Multimode Conditional Displacements

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Motivation

- SNAP Gates take time $\approx 2\pi/\chi$ where $\chi \approx$ MHz is dispersive coupling strength.
- Reducing Gate time $\rightarrow$ Increasing χ $\rightarrow$Reducing lifetime of cavity
- ECD Idea: Keep $\chi \approx 10$ kHz small; But enhance it by displacing cavity (α_0) far from origin
- Effective Gate time $1/\chi\alpha_0$ where $\alpha_0 \gg 1$; Faster, low noise gates

Achieving Conditional Displacements

Starting Point: $H/\hbar = \omega_c a^\dagger a + \omega_q \frac{\sigma_z}{2} + \chi a^\dagger a \frac{\sigma_z}{2} + H_{drive}$

Using **frame transformations**, our objective is to **isolate** the following term from the ac-Stark Shift

$$\tilde{H} = \chi(\alpha a^\dagger + \alpha^\star a)\sigma_i$$

where α is the displacement of the cavity mode. With such a term, we can realize a conditional displacement as follows

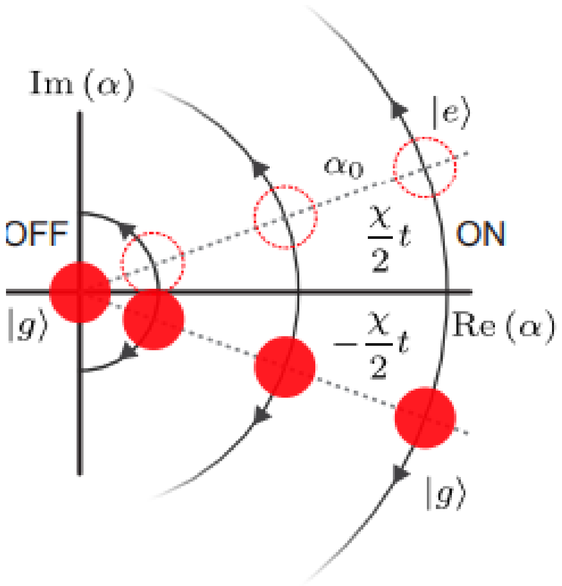
$$e^{-i(\chi(\alpha a^\dagger + \alpha^\star a)\sigma_i)t} \xleftrightarrow{\beta = -i\,\chi\alpha\,t} e^{(\beta a^\dagger - \beta^\star a)\sigma_i}$$

Conditional Displacements

$$\chi\alpha\sigma_z \quad (a + a^\dagger)$$

$$\boldsymbol{F} \;\; * \;\; \boldsymbol{x}$$

Force Displacement



A. Eickbusch, ..., R. Schoelkopf, M. Devoret. *ArXiv preprint arXiv:2111.06414 (2021)*

Dealing with Unwanted Terms I

1. Rotating Frames of oscillator and the qubit
2. Displacement transformation $D^{+}(\alpha(t)) = e^{\alpha(t)a^{+} - \alpha^{*}(t)a}$

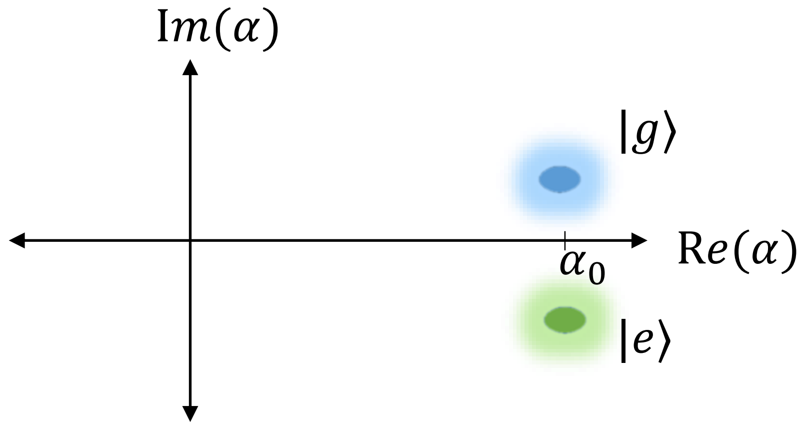
which renders $a \rightarrow a + \alpha(t)$

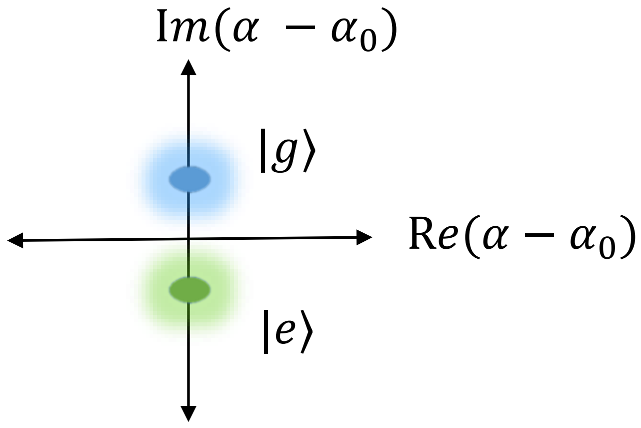
$$H_{disp} = D^{\dagger} H_{rot} D - i\dot{D}^{\dagger} D$$
$$= D^{\dagger} H_{rot} D + i(\dot{\alpha}^{\star} a - \dot{\alpha} a^{\dagger})$$

Cancel terms linear in $\boldsymbol{a}, \boldsymbol{a}^{+}$, such as the oscillator drive $\epsilon(t)a^{+} + \epsilon^{*}(t)a$, by picking the appropriate time dependent displacement frame

$$\dot{\alpha} = -i\epsilon(t) \qquad \dot{\alpha}^{*} = i\epsilon^{*}(t)$$

Implication: Disp. Frame Simulations





Lab Frame

- Large Displacement
- Number of photons $n = |\alpha_0|^2 \approx 900$
- Intractable simulations

Displaced Frame

- Size of Conditional Displacement ($|\alpha_g - \alpha_e| \leq 5$)
- Number of photons $n = |\alpha_g - \alpha_e|^2 \approx 25$
- Tractable simulations

Dealing with Unwanted Terms II

The **displaced frame** transformation, however, divides the **initial ac-Stark shift** term into the following 3 terms

$$\chi(a^\dagger + \alpha^*)(a + \alpha)\sigma_z$$

$$\downarrow$$

$$\chi a^\dagger a \sigma_z \quad + \quad \underbrace{\chi(\alpha a^\dagger + \alpha^* a)\sigma_z}_{\text{desired}} \quad + \quad \chi|\alpha|^2\sigma_z$$

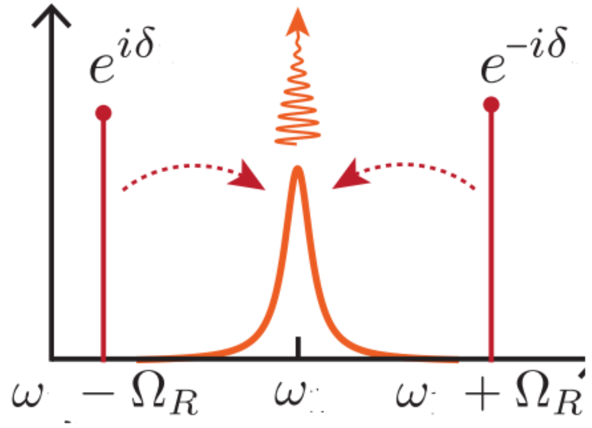
Sideband Drives

- Make terms **oscillate at different** frequencies
- Invoke RWA in a frame where only desired term is stationary

Echoed Cond. Displacements

- Terms have different no. of α's but only a single σ_z
- **Clever flipping of α and $\boldsymbol{\sigma_z}$** can echo out unwanted terms

Sideband Drives

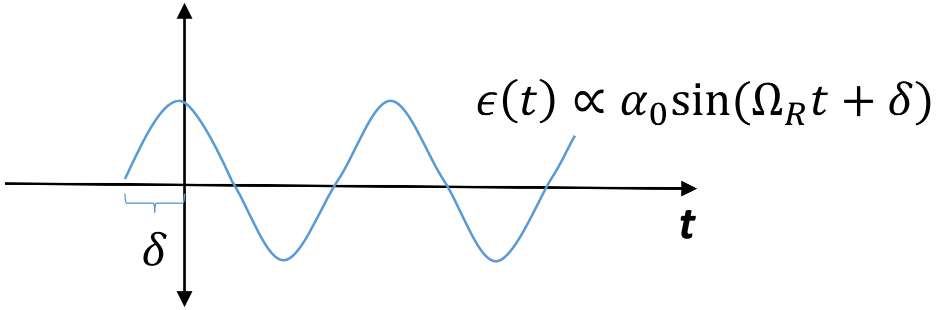


https://arxiv.org/pdf/1608.06652.pdf

Since α oscillatory,

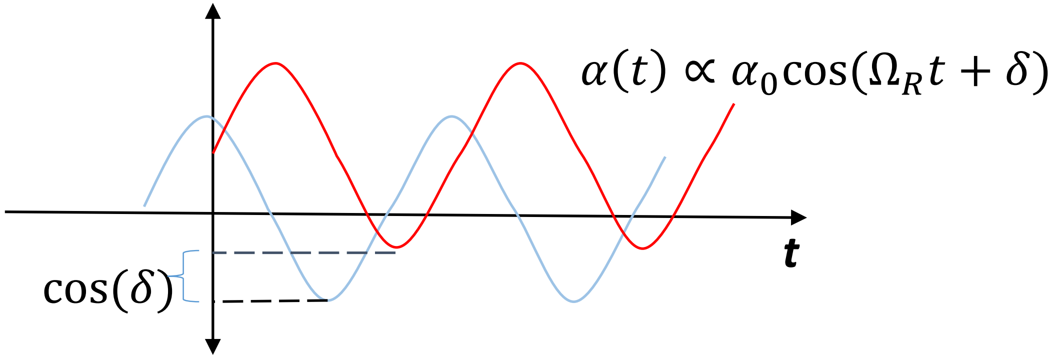
$$H = \chi a^{+} a \sigma_z + \chi(\alpha a^{+} + \alpha^{*} a)\sigma_z + \chi|\alpha|^2 \sigma_z + \Omega_R \sigma_x$$

$\omega = 0$ $\qquad$ $\omega = \Omega_R$ $\qquad$ $\omega = 2\Omega_R$



Frame Transformations:

1. $\sigma_x \leftrightarrow \sigma_z$ $\longrightarrow$ $\Omega_R \sigma_z$
2. Rotating Frame of the qubit $\qquad$ $\cancel{\Omega_R \sigma_z}$

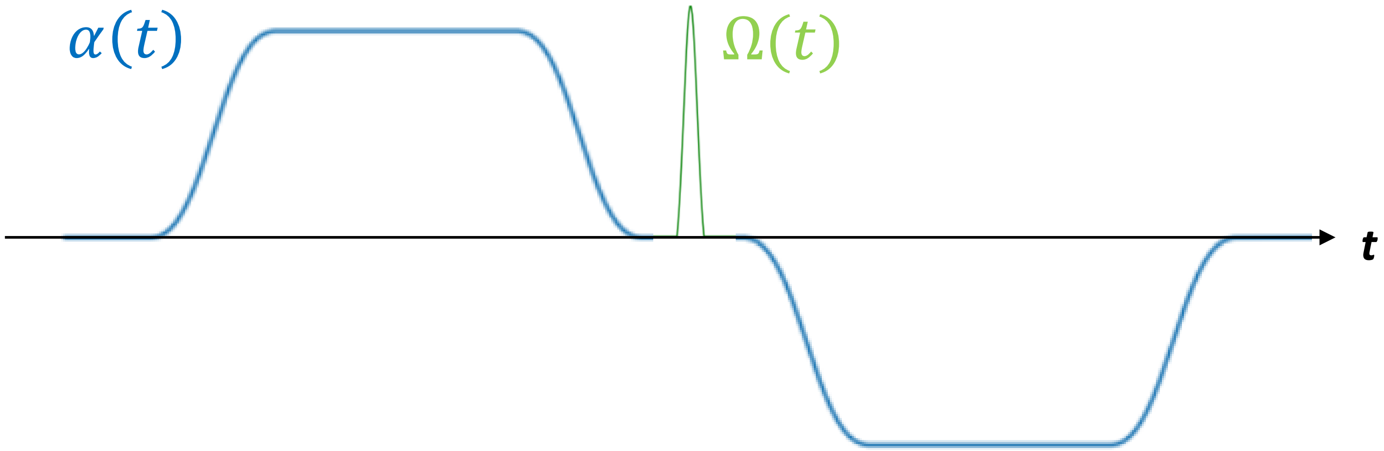


$$H = \chi \alpha_0 (a^{+} + a) \otimes (\sigma_x \cos\delta + \sigma_y \sin\delta) + \ldots$$

$\omega = 0$ $\qquad$ $\omega \geq \Omega_R$

Shay Hacohen-Gourgy, …, Irfan Siddiqi. Nature 538-7626 (2016).

Echoed Cond. Disp.



Not completely echoed out !

$$\chi a^\dagger a \sigma_z \qquad \xrightarrow{\text{Echo}} \qquad -\chi a^\dagger a \sigma_z$$

$$\chi(\alpha a^\dagger + \alpha^* a)\sigma_z \qquad \longrightarrow \qquad \chi(\alpha a^\dagger + \alpha^* a)\sigma_z$$

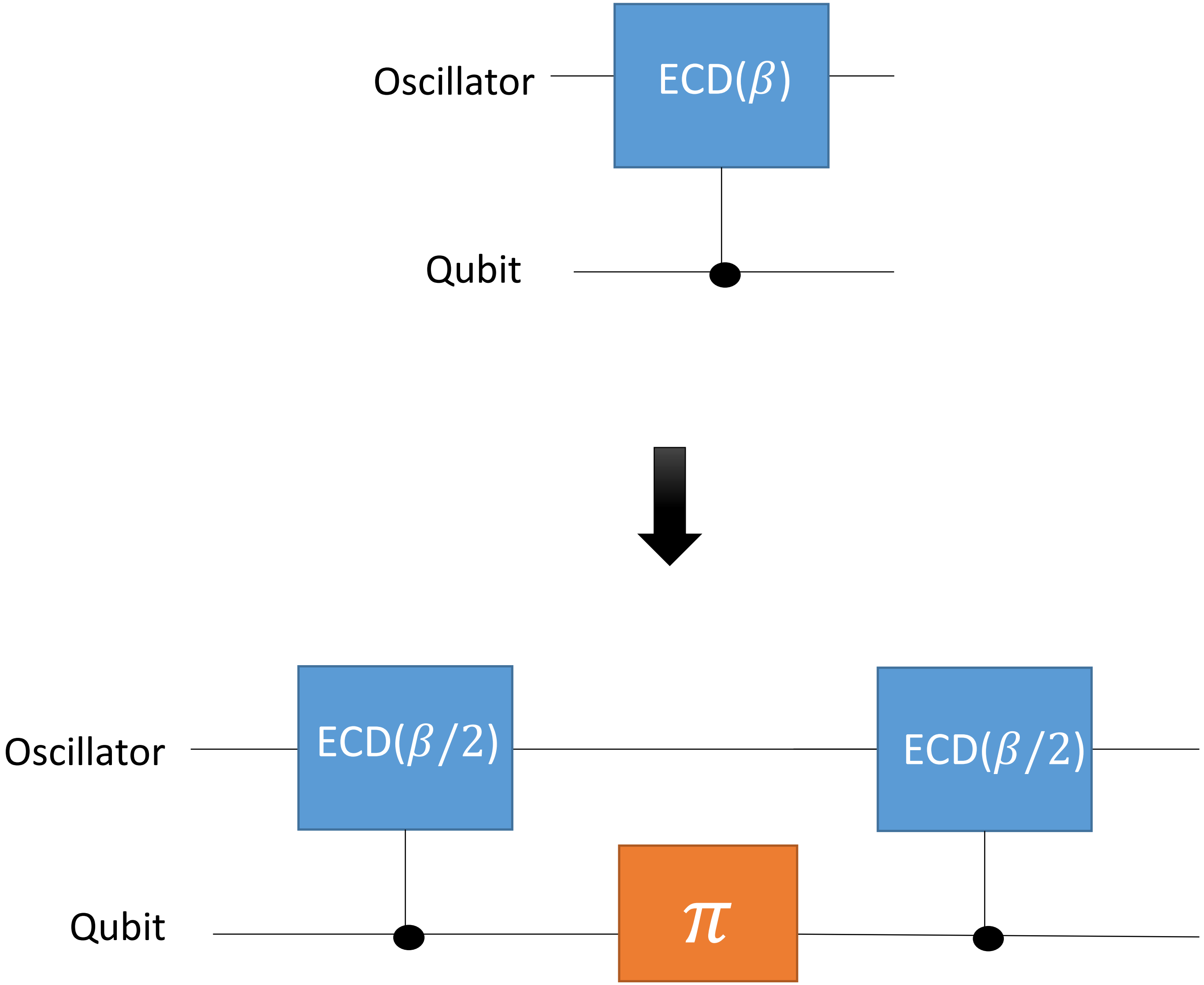
$$\chi|\alpha|^2\sigma_z \qquad \longrightarrow \qquad -\chi|\alpha|^2\sigma_z$$

A. Eickbusch, ..., R. Schoelkopf, M. Devoret. ArXiv preprint arXiv:2111.06414 (2021)

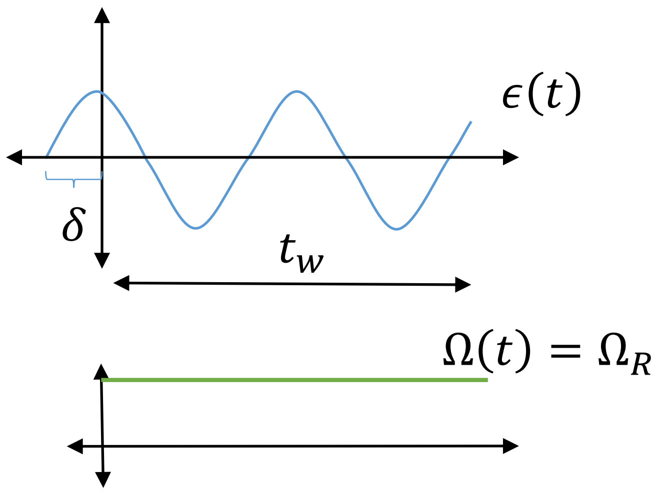
Idea: Meta Echoes

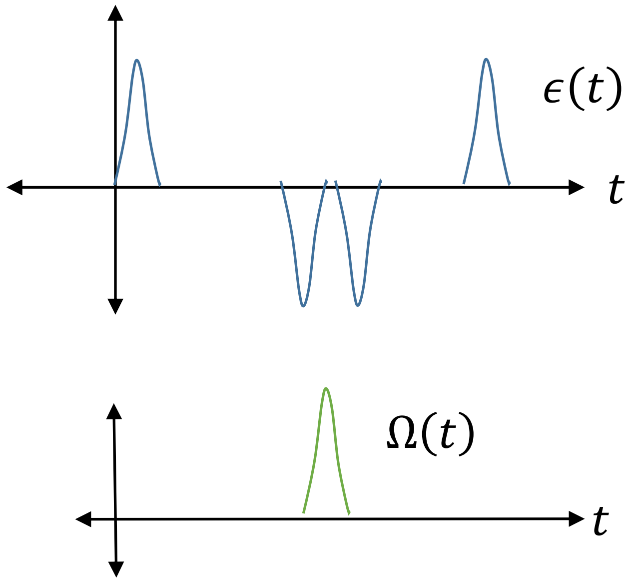
- Terms of form $\chi a^{+} a \sigma_z$ not completely echoed out by a single pi pulse since measurement of $a^{+}a$ does fluctuates

- So insert more pi pulses (qubit echoes) in the ECD pulse sequence



Comparison





Sideband Drives

- Oscillating $\epsilon(t), \alpha(t)$
- Continuous Rabi Driving on the qubit
- Ridding unwanted terms via **RWA**: Multiple Qubit Flips

Echoed Conditional Gates

- Single Oscillation of $\alpha(t)$
- Discrete Qubit pi pulses
- Ridding unwanted terms via **echoing**: Single Qubit Flip

What about Noise? Cavity Relaxation

Master Equation : $$\rho = -i\,[H,\rho] \; + \; \kappa\,\mathcal{D}[a]\rho$$

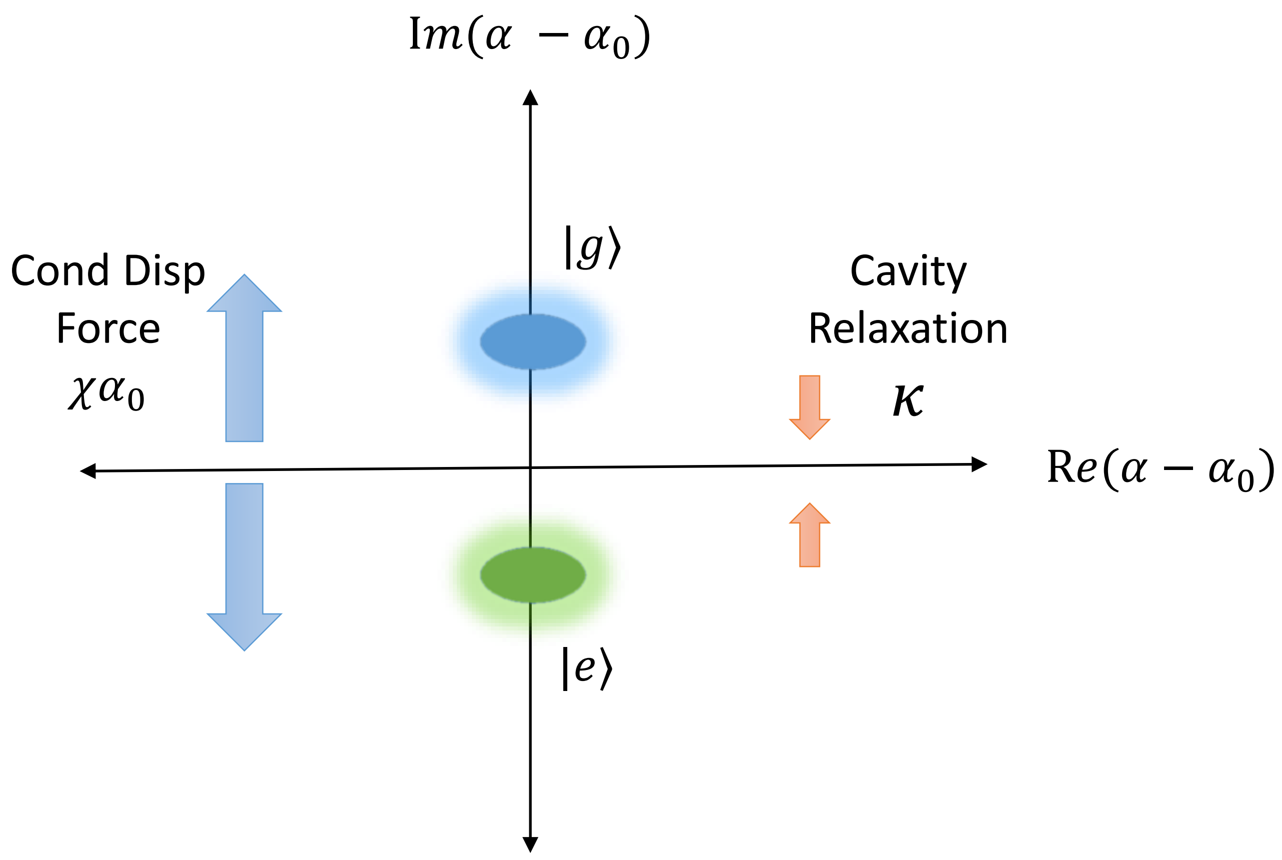
Displaced Frame Transf.

$$\kappa\mathcal{D}[a+\alpha]\,\tilde{\rho} = \kappa\mathcal{D}[a]\tilde{\rho} \; - \; i\,[\,\frac{i\kappa}{2}\left(\alpha^{\star}a - \alpha a^{\dagger}\right),\tilde{\rho}\,]$$

Remember

$$D(\alpha) = e^{\boldsymbol{\alpha a^{\dagger} - \alpha^{\star} a}}$$

What about Noise? Cavity Relaxation



Center of mass frame of Oscillator in its Phase Space

What about Noise? Cavity Relaxation

Master Equation : $$\rho = -i\,[H, \rho] + \kappa\,\mathcal{D}(a)\rho$$

Displaced Frame Transf.

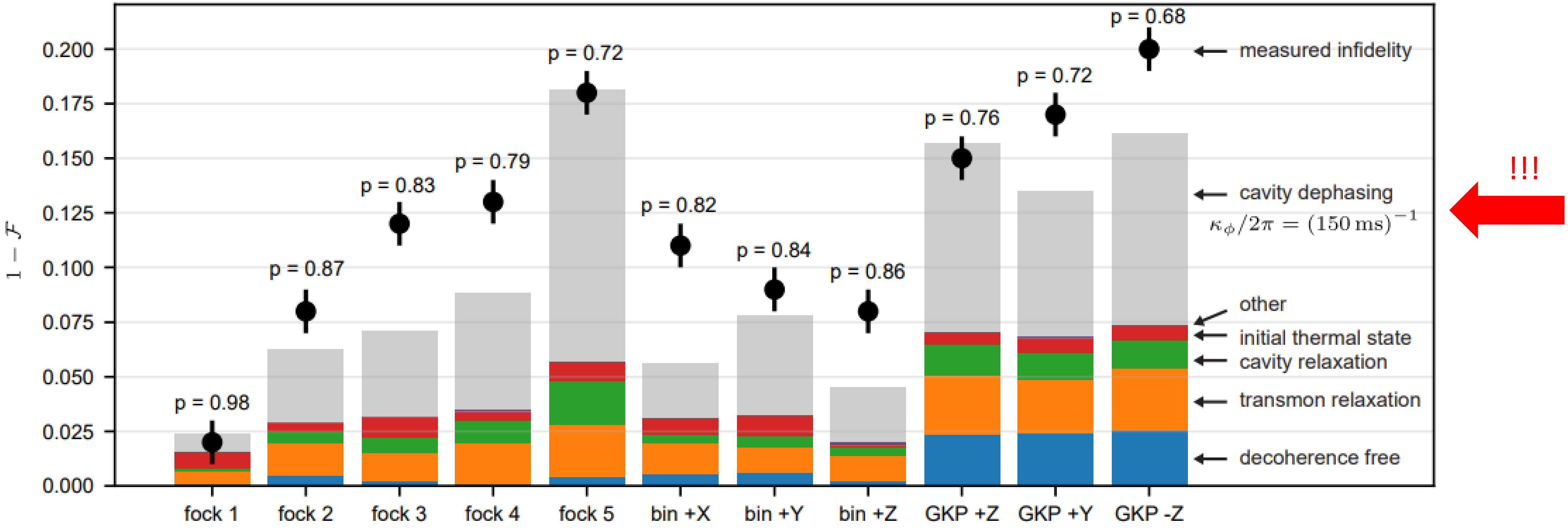
$$\kappa\mathcal{D}(a + \alpha)\,\tilde{\rho} = \kappa D(a)\tilde{\rho} \;-\; i\left[\frac{i\kappa}{2}\left(\alpha^{\star} a - \alpha a^{\dagger}\right), \tilde{\rho}\,\right]$$

New Term can be canceled in a similar way as the cavity drive as :

$$\dot{\alpha} = -i\epsilon(t) - \frac{\kappa\alpha}{2} \qquad \dot{\alpha}^{*} = i\epsilon^{*}(t) - \frac{\kappa\alpha^{*}}{2}$$

What about Noise? Cavity Dephasing

However, enhancement of cavity dephasing could not be avoided; proportional to $\kappa_\phi|\alpha|^2$



A. Eickbusch, ..., R. Schoelkopf, M. Devoret. ArXiv preprint arXiv:2111.06414 (2021)

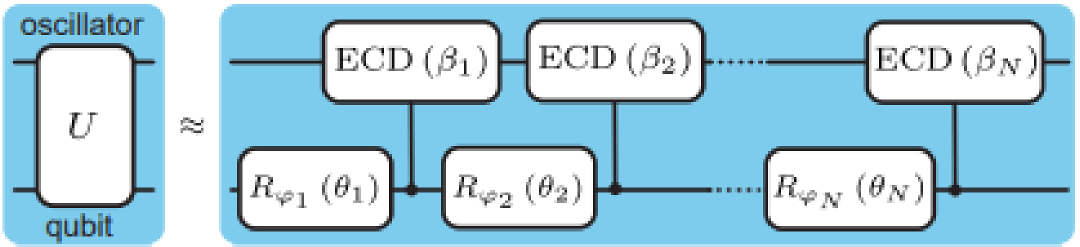
Implementation: Optimal Parameters

- ECD and Sideband Drives, by themselves, do not offer universal control of both oscillator and qubit
- Sol: Interleave parameterized qubit rotations between CD
- Gate times are dependent on # of layers to realize high fidelity gates

$$CD(\beta) = D\left(\frac{\beta}{2}\right)|g\rangle\langle g| + D\left(-\frac{\beta}{2}\right)|e\rangle\langle e|$$

$$R_\phi(\theta) = e^{-i\left(\frac{\theta}{2}\right)(\cos\phi\sigma_x + \sin\phi\sigma_y)}$$

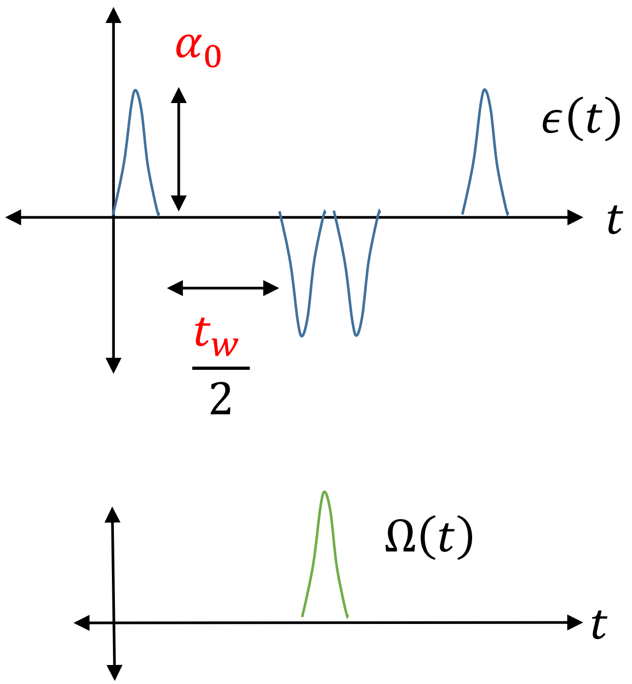
d



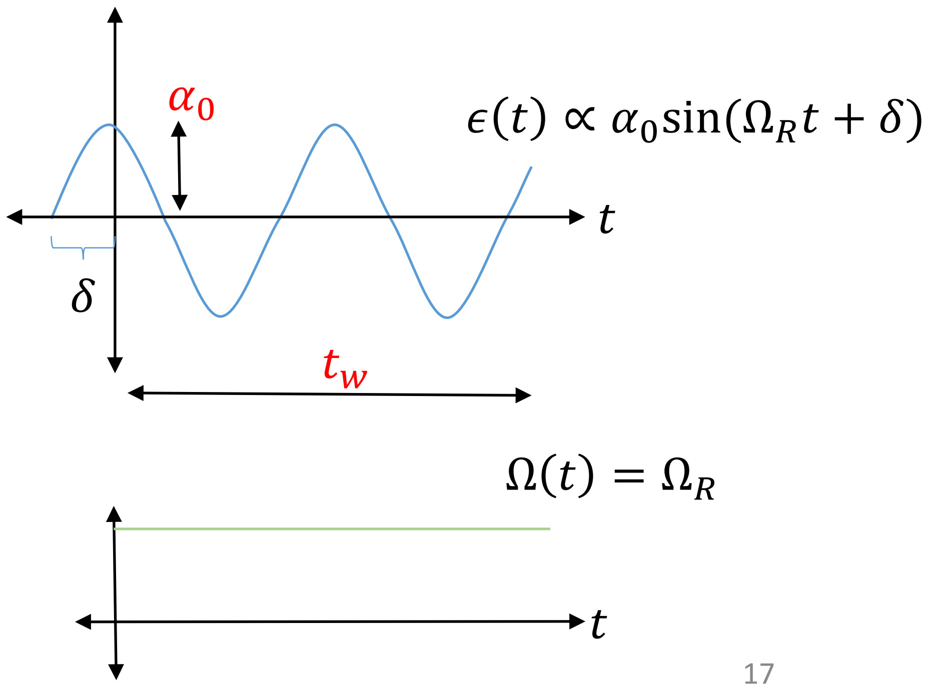
Implementation: Finding Pulses

Task: find wait time t_w and scale intermediate displacement $\alpha_0$such that $\chi\alpha_0 t_w = \beta$

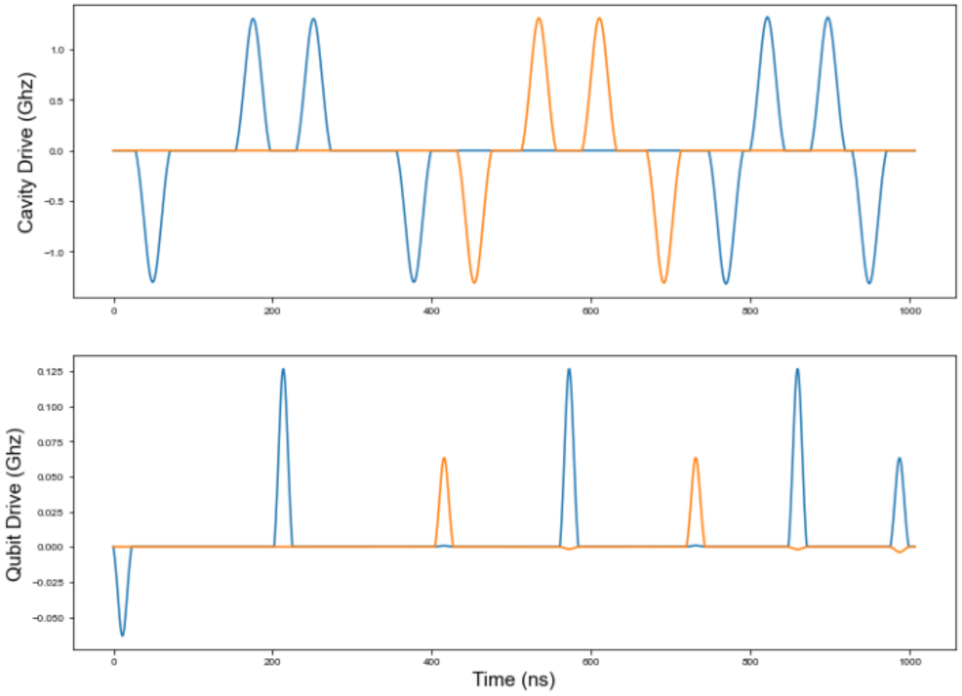
ECD

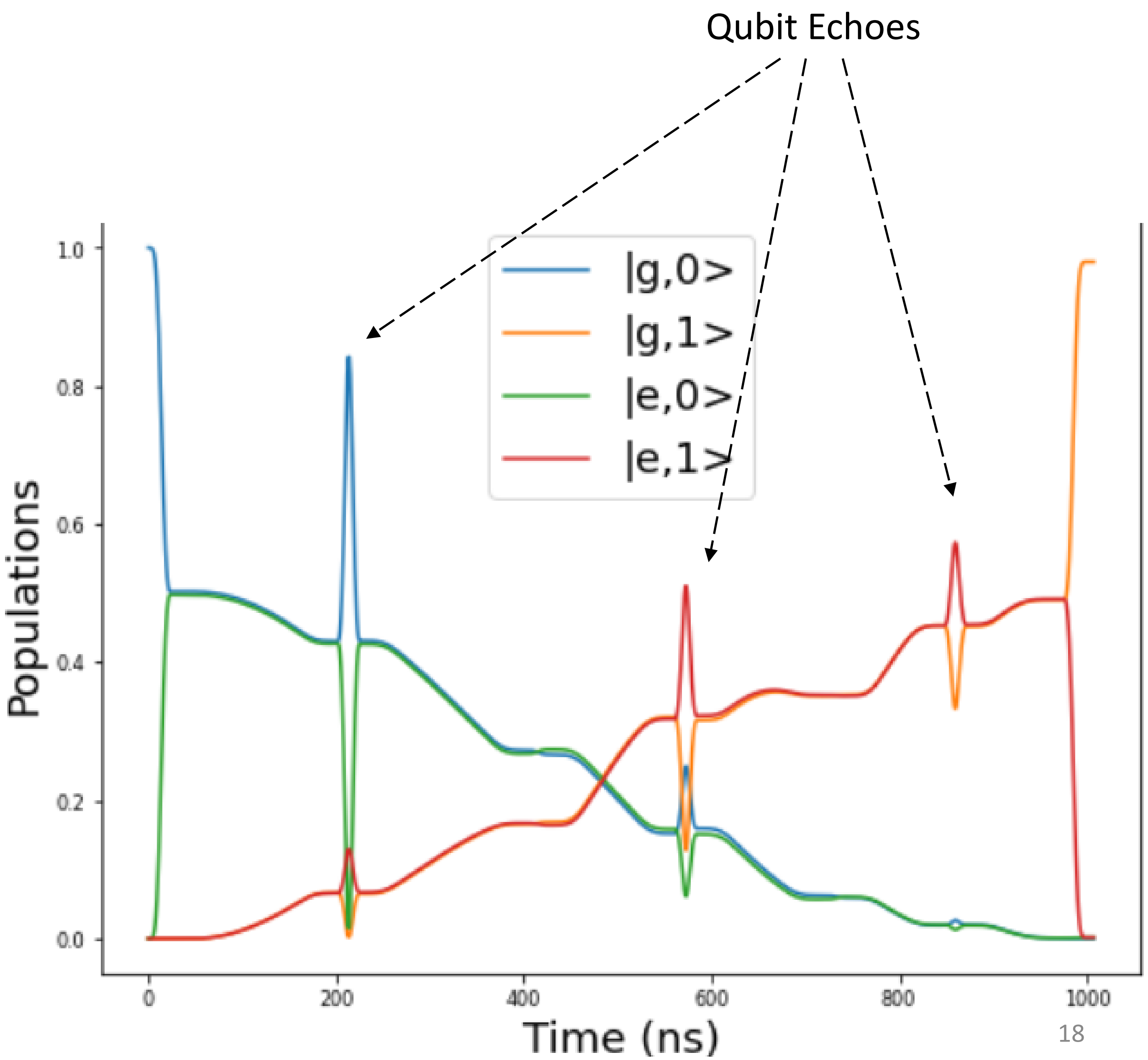


Sideband

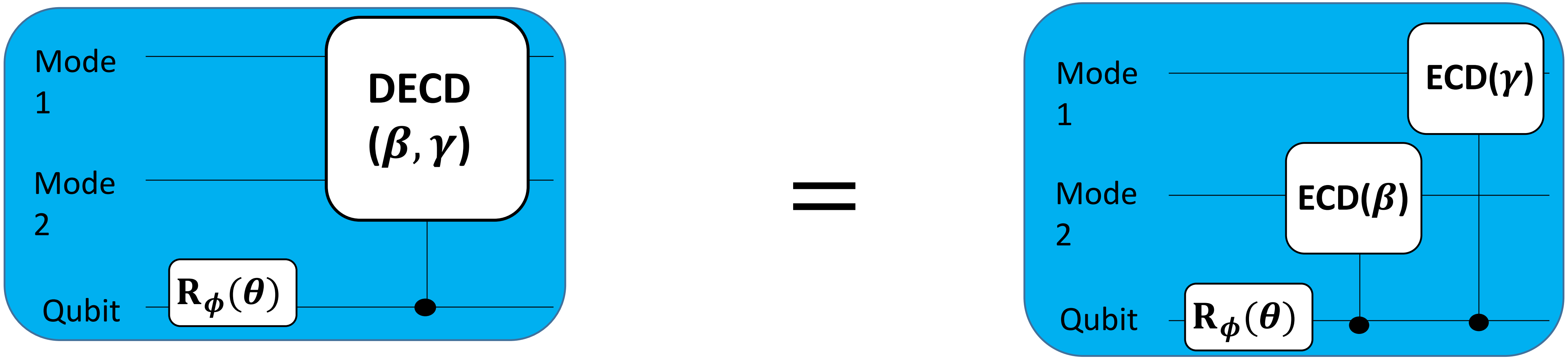


ECD: $|g0\rangle \rightarrow |g1\rangle$





Two Mode ECD

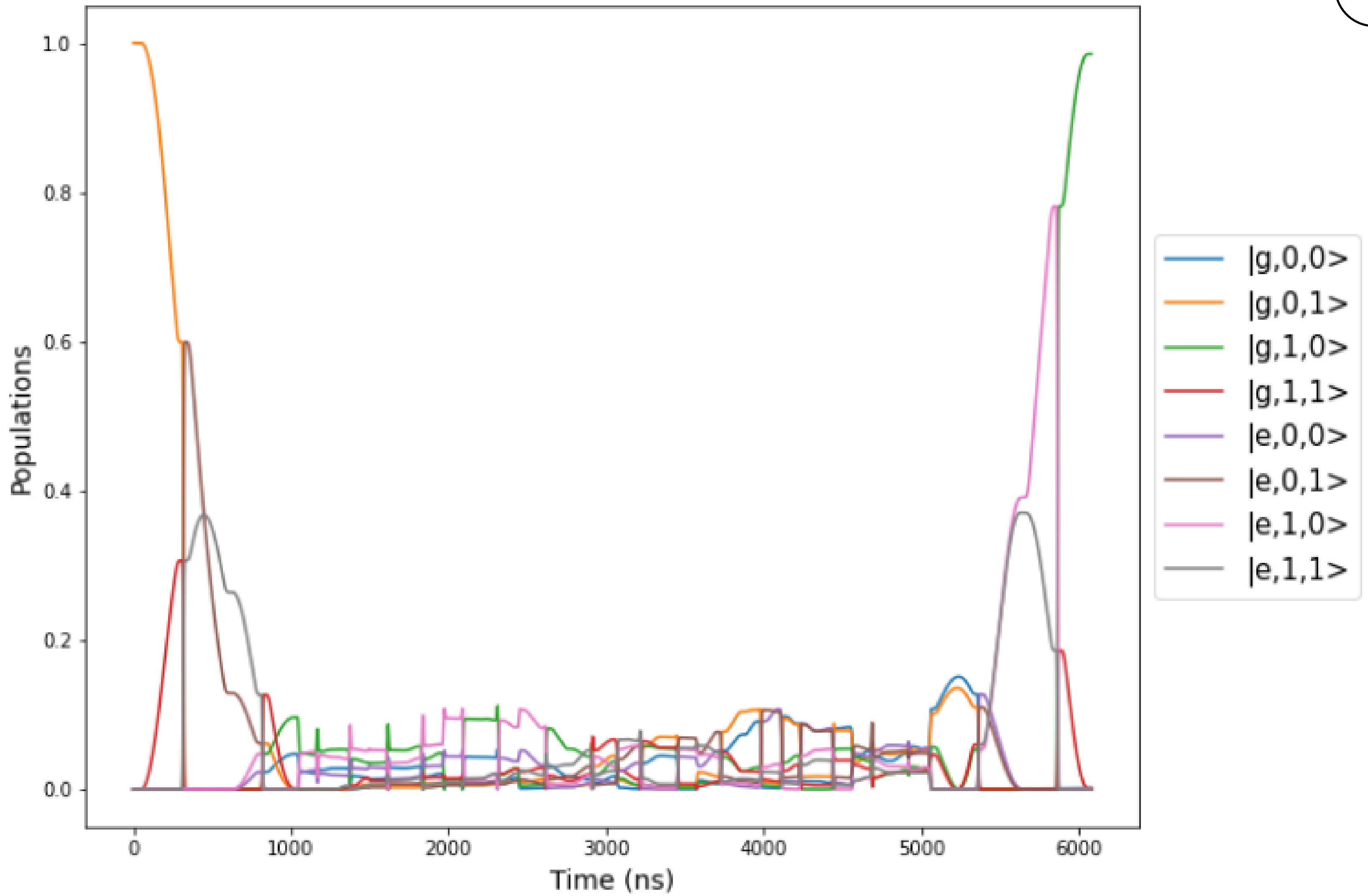


- Generalizing ECD gate to 2 modes
- Displacements on the two modes are not simultaneous (to avoid heating as observed in [*])

* Alec Eickbusch, Zhenghao Ding, …, Michel Devoret. W34. 00005. APS March Meeting (2022).

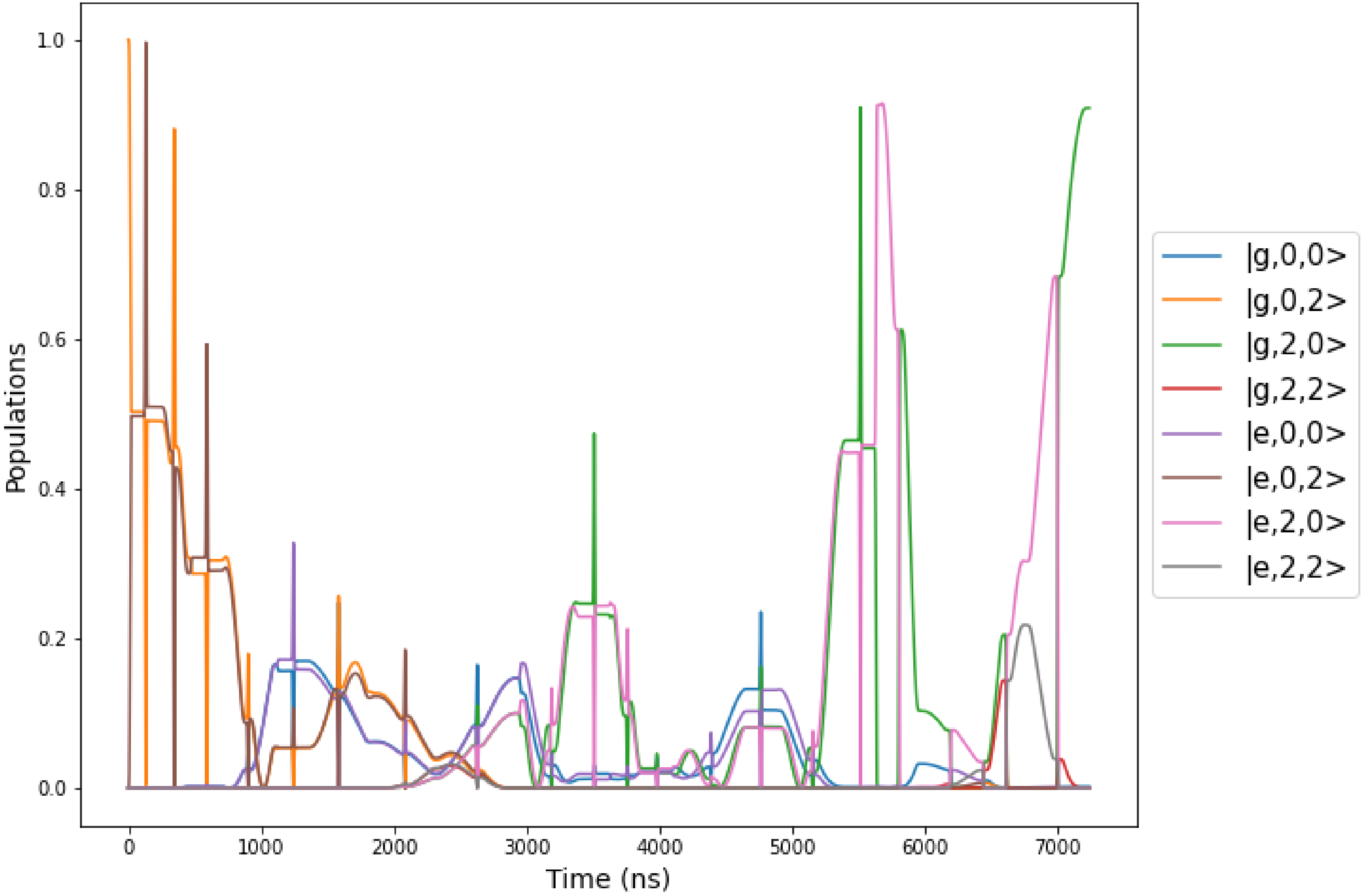
Two Mode ECD : $|g01\rangle \rightarrow |g10\rangle$

Batch Optimizer Fidelity: 0.995
Qutip Fidelity : 0.985
Layers: 10
Mode level Truncation : 20



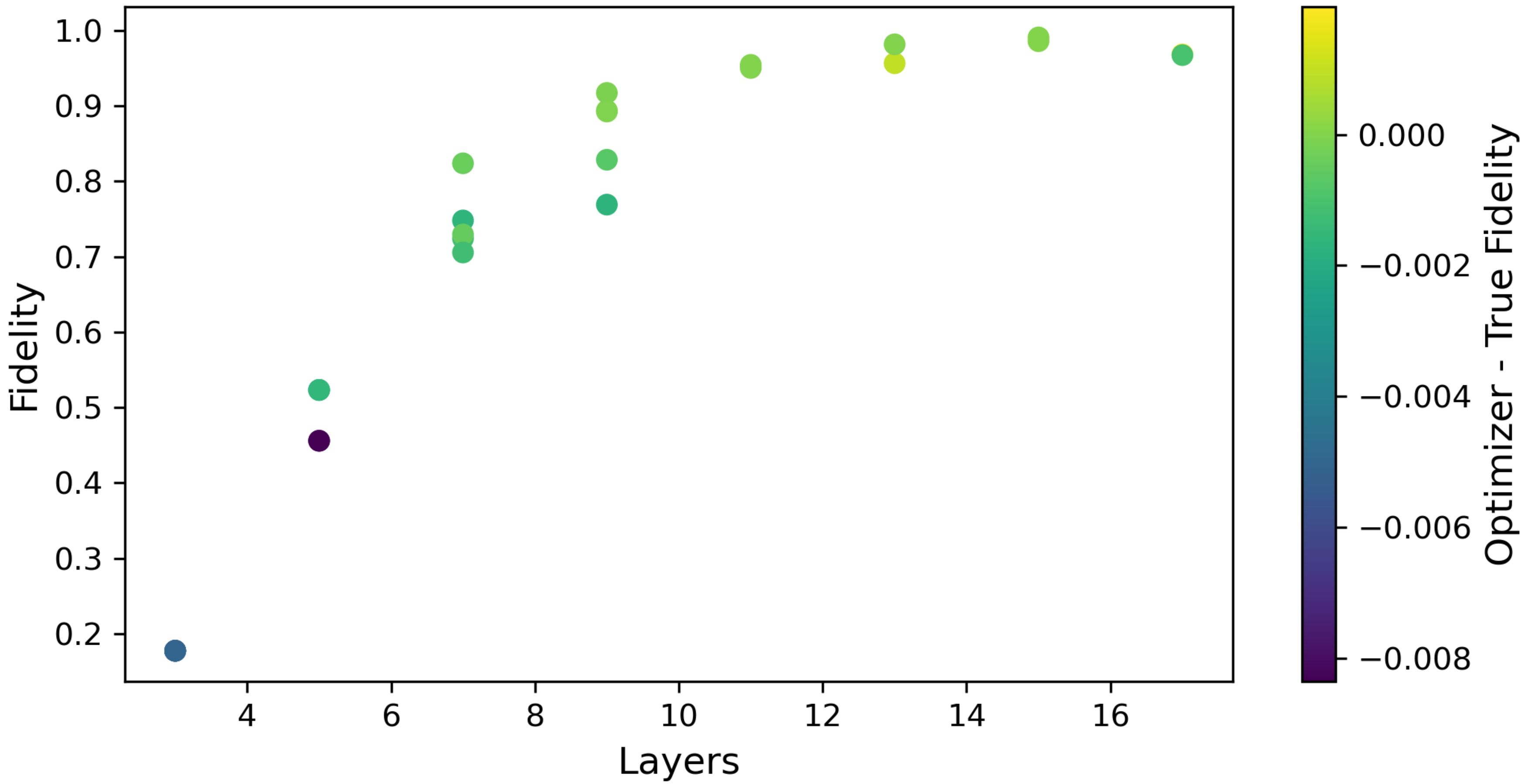
Two Mode ECD: $|g02\rangle \rightarrow |g20\rangle$

Batch Optimizer Fidelity: 0.921
Qutip Fidelity : 0.900
Layers: 10
Mode level Truncation : 20



Two Mode ECD: Simultaneous State Transfer

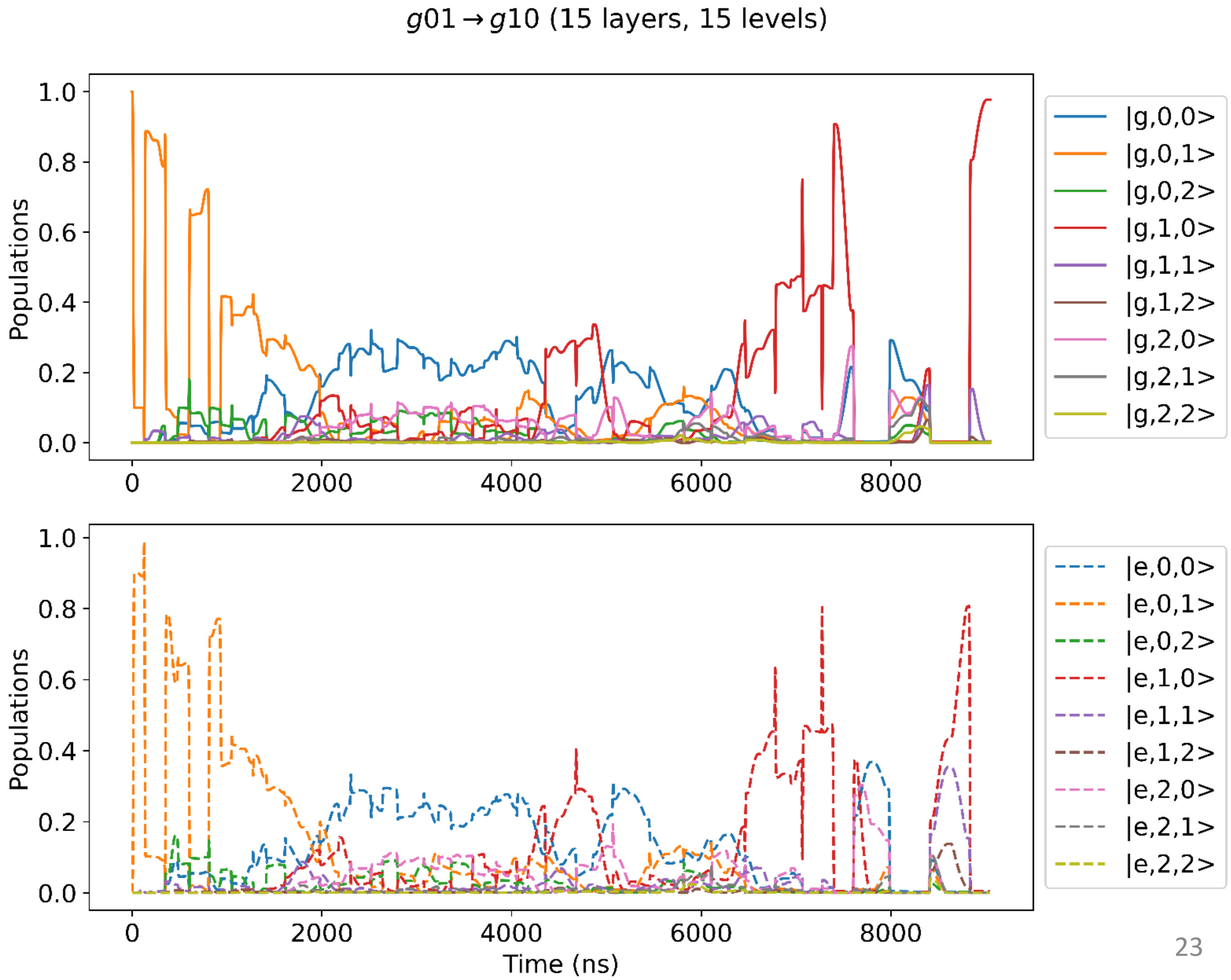
$\boldsymbol{g01} \rightarrow \boldsymbol{g10}$ and $\boldsymbol{g02} \rightarrow \boldsymbol{g20}$ (15 levels in each mode)



Two Mode ECD: Simultaneous State Transfer

- **Parameters Optimized** for both $g01 \rightarrow g10$ and $g02 \rightarrow g20$

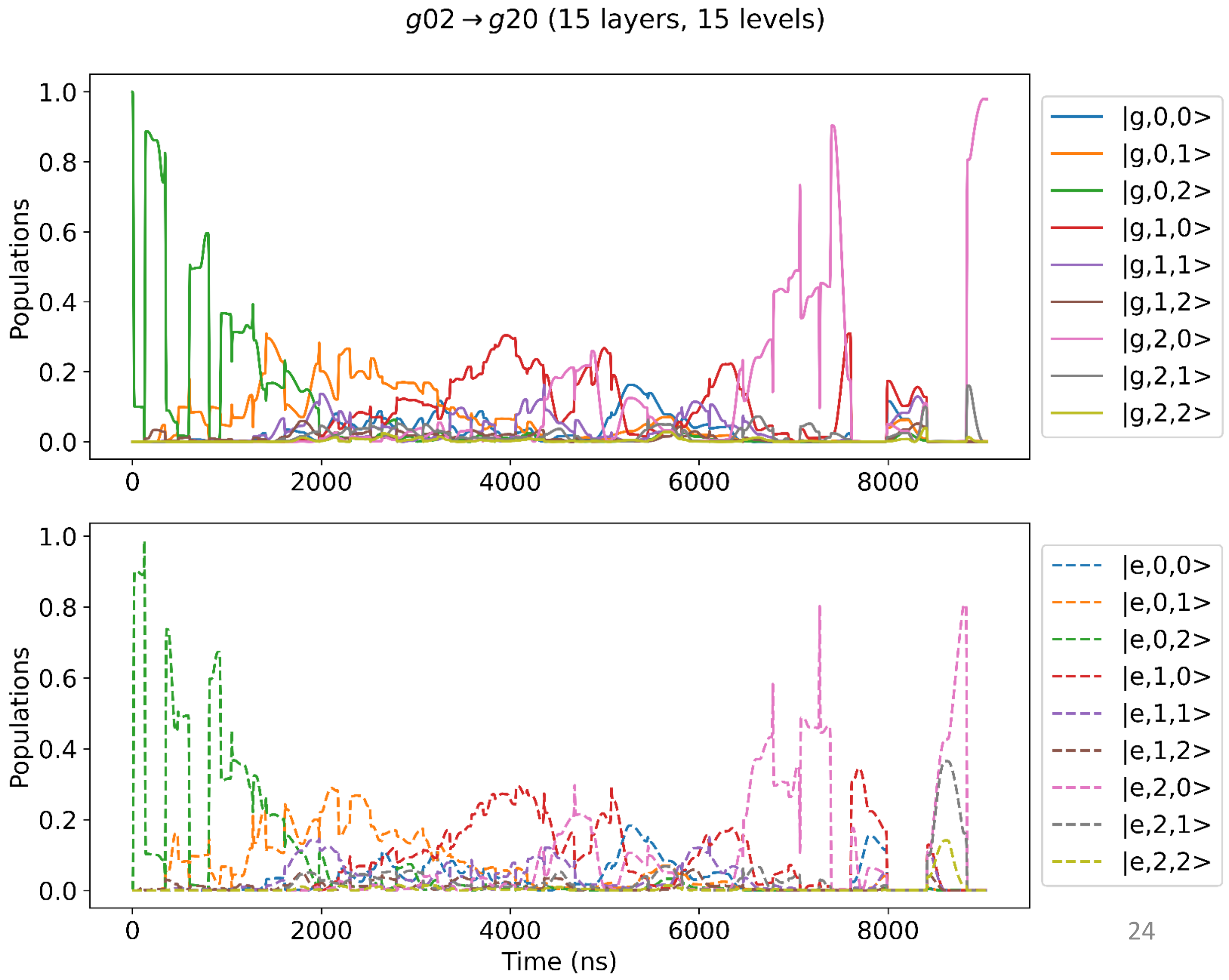
- **Qutip Simulation** of $g01 \rightarrow g10$



Two Mode ECD: Simultaneous State Transfer

- **Parameters Optimized** for both $g01 \rightarrow g10$ and $g02 \rightarrow g20$

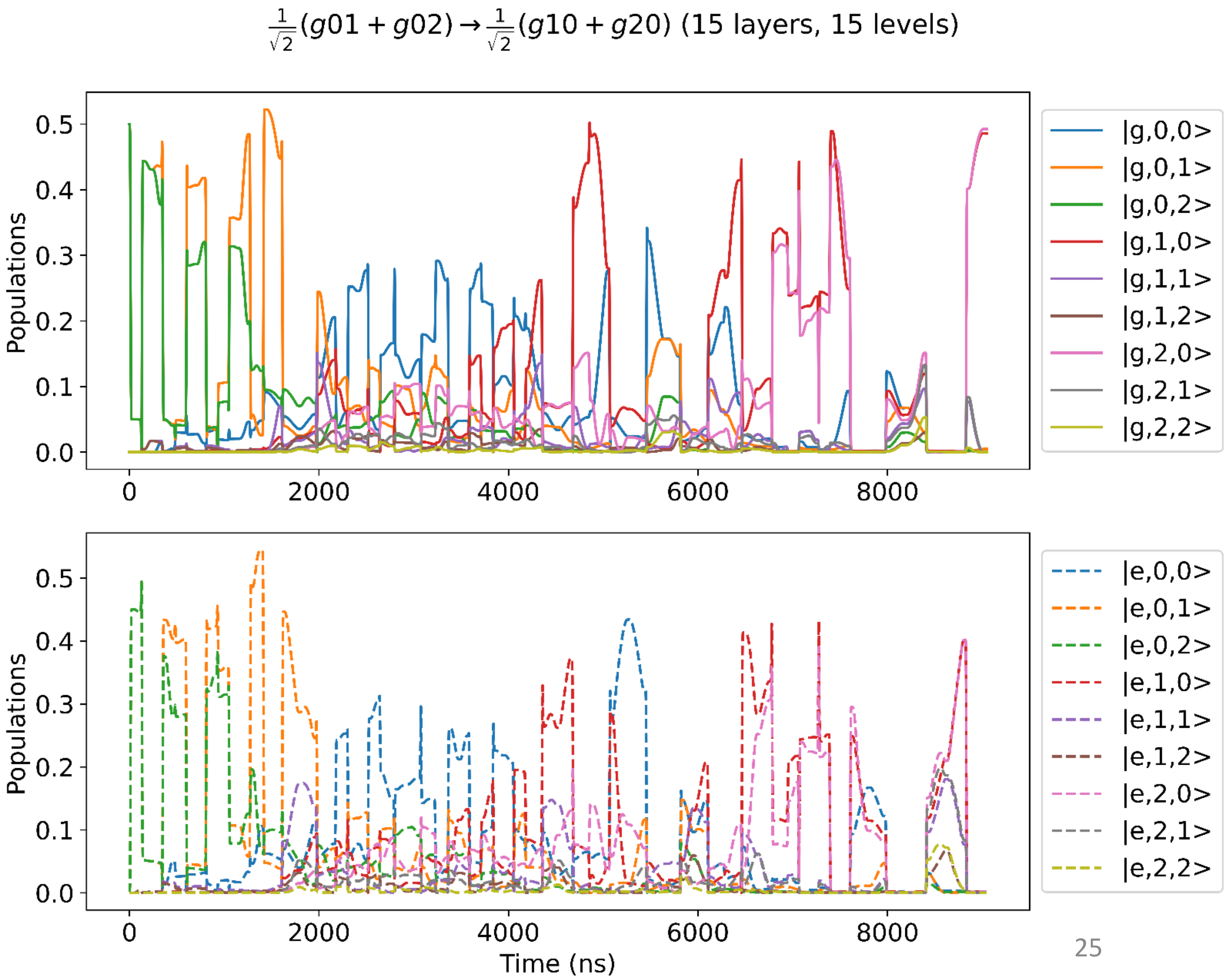
- **Qutip Simulation** of $g02 \rightarrow g20$



Two Mode ECD: Simultaneous State Transfer

- **Parameters Optimized** for both $g01 \rightarrow g10$ and $g02 \rightarrow g20$

- **Qutip Simulation** of $(g01 + g02)$ $\downarrow$ $(g10 + g20)$



Two Mode ECD : Unwanted Cross Kerr Terms

$$\chi_{ab} a^{\dagger} a\, b^{\dagger} b \xrightarrow{\text{Displaced Frame Transformation}} \chi_{ab}(a^{\dagger}+\alpha^{*})(a+\alpha)(b^{\dagger}+\beta^{*})(b+\beta)$$

Terms of form :

$\chi_{ab}\boldsymbol{\alpha\beta}\, a^{\dagger} b^{\dagger}$

$\chi_{ab}|\boldsymbol{\alpha}|^{2}\boldsymbol{\beta}\, b^{\dagger}$

$\chi_{ab}|\boldsymbol{\alpha}|^{2} b^{\dagger} b$

How to avoid :

α, β should not be simultaneously nonzero

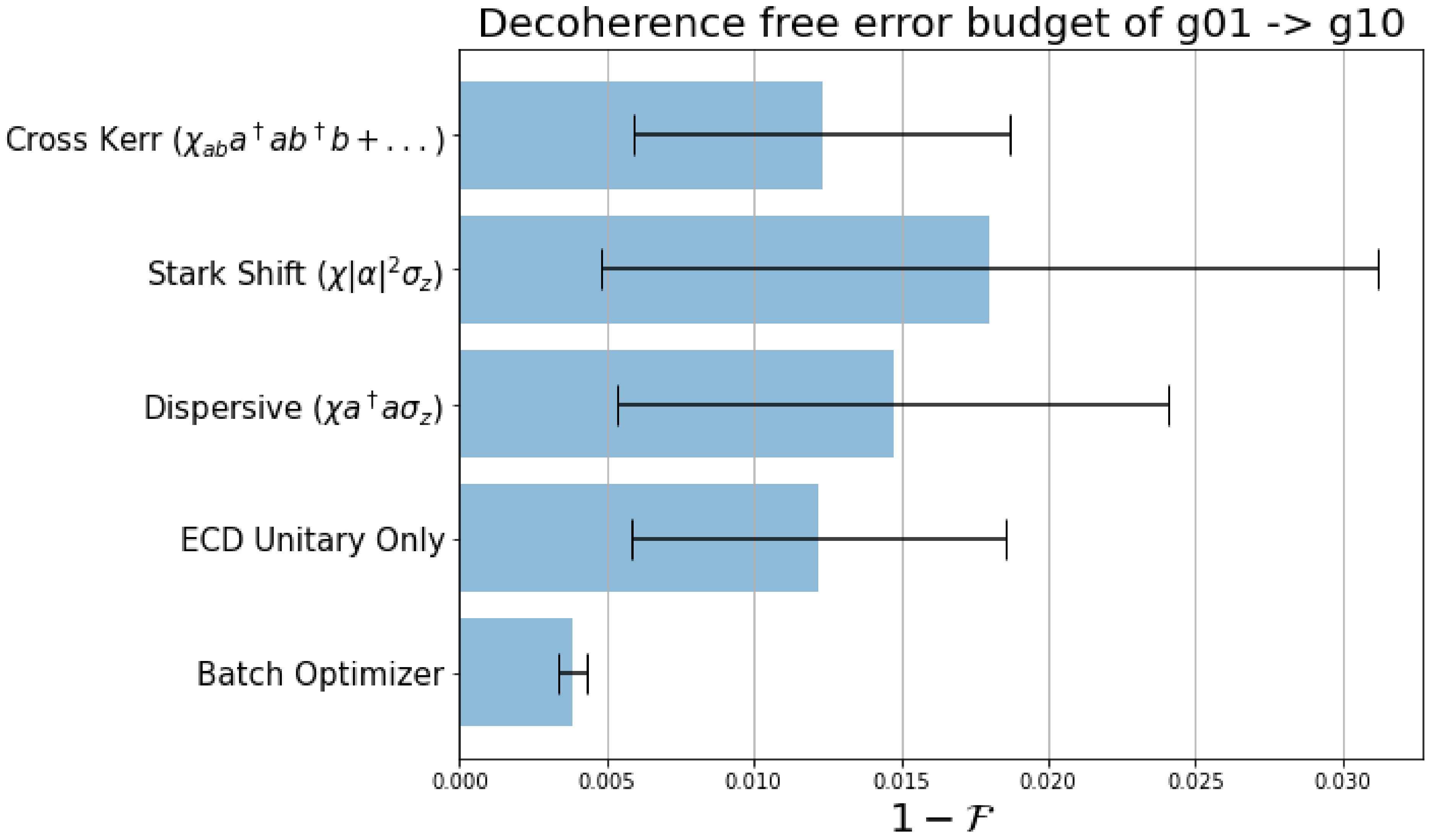
Echoed out when β flips

Make $\chi_{ab} \ll \chi_a, \chi_b \approx 10$ kHz

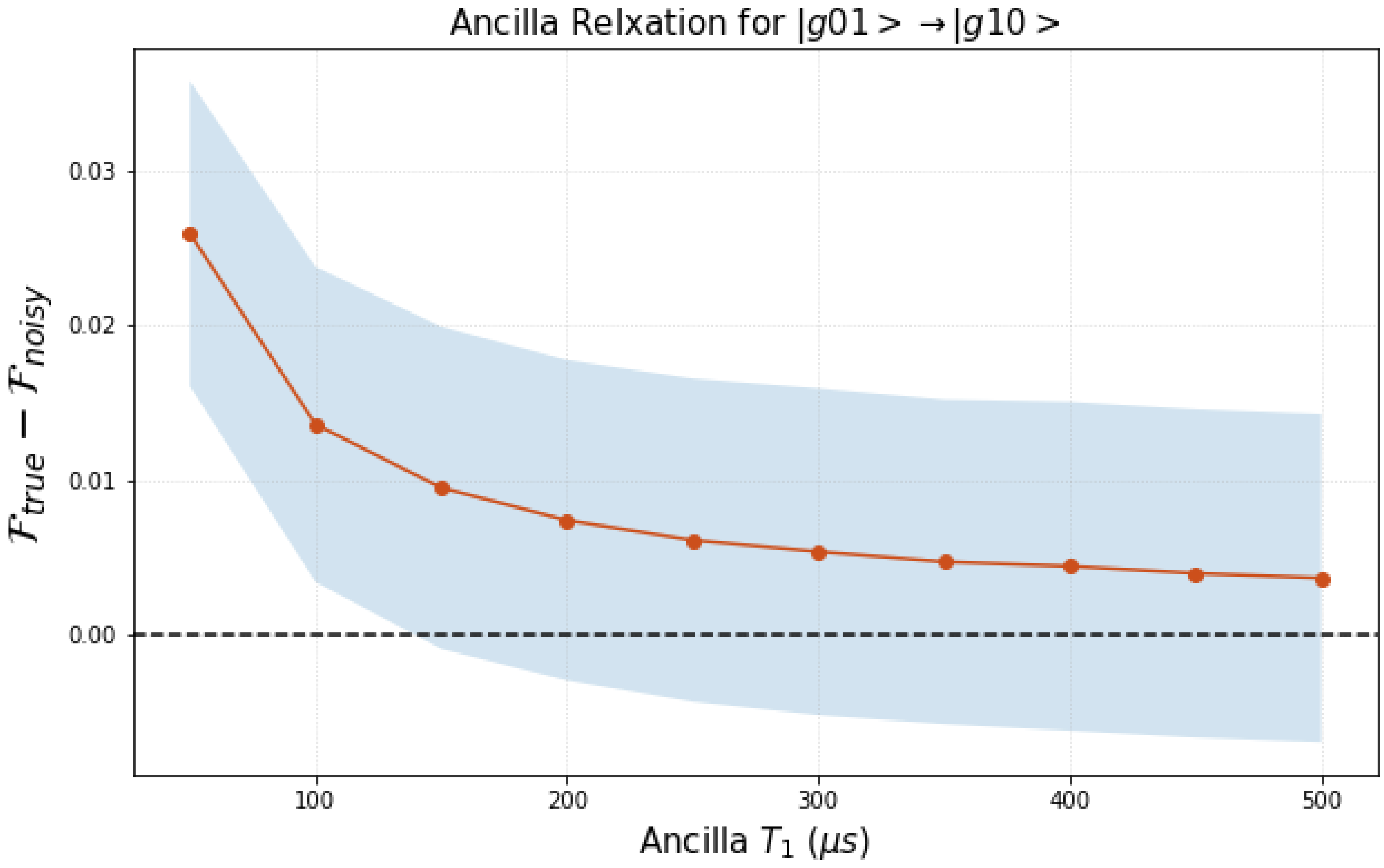
Note $\chi_{ab} = \sqrt{\kappa_a \kappa_b} = \frac{\chi_a \chi_b}{\alpha'} \approx 0.33$ Hz … good!

($\alpha' \leq 300$ MHz for transmons)

Two Mode ECD : QuTip Noise Simulations



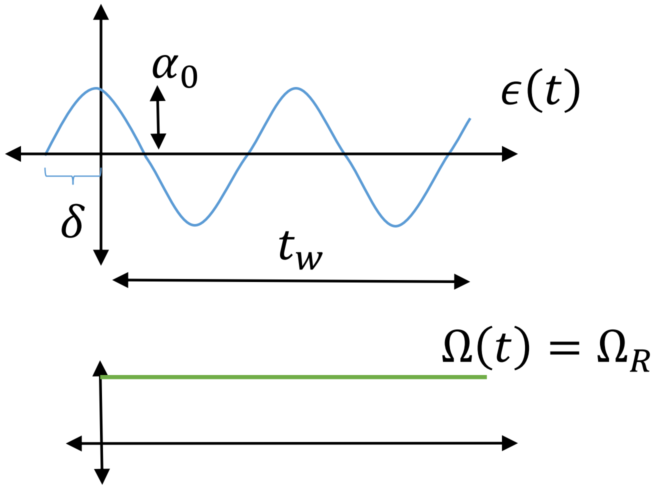
Two Mode ECD : QuTip Noise Simulations



Ancilla with better coherence times such as flux protected qubits may improve gate fidelities.

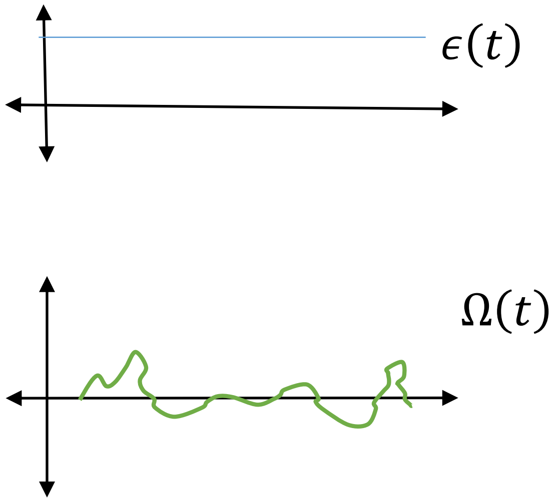
Circle Grape

Sideband Drives



- Changing $\epsilon(t)$
- Constant $\Omega(t)$

Circle Grape



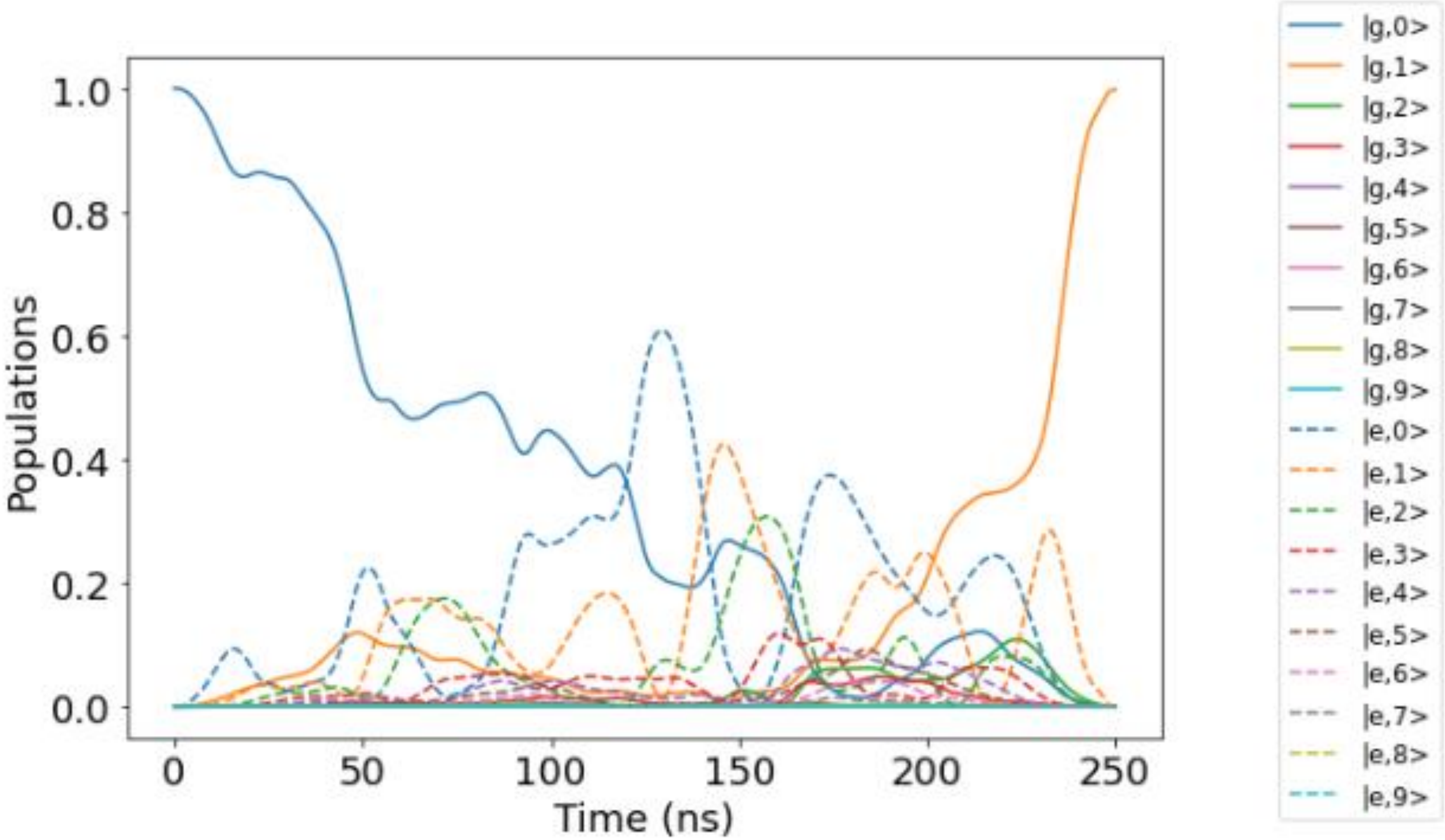
- Constat $\epsilon(t)$
- Changing $\Omega(t)$

$$H = \chi a^{+} a \sigma_z + \chi(\alpha_0 a^{+} + \alpha_0^{*} a)\sigma_z + \chi|\alpha_0|^2 \sigma_z + \Omega(t)\sigma_x$$

Sent to Optimizer

Similarly grap-ifying Sideband Drives?
Sending $\delta(t)$ to the optimizer

Circle Grape Results



Conclusion

- Conditional Displacement $\rightarrow$fast, high fidelity alternative to SNAP
- 3 ways to realize CD gates:
  1. Sideband Drives
  2. Echoed CD
  3. Circle GRAPE
- Challenges
  - Cavity Dephasing Error
  - Modeling Cavity Errors (Decay operators change in Dispersive Frame)
  - Optimizing Layer Count (ECD too slow compared to Circle GRAPE)

Extras

2 Comparison of ECD and Sideband Drive Method

1. Derivation of either method involves going into rotating frame of both qubit and cavity. And then finally a displaced frame transformation to obtain:

$$H = \underbrace{\chi a^\dagger a \sigma_z}_{\text{(i)}} + \underbrace{\chi(\alpha(t)a^\dagger + \alpha^*(t)a)\sigma_z}_{\text{(ii)}} + \underbrace{\chi|\alpha(t)|^2\sigma_z}_{\text{(iii)}} + \underbrace{\Delta_q\frac{\sigma_z}{2} + \Omega(t)\frac{\sigma_x}{2}}_{\text{Qubit Terms}} \tag{1}$$

   where $\alpha(t) \propto \cos(\omega t)$ for some frequency ω. In Siddiqi paper, $\omega = \Omega$ (Rabi frequency of qubit) which is direct consequence of sideband drives. In ECD, this oscillatory behavior is enforced by the pulse sequence of the cavity drive.

2. We can further simplify the Hamiltonian using the following trig identity

$$|\alpha(t)|^2 = \alpha_0^2\cos^2\omega t = \frac{\alpha_0^2}{2}(1+\cos(2\omega t)) \tag{2}$$

   [assuming α_0 to be real]. Then by choosing qubit detuning to be $\Delta_q = -\chi\alpha_0^2$, our Hamiltonian becomes:

$$H = \underbrace{\chi a^\dagger a \sigma_z}_{\text{(i)}} + \underbrace{\chi\alpha_0\cos(\omega t)(a^\dagger + a)\sigma_z}_{\text{(ii -osc)}} + + \underbrace{\frac{\chi}{2}\alpha_0^2\cos(2\omega t)\sigma_z}_{\text{(iii - osc)}} + \Omega(t)\frac{\sigma_x}{2} \tag{3}$$

3. The essential difference between the two approaches lies in how the qubit is used to isolate the generator of conditional displacement [term (ii)] in the hamiltonian above.

4. In the ECD scheme, the qubit drive flips σ_z exactly when sign of $\alpha(t)$ flips. Since this occurs at the halfway point, the unwanted terms (i) and (iii) are echoed out. The ECD hamiltonian is then

$$H_{ECD} = \chi\alpha(t)(a + a^\dagger)\sigma_z + \Omega(t)\frac{\sigma_x}{2} \tag{4}$$

where the latter term can be ignored since $\Omega(t) = 0$ for most of the time, as shown in Figure 4.

5. In Siddiqi's scheme, one realizes that the three terms (i), (ii -osc), (iii -osc) all rotate at different frequencies. Thus, we need to use the qubit to affect the rotation of the three terms in a way as to make only (ii-osc) stationary [and then invoke Rotating Wave Approximation to kill (i) and (iii -osc)].

6. Siddiqi does this by using first a Hadamard transformation $\sigma_z \leftrightarrow \sigma_x$ and then going into rotating frame of qubit drive (and thus removing qubit drive term) to obtain

$$H_{Sid} = \frac{\chi\alpha_0}{2}(a + a^\dagger)\sigma_\delta \tag{5}$$

where $\sigma_\delta = \cos(\delta)\sigma_x + \sin(\delta)\sigma_y$ and δ is phase difference between the 2 sideband drives. Note that terms (i) and (iii-osc) are rotating at frequency $\geq \Omega$, as a result of this transformation. Thus, invoking RWA, their effects are washed out.

7. To give ECD hamiltonian the σ_δ factor instead of just σ_z, we can go through the same transformations as above to obtain

$$H_{\tilde{ECD}} = \frac{\chi\alpha_0}{2}(a + a^\dagger)\sigma_{\tilde{\delta}} \tag{6}$$

where $\tilde{\delta} = \Omega(t) \times t$. For this evolution to occur, cavity drive should have the same form as in regular ECD. But the essential difference lies in qubit drive which should remain on (depending on required target$\tilde{\delta}$) and get a phase shift of π when $\alpha(t)$ changes sign at the middle of the pulse sequence. As shown in Figure 7, this phase shift will flip $\sigma_{\tilde{\delta}}$ in the same way as π pulse flips σ_z in regular ECD.

In skipping over steps, I have inevitably ignored a complication. Let's go more in detail here; after the Hadamard transformation, we obtain the following hamiltonian

$$H_{had} = \chi\alpha(t)(a + a^\dagger)\sigma_x + \Omega(t)\sigma_z \tag{7}$$

ignoring unwanted terms and factors of 2. When we then perform rotating frame transformation using $U = e^{-i\Omega(t)\sigma_z t}$, we obtain

$$
\begin{aligned}
\tilde{H_{had}} &= U H_{had} U^\dagger - i\dot{U}U^\dagger \\
&= \chi\alpha(t)(a + a^\dagger)\sigma_\delta + \Omega(t)\sigma_z - (\dot{\Omega}t + \Omega)\sigma_z \qquad (8) \\
&= \chi\alpha(t)(a + a^\dagger)\sigma_{\delta(t)} - \dot{\Omega}t\sigma_z
\end{aligned}
$$

Even if I make $\Omega(t)t = \delta$ constant, I will still not have isolated the conditional displacement term:

$$
\tilde{H_{had}} = \chi\alpha(t)(a + a^\dagger)\sigma_\delta - \Omega(t)\sigma_z \qquad (9)
$$

A possible rebuttal may be that $\chi\alpha(t) \gg \Omega(t) = \delta/t$. But this unwanted qubit term problem remains.

8. This change of σ_δ term in ECD hamiltonian offers us 1 **possible advantage**: no need for qubit rotations for universal control of cavity+qubit system. In regular ECD, for universal gate set, we needed extra qubit rotations (in addition to ECD gates). This may not be required if ECD hamiltonian has variable σ_δ term.

Proof: The generators for New-ECD(β, δ) are $\{q\sigma_x, q\sigma_y, p\sigma_x, p\sigma_y\}$. What needs to be shown is that, using this set of generators, we can reproduce the set of generators for ECD(β) and $R_\phi(\theta)$ and then the remaining proof will be same as that in Alec's paper.

Note $[q^2\sigma_x, p\sigma_x] \propto q$ and then $[p\sigma_x, q] \propto \sigma_x$. Similarly we can create σ_y and then combine the two pauli operators to produce σ_z. The final step is to recognize $[p\sigma_x, \sigma_y] \propto p\sigma_z$ and similarly we can produce $q\sigma_z$.

Note that the above set of generators $\{q\sigma_z, p\sigma_z, \sigma_x, \sigma_y\}$ is same as that for regular ECD(β) and $R_\phi(\theta)$ which Alec proved is universal for qubit and cavity.

This universality claim may be false. An unstated assumption in this proof lies in the first step of the second paragraph above, where I have ignored how to produce the term $q^2\sigma_x$. While its possible to create terms of the form $[q\sigma_x, q\sigma_y] \propto q^2\sigma_z$ and $[q\sigma_x, p\sigma_y] \propto \sigma_z$, it is unclear how to generate other pauli terms, σ_x and σ_y, using these commutation relations.

Another way to see this un-universality of New-ECD(β, δ) is that it only relies on 2 degrees of freedom– one for displacement and one for qubit rotations– while the original technique relies on 3 degrees of freedom – one for displacement and *two* for qubit rotations. Alternatively, for universal control of qubit, the $\sigma_\delta = \cos(\delta)\sigma_x + \sin(\delta)\sigma_y$ lacks σ_z. Thus New-ECD(β, δ) by itself may not be universal.

9. Is Siddiqi approach equivalent to ECD in the limit of small target displacements? While both approaches have similar effects on qubit population and $\alpha(t)$ in this limit, the pulse sequences for cavity and qubit drives are very different, as shown in Figure 9.

   ECD cavity drive consists of 1 sine wave followed by a flipped sine wave. This is created with an eye towards echoing out the familiar terms (i), (iii). Qubit drive is essentially non-existent except for the pi pulse at the midpoint.

   Siddiqi's cavity drive is simpler in that it is just an oscillating cosine wave (in rotating frame) with angular frequency Ω_R. And, his qubit drive is just a constant Rabi drive.