$\int_S f \, \mathrm{d}\mu.$ QF QUANTUM FORMALISM quantumformalism.com

Homework 3

Directions: Answer the following questions. You are encouraged to work together, join the discussion sessions, use discord, and ask me questions!

1. If $\mu_1, \ldots, \mu_n$ are measures on $(X, \mathcal{M})$, and $a_1, \ldots, a_n \in [0, \infty)$, then $\sum_{j=1}^{n} a_j \mu_j$ is a measure on $(X, \mathcal{M})$.

2. Suppose $(X, \mathcal{M}, \mu)$ is a measure space and that $E, F \in \mathcal{M}$. Show that $\mu(E) + \mu(F) = \mu(E \cup F) + \mu(E \cap F)$.

3. Let X be an uncountable set and let $\mathcal{A}$ be the collection of subsets A of X such that either A or A^c is countable. Define $\mu(A) = 0$ if A is countable and $\mu(A) = 1$ is A is uncountable. Prove that μ is a measure.

4. Let X be a set with σ-algebra $\mathcal{M}$. We say μ is a **finitely additive measure** if $\mu : \mathcal{M} \to [0, \infty]$ such that $\mu(\emptyset) = 0$ and given any finite collection of disjoint sets $E_1, \ldots, E_n \in \mathcal{M}$, $\mu(\cup_{j=1}^{n} E_j) = \sum_{j=1}^{n} \mu(E_j)$. Note that every measure is finitely additive.

   a) Suppose that μ is a finitely additive measure on $(X, \mathcal{M})$ and μ is continuous from below. Prove that μ is a measure.

   b) Suppose that μ is a finitely additive measure on $(X, \mathcal{M})$, $\mu(X) < \infty$ and μ is continuous from above. Prove that μ is a measure.
