Fast Control of Multimode Cavities with Conditional Displacements

Eesh Gupta, S. Chakram, ...

Flux transformers to introduce Static Magnetic Fields

Outline

Challenges in Cavity Control

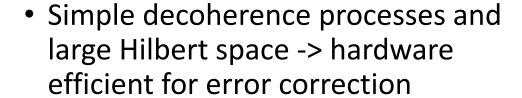
Multimode Echoed Conditional Displacements

Alternative Schemes

^{**} Ma, Wen-Long, et al. *Science Bulletin* 66.17 (2021): 1789-1805.

Why 3D Cavities?

 High Coherence times compared to transmons *



 Universal control achieved via coupling to transmons





* Romanenko, A., et al. Physical Review Applied 13.3 (2020): 034032.



Goal: Enact high-speed and low-error gates on cavity modes coupled to ancilla

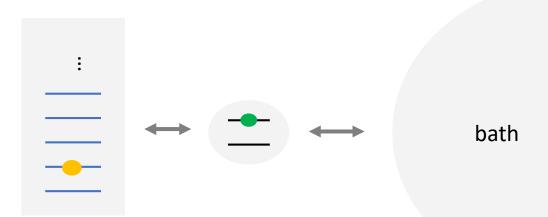
Challenges in Cavity Control

Coupling to lossy ancilla > source of error for high-Q cavities!

Transmon Relaxation

 Gate Errors dominated by ancilla coherence

•
$$\epsilon \sim \frac{\gamma_q}{\chi} = \frac{1}{(\chi T_q)}$$



• Mitigate by strong $\chi \gg \gamma_q {\sim} 10 \ \mathrm{kHz}$

Challenges in Cavity Control

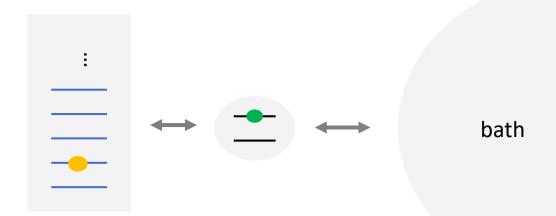
Can use finite-level ancilla for universal control

Inverse Purcell Effect

 Coupling to lossy ancilla reduces mode coherence

•
$$T_1^{cav} \leq \frac{\Delta^2}{g^2} T_1^q \sim \frac{2\alpha}{\chi} T_1^q$$

• Mitigate by weak $\chi \sim 10~\mathrm{kHz}$



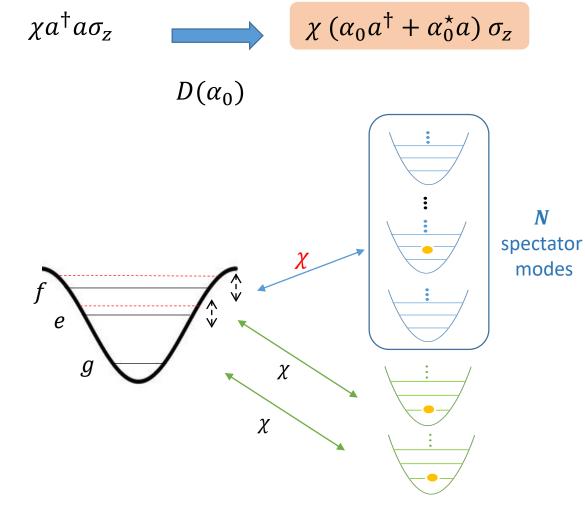
Conditional Displacement Gates

- Use Large Displacements to enhance effective interaction strength
- Weak $\chi/2\pi \sim 30$ kHz but strong $\chi\alpha_0/2\pi \sim 1$ MHz

Crosstalk Errors

 Dispersive shift causes frequency shifts in transmon from photons in spectator modes

$$-\epsilon_{coh} = \frac{N\chi}{\chi\alpha_0} = \frac{N}{\alpha_0}$$



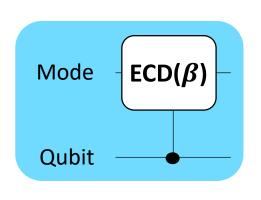
- Hacohen-Gourgy, S., Martin, L., Flurin, E. et al. Nature **538**, 491–494 (2016).
- Eickbusch, A., Sivak, V., Ding, A.Z. et al. *Nat. Phys.* 18, 1464–1469 (2022)

Echoed Conditional Gates

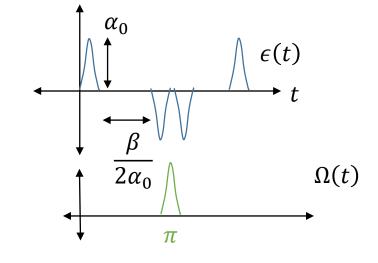
$$\chi(a^{\dagger} + \alpha^{*})(a + \alpha)\sigma_{z}$$

$$=$$

$$\chi a^{\dagger} a \sigma_{z} + \chi(\alpha a^{\dagger} + \alpha^{*} a) \sigma_{z} + \chi(\alpha | \alpha^{2} \sigma_{z})$$



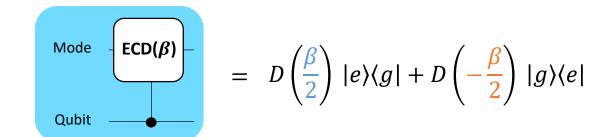
$$= D\left(\frac{\beta}{2}\right) |e\rangle\langle g| + D\left(-\frac{\beta}{2}\right) |g\rangle\langle e| =$$

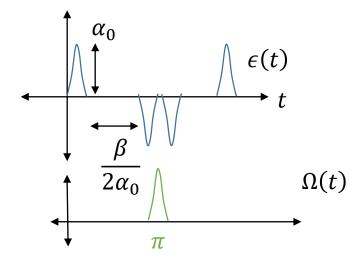


Echoed Conditional Displacements

 Symmetric pulse sequences to isolate conditional displacement term in Hamiltonian.

• Universal control of single mode coupled to qubit with gate set $\{ECD(\beta), R_{\phi}(\theta)\}$



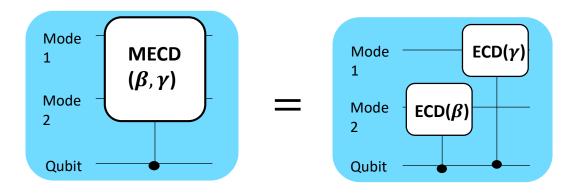


Multimode ECD

Sequentially acting ECD Gates on each mode

 Driving modes one at a time to avoid heating effects and amplifying cross kerr interaction. [1][2]

 Instead of enhanced cross kerr interaction, rely on JC-like interaction between 'tunable qubit' and modes





$$H_{\text{had}} = \sum_{i=1,2} \chi_i \alpha_i (a_i^{\dagger} + a_i) \sigma_x + \Omega(t) \sigma_z$$

Jaynes Cummings
Interaction

Tunable Qubit

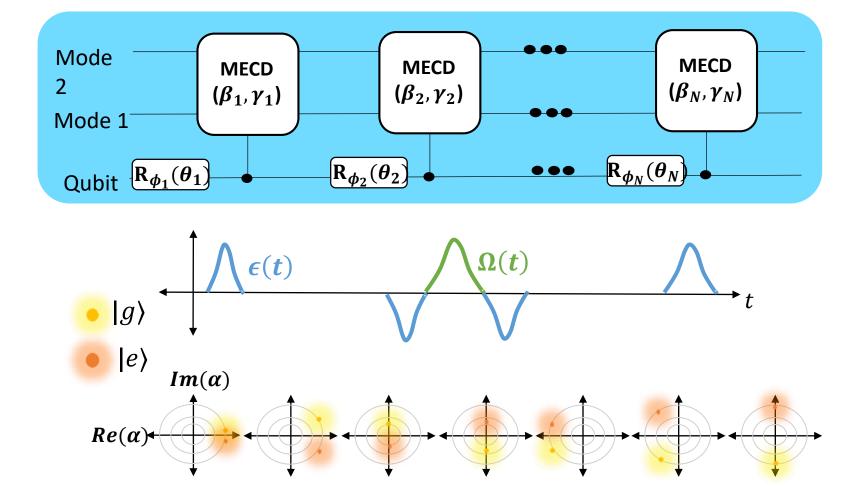
^[1] Eickbusch, Alec, et al. W34. 00005. APS March Meeting (2022).

^[2] Diringer, Asaf A., et al. arXiv preprint arXiv:2301.09831 (2023).

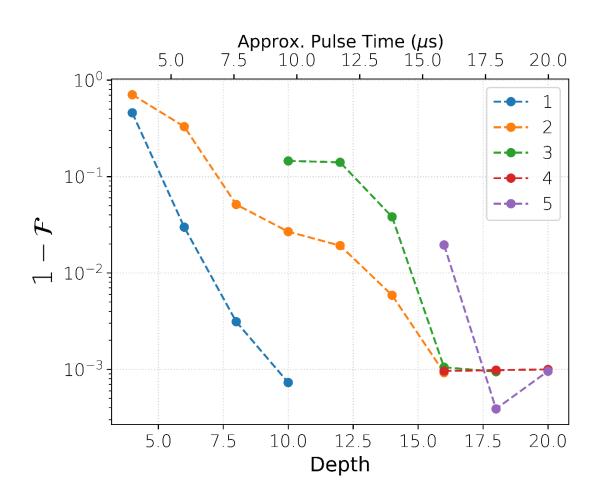
Workflow

Parameter Optimization

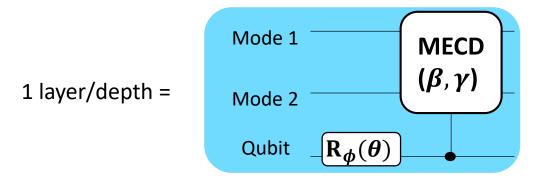
Pulse Optimization



Two Mode ECD: State Transfer

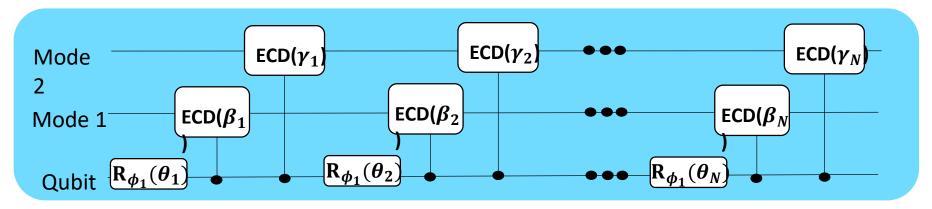


$$g \otimes |0n\rangle \rightarrow g \otimes |n0\rangle$$



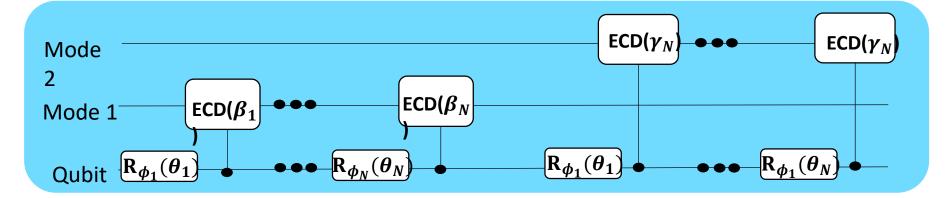
Two Mode ECD: State Transfer

$$g \otimes (|n0\rangle \rightarrow |0n\rangle)$$

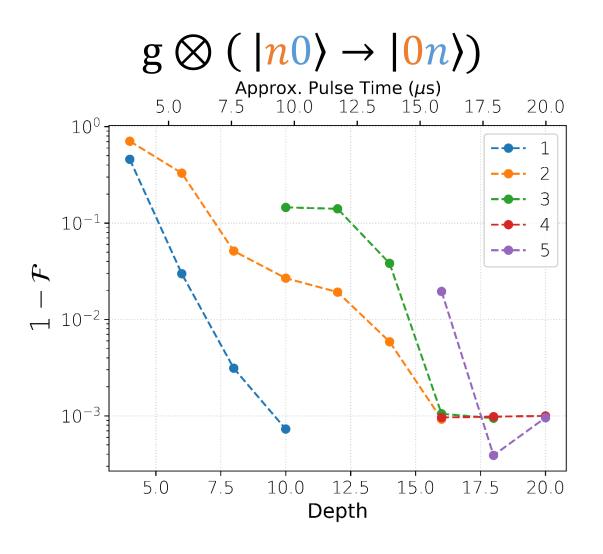


$$g \otimes (|n\rangle \rightarrow |0\rangle)$$

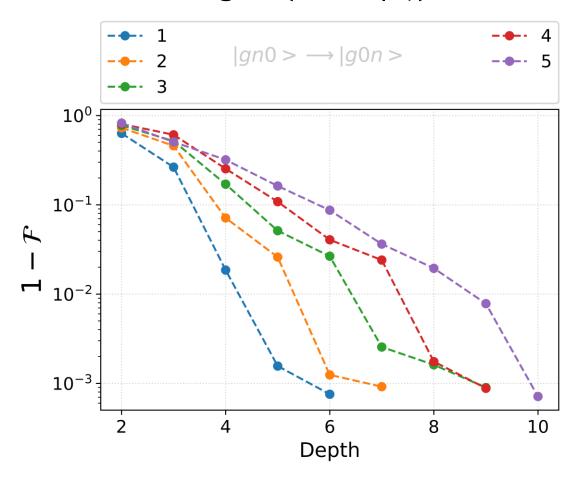
$$g \otimes (|0\rangle \rightarrow |n\rangle)$$



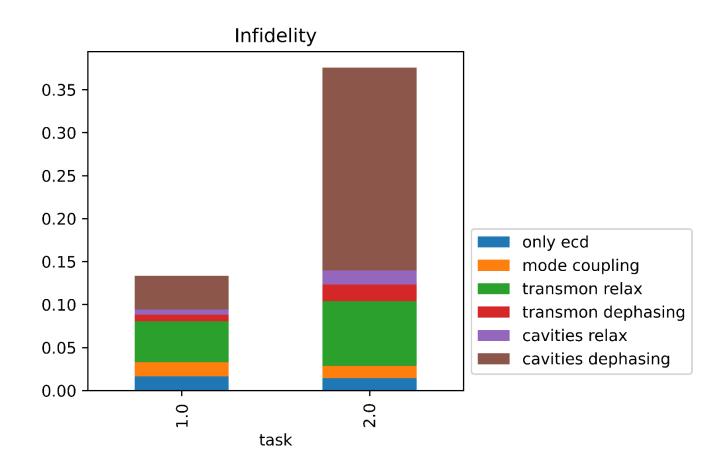
Two Mode ECD: State Transfer



$$g \otimes (|0\rangle \rightarrow |n\rangle)$$
$$g \otimes (|n\rangle \rightarrow |0\rangle)$$



Multimode ECD: Error Budget



- $T_{1,q} = 85 \,\mu\text{s}$
- $T_{2,q} = 98 \,\mu s$
- $T_{1,c} = 2 \, ms$
- $T_{\phi,c} = 150 \ ms \ *$

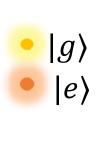
^{*} A. Eickbusch, ..., R. Schoelkopf, M. Devoret. ArXiv preprint arXiv:2111.06414 (2021)

Circle Grape

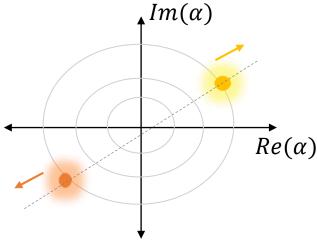
Qubit Drive Optimized

 Cavity mode always driven far from origin in phase space

Cavity drives detuned



$$H = \chi (\alpha a^{\dagger} + \alpha^* a) \sigma_z$$

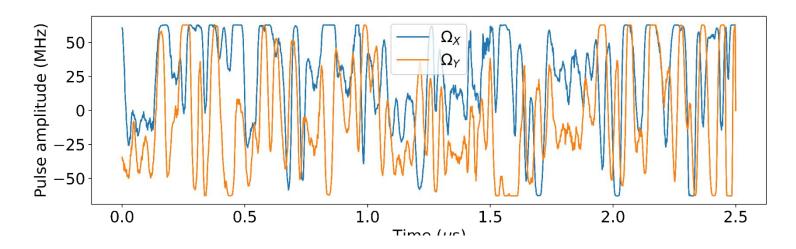


$$H = \Delta_c a^{\dagger} a + \chi (\alpha a^{\dagger} + \alpha^* a) \sigma_z$$

 $\operatorname{Im}(\alpha)$ $\operatorname{Re}(\alpha)$

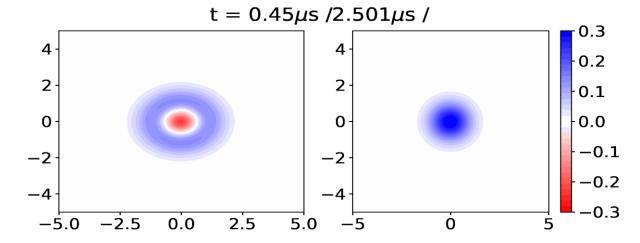
Rotate in circle because of detuning; is this a spiral?

Example: $|g10\rangle \rightarrow |g01\rangle$



Simulation Parameters

- $\Delta_c = 10 \text{ Mhz}$
- $|\Omega| = 10 \text{ Mhz}$
- $\chi_1, \chi_2/2\pi = 33 \text{ kHz}$
- $\alpha_1, \alpha_2 = 30$



Sideband Drives Method

Since α oscillatory,

$$H = \chi a^{+} a \sigma_{z} + \chi (\alpha a^{+} + \alpha^{*} a) \sigma_{z} + \chi |\alpha|^{2} \sigma_{z} + \Omega_{R} \sigma_{x}$$

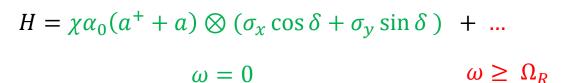
$$\omega = 0 \qquad \omega = \Omega_{R} \qquad \omega = 2\Omega_{R}$$

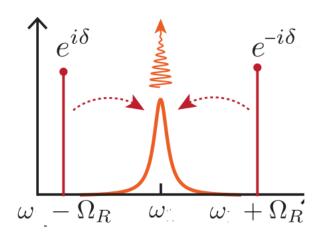
Frame Transformations:

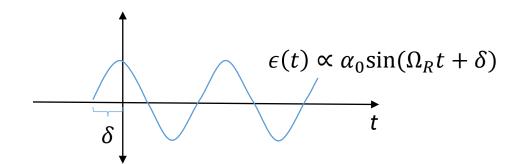
1.
$$\sigma_x \leftrightarrow \sigma_z$$
 $\Omega_R \sigma_z$

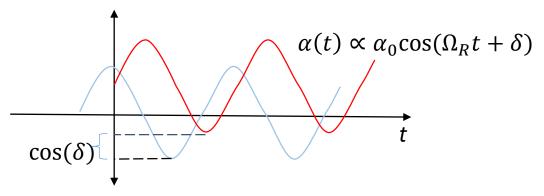
2. Rotating Frame of the qubit





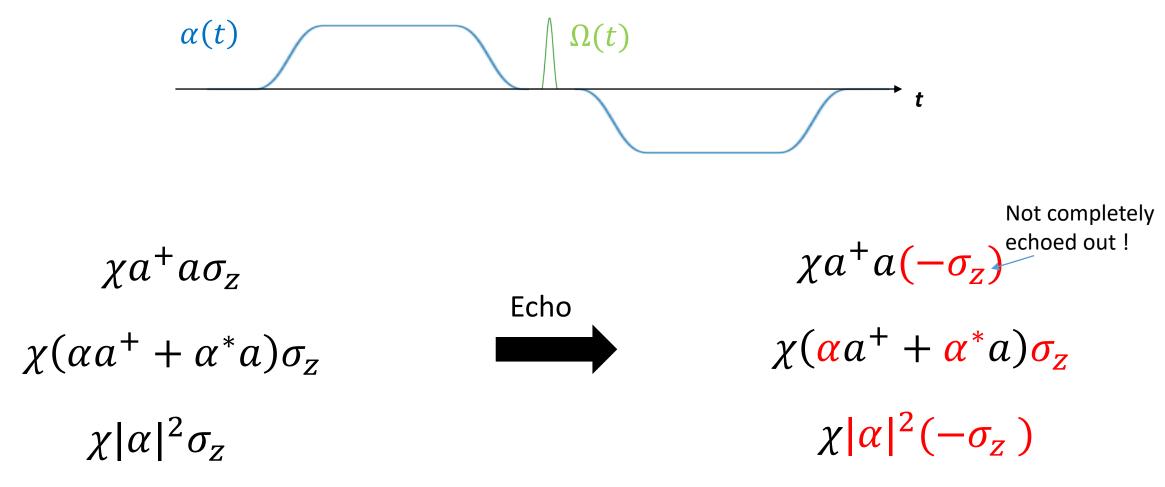






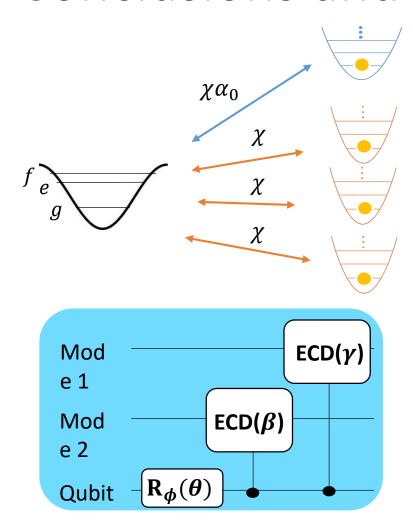
Shay Hacohen-Gourgy, ..., Irfan Siddiqi. Nature 538-7626 (2016).

Echoing in ECD



A. Eickbusch, ..., R. Schoelkopf, M. Devoret. ArXiv preprint arXiv:2111.06414 (2021)

Conclusions and Future Work



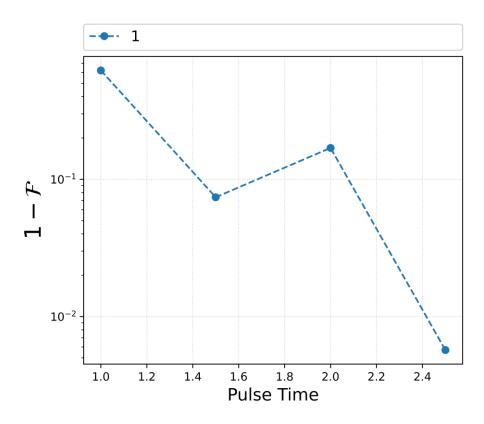
Suppression of cross-talk errors

 Achieve >0.999 fidelity for fock state transfer using Double ECD

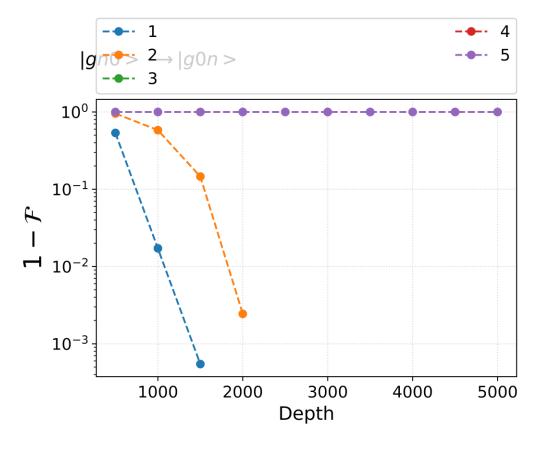
- Future Work:
 - Unite ECD with Sidebands scheme and CNOD

Circle Grape Results

$$g \otimes (|n0\rangle \rightarrow |0n\rangle)$$



$$g \otimes (|0\rangle \to |n\rangle)$$
$$g \otimes (|n\rangle \to |0\rangle)$$



Uniting with other schemes: Dealing with Unwanted Terms

The **displaced frame** transformation, however, divides the **initial ac-Stark shift** term into the following 3 terms

$$\chi(a^{\dagger} + \alpha^{*})(a + \alpha)\sigma_{z}$$

$$\downarrow$$

$$\chi a^{\dagger} a \sigma_{z} + \chi(\alpha a^{\dagger} + \alpha^{*} a)\sigma_{z} + \chi|\alpha|^{2}\sigma_{z}$$
desired

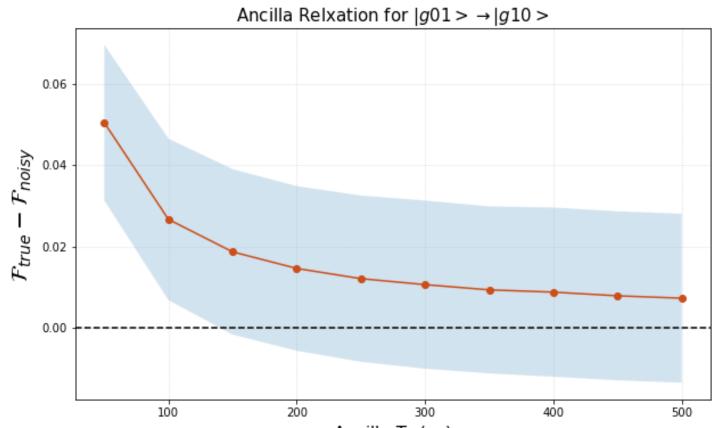
Sideband Drives

- Make terms oscillate at different frequencies
- Invoke RWA in a frame where only desired term is stationary

Echoed Cond. Displacements

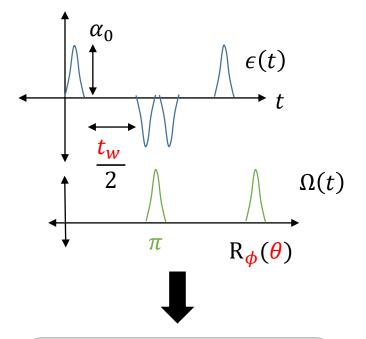
- Terms have different no. of α 's but only a single σ_z
- Clever flipping of α and σ_z can echo out unwanted terms

Transmon Relaxation



• Reduction in Errors if use better qubits i.e. fluxonium

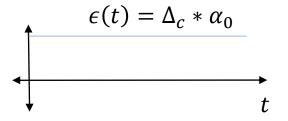
ECD



Optimizer

$$ec{eta} = lpha_0 \overrightarrow{t_w} \quad ec{\phi}$$
 , $ec{ heta}$

Circle Grape



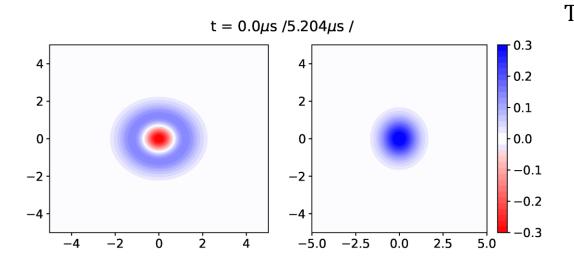


Optimizer

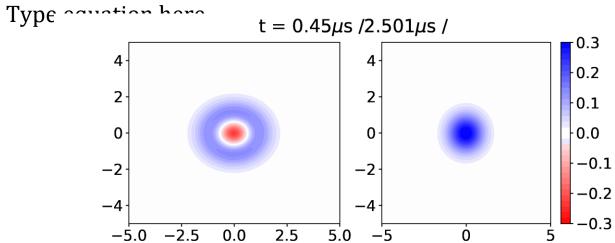
$$\Omega_{\chi}(t)$$
, $\Omega_{y}(t)$

Comparing Grape and MECD

ECD



Circle Grape



Motivation

Goal: Enact gates on cavity modes

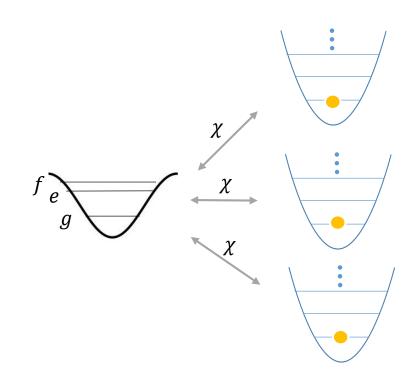
• Typical Schemes use $\chi a^\dagger a \sigma_z$

Increase χ for faster gates

 Coupling to the lossy ancilla reduces mode coherence

$$T_1^{cav} \le \frac{\Delta^2}{g^2} T_1^q \sim \frac{2\alpha}{\chi} T_1^q$$

Decrease χ for better mode coherence

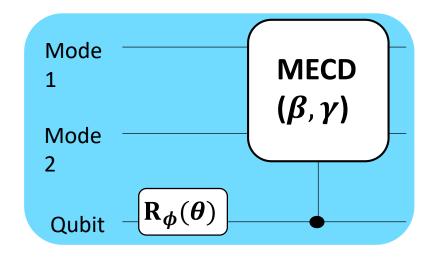


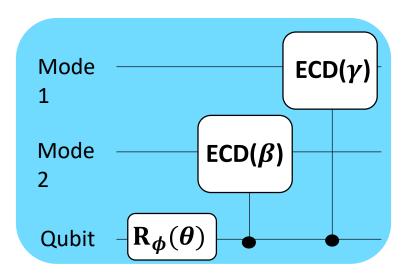
Circle Grape

$$H = \chi a^+ a \sigma_z + \chi (\alpha_0 a^+ + \alpha_0^* a) \sigma_z + \chi |\alpha_0|^2 \sigma_z + \Omega(t) \sigma_\chi$$
 Sent to Optimizer

- Continuous version
- Currently uses simulatenous drives
- Phase Space Dynamics

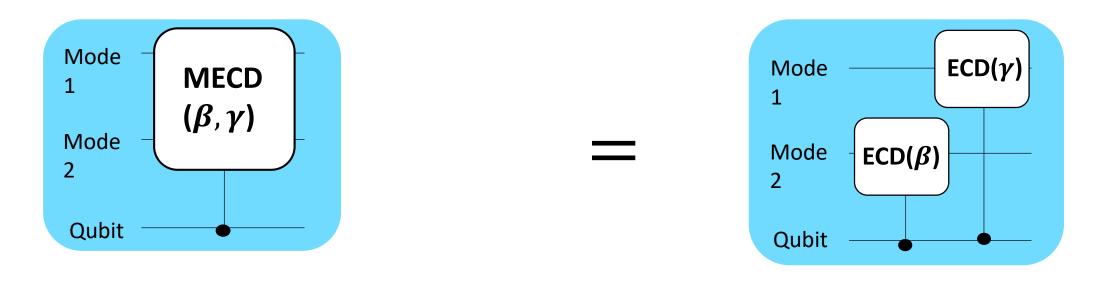
Multimode ECD





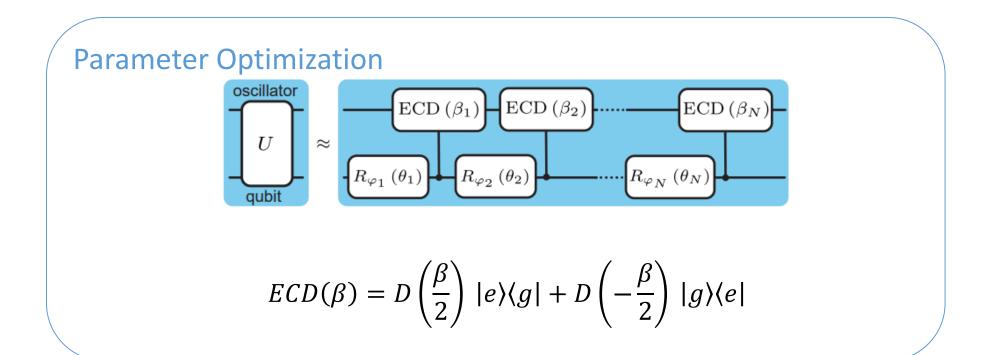
$$MECD(\beta, \gamma) = \begin{pmatrix} D_1 \left(-\frac{\beta}{2} \right) D_2 \left(\frac{\gamma}{2} \right) & 0 \\ 0 & D_1 \left(\frac{\beta}{2} \right) D_2 \left(-\frac{\gamma}{2} \right) \end{pmatrix}$$

Multimode ECD



- Universal Control for Two Modes
- Asimultaneous drives to prevent heating of modes [1,2] and amplification of cross kerr terms
 - [1] Eickbusch, Alec, et al. W34. 00005. APS March Meeting (2022).
 - [2] Diringer, Asaf A., et al. arXiv preprint arXiv:2301.09831 (2023).

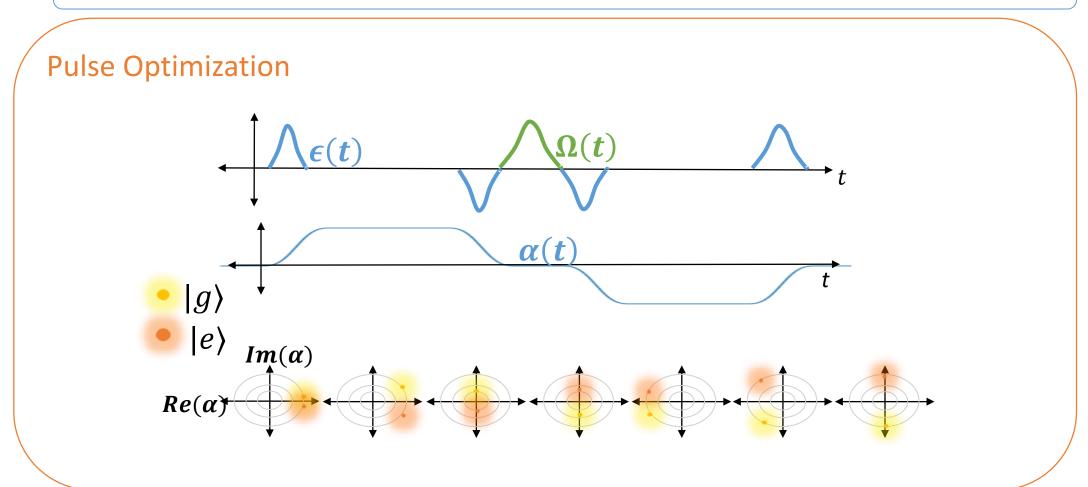
Prev. Work: Echoed Cond. Disp.



Pulse Optimization

Prev. Work: Echoed Cond. Disp.

Parameter Optimization



Two Mode ECD: Unwanted Cross Kerr Terms

$$\chi_{ab}a^+ab^+b$$

Displaced Frame Transformation

$$\chi_{ab}(a^+ + \alpha^*)(a + \alpha)(b^+ + \beta^*)(b + \beta)$$

Terms of form:

 $\chi_{ab} \alpha \beta a^+ b^+$

 $\chi_{ab} |\alpha|^2 \beta b^+$

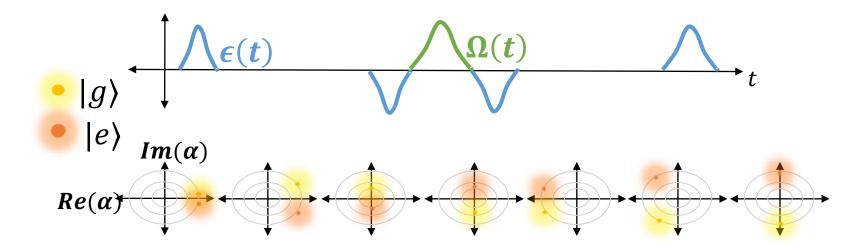
 $\chi_{ab} |\alpha|^2 b^+ b$

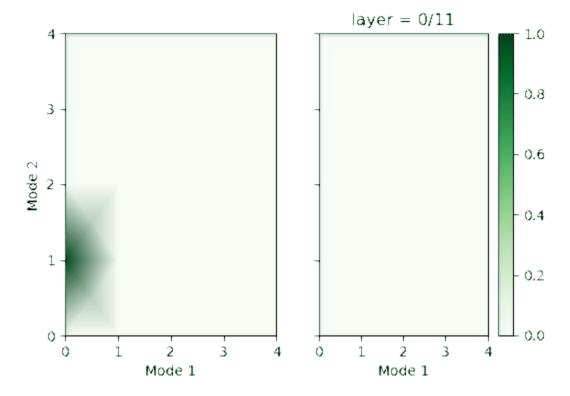
How to avoid:

 α, β should not be simultaneously nonzero

Echoed out when β flips

$$\begin{aligned} \text{Make } \chi_{ab} \ll \chi_a, \chi_b \approx 10 \text{ kHz} \\ \text{Note } \chi_{ab} = \sqrt{\kappa_a \kappa_b} = \frac{\chi_a \chi_b}{\alpha'} \approx 0.33 \text{ Hz ... good!} \\ (\alpha' \leq 300 \text{ MHz for transmons)} \end{aligned}$$





Large Displacements

Conditional Displacement Gates:

- Use Large Displacements to enhance effective interaction strength

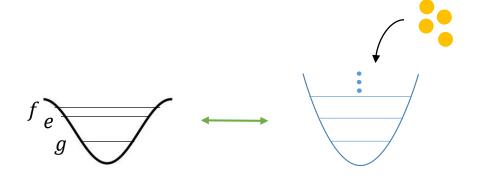
$$\chi a^{\dagger} a \sigma_{z} \longrightarrow \chi (\alpha_{0} a^{\dagger} + \alpha_{0}^{\star} a) \sigma_{z}$$

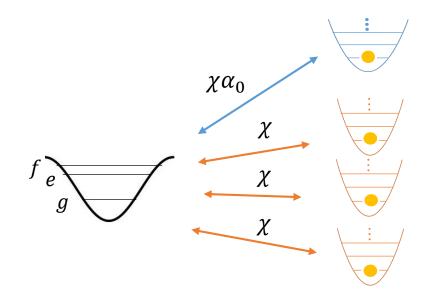
$$D(\alpha_{0})$$

Advantage in Multimode Context:

Gate Speed
$$g_{gate} = \chi \alpha_0$$

Coherent Errors: $\epsilon_{coh} = \frac{N\chi}{g_{gate}} = \frac{N}{\alpha_0}$





N non-target modes

- Hacohen-Gourgy, S., Martin, L., Flurin, E. et al. Nature 538, 491–494 (2016).
- Eickbusch, A., Sivak, V., Ding, A.Z. et al. Nat. Phys. 18, 1464–1469 (2022)