Multimode Conditional Displacements

Motivation

- SNAP Gates take time $\approx 2\pi/\chi$ where $\chi \approx$ MHz is dispersive coupling strength.
- Reducing Gate time $\rightarrow$ Increasing χ $\rightarrow$Reducing lifetime of cavity
- ECD Idea: Keep $\chi \approx 10$ kHz small; But enhance it by displacing cavity (α_0) far from origin
- Effective Gate time $1/\chi\alpha_0$ where $\alpha_0 \gg 1$

Achieving Conditional Displacements

Starting Point: $$H/\hbar = \omega_c a^\dagger a + \omega_q \frac{\sigma_z}{2} + \chi a^\dagger a \frac{\sigma_z}{2} + H_{drive}$$

Using **frame transformations**, our objective is to **isolate** the following term from the ac-Stark Shift

$$\tilde{H} = \chi(\alpha a^\dagger + \alpha^\star a)\sigma_i$$

where α is the displacement of the cavity mode. With such a term, we can realize a conditional displacement as follows

$$e^{-i(\chi(\alpha a^\dagger + \alpha^\star a)\sigma_i)t} \xleftrightarrow{\beta = -i\,\chi\alpha\,t} e^{(\beta a^\dagger - \beta^\star a)\sigma_i}$$

Dealing with Unwanted Terms I

1. Rotating Frames of oscillator and the qubit
2. Displacement transformation $D^{+}(\alpha(t)) = e^{\alpha^{*}(t)a - \alpha(t)a^{+}}$

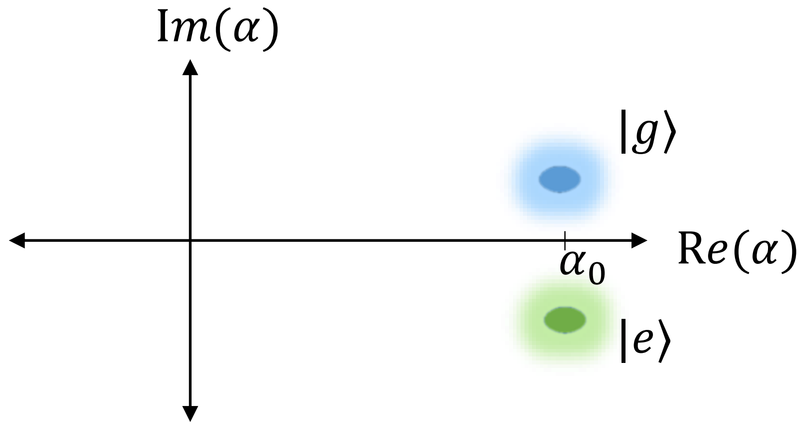
which renders $a \rightarrow a + \alpha(t)$

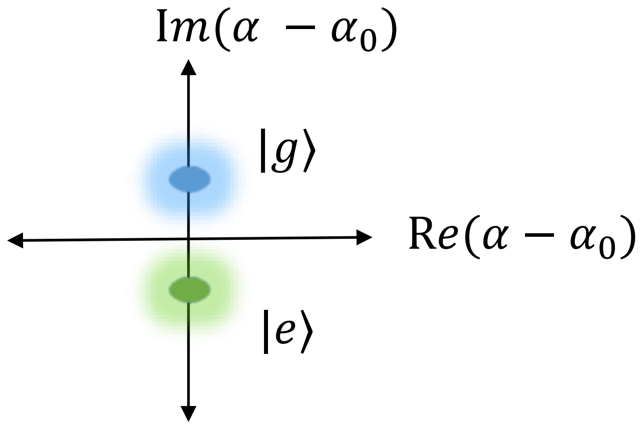
$$
\begin{aligned}
H_{disp} &= D^{\dagger} H_{rot} D - i\dot{D}^{\dagger} D \\
&= D^{\dagger} H_{rot} D + i(\dot{\alpha}^{\star} a - \dot{\alpha} a^{\dagger})
\end{aligned}
$$

Cancel terms linear in $\boldsymbol{a}, \boldsymbol{a}^{+}$, such as the oscillator drive $\epsilon(t)a^{+} + \epsilon^{*}(t)a$, by picking the appropriate time dependent displacement frame

$$\boxed{\dot{\alpha} = -i\epsilon(t) \qquad \dot{\alpha}^{*} = i\epsilon^{*}(t)}$$

Implication: Disp. Frame Simulations





Lab Frame

- Large Displacement
- Number of photons $n = |\alpha_0|^2 \approx 900$
- Intractable simulations

Displaced Frame

- Size of Conditional Displacement ($|\alpha_g - \alpha_e| \leq 5$)
- Number of photons $n = \left|\alpha_g - \alpha_e\right|^2 \approx 25$
- Tractable simulations

Dealing with Unwanted Terms II

The **displaced frame** transformation, however, divides the **initial ac-Stark shift** term into the following 3 terms

$$\chi(a^\dagger + \alpha^*)(a + \alpha)\sigma_z$$

$$\downarrow$$

$$\chi a^\dagger a \sigma_z \quad + \quad \underbrace{\chi(\alpha a^\dagger + \alpha^* a)\sigma_z}_{\text{desired}} \quad + \quad \chi|\alpha|^2\sigma_z$$

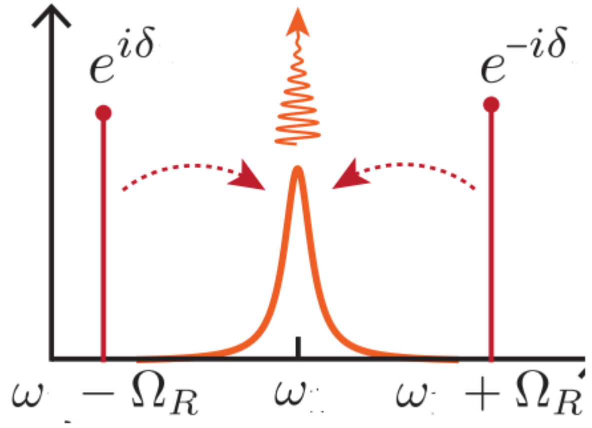
Sideband Drives

- Make terms **oscillate at different** frequencies
- Invoke RWA in a frame where only desired term is stationary

Echoed Cond. Displacements

- Terms have different no. of α's but only a single σ_z
- **Clever flipping of α and $\boldsymbol{\sigma_z}$** can echo out unwanted terms

Sideband Drives

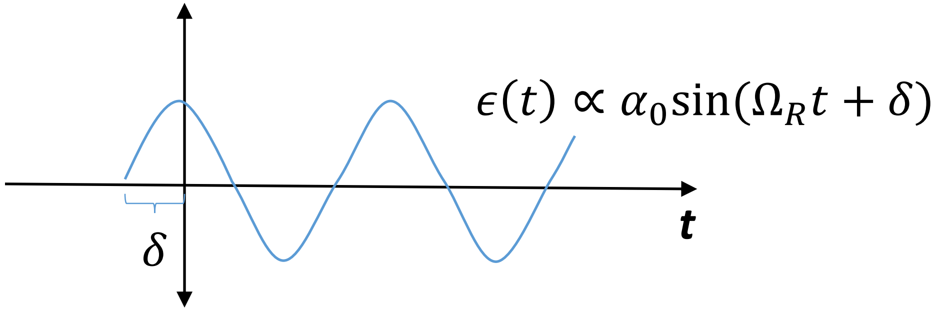


https://arxiv.org/pdf/1608.06652.pdf

Since α oscillatory,

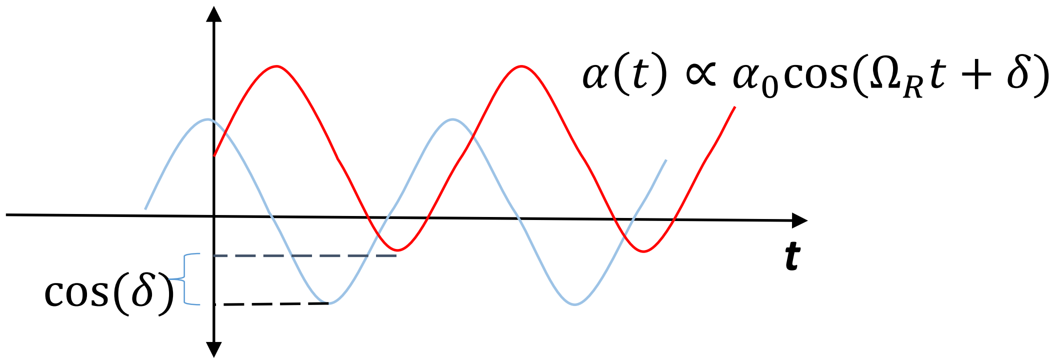
$$H = \chi a^{+} a \sigma_z + \chi(\alpha a^{+} + \alpha^{*} a)\sigma_z + \chi|\alpha|^2 \sigma_z + \Omega_R \sigma_x$$

$\omega = 0$ $\quad$ $\omega = \Omega_R$ $\quad$ $\omega = 2\Omega_R$



Frame Transformations:

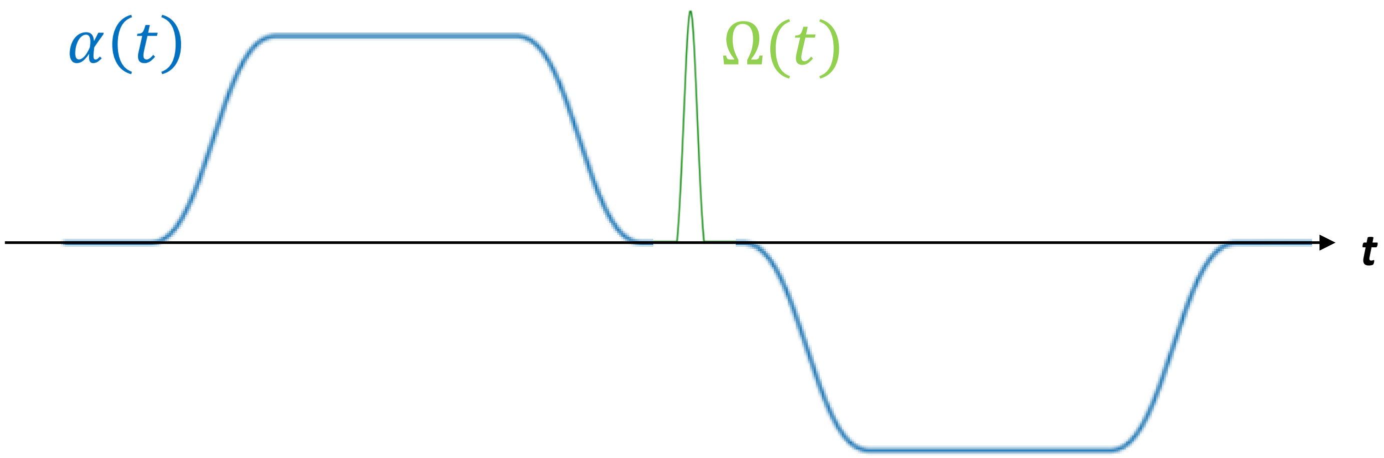
1. $\sigma_x \leftrightarrow \sigma_z$ $\longrightarrow$ $\Omega_R \sigma_z$
2. Rotating Frame of the qubit $\quad$ $\cancel{\Omega_R \sigma_z}$



$$H = \chi \alpha_0 (a^{+} + a) \otimes (\sigma_x \cos\delta + \sigma_y \sin\delta) + \ldots$$

$\omega = 0$ $\quad$ $\omega \geq \Omega_R$

Echoed Cond. Disp.



$$\chi a^+ a \sigma_z$$

$$\chi(\alpha a^+ + \alpha^* a)\sigma_z$$

$$\chi|\alpha|^2\sigma_z$$

Echo

$$-\chi a^+ a \sigma_z$$

Not completely echoed out !

$$\chi(\alpha a^+ + \alpha^* a)\sigma_z$$

$$-\chi|\alpha|^2\sigma_z$$

Comparison

Sideband Drives

- Oscillating $\epsilon(t), \alpha(t)$
- Continuous Rabi Driving on the qubit
- Rotating Wave Approximation: $e^{i\Omega t}$

Echoed Conditional Gates

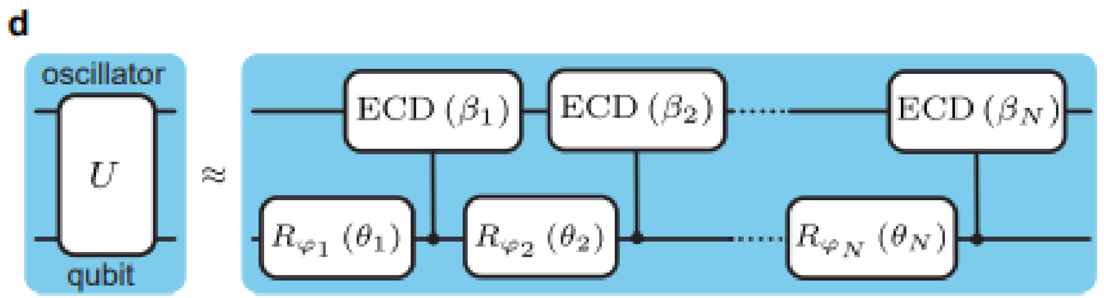
- Oscillating, yet stretched $\epsilon(t), \alpha(t)$
- Discrete Qubit pi pulses
- Flipping sign- not fully rotating but hopping on the complex plane (if think of coefficients of unwanted terms as step function)

Implementation: Optimal Parameters

- ECD and Sideband Drives, by themselves, do not offer universal control of both oscillator and qubit
- Sol: Interleave parameterized qubit rotations between CD
- Gate times are dependent on # of layers to realize high fidelity gates

$$CD(\beta) = D\left(\frac{\beta}{2}\right)|g\rangle\langle g| + D\left(-\frac{\beta}{2}\right)|e\rangle\langle e|$$

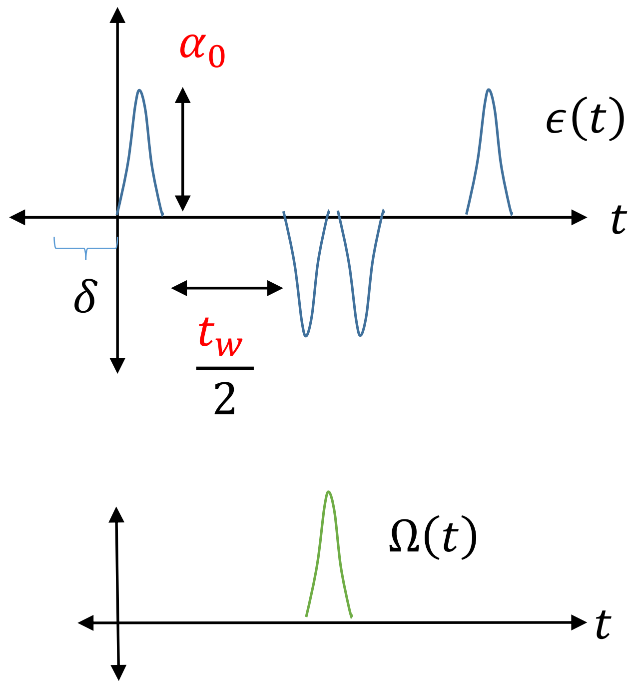
$$R_{\phi}(\theta) = e^{-i\left(\frac{\theta}{2}\right)(\cos\phi\sigma_x + \sin\phi\sigma_y)}$$



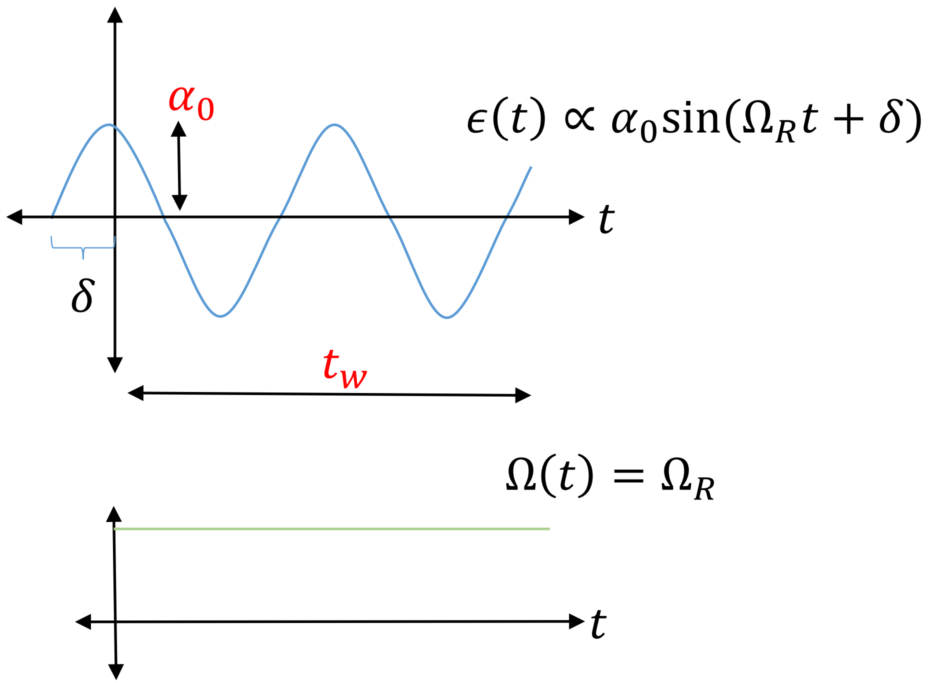
Implementation: Finding Pulses

Task: find wait time t_w and scale intermediate displacement $\alpha_0$such that $\chi\alpha_0 t_w = \beta$

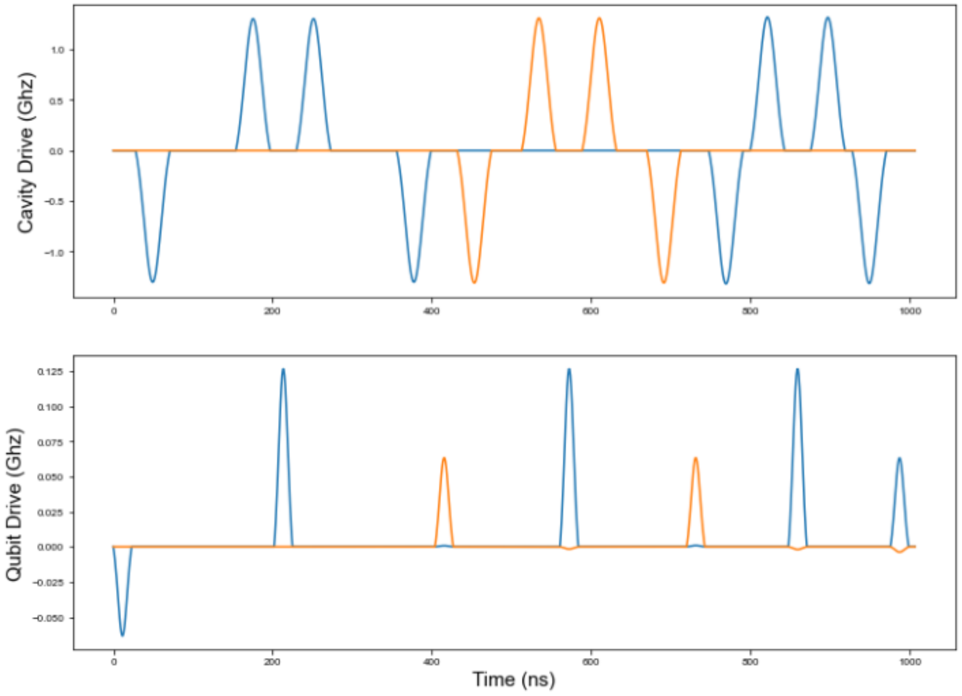
ECD

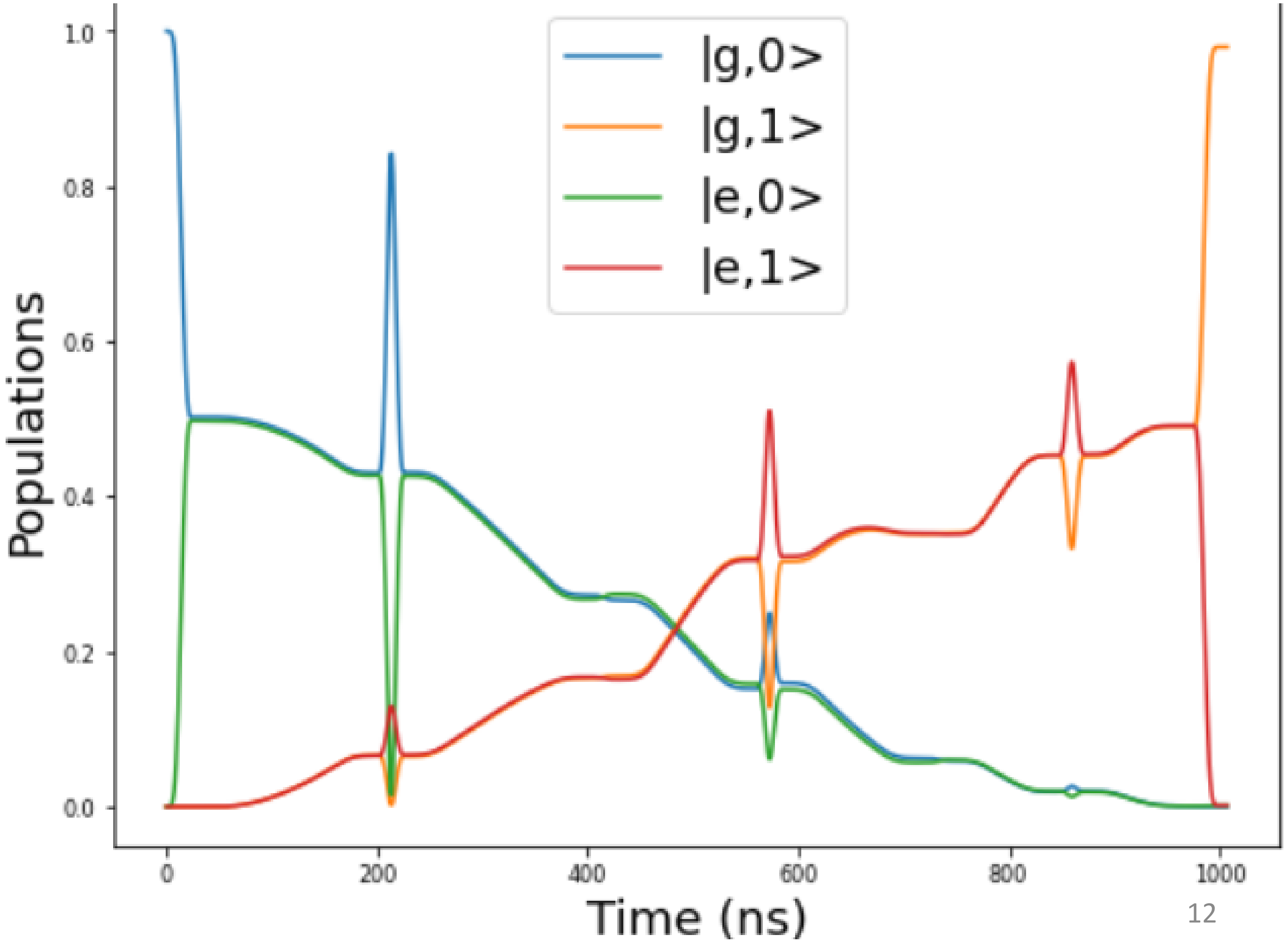


Sideband



ECD: $|g0\rangle \rightarrow |g1\rangle$

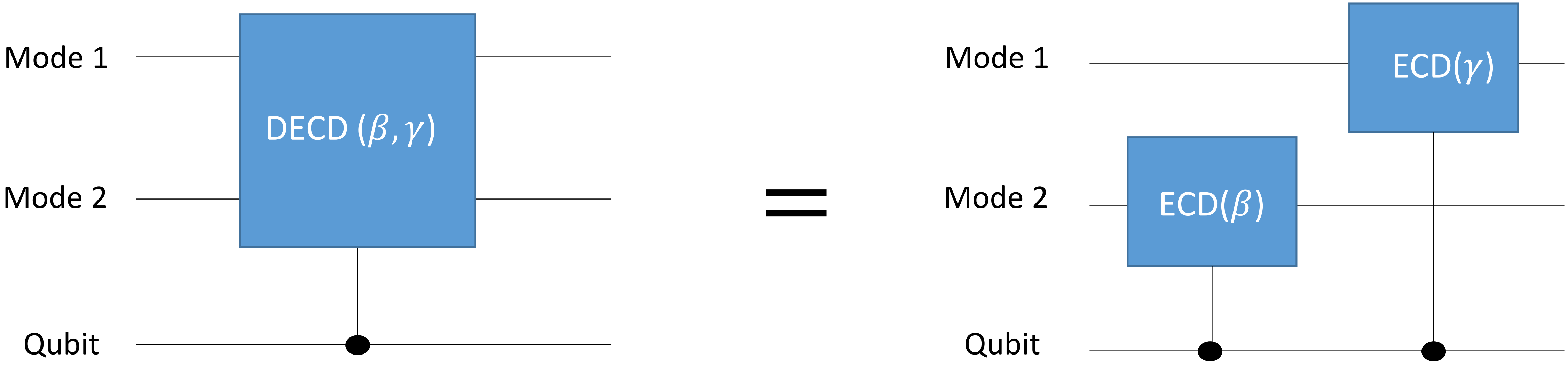




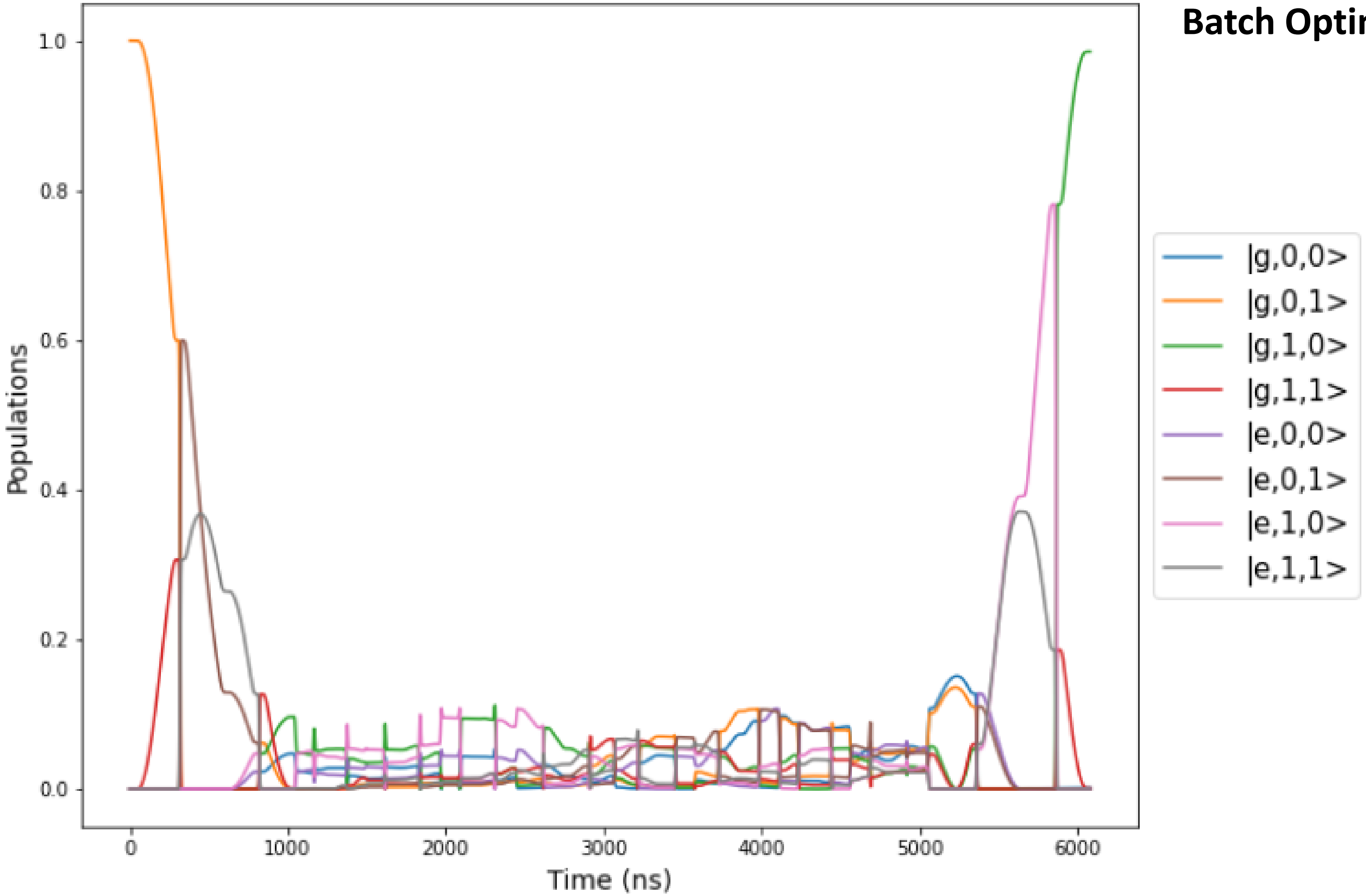
Two Mode ECD

- Generalize ECD idea to two modes coupled to an ancilla qubit

- Layer Reduction Strategies needed



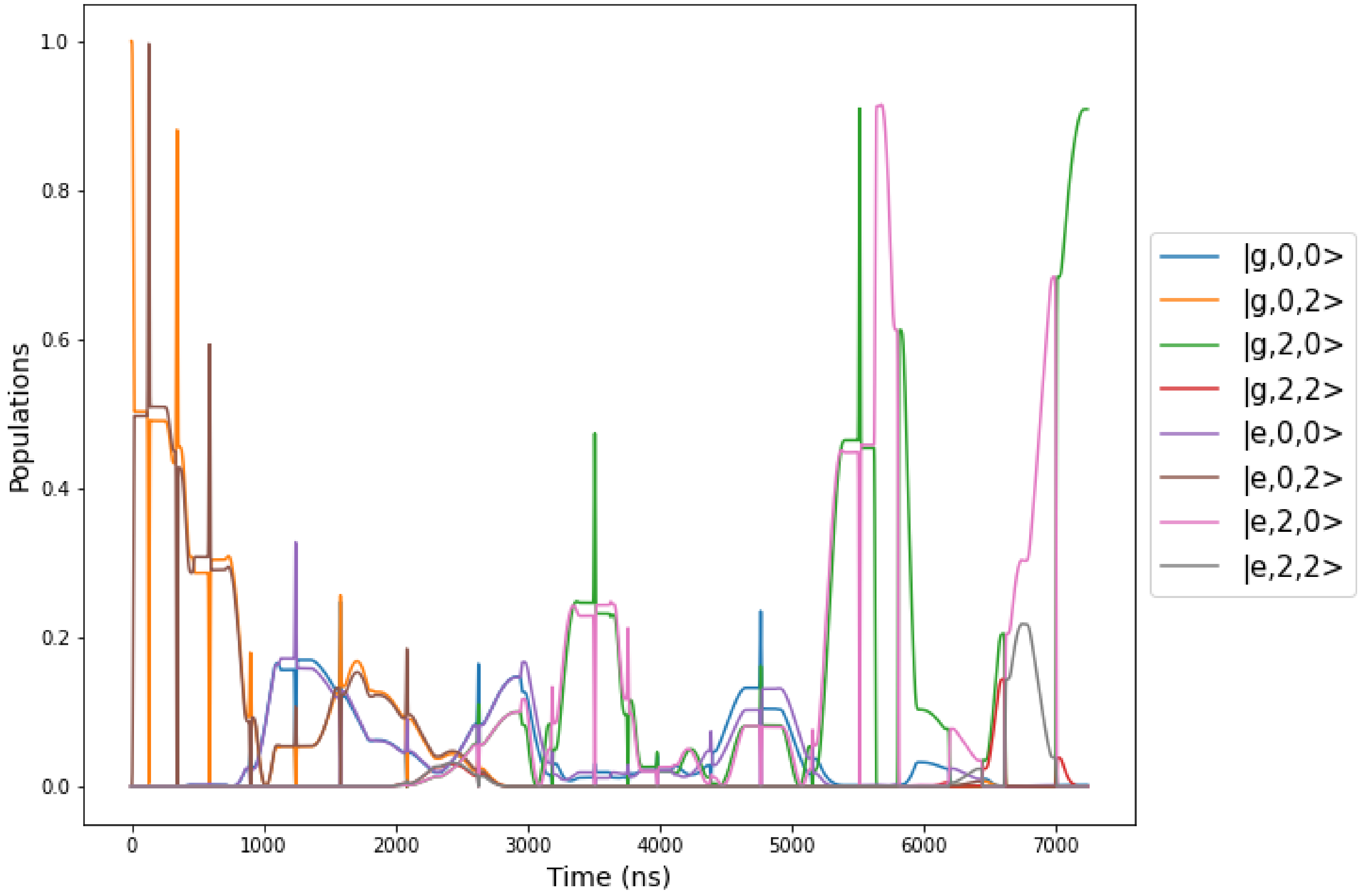
Two Mode ECD : $|g01\rangle \rightarrow |g10\rangle$



Batch Optimizer Fidelity: 0.995308

Two Mode ECD: $|g02\rangle \rightarrow |g20\rangle$

Batch Optimizer Fidelity: 0.9219278



Two Mode ECD : Why not Simultaneous Displacements?

$$\eta a^{+}a\, b^{+}b \xrightarrow{\text{Displaced Frame Transformation}} \eta(a^{+}+\alpha^{*})(a+\alpha)(b^{+}+\beta^{*})(b+\beta)$$

- Produces terms like $\eta\boldsymbol{\alpha\beta}\, a^{+}b^{+}$ which can be active if α, β are simultaneously nonzero
- Even if we avoid simultaneous displacements, this still leaves us with terms such as $\eta|\boldsymbol{\alpha}|^{\mathbf{2}}b^{+}b$ and $\eta\boldsymbol{\alpha}\, a^{+}b^{+}b$
- Thus need small $\eta = 2\sqrt{\chi_a\chi_b}$

Two Mode ECD

Meta Echoes

Grape

Enter Circle Grape

Sideband Drives
(Grap-ified)

- Optimizing over $\delta(t)$ and $\alpha(t)$ --- changing nature of oscillator drive but keeping qubit drive constant

Circle Grape

- Optimizing over $\Omega(t)$ while applying a constant cavity drive.
- These 2 approaches are then opposites