

Multimode Conditional Displacements

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Motivation

- SNAP Gates take time $\approx 2\pi/\chi$ where $\chi \approx \text{MHz}$ is dispersive coupling strength.
- Reducing Gate time \rightarrow Increasing $\chi \rightarrow$ Reducing lifetime of cavity
- ECD Idea: Keep $\chi \approx 10 \text{ kHz}$ small; But enhance it by displacing cavity (α_0) far from origin
- Effective Gate time $1/\chi\alpha_0$ where $\alpha_0 \gg 1$

Achieving Conditional Displacements

Starting Point: $H/\hbar = \omega_c a^\dagger a + \omega_q \frac{\sigma_z}{2} + \chi a^\dagger a \frac{\sigma_z}{2} + H_{drive}$

Using **frame transformations**, our objective is to **isolate** the following term from the ac-Stark Shift

$$\tilde{H} = \chi(\alpha a^\dagger + \alpha^* a)\sigma_i$$

where α is the displacement of the cavity mode. With such a term, we can realize a conditional displacement as follows

$$e^{-i(\chi(\alpha a^\dagger + \alpha^* a)\sigma_i)t} \quad \xleftrightarrow{\beta = -i \chi \alpha t} \quad e^{(\beta a^\dagger - \beta^* a)\sigma_i}$$

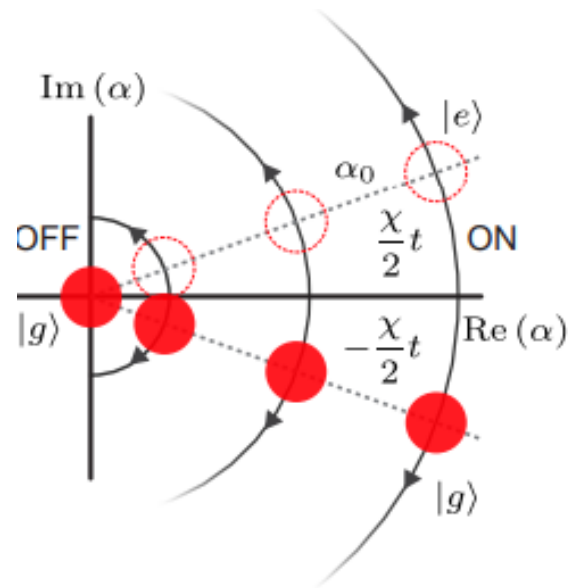
Conditional Displacements

$$\chi\alpha\sigma_z \quad (a + a^\dagger)$$

$$\mathbf{F} * \mathbf{x}$$

Force

Displacement



Dealing with Unwanted Terms I

1. Rotating Frames of oscillator and the qubit
2. Displacement transformation $D^\dagger(\alpha(t)) = e^{\alpha^*(t)a - \alpha(t)a^\dagger}$
which renders $a \rightarrow a + \alpha(t)$

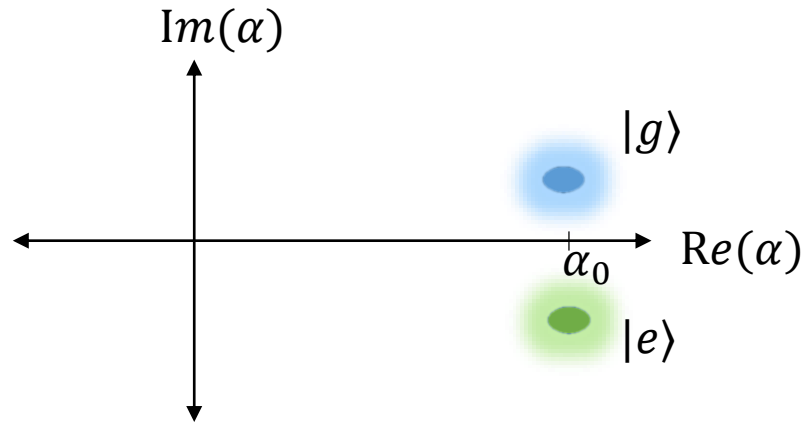
$$\begin{aligned} H_{disp} &= D^\dagger H_{rot} D - i\dot{D}^\dagger D \\ &= D^\dagger H_{rot} D + i(\dot{\alpha}^* a - \dot{\alpha} a^\dagger) \end{aligned}$$

Cancel terms linear in a, a^\dagger , such as the oscillator drive $\epsilon(t)a^\dagger + \epsilon^*(t)a$, by picking the appropriate time dependent displacement frame

$$\dot{\alpha} = -i\epsilon(t)$$

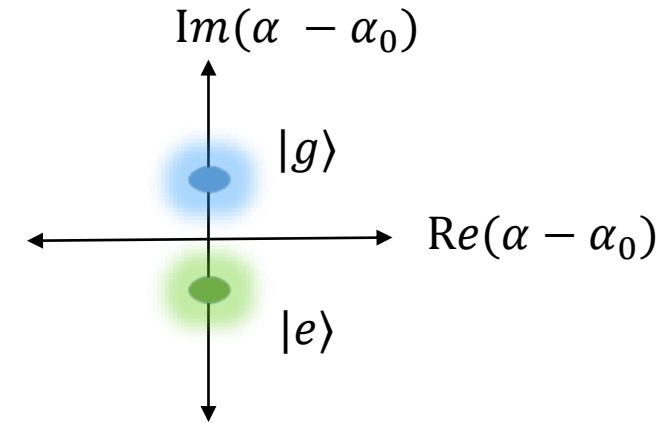
$$\dot{\alpha}^* = i\epsilon^*(t)$$

Implication: Disp. Frame Simulations



Lab Frame

- Large Displacement
- Number of photons $n = |\alpha_0|^2 \approx 900$
- Intractable simulations



Displaced Frame

- Size of Conditional Displacement ($|\alpha_g - \alpha_e| \leq 5$)
- Number of photons $n = |\alpha_g - \alpha_e|^2 \approx 25$
- Tractable simulations

Dealing with Unwanted Terms II

The **displaced frame** transformation, however, divides the **initial ac-Stark shift** term into the following 3 terms

$$\begin{array}{c} \chi(a^\dagger + \alpha^*)(a + \alpha)\sigma_z \\ \downarrow \\ \chi a^\dagger a \sigma_z + \underbrace{\chi(\alpha a^\dagger + \alpha^* a)\sigma_z}_{\text{desired}} + \chi|\alpha|^2\sigma_z \end{array}$$

Sideband Drives

- Make terms **oscillate at different** frequencies
- Invoke RWA in a frame where only desired term is stationary

Echoed Cond. Displacements

- Terms have different no. of α 's but only a single σ_z
- **Clever flipping of α and σ_z** can echo out unwanted terms

Sideband Drives

Since α oscillatory,

$$H = \chi a^\dagger a \sigma_z + \chi(\alpha a^\dagger + \alpha^* a) \sigma_z + \chi |\alpha|^2 \sigma_z + \Omega_R \sigma_x$$

$$\omega = 0$$

$$\omega = \Omega_R$$

$$\omega = 2\Omega_R$$

Frame Transformations:

$$1. \quad \sigma_x \leftrightarrow \sigma_z \quad \longrightarrow$$

$$\Omega_R \sigma_z$$

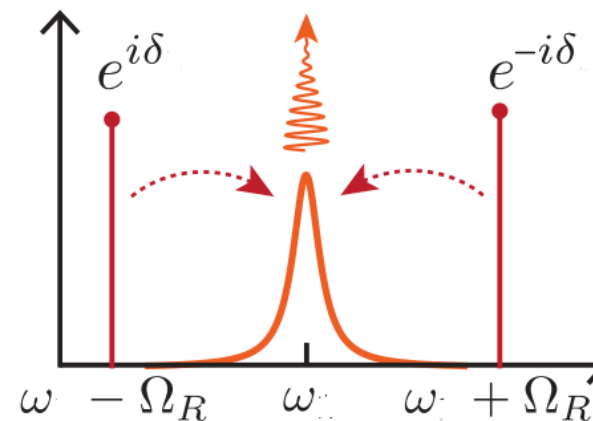
2. Rotating Frame of the qubit

$$\cancel{\Omega_R \sigma_z}$$

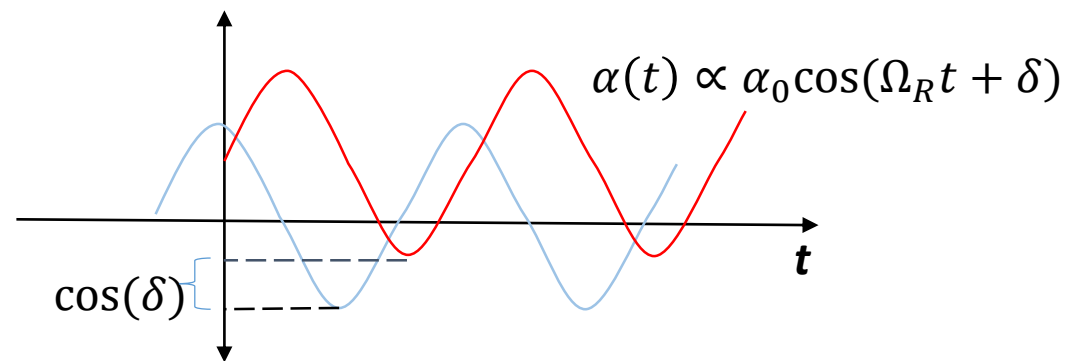
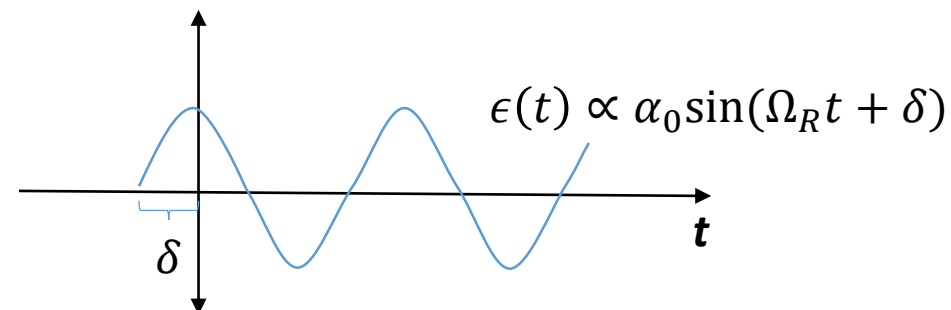
$$H = \chi \alpha_0 (a^\dagger + a) \otimes (\sigma_x \cos \delta + \sigma_y \sin \delta) + \dots$$

$$\omega = 0$$

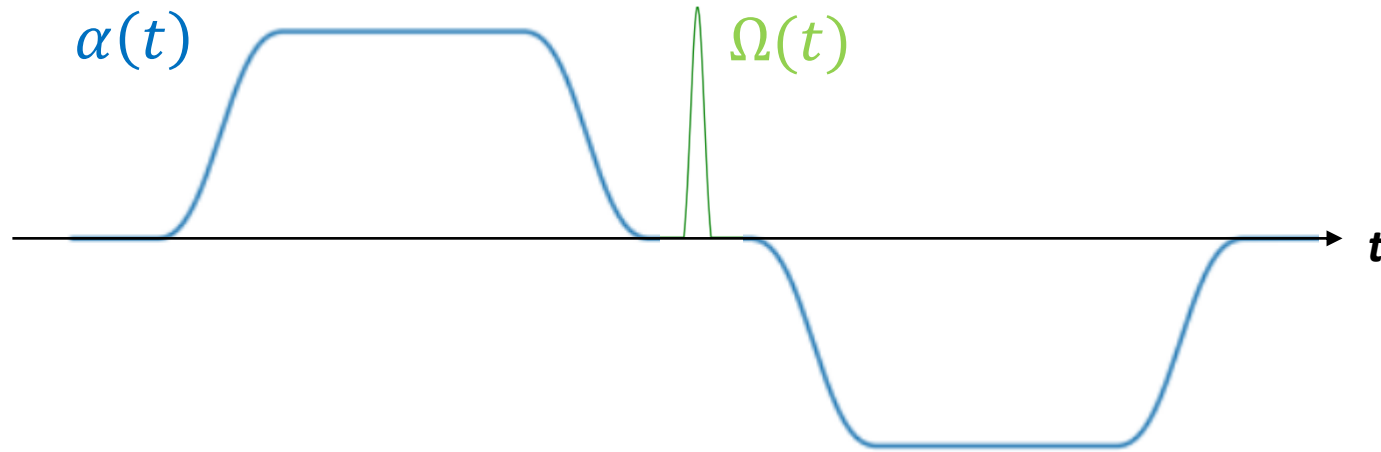
$$\omega \geq \Omega_R$$



<https://arxiv.org/pdf/1608.06652.pdf>



Echoed Cond. Disp.



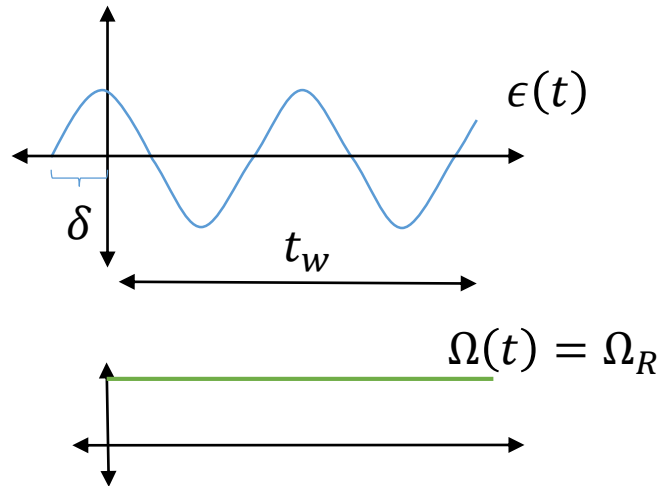
$$\begin{aligned} &\chi a^\dagger a \sigma_z \\ &\chi(\alpha a^\dagger + \alpha^* a) \sigma_z \\ &\chi |\alpha|^2 \sigma_z \end{aligned}$$

Echo


$$\begin{aligned} &-\chi a^\dagger a \sigma_z \\ &\chi(\alpha a^\dagger + \alpha^* a) \sigma_z \\ &-\chi |\alpha|^2 \sigma_z \end{aligned}$$

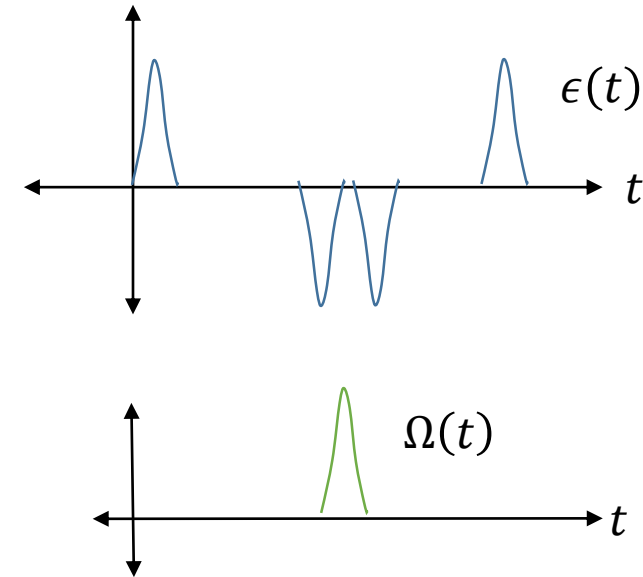
Not completely
echoed out !

Comparison



Sideband Drives

- Oscillating $\epsilon(t), \alpha(t)$
- Continuous Rabi Driving on the qubit
- Ridding unwanted terms via **RWA**:
 $e^{i\Omega t}$



Echoed Conditional Gates

- Single Oscillation of $\alpha(t)$
- Discrete Qubit pi pulses
- Ridding unwanted terms via **echoing**:
Step Function

What about Noise? Cavity Relaxation

Master Equation : $\dot{\rho} = -i [H, \rho] + \kappa \mathcal{D}(a)\rho$

Displaced Frame Transf.



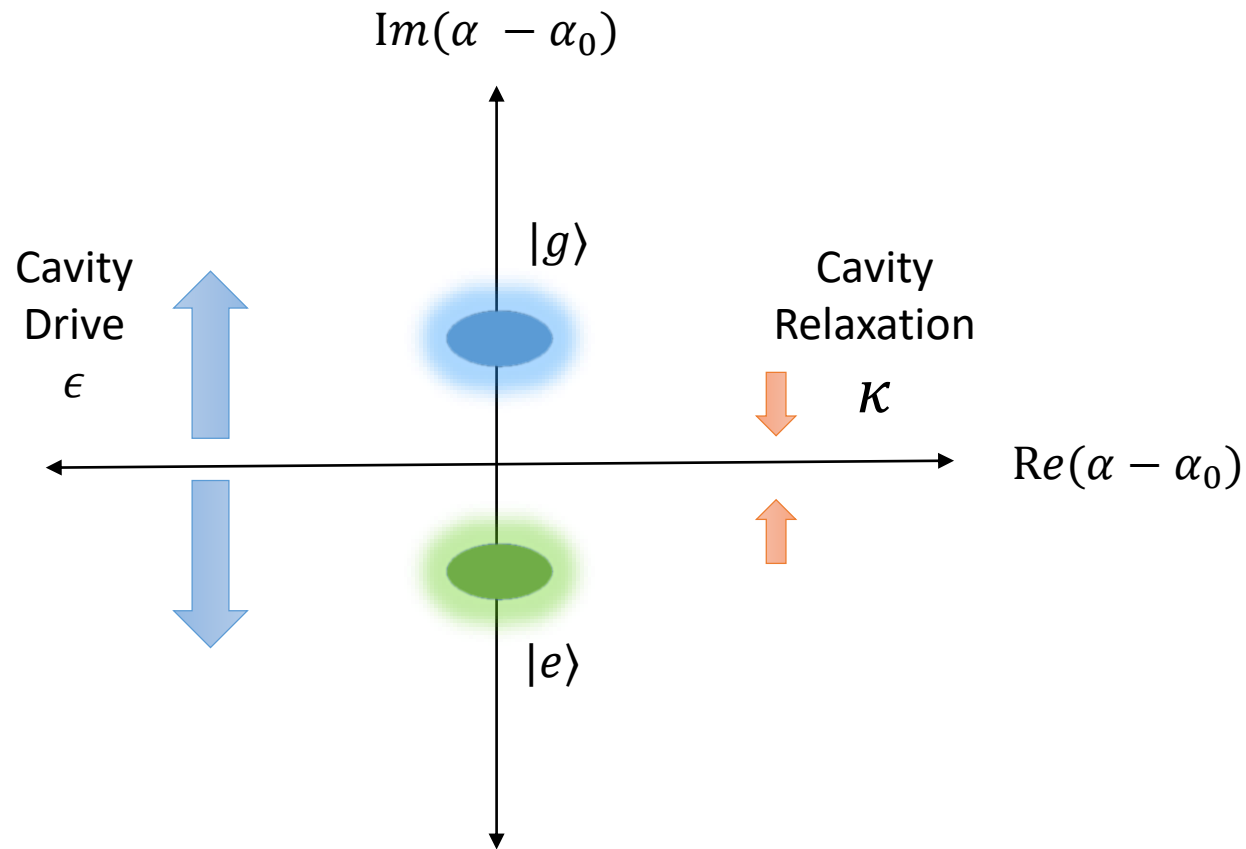
$$\kappa \mathcal{D}(a + \alpha) \tilde{\rho} = \kappa \mathcal{D}(a) \tilde{\rho} - i \left[\frac{i\kappa}{2} (\alpha^* a - \alpha a^\dagger), \tilde{\rho} \right]$$

New Term can be canceled in a similar way as the cavity drive as :

$$\dot{\alpha} = -i\epsilon(t) - \frac{\kappa\alpha}{2}$$

$$\dot{\alpha}^* = i\epsilon^*(t) - \frac{\kappa\alpha^*}{2}$$

What about Noise? Cavity Relaxation and Dephasing



Center of mass frame of Oscillator in its Phase Space

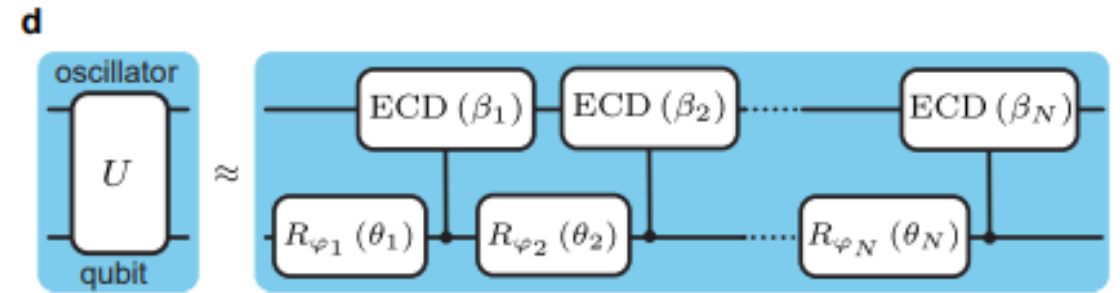
- However, enhancement of cavity dephasing could not be avoided.
- Dominant term proportional to $\kappa_\phi |\alpha|^2$
- Inbuilt echoes in ECD may reduce some damage.

Implementation: Optimal Parameters

- ECD and Sideband Drives, by themselves, do not offer universal control of both oscillator and qubit
- Sol: Interleave parameterized qubit rotations between CD
- Gate times are dependent on # of layers to realize high fidelity gates

$$CD(\beta) = D\left(\frac{\beta}{2}\right)|g\rangle\langle g| + D\left(-\frac{\beta}{2}\right)|e\rangle\langle e|$$

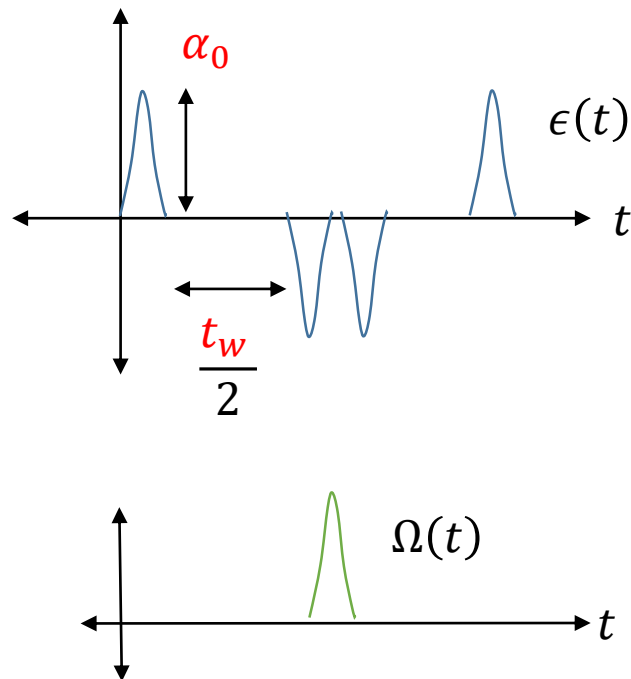
$$R_\phi(\theta) = e^{-i\left(\frac{\theta}{2}\right)(\cos \phi \sigma_x + \sin \phi \sigma_y)}$$



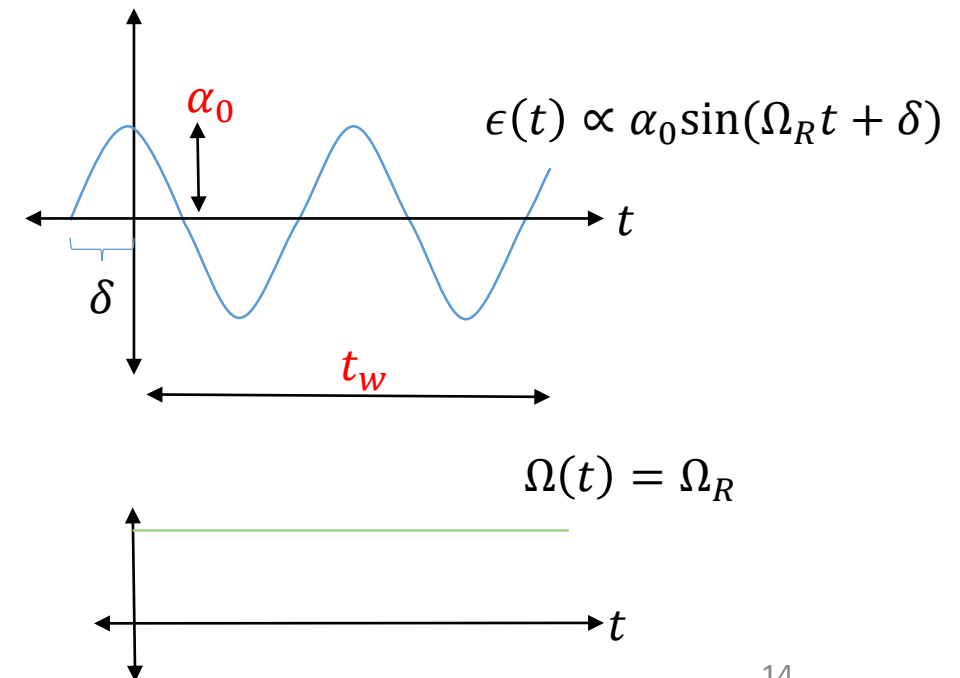
Implementation: Finding Pulses

Task: find wait time t_w and scale intermediate displacement α_0 such that $\chi \alpha_0 t_w = \beta$

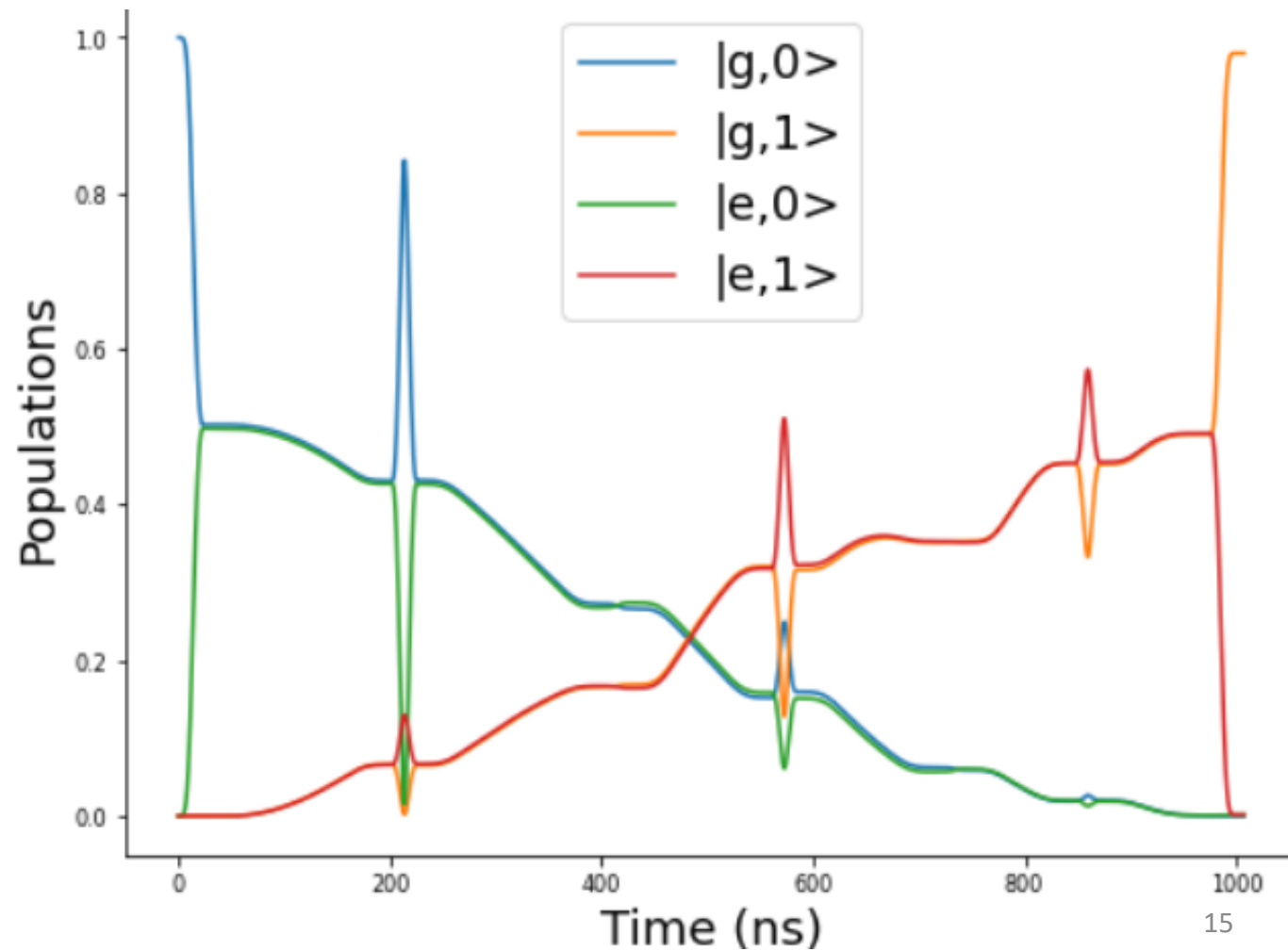
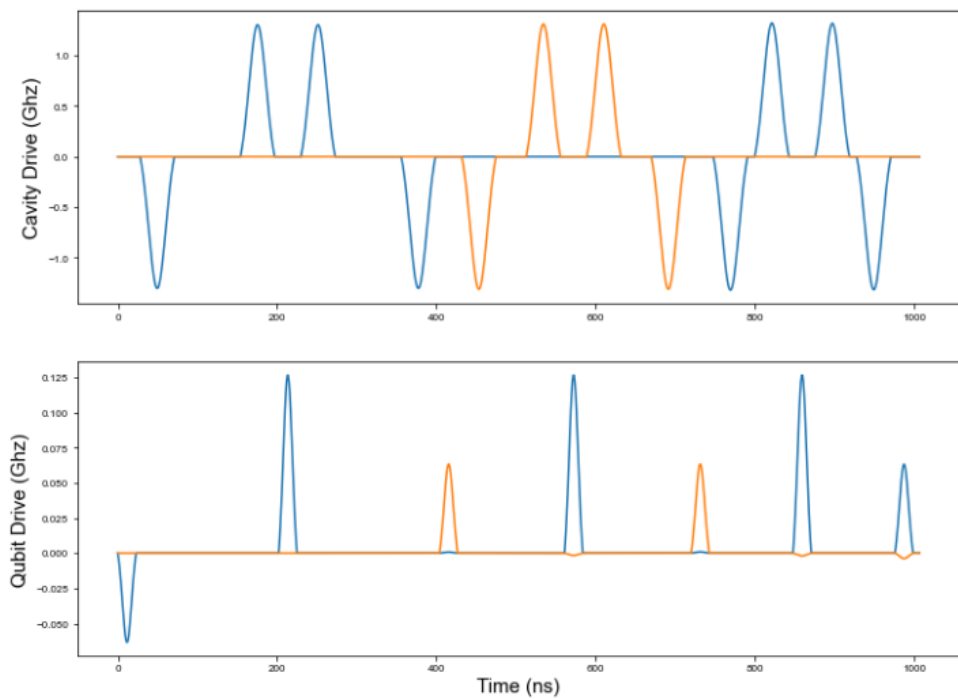
ECD



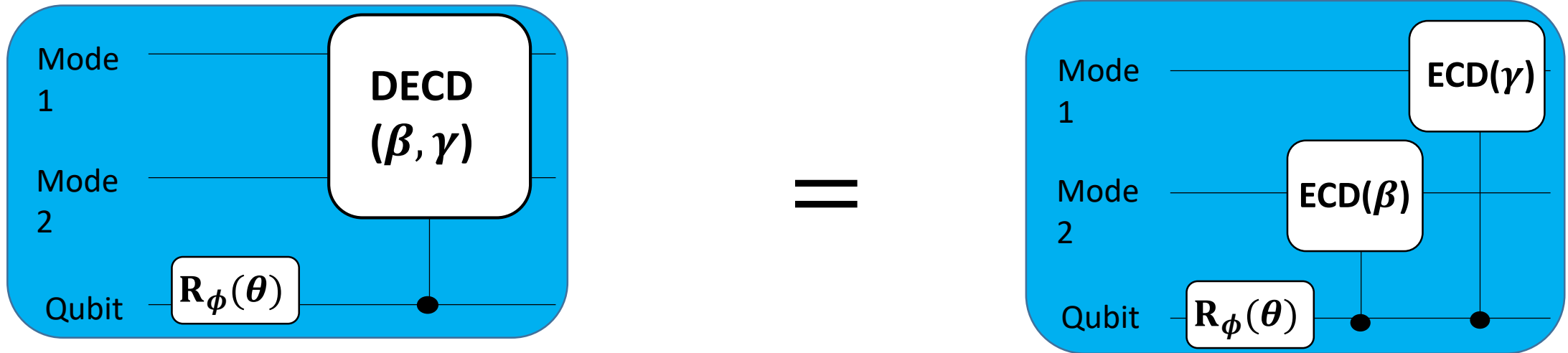
Sideband



ECD: $|g0\rangle \rightarrow |g1\rangle$



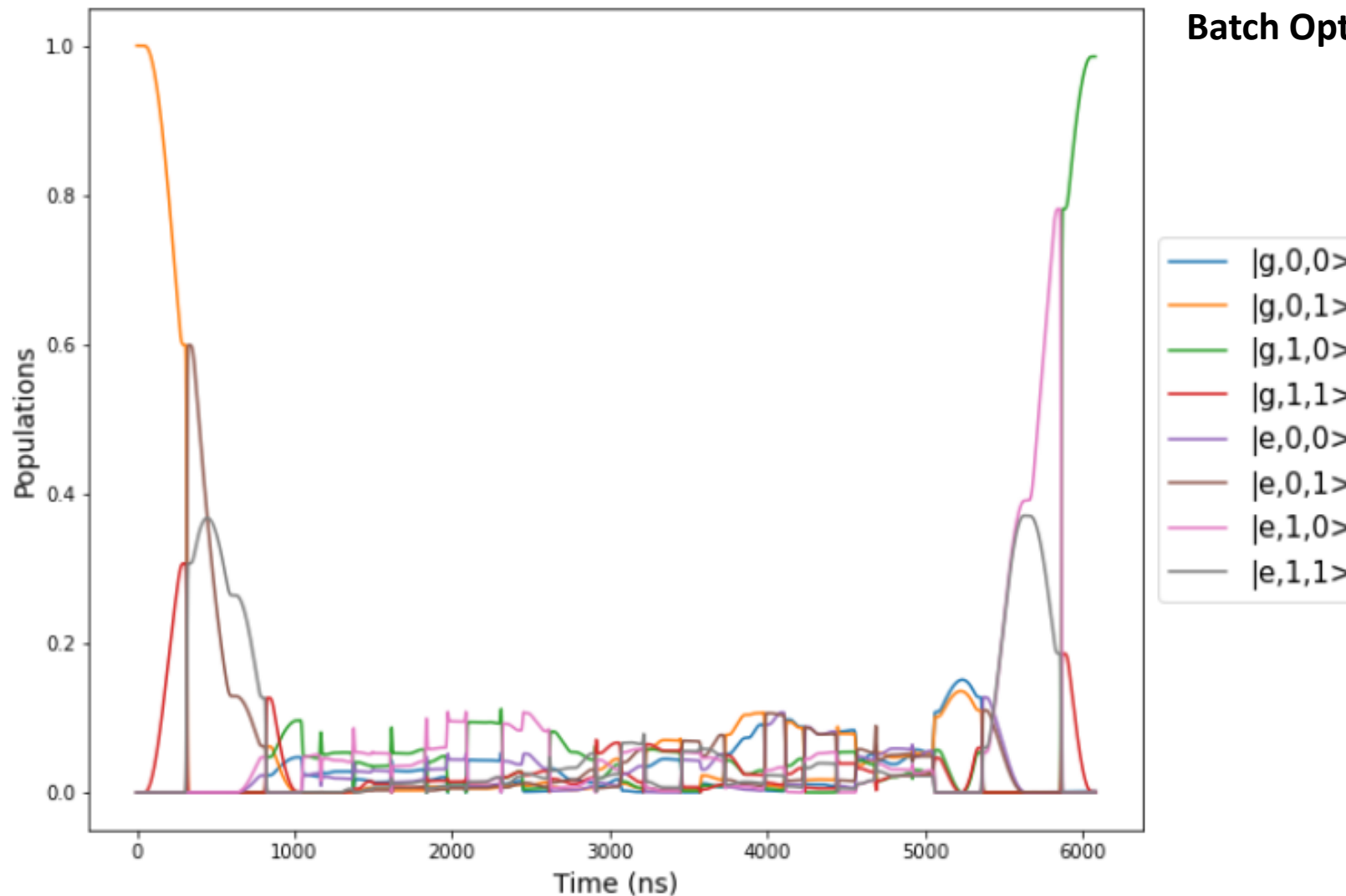
Two Mode ECD



- Generalizing ECD gate to 2 modes
- Displacements on the two modes are not simultaneous (to avoid heating as observed in [*])

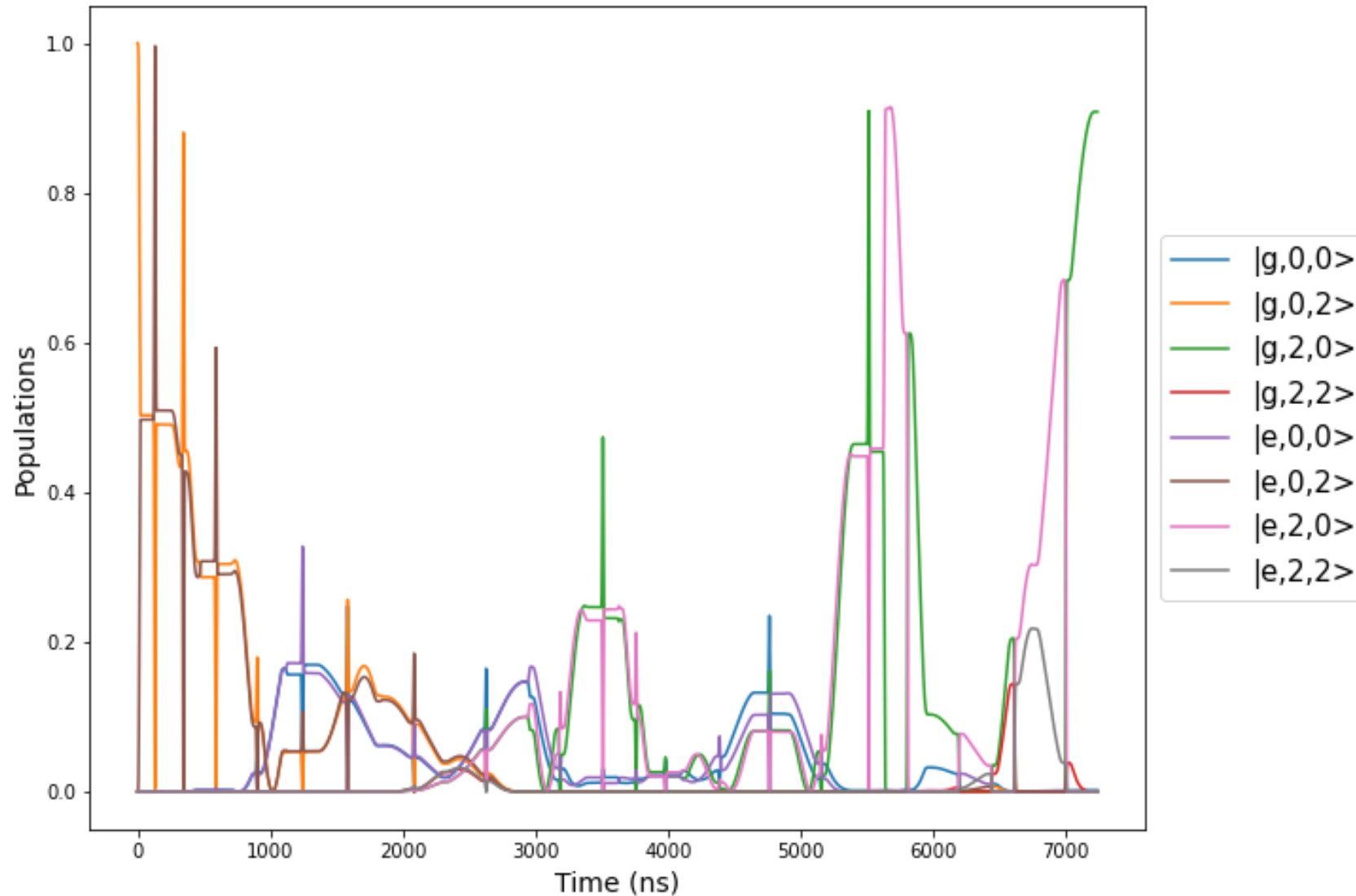
* Alec Eickbusch, Zhenghao Ding, ..., Michel Devoret. W34. 00005. APS March Meeting (2022).

Two Mode ECD : $|g01\rangle \rightarrow |g10\rangle$



Batch Optimizer Fidelity: 0.995308

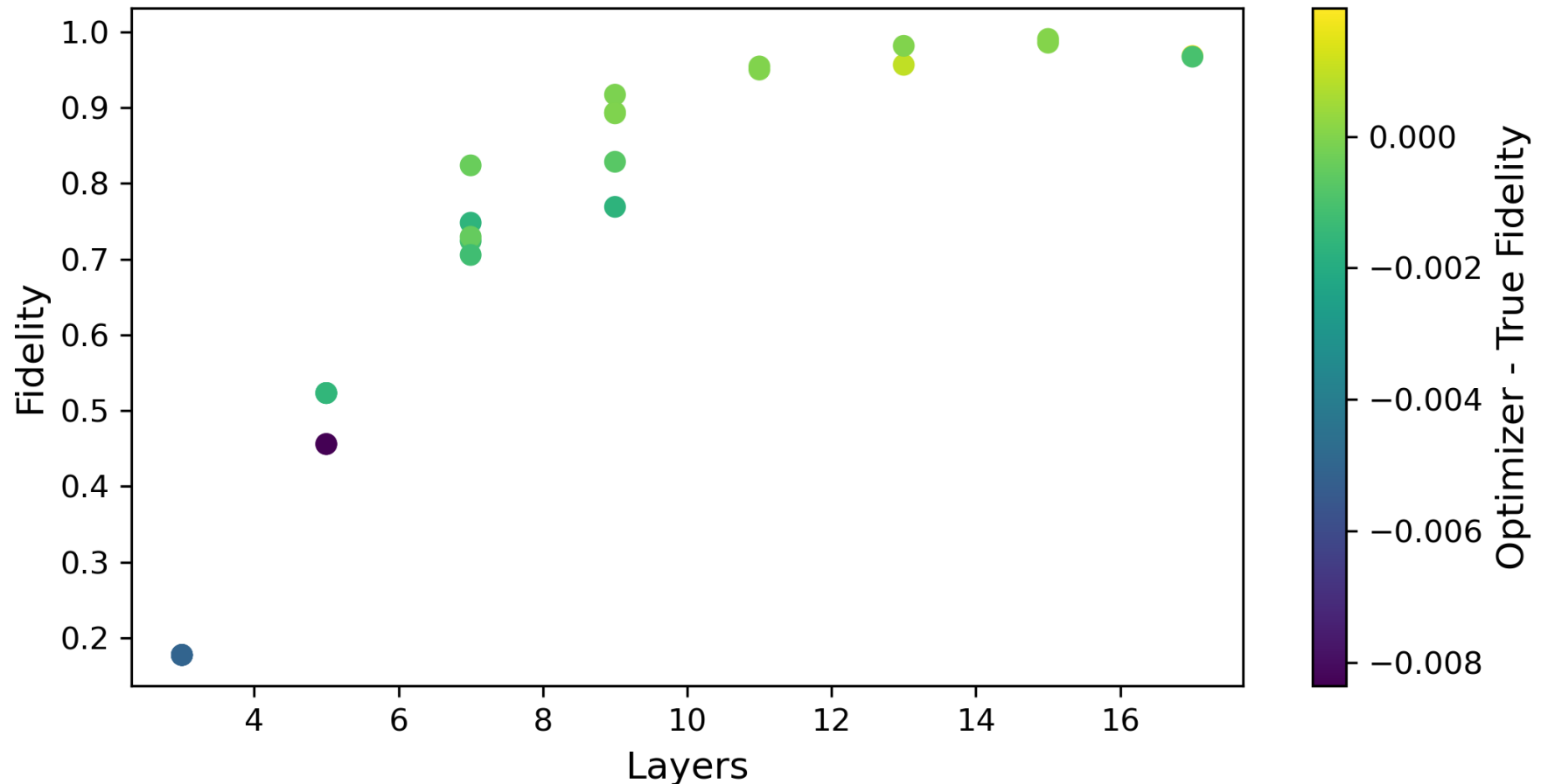
Two Mode ECD: $|g02\rangle \rightarrow |g20\rangle$



Batch Optimizer Fidelity: 0.92192787

Two Mode ECD: Simultaneous State Transfer

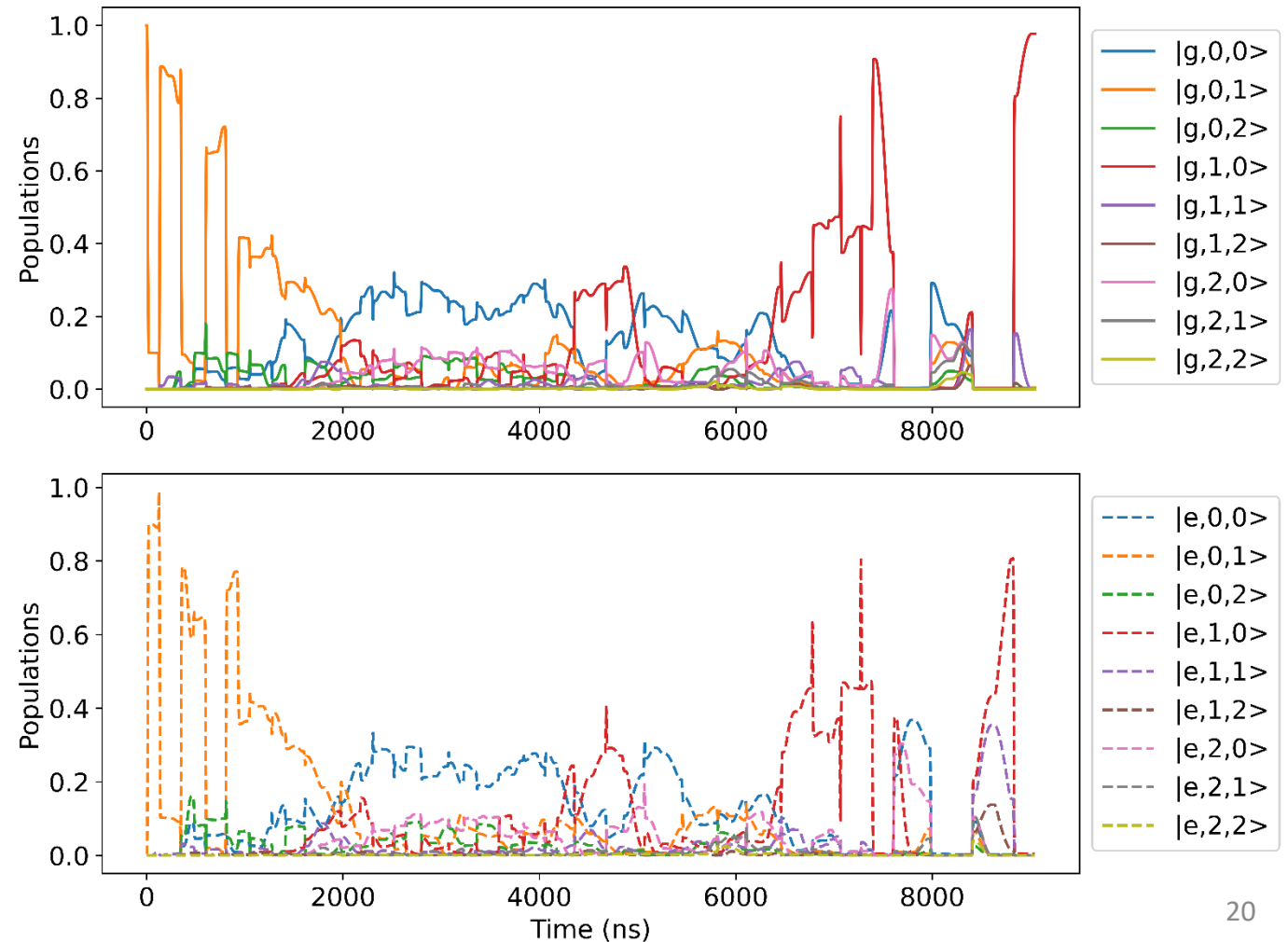
$g01 \rightarrow g10$ and $g02 \rightarrow g20$ (15 levels in each mode)



Two Mode ECD: Simultaneous State Transfer

- Parameters
Optimized for both
 $g_{01} \rightarrow g_{10}$ and
 $g_{02} \rightarrow g_{20}$
- Qutip Simulation of
 $g_{01} \rightarrow g_{10}$

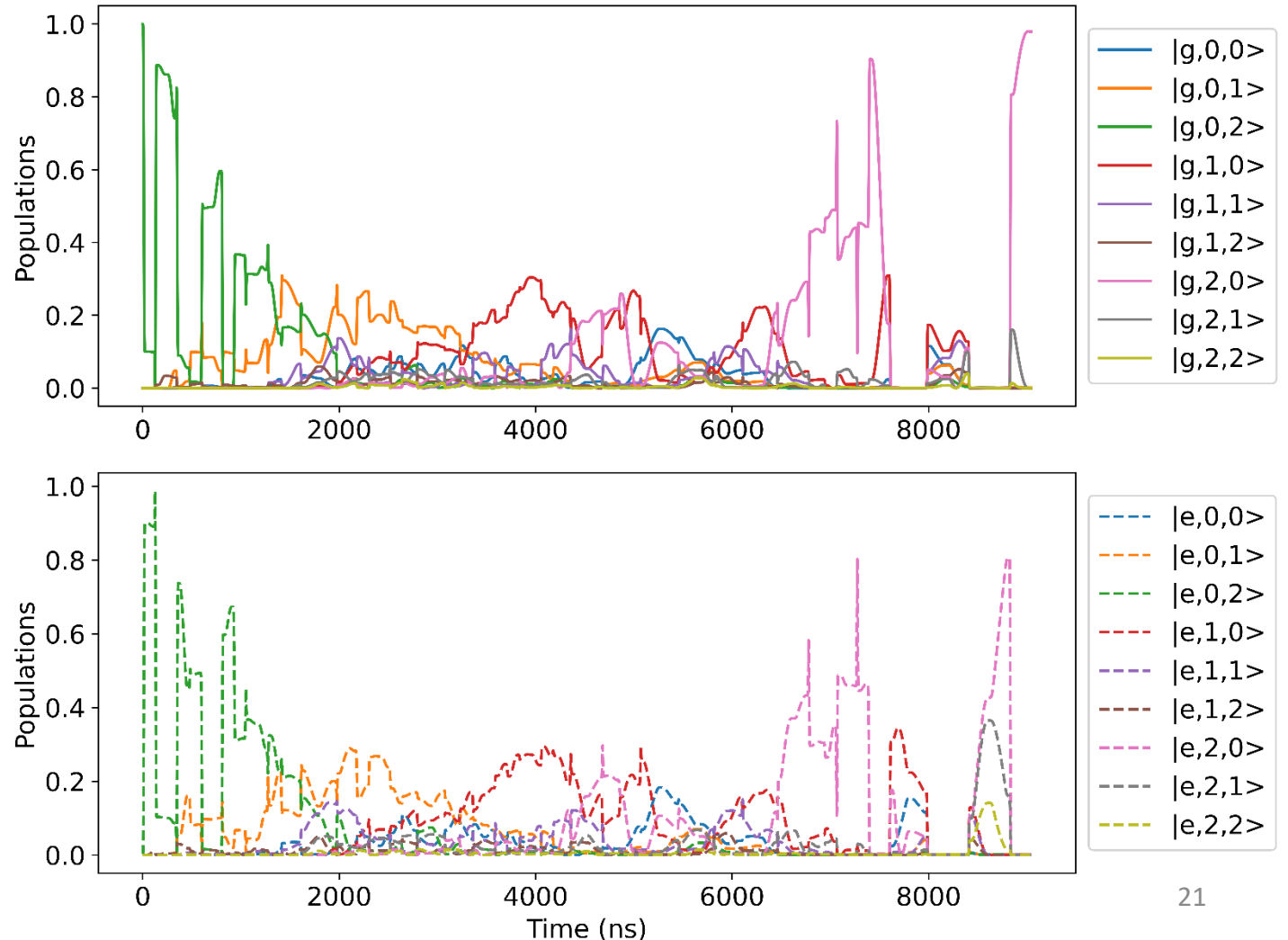
$g_{01} \rightarrow g_{10}$ (15 layers, 15 levels)



Two Mode ECD: Simultaneous State Transfer

$g_{02} \rightarrow g_{20}$ (15 layers, 15 levels)

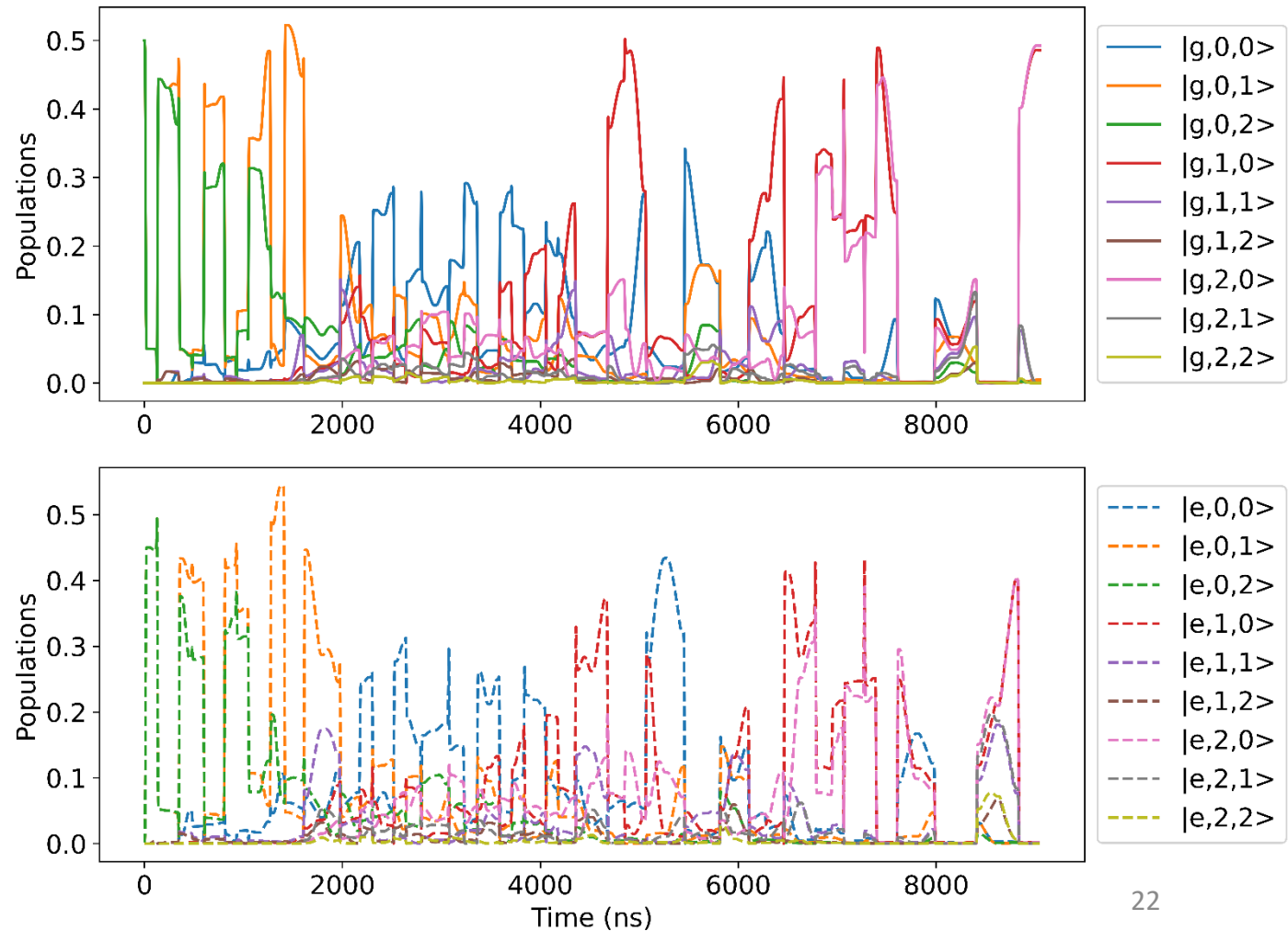
- Parameters
Optimized for both
 $g_{01} \rightarrow g_{10}$ and
 $g_{02} \rightarrow g_{20}$
- Qutip Simulation of
 $g_{02} \rightarrow g_{20}$



Two Mode ECD: Simultaneous State Transfer

$$\frac{1}{\sqrt{2}}(g_{01} + g_{02}) \rightarrow \frac{1}{\sqrt{2}}(g_{10} + g_{20}) \text{ (15 layers, 15 levels)}$$

- Parameters
Optimized for both
 $g_{01} \rightarrow g_{10}$ and
 $g_{02} \rightarrow g_{20}$
- Qutip Simulation of
 $(g_{01} + g_{02})$
↓
 $(g_{10} + g_{20})$



Two Mode ECD : Unwanted Cross Kerr Terms

$$\chi_{ab} a^+ a b^+ b \xrightarrow{\text{Displaced Frame Transformation}} \chi_{ab} (a^+ + \alpha^*)(a + \alpha)(b^+ + \beta^*)(b + \beta)$$

Terms of form :

$$\chi_{ab} \alpha \beta a^+ b^+$$

$$\chi_{ab} |\alpha|^2 \beta b^+$$

$$\chi_{ab} |\alpha|^2 b^+ b$$

How to avoid :

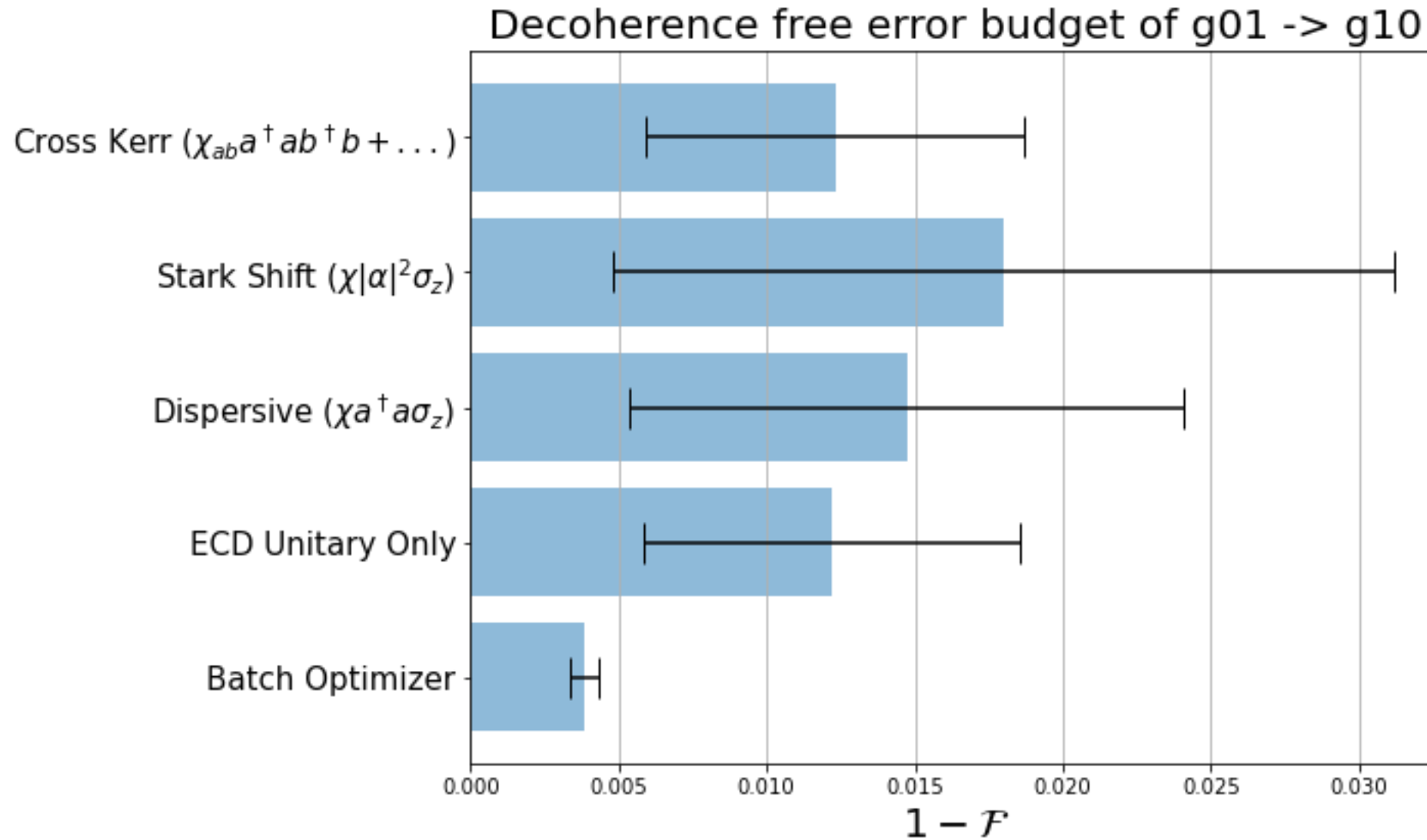
α, β should not be simultaneously nonzero

Echoed out when β flips

Make $\chi_{ab} \ll \chi_a, \chi_b \approx 10$ kHz

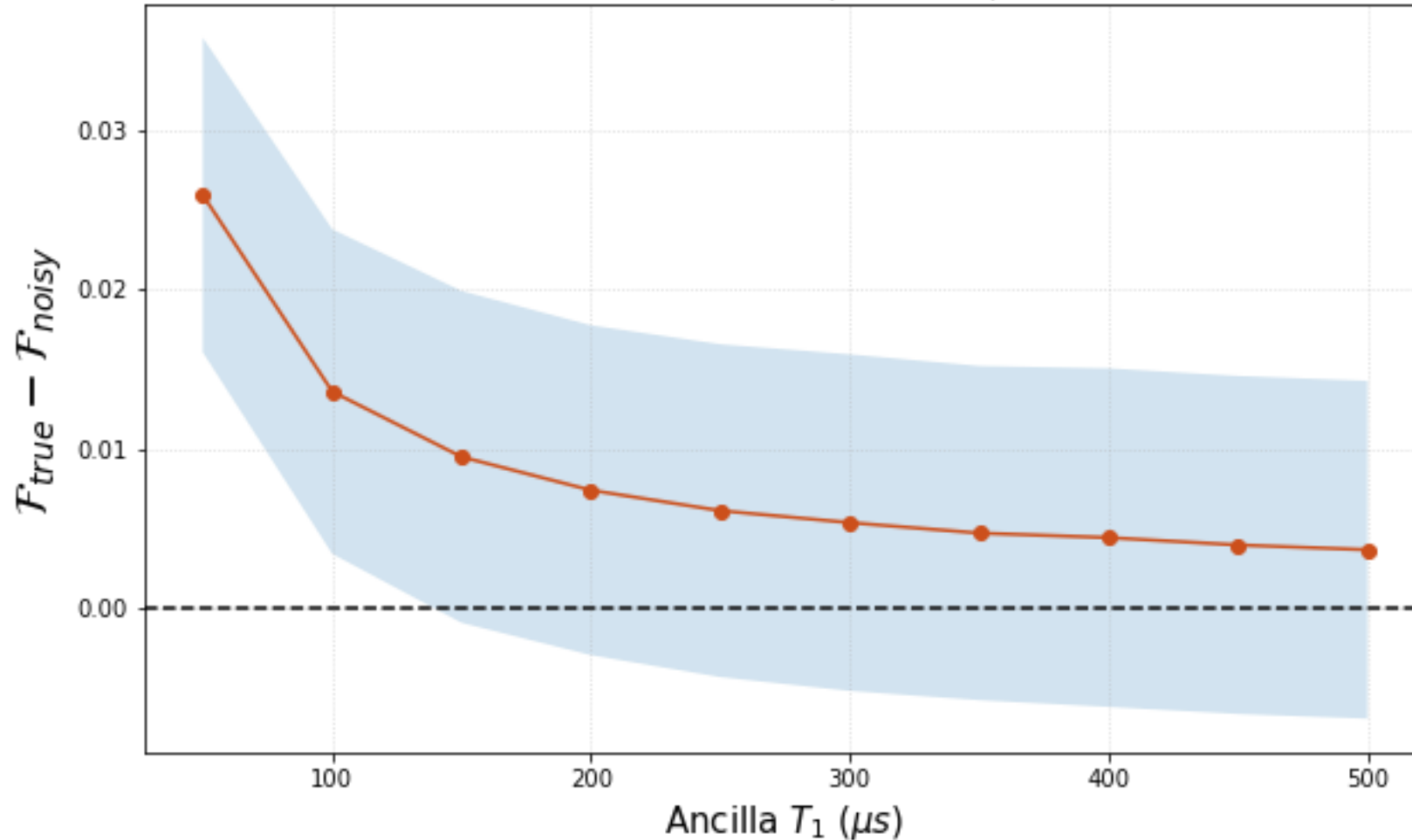
Note $\chi_{ab} = \sqrt{\kappa_a \kappa_b} = \frac{\chi_a \chi_b}{\alpha'} \approx 0.33$ Hz ... good!
 ($\alpha' \leq 300$ MHz for transmons)

Two Mode ECD : QuTip Noise Simulations



Two Mode ECD : QuTip Noise Simulations

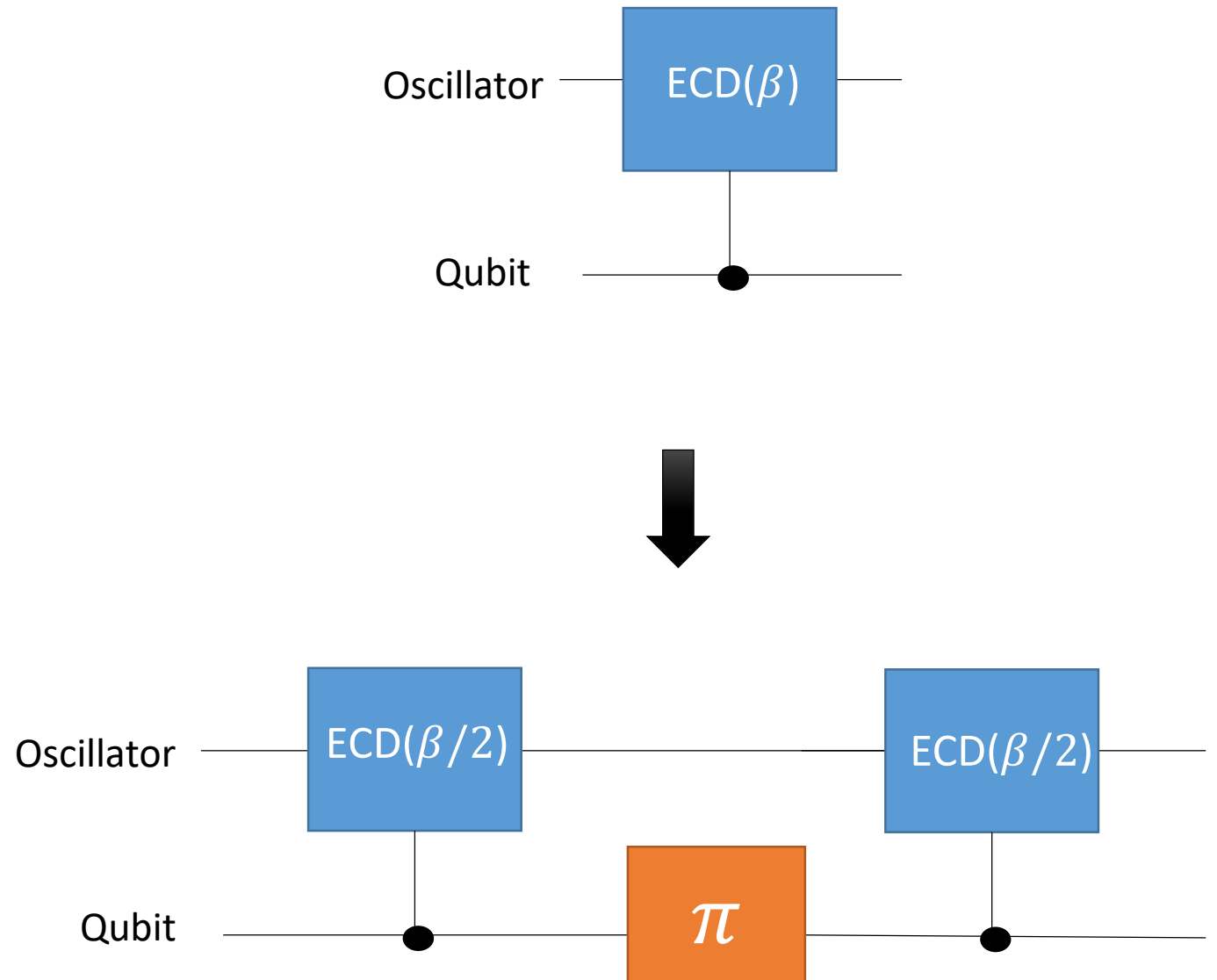
Ancilla Relaxation for $|g01\rangle \rightarrow |g10\rangle$



Ancilla with better coherence times such as flux protected qubits may improve gate fidelities.

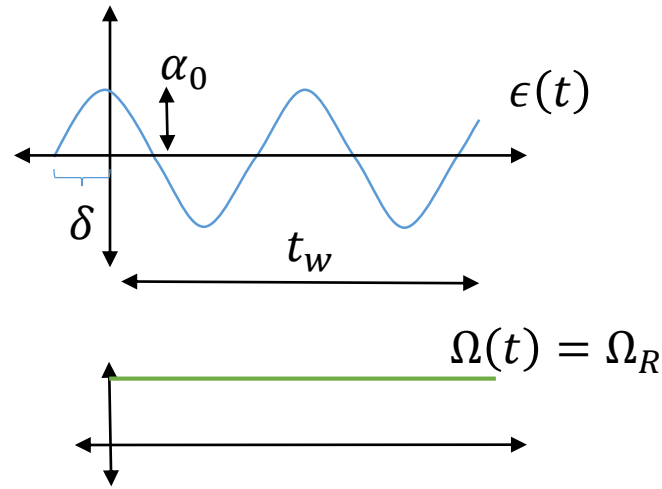
Meta Echoes

- Terms of form $\chi a^\dagger a \sigma_z$ not completely echoed out by a single pi pulse since measurement of $a^\dagger a$ does not always yield $|\alpha|^2$
- So insert more pi pulses (qubit echoes) in the ECD pulse sequence



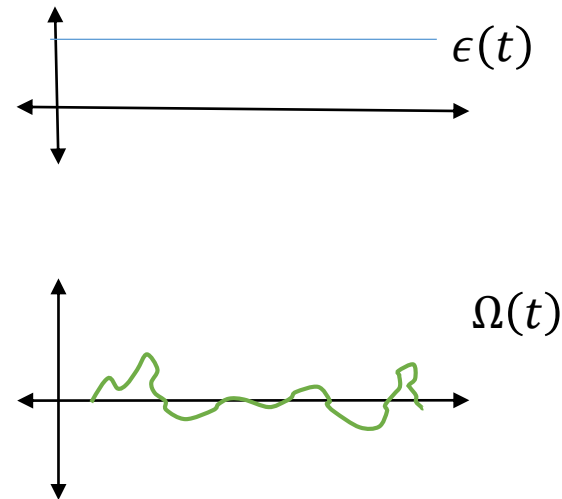
Circle Grape

Sideband Drives



- Changing $\epsilon(t)$
- Constant $\Omega(t)$

Circle Grape



- Constant $\epsilon(t)$
- Changing $\Omega(t)$

$$H = \chi a^\dagger a \sigma_z + \chi (\alpha_0 a^\dagger + \alpha_0^* a) \sigma_z + \chi |\alpha_0|^2 \sigma_z + \Omega(t) \sigma_x$$

Sent to Optimizer

Similarly grap-ifying Sideband Drives?

Sending $\delta(t)$ to the optimizer

Circle Grape

