomorphism.
- Suppose that $f_1 : X \longrightarrow Y$ is just a homeomorphism and $f_2 : Y \longrightarrow Z$ is a diffeomorphism. Is the composition $f_2 \circ f_1$ a diffeomorphism?
- Suppose now that $f_1 : X \longrightarrow Y$ is a diffeomorphism and $f_2 : Y \longrightarrow Z$ is just a homeomorphims. Is the composition $f_2 \circ f_1$ also a diffeomorphism?
- Let (U, ϕ) be a smooth chart on X. Prove that $\phi : U \longrightarrow \phi(U) \subseteq \mathbb{R}^n$ is indeed a diffeomorphism.

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The Diffeomorphism Group

Definition (1.3)

Let X be an $n-$ dimensional smooth manifold. We define the set $Diff(X) = \{f : X \longrightarrow X \mid f \text{ is a diffeomorphism }\}$

- In the next modules we'll see that $Diff(X)$ can also be equipped with very interesting structures including topological ones!

Proposition (1.0)

Let X be an $n-$ dimensional smooth manifold. Then $Diff(X)$ forms a group under the composition of maps $\circ$ and the group identity is $id_X : X \longrightarrow X$.

Proof : Try prove it yourself?!

- In the future, we'll cover the notion of 'homotopy type' of $Diff(X)$. In some cases this is a hard problem, for example with $Diff(S^4)$!

Theorem (1.1)

Let X and Y be two smooth manifolds. Then $Diff(X)$ is isomorphic (in the group sense) to $Diff(Y)$ iff X is diffeomorphic to Y.

Proof : Search Filipkiewicz's theorem or try prove it?!

- Is the other way round true i.e. X is diffeomorphic to Y iff $Diff(X)$ is isomorphic (in the group sense) to $Diff(Y)$?

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Diffeomorphism Group Challenge

- Try gain some intuition on what $Diff(X)$ does for the following concrete smooth manifolds:
  1. $\mathbb{R}^3$ or even $\mathbb{R}^n$ in general.
  2. The sphere S^2.
  3. The circle S^1.
- The set of homeomorphims $Homeo(X) = \{g : X \longrightarrow X \mid g$ is a homeomorphism $\}$ is a group too! Which of the following inclusions is true:
  1. $Homeo(X) \subseteq Diff(X)$ i.e. $Homeo(X)$ is a subgroup of $Diff(X)$.
  2. $Diff(X) \subseteq Homeo(X)$ i.e. $Diff(X)$ is a subgroup of $Homeo(X)$.
- Does the Filipkiewicz's theorem imply that $Diff(X)$ is an algebraic invariant for X i.e. X is diffeomorphic to Y iff $Diff(X)$ is isomorphic (in the group sense) to $Diff(Y)$?

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The Notion of Exotic Structures

- **Natural question**: how many different smooth structures can we equip an $n-$ dimensional topological manifold X? To make things more interesting and familiar, we'll consider the following use cases:

  1. $X = \mathbb{R}^n$ with the standard smooth structure i.e. the structure generated by the smooth atlas $\mathcal{A} = \{(\mathbb{R}^n, id_{\mathbb{R}^n})\}$.
  2. $X = S^n$ with the smooth structure generated by the smooth atlas given by the two stereographic projection charts on the $n-$ sphere i.e. $\{(S^n \setminus (0, 0, \ldots, 1), \phi_1), (S^n \setminus (0, 0, \ldots, -1), \phi_2)\}$ where ϕ_1 is the projection from the north pole and ϕ_2 is the projection from the south pole.

- Then the natural question above can be rephrased as: how many different smooth structures other than the standard ones are there for $\mathbb{R}^n$ and S^n?

- **Note:** Smooth structures that are different from the 'standard ones' as defined above are called 'Exotic Smooth Structures'!

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Exotic Smooth Structures on $\mathbb{R}^n$

- For dimensions $1, 2$ and 3, there are no exotic smooth structures! So up to diffeomorphism, there is only one smooth structure in these dimensions i.e. all smooth manifolds of dimension $1, 2$ and 3 constructed from the underlying standard topological manifold structures of $\mathbb{R}, \mathbb{R}^2$ and $\mathbb{R}^3$ are diffeomorphic to the smooth manifolds generated by the standard smooth structures on $\mathbb{R}, \mathbb{R}^2$ and $\mathbb{R}^3$ respectively.

  An equivalent way to say the above is that: any topological manifold of dimension $1, 2$ or 3 that is homeomorphic to $\mathbb{R}, \mathbb{R}^2$ or $\mathbb{R}^3$ respectively, must be diffeomorphic to to the smooth manifold generated by the standard smooth structure on $\mathbb{R}, \mathbb{R}^2$ or $\mathbb{R}^3$ respectively.

- For dimension $n > 4$, there are no exotic structures! Hence, up to diffeomorphism, there is a unique smooth structure on $\mathbb{R}^n$ for $n > 4$.

- For dimension 4 , surprisingly there is an uncountable number of exotic smooth structures that we can equip $\mathbb{R}^4$ with! Hence, $\mathbb{R}^4$ can generate an uncountanble number of smooth manifolds that are not diffeomorphic to the smooth manifold generated by the standard smooth structure on $\mathbb{R}^4$!

- **Curious questions to our friends in physics:** Which smooth structures on $\mathbb{R}^4$ is the right model for space-time??!

Smooth Structures on S^n

- For dimensions $n \leq 3$ there are no exotic structures! So up to diffeomorphism, S^1, S^2 and S^3 have unique smooth structures.

- For dimension $n = 4$, surprisingly it's an open problem whether the $4-$ sphere S^4 has any exotic structure! This is known as the 'Smooth Poincaré Conjecture'.

- For dimension $n \geq 5$, there is either none or if there is, it must be only finitely many exotic smooth structures on the $n-$ sphere (Kervaire & Milnor)! But exactly how many??! Well, the following partial results:

    1. For dimension 7, there are 28 exotic structures for the $7-$ sphere S^7 (Milnor).

    2. For dimension $5 \leq n \leq 61$, the only $n-$ spheres with no exotic structures are; S^5, S^6, S^{12} and S^{61}. Hence, all the others in the range have exotic structures. For example, S^{15} has 16256 exotic structures!

- It is a conjecture that for $n \geq 5$, the only $n-$ spheres without exotic structures are; S^5, S^6, S^{12} and S^{61}!

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We gratefully acknowledge support from the Simons Foundation and member institutions.

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Mathematics > Geometric Topology

[Submitted on 28 Jun 2009 (v1), last revised 17 Sep 2009 (this version, v2)]

Man and machine thinking about the smooth 4-dimensional Poincaré conjecture

Michael Freedman, Robert Gompf, Scott Morrison, Kevin Walker

While topologists have had possession of possible counterexamples to the smooth 4-dimensional Poincaré conjecture (SPC4) for over 30 years, until recently no invariant has existed which could potentially distinguish these examples from the standard 4-sphere. Rasmussen's s-invariant, a slice obstruction within the general framework of Khovanov homology, changes this state of affairs. We studied a class of knots K for which nonzero s(K) would yield a counterexample to SPC4. Computations are extremely costly and we had only completed two tests for those K, with the computations showing that s was 0, when a landmark posting of Akbulut (arXiv:0907.0136) altered the terrain. His posting, appearing only six days after our initial posting, proved that the family of ``Cappell--Shaneson'' homotopy spheres that we had geared up to study were in fact all standard. The method we describe remains viable but will have to be applied to other examples. Akbulut's work makes SPC4 seem more plausible, and in another section of this paper we explain that SPC4 is equivalent to an appropriate generalization of Property R (``in S^3, only an unknot can yield S^1 x S^2 under surgery''). We hope that this observation, and the rich relations between Property R and ideas such as taut foliations, contact geometry, and Heegaard Floer homology, will encourage 3-manifold topologists to look at SPC4.

Comments:	37 pages; changes reflecting that the integer family of Cappell-Shaneson spheres are now known to be standard (arXiv:0907.0136)
Subjects:	**Geometric Topology (math.GT)**; Quantum Algebra (math.QA)
MSC classes:	57R60, 57N13, 57M25
Journal reference:	Quantum Topology, Volume 1, Issue 2 (2010), pp. 171-208
DOI:	10.4171/QT/5
Cite as:	arXiv:0906.5177 **[math.GT]**
	(or arXiv:0906.5177v2 **[math.GT]** for this version)

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Recommended Reading I

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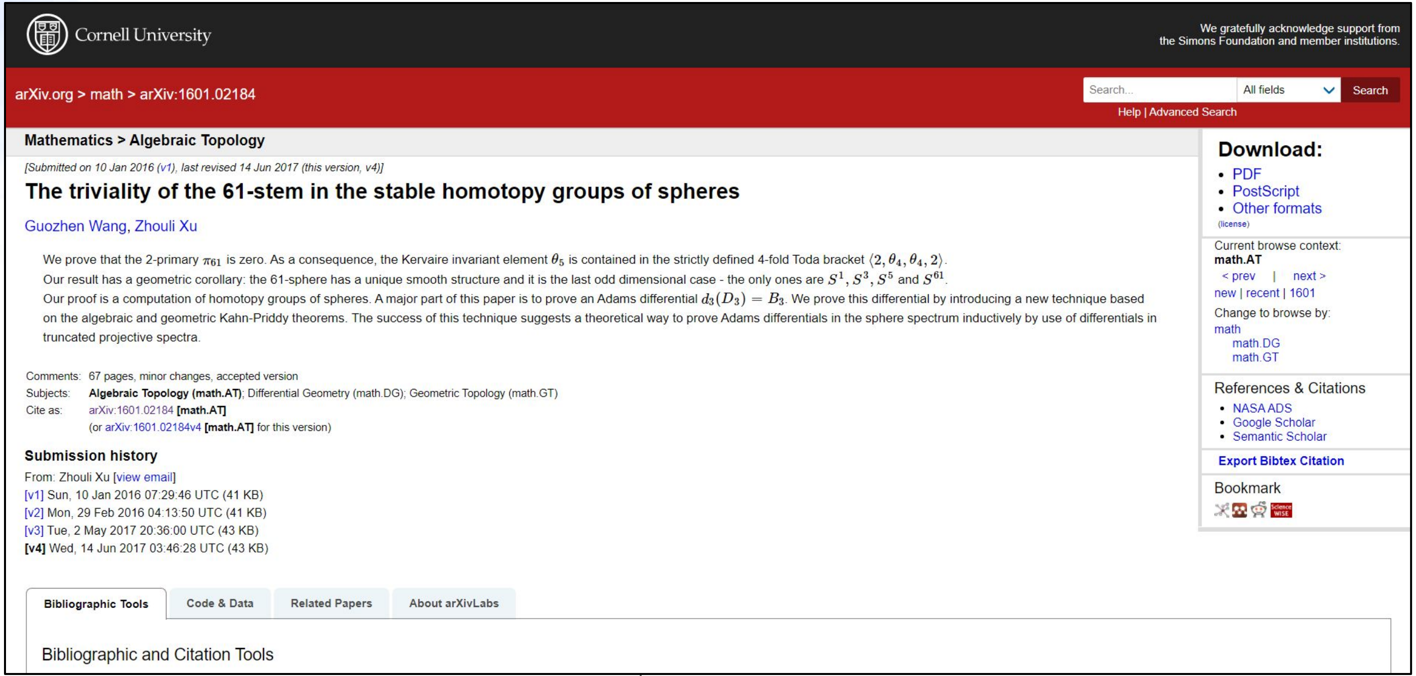
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We gratefully acknowledge support from the Simons Foundation and member institutions.

arXiv.org > math > arXiv:1601.02184

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Mathematics > Algebraic Topology

[Submitted on 10 Jan 2016 (v1), last revised 14 Jun 2017 (this version, v4)]

The triviality of the 61-stem in the stable homotopy groups of spheres

Guozhen Wang, Zhouli Xu

We prove that the 2-primary π_{61} is zero. As a consequence, the Kervaire invariant element θ_5 is contained in the strictly defined 4-fold Toda bracket $\langle 2, \theta_4, \theta_4, 2 \rangle$.
Our result has a geometric corollary: the 61-sphere has a unique smooth structure and it is the last odd dimensional case - the only ones are S^1, S^3, S^5 and S^{61}.
Our proof is a computation of homotopy groups of spheres. A major part of this paper is to prove an Adams differential $d_3(D_3) = B_3$. We prove this differential by introducing a new technique based on the algebraic and geometric Kahn-Priddy theorems. The success of this technique suggests a theoretical way to prove Adams differentials in the sphere spectrum inductively by use of differentials in truncated projective spectra.

Comments: 67 pages, minor changes, accepted version
Subjects: **Algebraic Topology (math.AT)**; Differential Geometry (math.DG); Geometric Topology (math.GT)
Cite as: arXiv:1601.02184 **[math.AT]**
(or arXiv:1601.02184v4 **[math.AT]** for this version)

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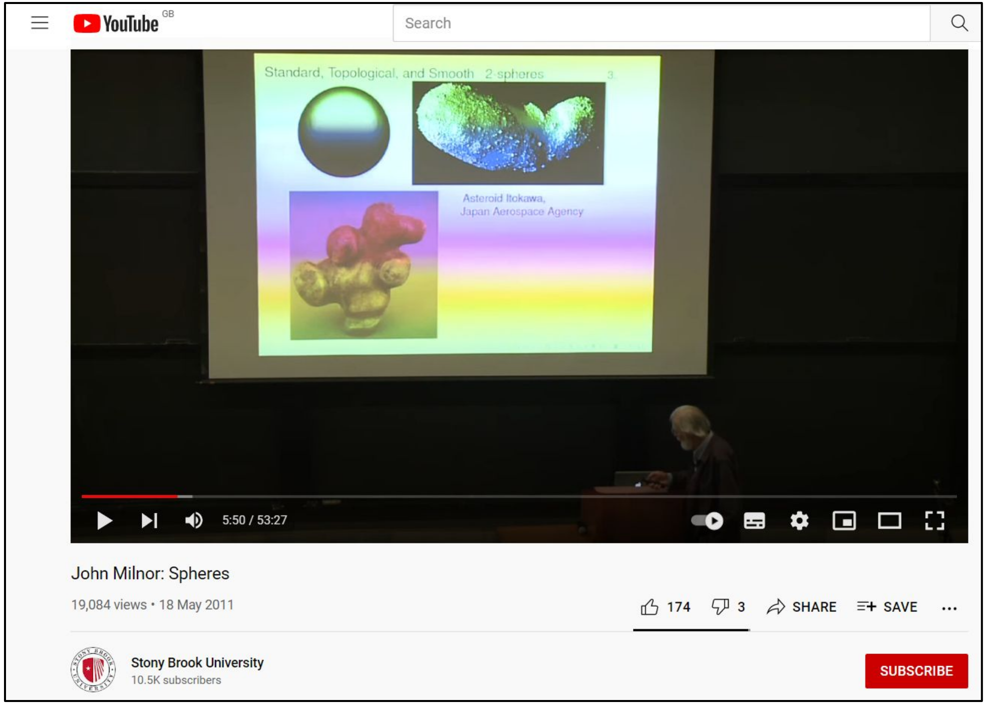

Video Presentation I

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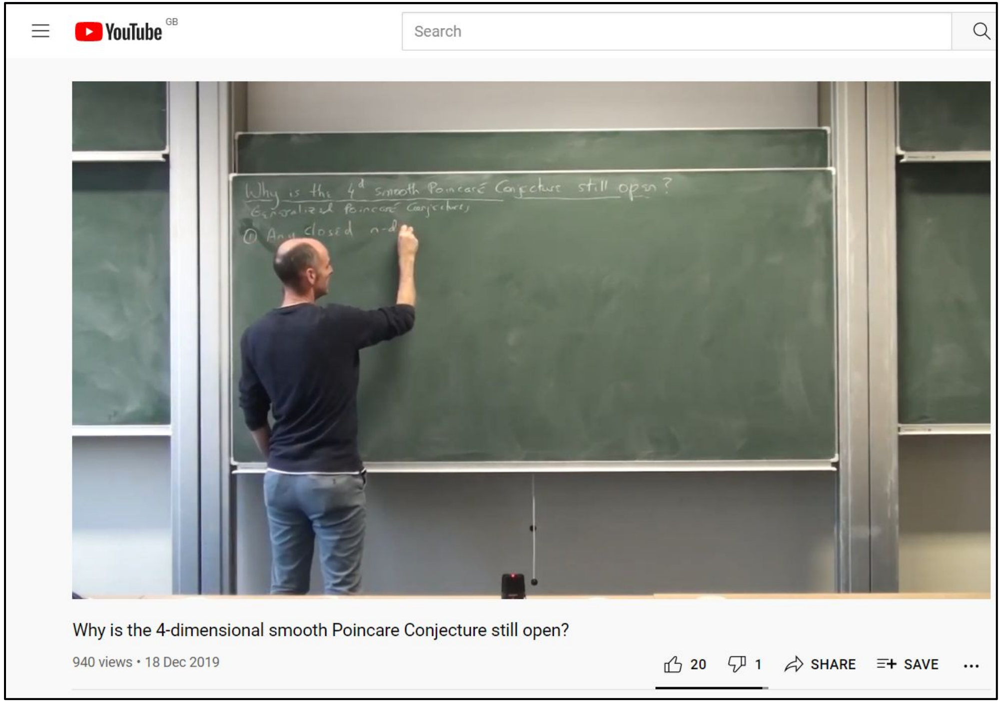
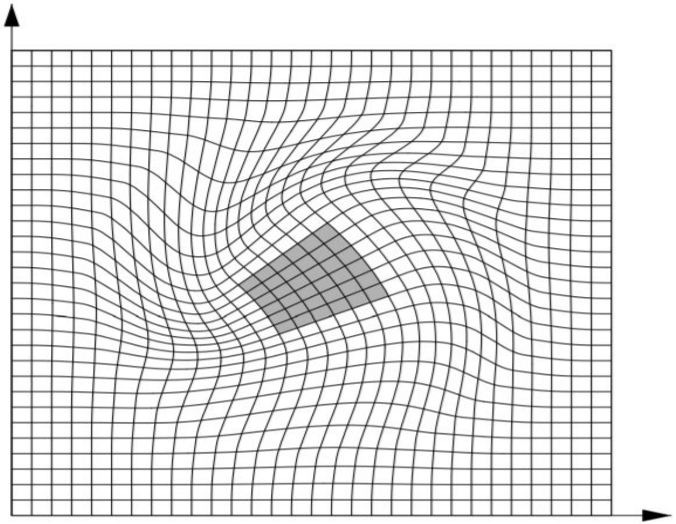


Video Presentation II

Multivariable Calculus



4.6 Higher Order Derivatives

Partial differentiation can be carried out more than once on nice enough functions. For example if

$$f(x,y) = e^{x \sin y}$$

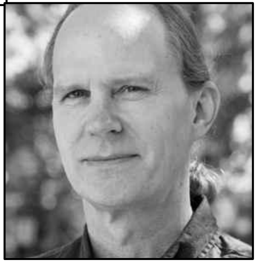
then

$$D_1 f(x,y) = \sin y e^{x \sin y}, \quad D_2 f(x,y) = x \cos y e^{x \sin y}.$$

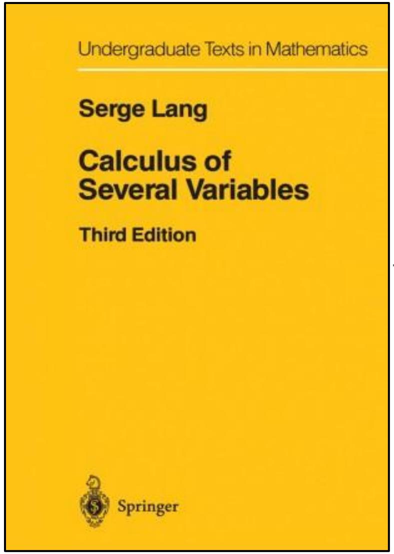
Taking partial derivatives again yields

$$\begin{aligned}
D_1 D_1 f(x,y) &= \sin^2 y e^{x \sin y}, \\
D_1 D_2 f(x,y) &= \cos y e^{x \sin y} + x \sin y \cos y e^{x \sin y}, \\
D_2 D_1 f(x,y) &= \cos y e^{x \sin y} + x \sin y \cos y e^{x \sin y} = D_1 D_2 f(x,y), \\
D_2 D_2 f(x,y) &= -x \sin y e^{x \sin y} + x^2 \cos^2 y e^{x \sin y},
\end{aligned}$$

and some partial derivatives of these in turn are,



Prof. Jerry Shurman



CHAPTER VI

Higher Derivatives

In this chapter, we discuss two things which are of independent interest. First, we define partial differential operators (with constant coefficients). It is very useful to have facility in working with these formally.

Secondly, we apply them to the derivation of Taylor's formula for functions of several variables, which will be very similar to the formula for one variable. The formula, as before, tells us how to approximate a function by means of polynomials. In the present theory, these polynomials involve several variables, of course. We shall see that they are hardly more difficult to handle than polynomials in one variable in the matters under consideration.

The proof that the partial derivatives commute is tricky. It can be omitted without harm in a class allergic to theory, because the technique involved never reappears in the rest of this book.

§1. *Repeated partial derivatives*

Let f be a function of two variables, defined on an open set U in 2-space. Assume that its first partial derivative exists. Then $D_1 f$ (which we also write $\partial f/\partial x$ if x is the first variable) is a function defined on U. We may then ask for its first or second partial derivatives, i.e. we may form $D_2 D_1 f$ or $D_1 D_1 f$ if these exist. Similarly, if $D_2 f$ exists, and if the first partial derivative of $D_2 f$ exists, we may form $D_1 D_2 f$.

Suppose that we write f in terms of the two variables (x, y). Then we can write

$$D_1 D_2 f(x, y) = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = (D_1(D_2 f))(x, y),$$

and

$$D_2 D_1 f(x, y) = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = (D_2(D_1 f))(x, y).$$

For example, let $f(x, y) = \sin(xy)$. Then

$$\frac{\partial f}{\partial x} = y\cos(xy) \quad \text{and} \quad \frac{\partial f}{\partial y} = x\cos(xy).$$

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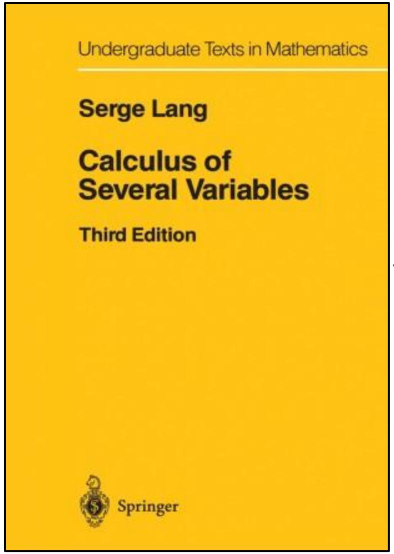

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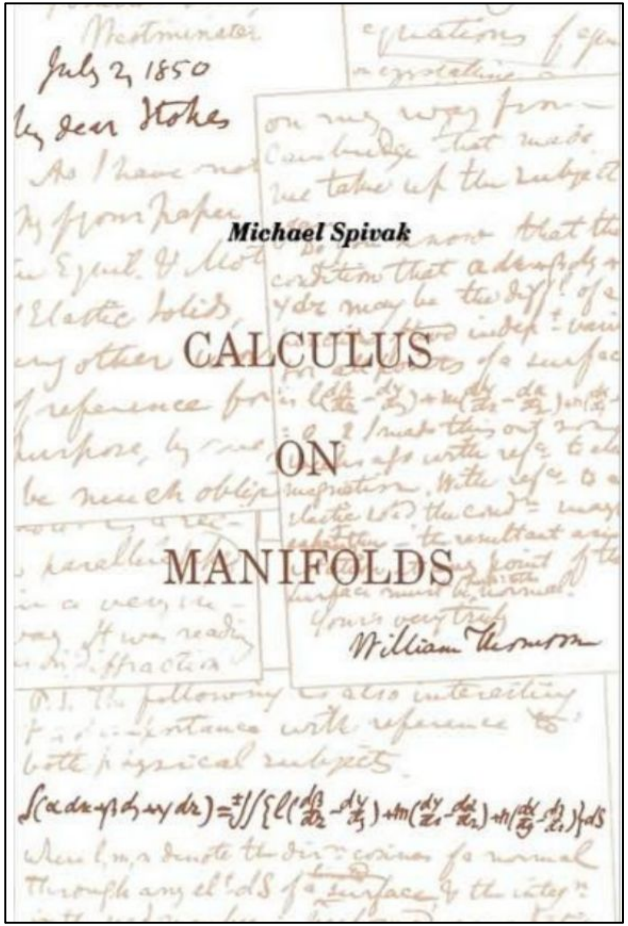
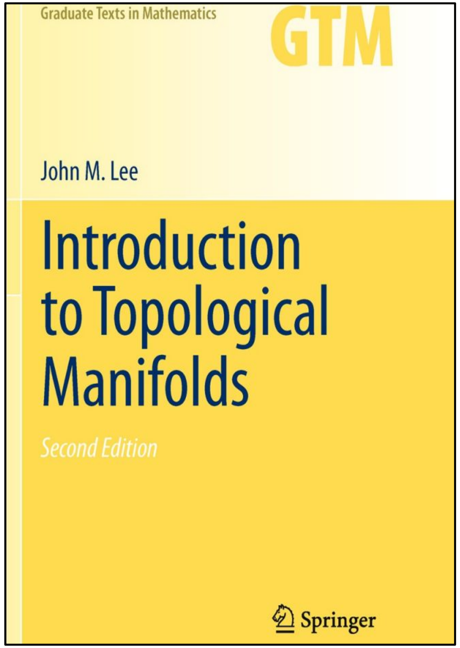
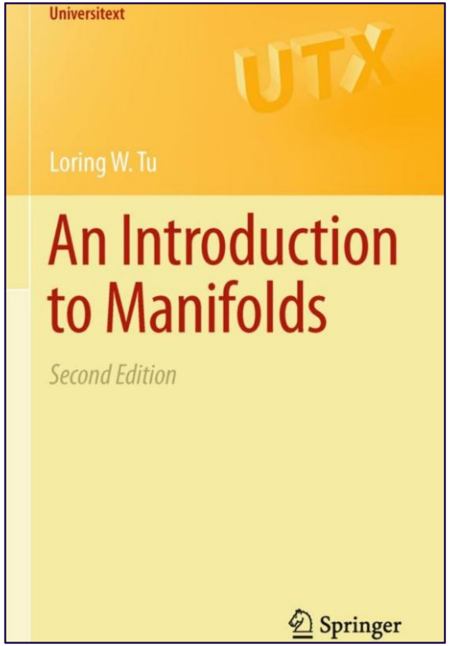
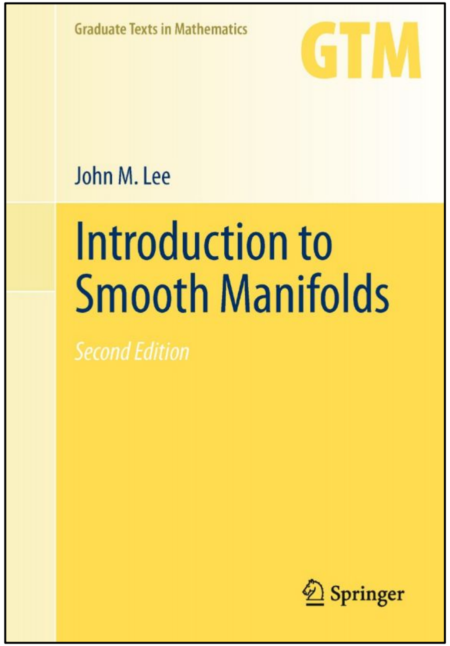

Michael Spivak
CALCULUS
ON
MANIFOLDS





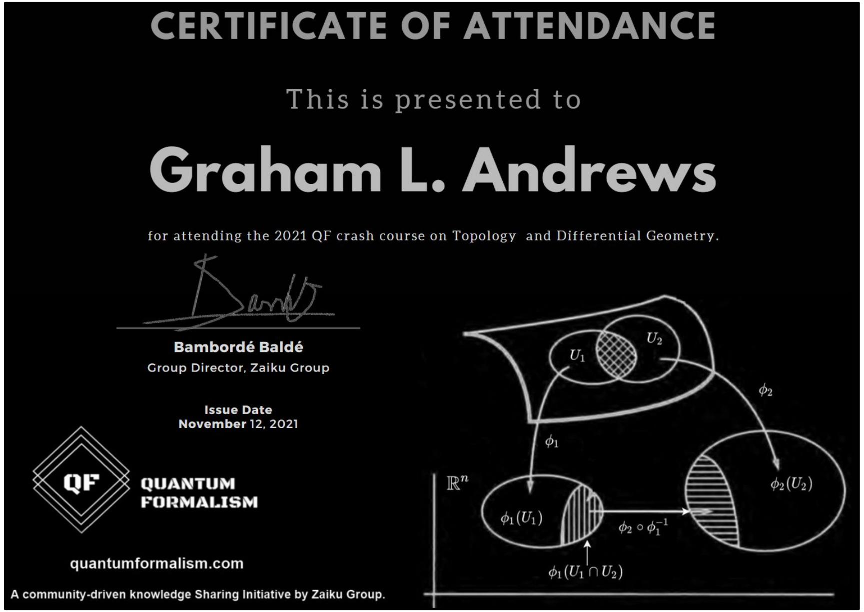


Topological & Diff. Manifolds

About Certification of Attendance

For those confirming that they've audited the entire crash course via the live lectures or the YouTube playlist. I'll take the confirmation at face value! A PDF only certificate will be issued.

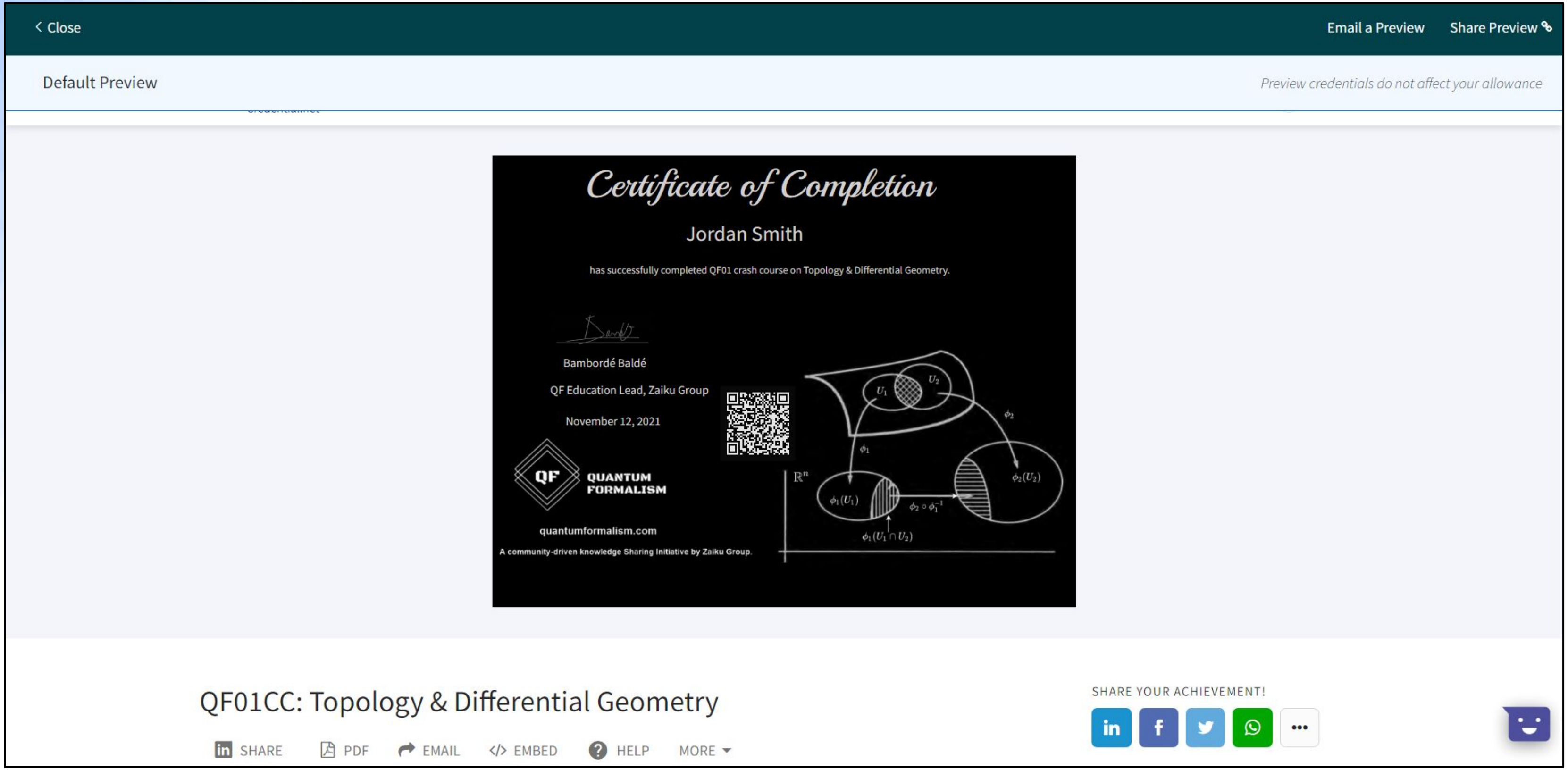
Certification of Attendance (Preview)



About Certification of Completion

For those who can prove that they've got the basic knowledge that was covered in the manifold section i.e. know the basics of topological manifolds and most importantly, the basics of smooth manifolds. A fancy digital certificate will be issued using Accredible!

Certification of Completion (Preview)



QUANTUM FORMALISM

GitHub: github.com/quantumformalism

YouTube: youtube.com/ZaikuGroup

Discord: discord.gg/SPcmcsXMD2

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