Error Mitigation

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Contents

1 Introduction

Isolating physical quantum systems is hard. For this reason, most quantum systems are considered *open* in the sense that the system inevitably interacts with its external environment. To illustrate, consider a qubit represented by 2 discrete states of an electron. Being a charged particle, that electron is naturally bound to interact with other charged particles and surrounding electromagnetic fields. This means that the *environment* can affect the state of the electron as well as the qubit it represents. These uncontrollable effects are called *noise* and they can significantly affect our chemistry computations. To understand the nature of such quantum noise, we will first turn our attentention to classical noise.

2 Classical Noise

Imagine a bit inside a computer hard-drive with external magnetic fields. These magnetic fields have potential to change the state of the bit with a probability of let' say p. Let i_0, i_1 be initial probabilities of the bit being in state 0 and 1 respectively. Similarly, let j_1, j_2 be the new probabilities after the bit interacts with the magnetic field. Then, we can represent such *noisy* event as

$$\begin{bmatrix} j_0 \\ j_1 \end{bmatrix} = \begin{bmatrix} 1-p & p \\ p & 1-p \end{bmatrix} \begin{bmatrix} i_0 \\ i_1 \end{bmatrix}.$$

$$\begin{bmatrix} j_0 \\ j_1 \end{bmatrix} = \begin{bmatrix} (1-p)i_0 + pi_1 \\ pi_0 + (1-p)i_1 \end{bmatrix}$$

We can make sense of this equation by looking at the equality $j_0 = (1-p)i_0 + pi_1$. Here, j_0 denotes the final probability of the bit being in state 0. If the probability of initial state being 0 is i_0, then the probability of getting state 0 after interaction is $(1-p)i_0$. In this case, we multiplied the probability of no-bit-flip $1-p$ with i_0. On the other hand, if the probability of initial state being 1 is i_1, then the probability of getting state 0 after interaction is the product of bit-flip probability p and i_1. Thus, the probability of getting the final state of 0 i.e. j_0 is simply the sum of $(1-p)i_0$ and pi_1. Now, we can write the above equations more succintly as

$$\vec{j} = \hat{E}\vec{i}$$

where $\hat{E}$ is called the evolution matrix (or the *noise* matrix), and $\vec{i}, \vec{j}$ are the initial and final probability distributions respectively. Then, the final state of the system $\vec{j}$ is said to be "linearly" related to the initial state of the system $\vec{i}$.

Note that for the *noise* matrix to describe such a linear transformation, it has to abide by 2 rules:

- *Positivity*: All entries of E must be non-negative. If E has negative entries, then the vector $E\vec{q}$ will have negative components i.e. negative probabilities. That would be non-sensical.

- *Completeness*: The entries in each column of E must add up to 1. Suppose $\vec{i} = \begin{bmatrix} x \\ y \end{bmatrix}$ and $E = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$ Then

$$\vec{j} = E\vec{i} = \begin{bmatrix} ax + cy \\ bx + dy \end{bmatrix}$$

  For $\vec{i}, \vec{j}$ to be valid probability distribution, their components must add up to 1. Then we can assume that $x + y = 1$, and $ax + cy + bx + dy = 1$ or $(a + b)x + (c + d)y = 1$. For the latter equation to be true for any non-trivial inputs x, y such that $x + y = 1$, both coefficients $a + b$ and $c + d$ must be equal to 1. Since these coefficients represent the sum of entries of each column of E, we conclude that the sum of entries of each column of E must be equal to 1.

2.1 Markov Process

Earlier we only looked at one "noise event". Now suppose we have 2 *noisy* gates A and B. An important assumption that we can make here is whether gate A works correctly is independent of whether gate B works correctly. That is, the *noisiness* of gate A in independent of the *noisiness* of gate B. This assumption can be *physically reasonable* in cases such as the one where gate A and B are placed a significant distance apart from each other. Then the process of gate A and B being applied in any order is known as Markov process.

With each noise event being indpendent and being described by a linear transformation, then the final state after a multiple noise events is still linearly related to the initial state.

3 Quantum Operations

3.1 Operator Sum Representation

In order to describe "noise events", we must first develop a mathematical tools to describe operations on *open* quantum systems. To establish these tools, we would need some *reasonable* assumptions about the system and the environment.

1. The principal quantum system and environment will be contained a *closed* system. A concern that may arise here is that the environment has infinite degrees of freedom. However, if our quantum system has d degrees of freedom, then it is safe enough to assume that the environment has at most d^2 degrees of freedom.

2. The principal quantum system ρ and the environment ρ_{env} start out in a product state. This is generally not true as the quantum system and the environment constantly interact with each other. The resultant correlations may give rise to various entangled states. As we will find out

later, the mathematical formalism will work even if the system and the environment do not start out in a product state.

3.