Master Equation

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1 Background

1.1 Closed Quantum System

A closed system is one in which we consider our system (let's say an electron) to be the only entity in the universe. There is no environment or other electrons with which our electron can interact. Then if the initial state of our electron is $|\psi(0)\rangle$, then the state of the electron $|\psi(t)\rangle$ at any other time t in the future is completely determined by $|\psi(0)\rangle$. In other words, no information about the quantum system "leaks out" during the time evolution. Such deterministic evolution of states (aka unitary time evolution) allows us to compute the probabilities of the outcomes of later experiments.

1.2 Open Quantum System

In open quantum systems, however, we do recognize that our electron is not the only entity in the universe. There are other particles and fields that our electron could interact with. These interactions may result in dissipation of energy and hence information.

In such quantum zoo, we cannot expect our quantum system to evolve in unitary fashion. That is, information will "leak out" and time evolution will be a little complicated. Let's say:

Electron : A

Environment : B

State of the electron: $\rho_A(t)$

State of the environment: $\rho_B(t)$

State of the combined system: $\rho_{AB}(t)$

Now, the hamiltonian of entire system is

$$H = H_A + H_B + H_{AB}$$

where H_A only acts on system A, H_B only acts on system B and H_{AB} is the the interaction hamiltonian.

Even though we are taking pains to define the state of the environment, we are only concerned with the dynamics of our electron or system A. With this is mind, our objectives in studying open quantum systems is twofold:

1. Extract the evolution of $\rho_A(t)$ from the quantum equation of motion of the combined system.

$$i\hbar \frac{d}{dt}\rho_{AB}(t) = [H_{AB}, \rho_{AB}(t)]$$

2. Do so in a way that guarantees the properties of $\rho_A(t)$ as a density operator (eg. positivity) are preserved in any moment of time.

2   Approximations

2.1   Born-Markov Approximation

It is worth talking about time scales when discussing correlations between the system and environment. Suppose that before time $t = 0$, there exist no correlations between the electron and the environment. Now at time $t = 0$ suppose the electron emits a photon into the environemnt. Now this photon diappears dramatically fast, and the environment returns quickly to its initial state. This example shows that the environment is quick to recover from any internal correlations due to interaction with the system. On the other hand, the system is relatively slow to recover from these "perturbations" or changes. If the system and environment exchange energy, then it will take system A a longer time to return back to equilibrium than the environment.

Let τ_B be the relaxation time for the environment and T_A be the relaxation time for the system. Now suppose we are taking snapshots of changes to the state of the system $\Delta\rho_A(t)$ every Δt units of time. Here we will assume that

$$\tau_B << \Delta t << T_A$$

What does this assumption mean? Suppose the time is $t = 0$ are correlations are turned on. By the time we take a snapshot, the environment has returned to its initial equilibrium state. However, the state of the system is still in the process of "recovering" from the correlations.