total Hamiltonian

Perturbation

Suppose $H(t) = H^{(0)} + \delta H(t)$

$H^{(0)}$: Time independent Hamiltonian

Now let's say you have some state $|\psi(t)\rangle$ in the schrodinger picture. That is,

$$|\psi(t)\rangle = U_H |\psi(0)\rangle$$

U_H: Unitary corresponding to total Hamiltonian

$|\psi(0)\rangle$: initial state

If we want to bring $|\psi(t)\rangle$ to interaction picture, then

$$|\psi(t)\rangle_I = U^{\dagger}_{H^{(0)}} |\psi(t)\rangle$$

$U^{\dagger}_{H^{(0)}}$: Unitary corresponding to $H^{(0)}$

This makes sense because if $\delta H(t) = 0$, then $H = H^{(0)}$. So,

$$|\psi(t)\rangle = U_H |\psi(0)\rangle = U_{H^{(0)}} |\psi(0)\rangle.$$

Then the state in interaction picture becomes constant

$$|\psi(t)\rangle_I = U^{\dagger}_{H^{(0)}} |\psi(t)\rangle$$

$$= U^{\dagger}_{H^{(0)}} U_{H^{(0)}} |\psi(0)\rangle$$

$$= |\psi(0)\rangle$$