Module II Prerequisite Crash Course

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Module I (aka Foundation Module)

Rings and Fields 101 #1	Matrix Algebra 101 #2	Quantum Axioms & Operators 101 #4
Group Theory 101 #1	Linear Operators 101 #2	Finite dim. Hilbert Spaces 101 #2
Naive Set Theory Overview #1	Complex Vector spaces 101 #2	Matrix Groups 101: U(n) + SU(n) #2

#n is the number of live lectures

Module II Wish List?

1. More advanced mathematics than the foundation module.
2. Ideally, closer to the algebraic topics covered in the foundation module.
3. Relevant to pure mathematics, mathematical physics, quantum computing etc.

Module II Choice

Lie Groups, Lie Algebras & Representations

October - December?

25

Lie theory connects to almost *every* other branch of mathematics! It's almost absurdly well connected! Just off the top of my head:

- analysis (e.g. harmonic analysis and the Peter-Weyl theorem),
- algebraic topology (e.g. principal bundles and characteristic classes),
- algebraic geometry (e.g. algebraic groups and flag varieties),
- combinatorics (e.g. root systems and Coxeter groups),
- differential geometry (e.g. connections and Chern-Weil theory),
- number theory (e.g. automorphic forms and the Langlands program),
- low-dimensional topology (e.g. quantum groups and Chern-Simons theory),
- Riemannian geometry (e.g. holonomy and symmetric spaces),
- finite group theory (e.g. the finite simple groups of Lie type)...

And this is just mathematics. Lie theory is also hugely relevant to physics as well (which is related to some of the stuff above) since it's an important ingredient in the study of gauge theory; for example, the Standard Model is a gauge theory.

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edited Jun 13 '15 at 8:25

answered Jun 13 '15 at 4:47

Qiaochu Yuan
344k 38 711 1093

Interesting Applications (Maths & Physics)

Quantum Physics

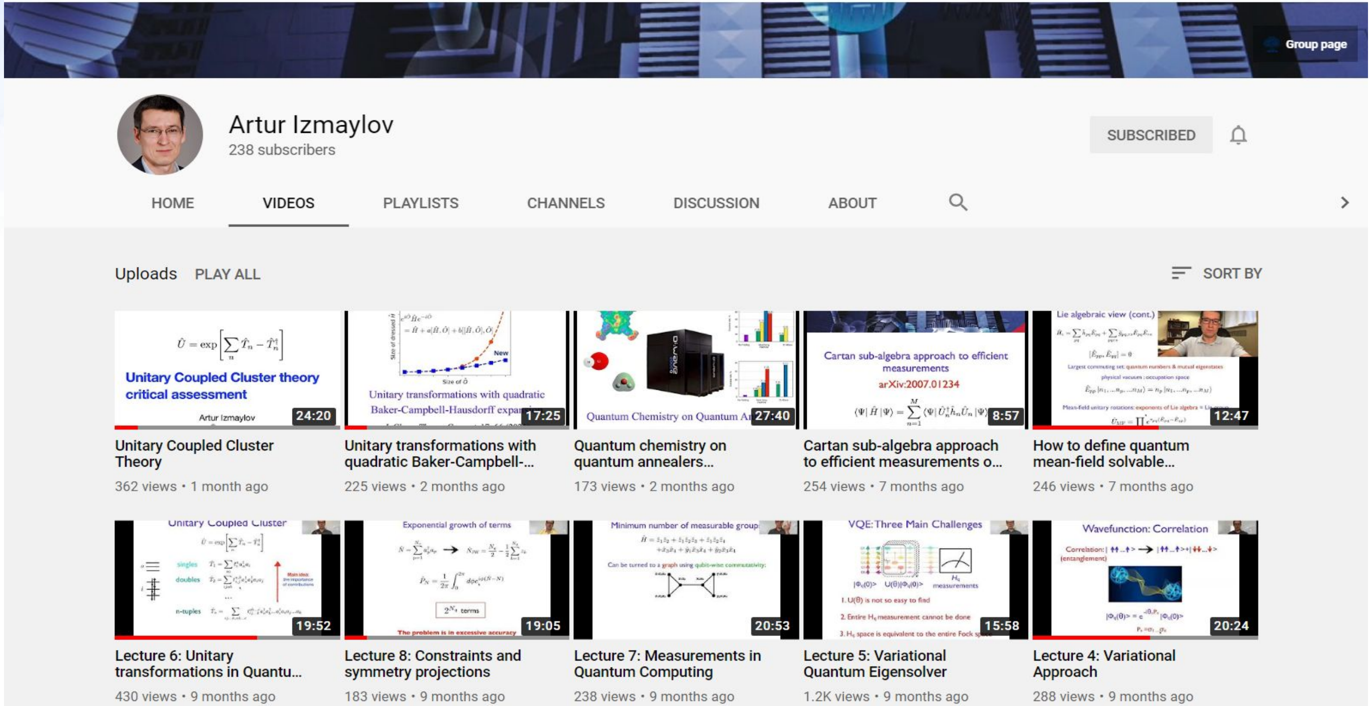
[Submitted on 13 Feb 2020 (v1), last revised 21 Nov 2020 (this version, v2)]

Unitary transformation of the electronic Hamiltonian with an exact quadratic truncation of the Baker-Campbell-Hausdorff expansion

Robert A. Lang, Ilya G. Ryabinkin, Artur F. Izmaylov

Application of current and near-term quantum hardware to the electronic structure problem is highly limited by qubit counts, coherence times, and gate fidelities. To address these restrictions within the variational quantum eigensolver (VQE) framework, many recent contributions have suggested dressing the electronic Hamiltonian to include a part of electron correlation, leaving the rest to be accounted by VQE state preparation. We present a new dressing scheme that combines preservation of the Hamiltonian hermiticity and an exact quadratic truncation of the Baker-Campbell-Hausdorff expansion. The new transformation is constructed as the exponent of an involutory linear combination (ILC) of anti-commuting Pauli products. It incorporates important strong correlation effects in the dressed Hamiltonian and can be viewed as a classical preprocessing step alleviating the resource requirements of the subsequent VQE application. The assessment of the new computational scheme for electronic structure of the LiH, H_2O, and N_2 molecules shows significant increase in efficiency compared to conventional qubit coupled cluster dressings.

Interesting Application (VQEs)



Interesting Application (VQEs)

Deep Learning on Lie Groups for Skeleton-based Action Recognition

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Abstract

In recent years, skeleton-based action recognition has become a popular 3D classification problem. State-of-the-art methods typically first represent each motion sequence as a high-dimensional trajectory on a Lie group with an additional dynamic time warping, and then shallowly learn favorable Lie group features. In this paper we incorporate the Lie group structure into a deep network architecture to learn more appropriate Lie group features for 3D action recognition. Within the network structure, we design rotation mapping layers to transform the input Lie group features into desirable ones, which are aligned better in the temporal domain. To reduce the high feature dimensionality, the architecture is equipped with rotation pooling layers for the elements on the Lie group. Furthermore, we propose a logarithm mapping layer to map the resulting manifold data into a tangent space that facilitates the application of regular output layers for the final classification. Evaluations of the proposed network for standard 3D human action recognition datasets clearly demonstrate its superiority over existing shallow Lie group feature learning methods as well as most conventional deep learning methods.

As studied in [41, 3, 42], Lie group feature learning methods often suffer from speed variations (i.e., temporal misalignment), which tend to deteriorate classification accuracy. To handle this issue, they typically employ dynamic time warping (DTW), as originally used in speech processing [30]. Unfortunately, such process costs additional time, and also results in a two-step system that typically performs worse than an end-to-end learning scheme. Moreover, such Lie group representations for action recognition tend to be extremely high-dimensional, in part because the features are extracted per skeletal segment and then stacked. As a result, any computation on such nonlinear trajectories is expensive and complicated. To address this problem, [41, 3, 42] attempt to first flatten the underlying manifold via tangent approximation or rolling maps, and then exploit SVM or PCA-like method to learn features in the resulting flattened space. Although these methods achieve some success, they merely adopt shallow linear learning schemes, yielding sub-optimal solutions on the specific nonlinear manifolds.

Deep neural networks have shown their great power in learning compact and discriminative representations for images and videos, thanks to their ability to perform nonlinear computations and the effectiveness of gradient descent training with backpropagation. This has motivated us to

Interesting Application (Deep Learning)

Lie group

From Wikipedia, the free encyclopedia

Not to be confused with Group of Lie type.

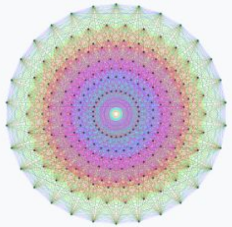
In mathematics, a **Lie group** (pronounced /liː/ "Lee") is a group that is also a differentiable manifold. A manifold is a space that locally resembles Euclidean space, whereas groups define the abstract, generic concept of multiplication and the taking of inverses (division). Combining these two ideas, one obtains a continuous group where points can be multiplied together, and their inverse can be taken. If, in addition, the multiplication and taking of inverses are defined to be smooth (differentiable), one obtains a Lie group.

Lie groups provide a natural model for the concept of continuous symmetry, a celebrated example of which is the rotational symmetry in three dimensions (given by the special orthogonal group $\mathrm{SO}(3)$). Lie groups are widely used in many parts of modern mathematics and physics.

Lie groups were first found by studying matrix subgroups G contained in $\mathrm{GL}_n(\mathbb{R})$ or $\mathrm{GL}_n(\mathbb{C})$, the groups of $n \times n$ invertible matrices over $\mathbb{R}$ or $\mathbb{C}$. These are now called the classical groups, as the concept has been extended far beyond these origins. Lie groups are named after Norwegian mathematician Sophus Lie (1842–1899), who laid the foundations of the theory of continuous transformation groups. Lie's original motivation for introducing Lie groups was to model the continuous symmetries of differential equations, in much the same way that finite groups are used in Galois theory to model the discrete symmetries of algebraic equations.

Contents [hide]

Lie groups

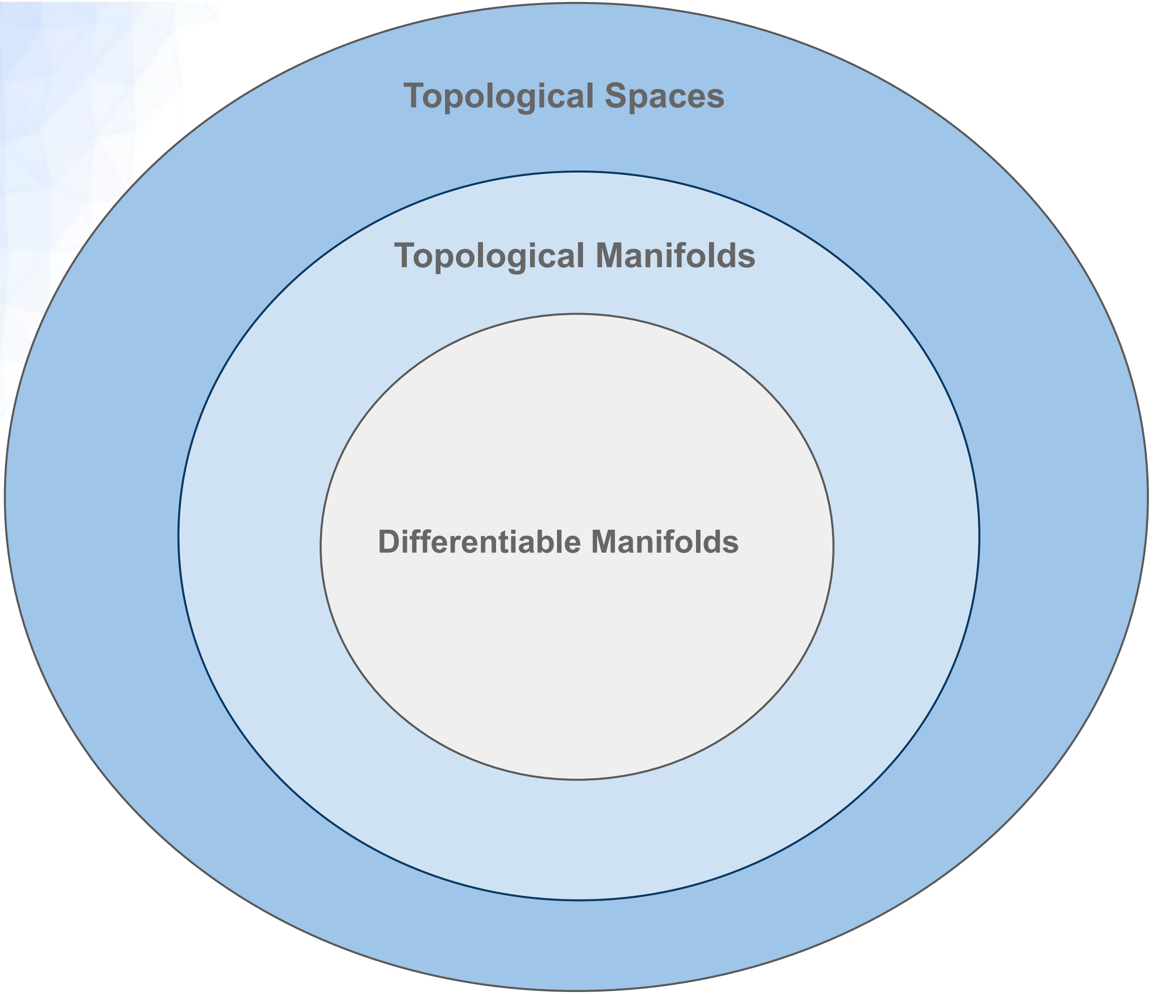


Classical groups	[show]
Simple Lie groups	[show]
Other Lie groups	[show]
Lie algebras	[show]
Semisimple Lie algebra	[show]
Representation theory	[show]
Lie groups in physics	[show]
Scientists	[show]

Glossary
Table of Lie groups

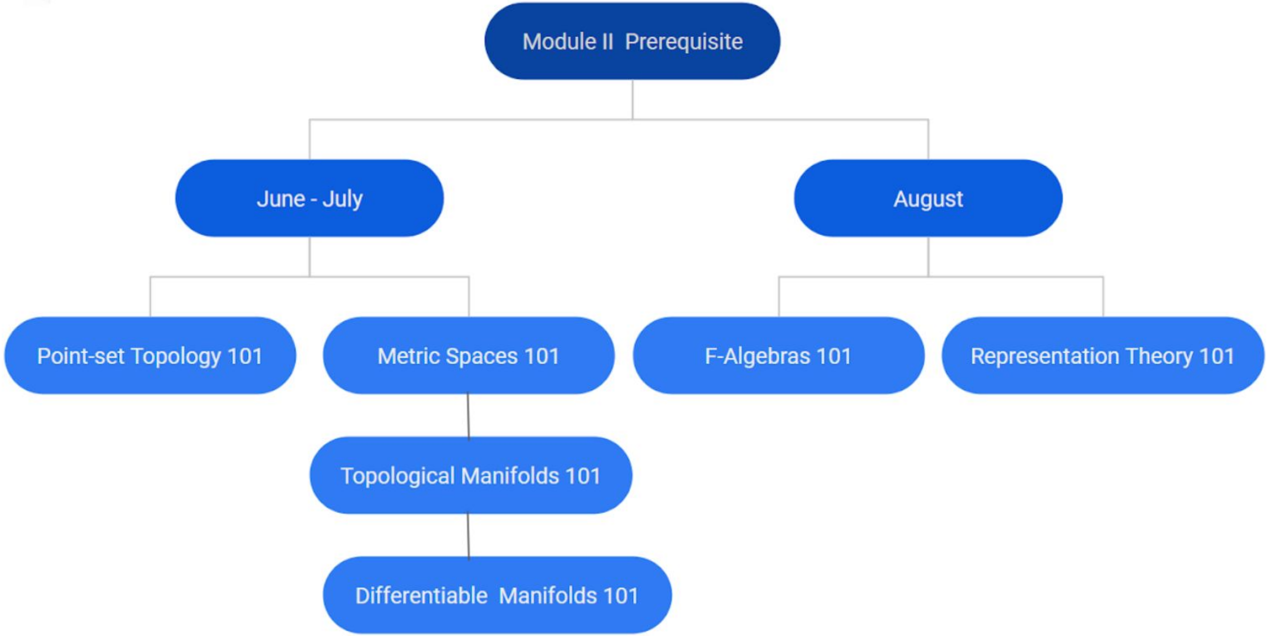
V · T · E

Algebraic structure → Group theory
Group theory



quantumformalism.com

Crash Course Plan



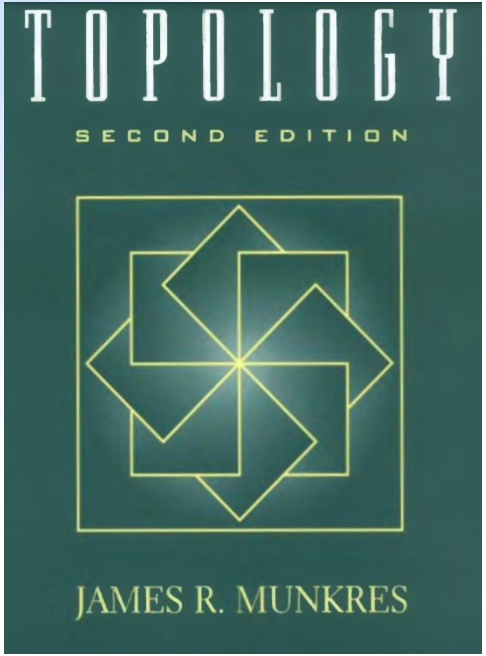
Baseline Knowledge Requirements

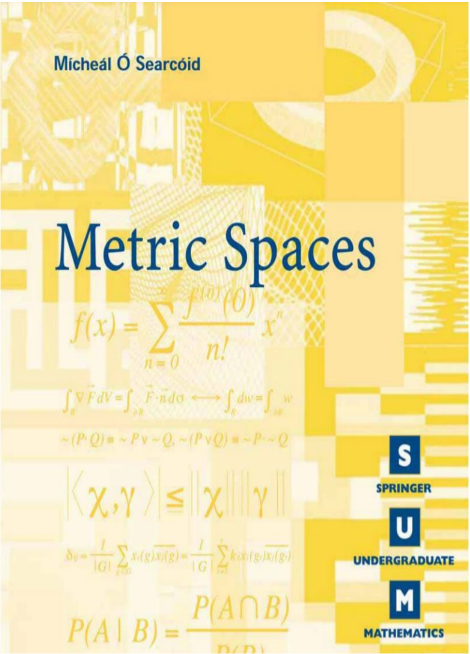
Must know the basic concepts of naive set theory including:

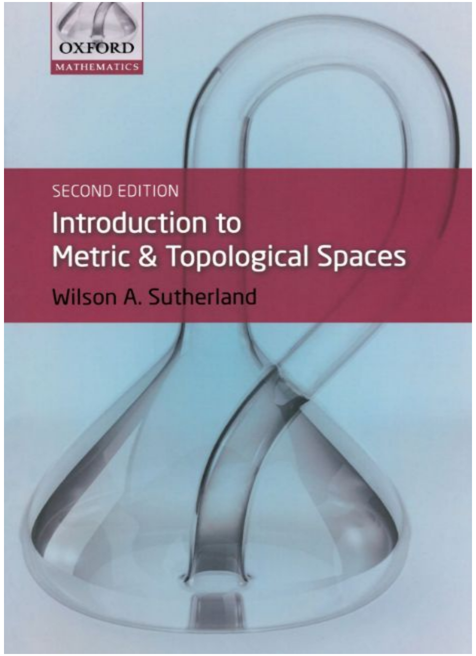
- The power set.
- Intersections and unions.
- Complements.
- Cartesian products.
- Indexing sets as well as their unions and intersections.
- Maps between sets including; pre-images, composition of maps, injectiveness, surjectiveness, bijection etc.

Source for revising:
https://github.com/quantumformalism/math-lectures/blob/master/foundation-module/lecture-01/SWDM.pdf

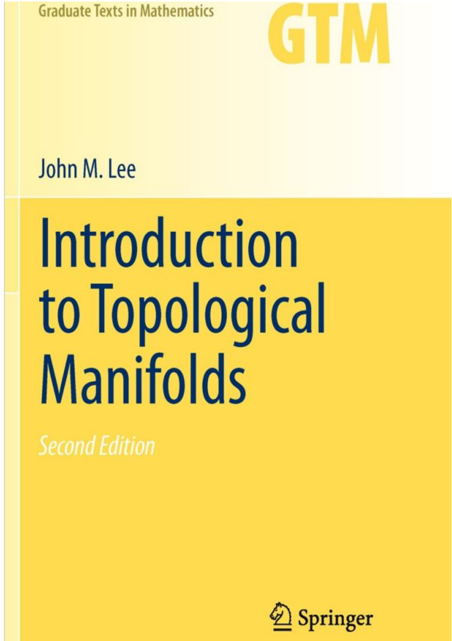
Recommended focus on pages **80 - 115 and pages **157 - 196****

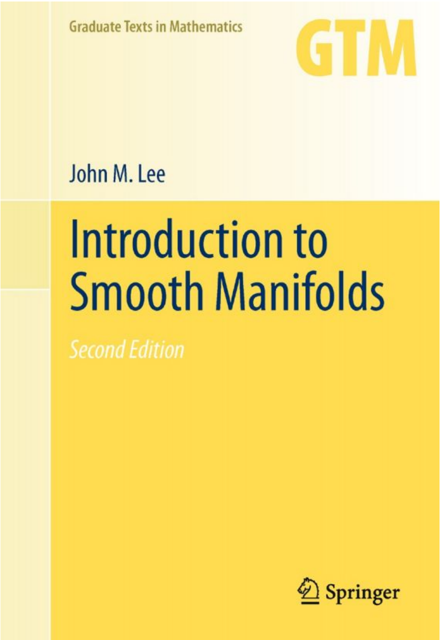






Point-set Topology & Metric Spaces





Topological & Diff. Manifolds

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