Multimode Conditional Displacements

Motivation

- SNAP Gates take time $\approx 2\pi/\chi$ where $\chi \approx$ MHz is dispersive coupling strength.
- Reducing Gate time $\rightarrow$ Increasing χ $\rightarrow$ Reducing lifetime of cavity
- ECD Idea: Keep $\chi \approx 10$ kHz small; But enhance it by displacing cavity (α_0) far from origin
- Effective Gate time $1/\chi\alpha_0$ where $\alpha_0 \gg 1$

Achieving Conditional Displacements

Starting Point:
$$H/\hbar = \omega_c a^\dagger a + \omega_q \frac{\sigma_z}{2} + \chi a^\dagger a \frac{\sigma_z}{2} + H_{drive}$$

Using **frame transformations**, our objective is to **isolate** the following term from the ac-Stark Shift

$$\tilde{H} = \chi(\alpha a^\dagger + \alpha^\star a)\sigma_i$$

where α is the displacement of the cavity mode. With such a term, we can realize a conditional displacement as follows

$$e^{-i(\chi(\alpha a^\dagger + \alpha^\star a)\sigma_i)t}$$

$$\xrightarrow{\beta = -i \chi \alpha t}$$

$$e^{(\beta a^\dagger - \beta^\star a)\sigma_i}$$

Dealing with Unwanted Terms I

1. Rotating Frames of oscillator and the qubit
2. Displacement transformation $D^+(\alpha(t)) = e^{\alpha^*(t)a - \alpha(t)a^+}$
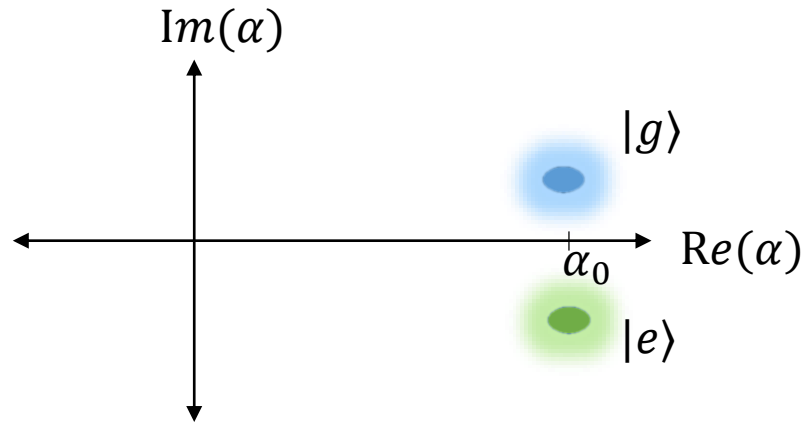   which renders $a \rightarrow a + \alpha(t)$

$$H_{disp} = D^\dagger H_{rot} D - i\dot{D}^\dagger D$$
$$= D^\dagger H_{rot} D + i(\dot{\alpha}^\star a - \dot{\alpha}a^\dagger)$$

Cancel terms linear in a, a^+, such as the oscillator drive $\epsilon(t)a^+ + \epsilon^*(t)a$, by picking the appropriate time dependent displacement frame
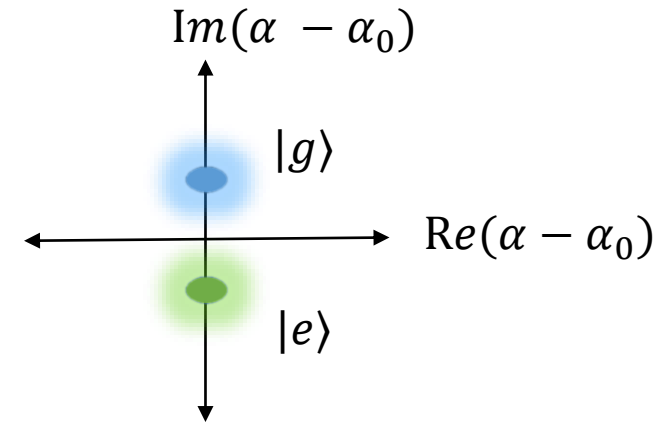
$$\dot{\alpha} = -i\epsilon(t) \qquad\qquad \dot{\alpha}^* = i\epsilon^*(t)$$

Implication: Disp. Frame Simulations

Lab Frame

- Large Displacement

- Number of photons $n = |\alpha_0|^2 \approx 900$

- Intractable simulations

Displaced Frame

- Size of Conditional Displacement ($|\alpha_g - \alpha_e| \leq 5$)

- Number of photons $n = |\alpha_g - \alpha_e|^2 \approx 25$

- Tractable simulations

Dealing with Unwanted Terms II

The **displaced frame** transformation, however, divides the **initial ac-Stark shift** term into the following 3 terms

$$\chi(a^\dagger + \alpha^*)(a + \alpha)\sigma_z$$

$$\downarrow$$

$$\chi a^\dagger a \sigma_z \quad + \quad \underbrace{\chi(\alpha a^\dagger + \alpha^* a)\sigma_z}_{\text{desired}} \quad + \quad \chi|\alpha|^2 \sigma_z$$

Sideband Drives
- Make terms **oscillate at different** frequencies
- Invoke RWA in a frame where only desired term is stationary

Echoed Cond. Displacements
- Terms have different no. of α's but only a single σ_z
- **Clever flipping of α and σ_z** can echo out unwanted terms

Sideband Drives

Since α oscillatory,

$$H = \chi a^+ a \sigma_z + \chi(\alpha a^+ + \alpha^* a)\sigma_z + \chi|\alpha|^2 \sigma_z + \Omega_R \sigma_x$$

$\omega = 0$ $\omega = \Omega_R$ $\omega = 2\Omega_R$

$$\epsilon(t) \propto \alpha_0 \sin(\Omega_R t + \delta)$$

Frame Transformations:

1. $\sigma_x \leftrightarrow \sigma_z$ $\longrightarrow$ $\Omega_R \sigma_z$
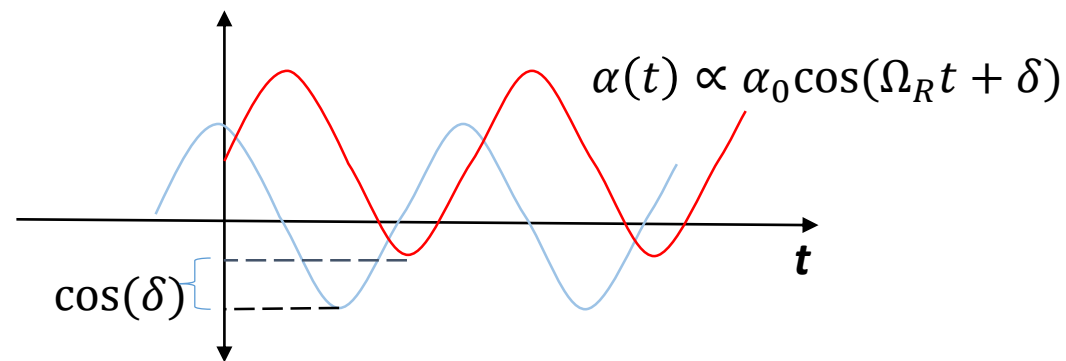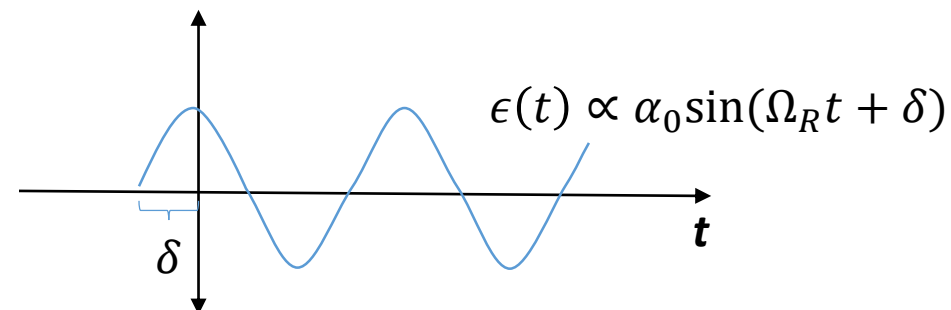
2.   Rotating Frame of the qubit $\cancel{\Omega_R \sigma_z}$

$$H = \chi\alpha_0(a^+ + a) \otimes (\sigma_x \cos\delta + \sigma_y \sin\delta) + \ldots$$
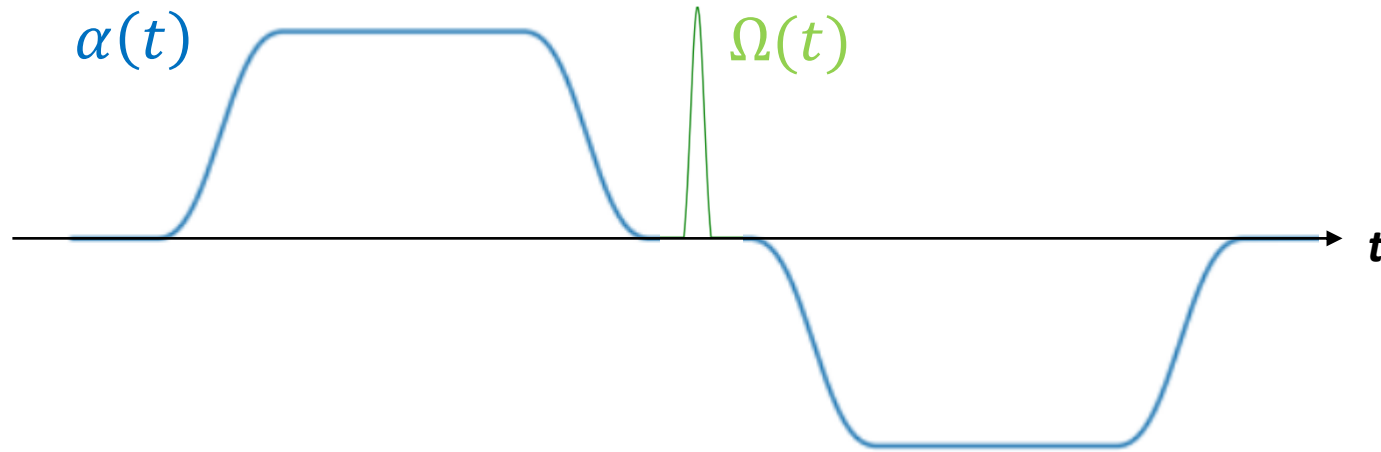
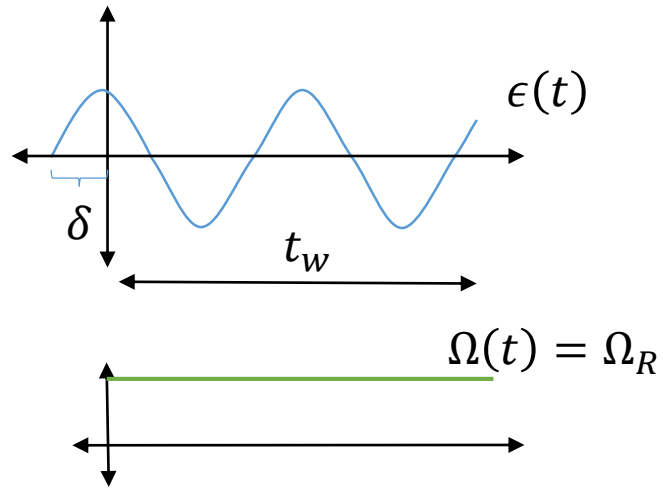$\omega = 0$ $\omega \geq \Omega_R$
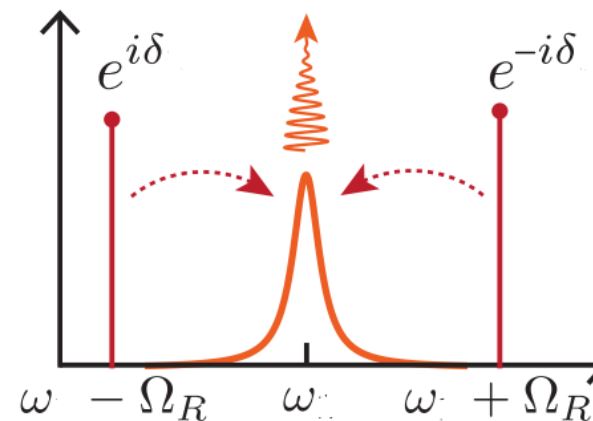
$$\alpha(t) \propto \alpha_0 \cos(\Omega_R t + \delta)$$

$\cos(\delta)$

Shay Hacohen-Gourgy, ..., Irfan Siddiqi. Nature 538-7626 (2016).

Echoed Cond. Disp.

$\alpha(t)$ $\Omega(t)$ t

$\chi a^+ a \sigma_z$ Not completely echoed out !

$\xrightarrow{\text{Echo}}$ $-\chi a^+ a \textcolor{red}{\sigma_z}$

$\chi(\alpha a^+ + \alpha^* a)\sigma_z$ $\chi(\textcolor{red}{\alpha} a^+ + \textcolor{red}{\alpha^*} a)\textcolor{red}{\sigma_z}$

$\chi|\alpha|^2 \sigma_z$ $-\chi|\textcolor{red}{\alpha}|^2 \textcolor{red}{\sigma_z}$

Comparison

Sideband Drives

- Oscillating $\epsilon(t), \alpha(t)$

- Continuous Rabi Driving on the qubit

- Ridding unwanted terms via **RWA**: $e^{i\Omega t}$
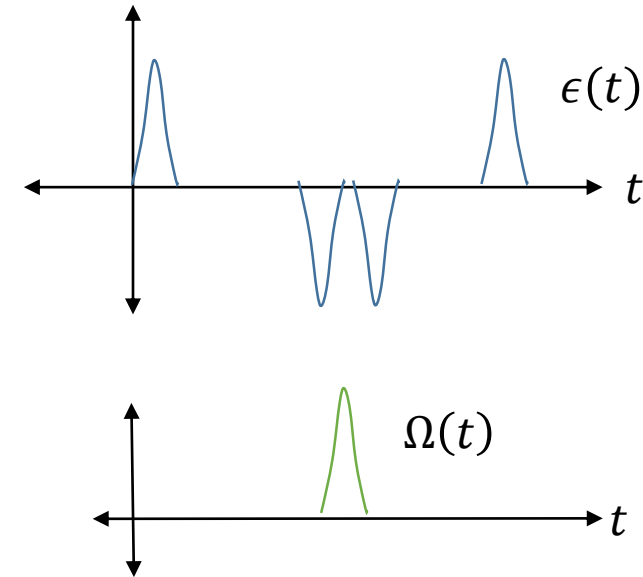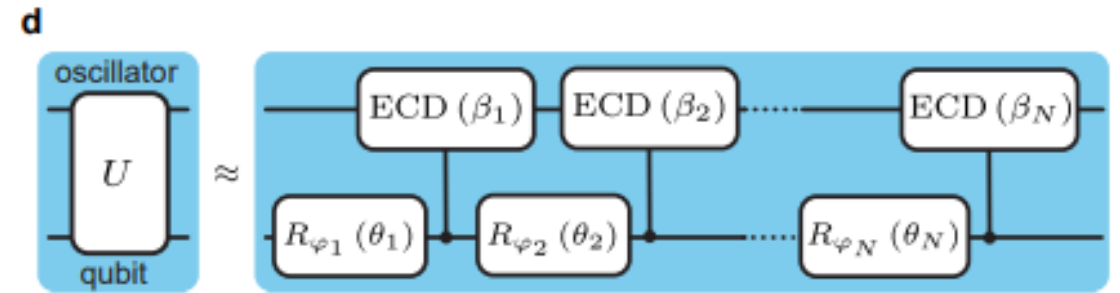
Echoed Conditional Gates

- Single Oscillation of $\alpha(t)$

- Discrete Qubit pi pulses

- Ridding unwanted terms via **echoing**: Step Function

Implementation: Optimal Parameters

- ECD and Sideband Drives, by themselves, do not offer universal control of both oscillator and qubit

- Sol: Interleave parameterized qubit rotations between CD

- Gate times are dependent on # of layers to realize high fidelity gates

$$CD(\beta) = D\left(\frac{\beta}{2}\right)|g\rangle\langle g| + D\left(-\frac{\beta}{2}\right)|e\rangle\langle e|$$
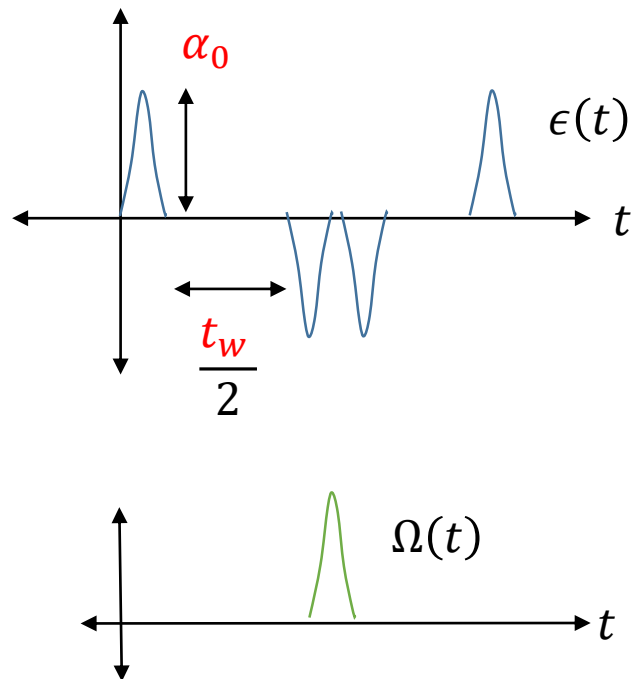
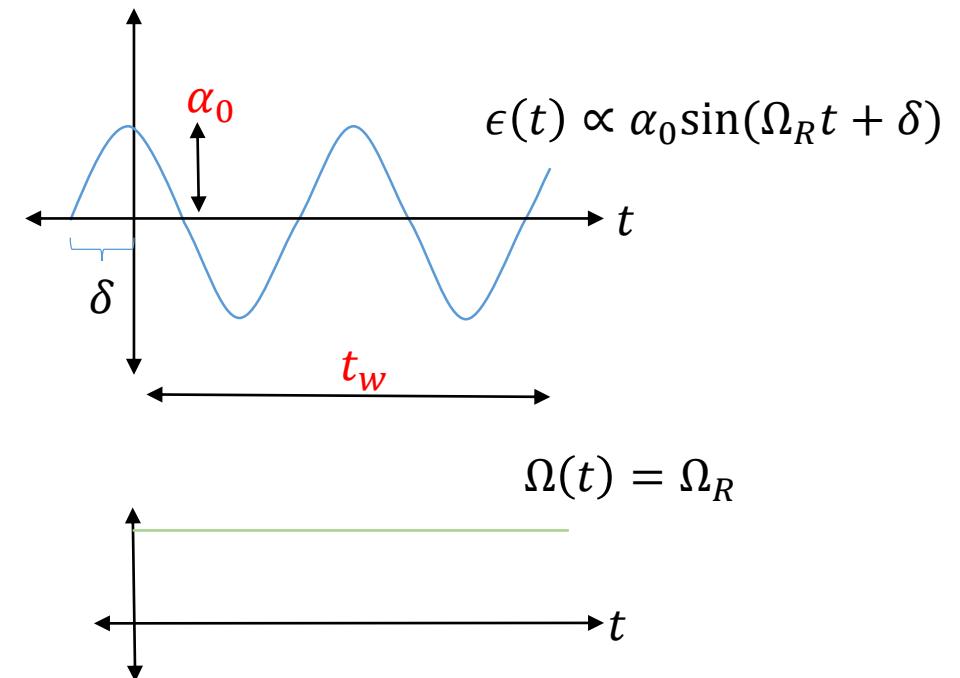$$R_\phi(\theta) = e^{-i\left(\frac{\theta}{2}\right)(\cos\phi\,\sigma_x + \sin\phi\,\sigma_y)}$$

Implementation: Finding Pulses

Task: find wait time t_w and scale intermediate displacement α_0 such that $\chi \alpha_0 t_w = \beta$
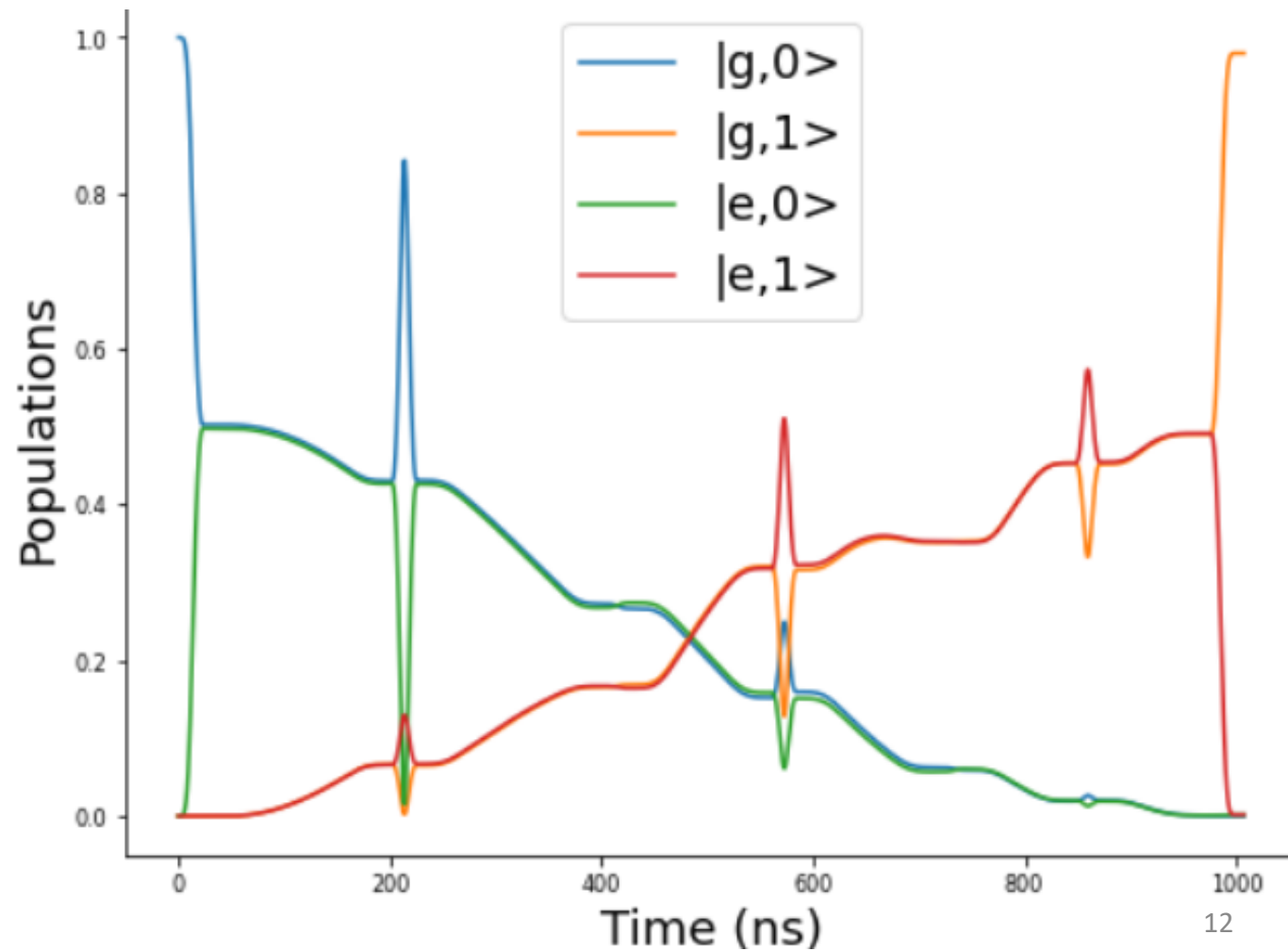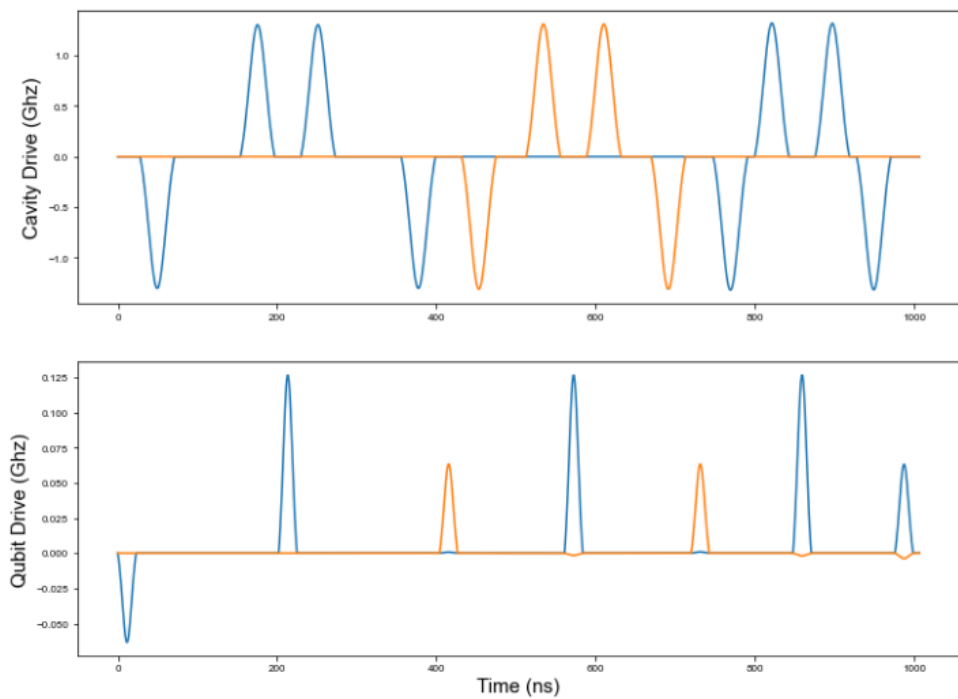
ECD

Sideband
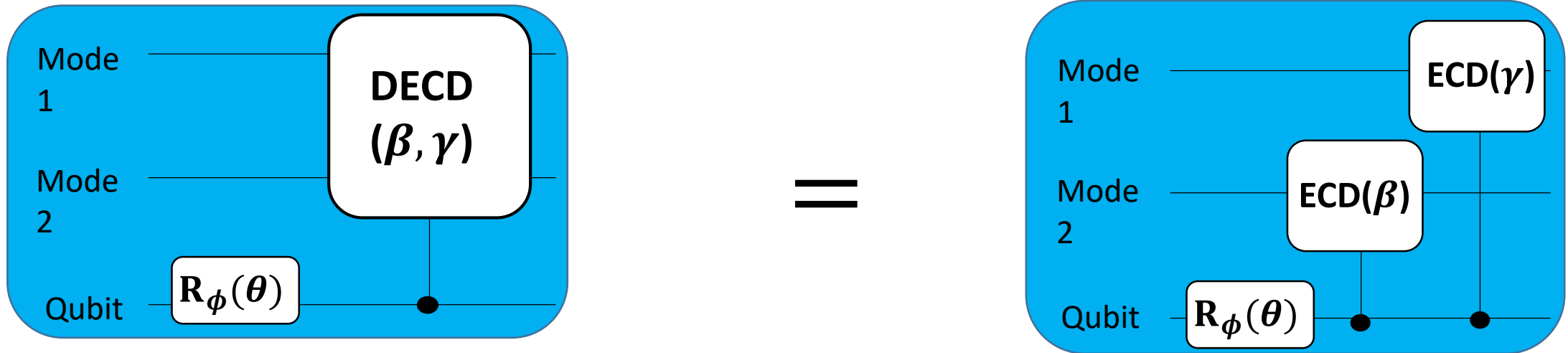
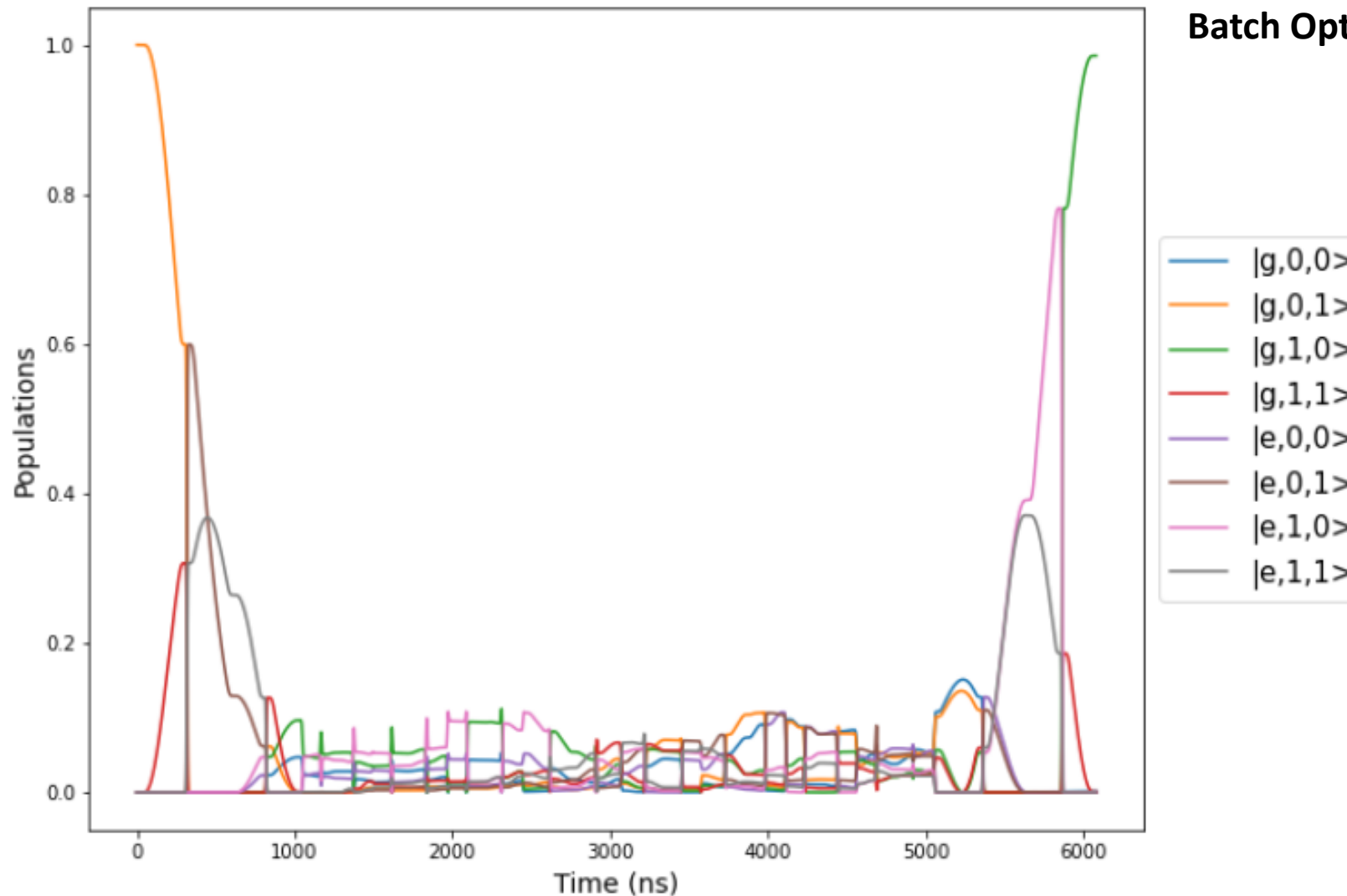# ECD: $|g0\rangle \rightarrow |g1\rangle$

Two Mode ECD

- Generalizing ECD gate to 2 modes
- Displacements on the two modes are not simultaneous (to avoid heating as observed in [*])
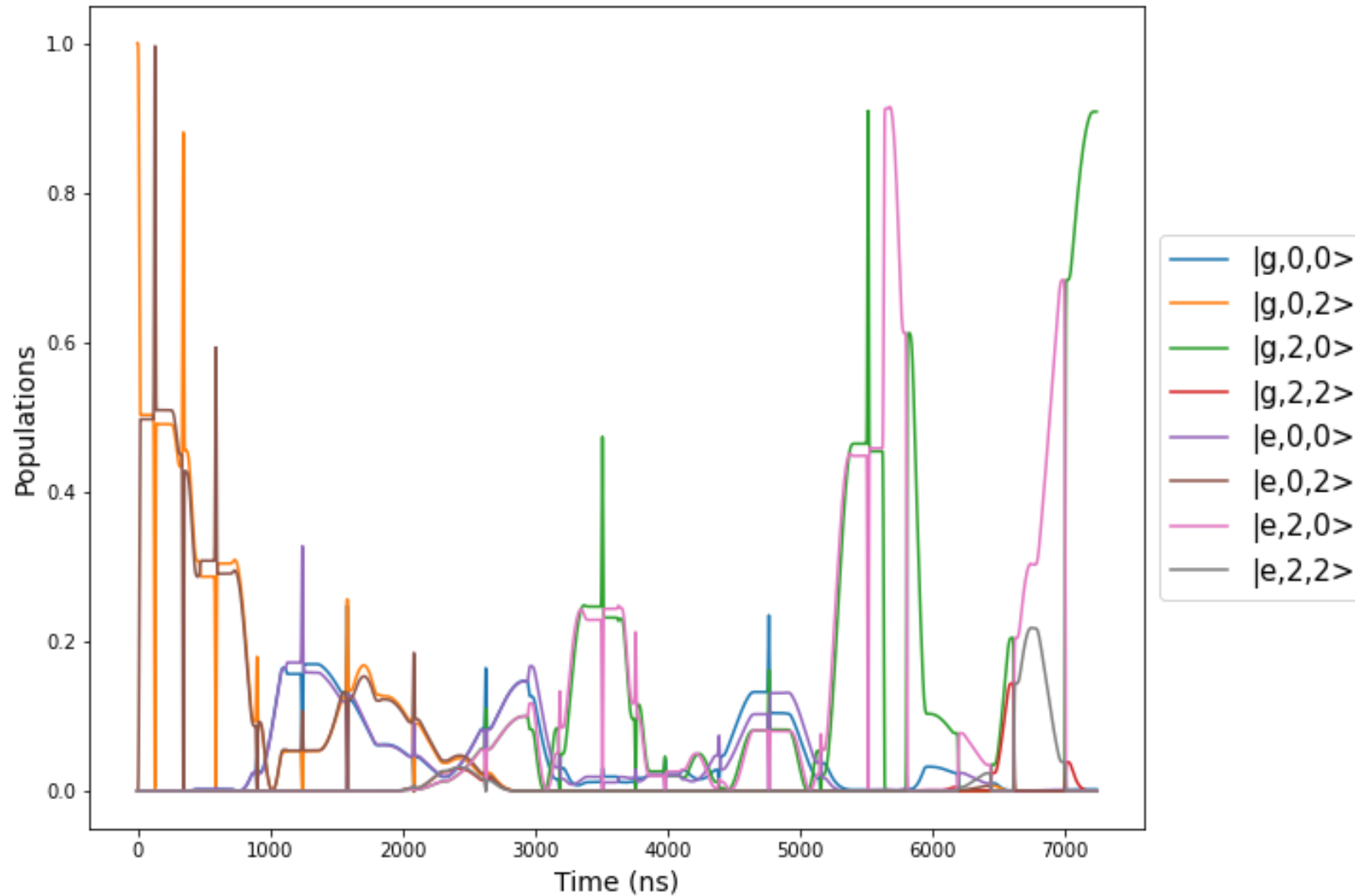
* Alec Eickbusch, Zhenghao Ding, …, Michel Devoret. W34. 00005. APS March Meeting (2022).

Two Mode ECD : $|g01\rangle \rightarrow |g10\rangle$

Batch Optimizer Fidelity: 0.995308

Two Mode ECD: $|g02\rangle \rightarrow |g20\rangle$

Batch Optimizer Fidelity: 0.92192787

Two Mode ECD : Unwanted Cross Kerr Terms

$$\chi_{ab} a^+ a \, b^+ b \xrightarrow{\text{Displaced Frame Transformation}} \chi_{ab}(a^+ + \alpha^*)(a + \alpha)(b^+ + \beta^*)(b + \beta)$$

Terms of form :

$\chi_{ab} \boldsymbol{\alpha\beta} \, a^+ b^+$

$\chi_{ab} |\boldsymbol{\alpha}|^2 \boldsymbol{\beta} \, b^+$

$\chi_{ab} |\boldsymbol{\alpha}|^2 b^+ b$
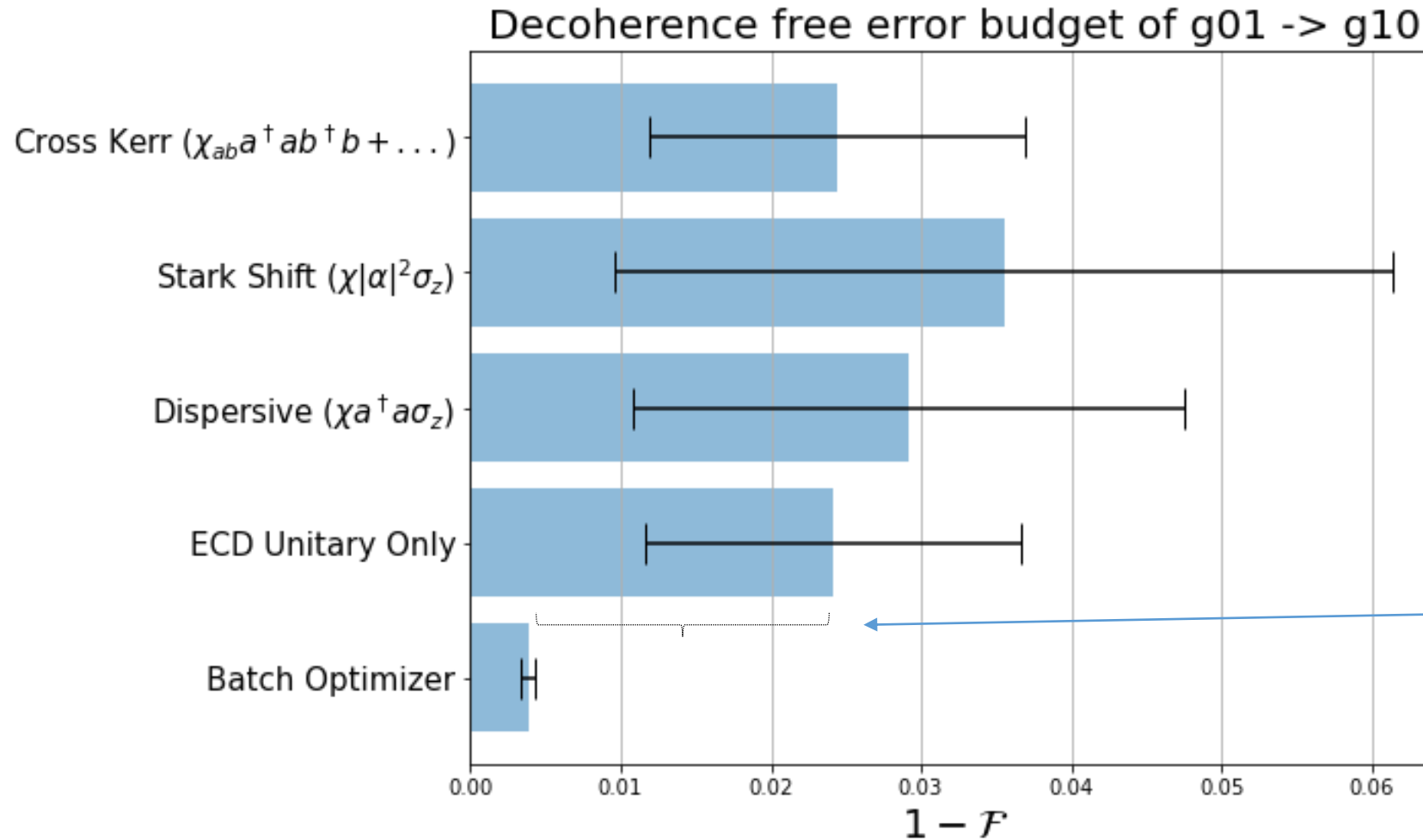
How to avoid :

α, β should not be simultaneously nonzero

Echoed out when β flips

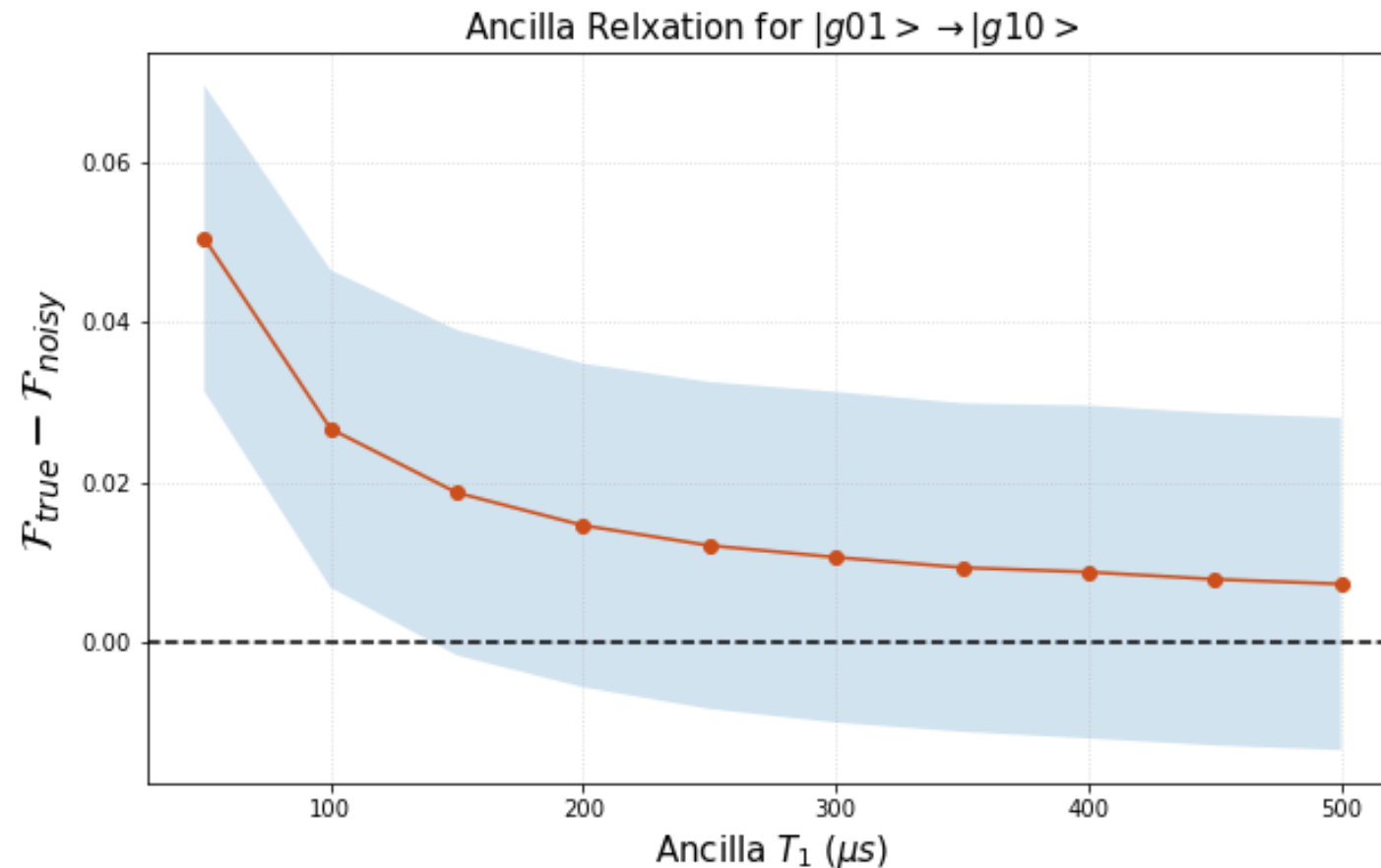Make $\chi_{ab} \ll \chi_a, \chi_b \approx 10$ kHz

Note $\chi_{ab} = \sqrt{\kappa_a \kappa_b} = \frac{\chi_a \chi_b}{\alpha'} \approx 0.33$ Hz ... good!

$(\alpha' \leq 300$ MHz for transmons$)$

Two Mode ECD : QuTip Noise Simulations

Decoherence free error budget of g01 -> g10

Qutip – Batch Optimizer Inconsistency; Likely due to reduced Hilbert Space of the cavity modes (n = 15 levels in each)
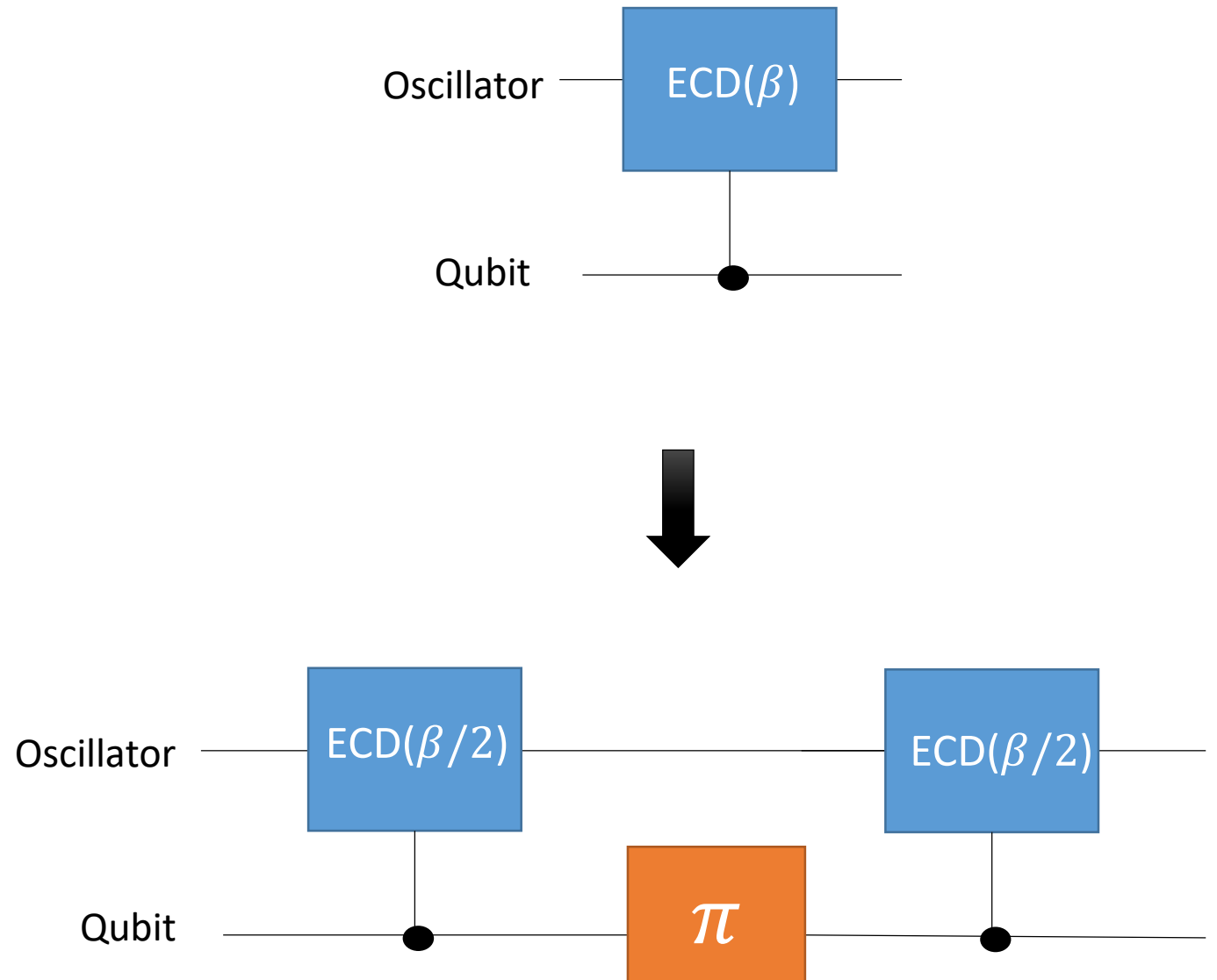
Two Mode ECD : QuTip Noise Simulations

Ancilla with better coherence times such as flux protected qubits may improve gate fidelities.
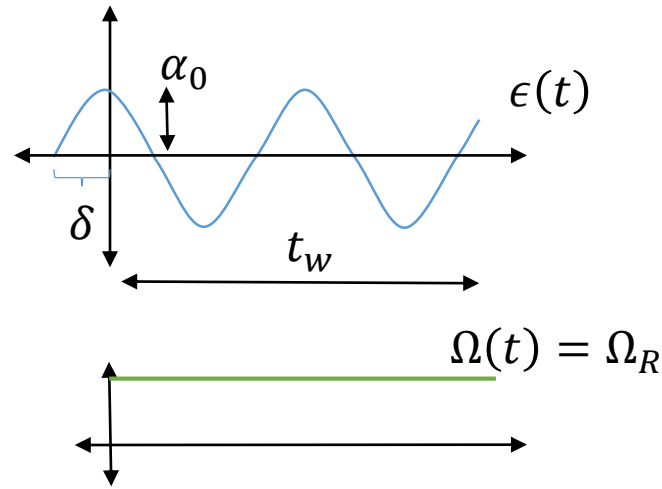
Meta Echoes

- Terms of form $\chi a^+ a \sigma_z$ not completely echoed out by a single pi pulse since measurement of $a^+ a$ does not always yield $|\alpha|^2$

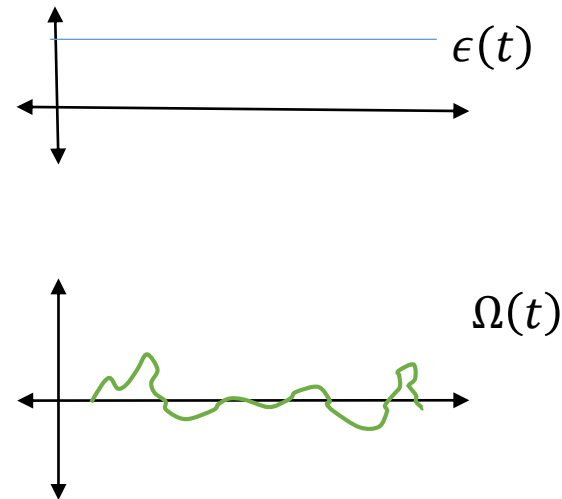- So insert more pi pulses (qubit echoes) in the ECD pulse sequence

Circle Grape

Sideband Drives

$\epsilon(t)$

α_0

δ

t_w

$\Omega(t) = \Omega_R$

- Changing $\epsilon(t)$
- Constant $\Omega(t)$

Circle Grape

$\epsilon(t)$

$\Omega(t)$

- Constat $\epsilon(t)$
- Changing $\Omega(t)$

$$H = \chi a^+ a \sigma_z + \chi(\alpha_0 a^+ + \alpha_0^* a)\sigma_z + \chi|\alpha_0|^2 \sigma_z + \Omega(t)\sigma_x$$

Sent to Optimizer

Similarly grap-ifying Sideband Drives?
Sending $\delta(t)$ to the optimizer

Circle Grape