(1) $v = 7.6$ km/s $\quad I_{sp} = 400$ sec

$g_0 = 9.8\ m/s^2$.

Now, $\Delta v = I_{sp} \cdot g_0 \cdot \ln\left(\frac{M_0}{m_f}\right)$

$$7.6 = 400 \cdot 9.8 \quad \ln\left(M_0/m_f\right)$$

$$\frac{76}{400 \times 98} = \ln\left(M_0/m_f\right)$$

$$\boxed{\frac{M_0}{m_f} = 1}$$

(2) $F_{thrust} = -c\frac{dm}{dt}$.

$$F_{net} = -c\frac{dm}{dt} - mg = m\,dv/dt$$

$$-c/m\,dm - g\,dt = dv$$

$$-c\ln\left|\frac{m_f}{m_0}\right| - gt = v(t) - u_0.$$

$$v(t) = u_0 + c\ln\left|\frac{m_0}{m_f}\right| - gt.$$

$$\boxed{v(t) = v_0 - c\ln|\mu| - gt.}$$

Now, let the burning rate of fuel is constant.

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$$K = -\frac{dm}{dt}$$

$$\int_0^{t_b} K\,dt = -\int_{m_0} dm \qquad +K t_b = -m' + m_0.$$

$$\boxed{K = \frac{m_0 - m'}{t_b}}$$

where t_b is the fuel burning time or the time in which it is placed in the orbit.

for $t < t_b$, $\int_0^t K\,dt = -\int_{m_0}^{m(t)} dm.$

$$\Rightarrow Kt = -m(t) + m_0.$$

$$(m_0 - m')\frac{t}{t_b} - m_0 = -m(t)$$

$$m(t) = m_0 - (m_0 - m')\,t/t_b.$$

now,
$$h = \int_0^t u\,dt$$

$$h = \int_0^t \left(-c \ln \frac{m(t)}{m_0} - g(t)\right) dt$$

$$h = -c\int_0^t \ln\left(\frac{m_0 - (m_0 - m')\,t/t_b}{m}\right) - g\int_0^t t\,dt$$

$$h = \Big[-ct \ln\left(m_0 - (m_0 - m')\,t/t_b\right) + ct + \frac{c m_0 t_b}{m_0 - m'} \ln\left(m_0 - (m_0 - m')\,t/t_b\right) - g\frac{t^2}{2}\Big]_0^t + ct \ln m_0.$$

$$h = -ct \ln\left(m_0 - (m_0 - m')\,t/t_b\right) + ct - \frac{gt^2}{2} + ct \ln m_0$$
$$+ \frac{c m_0 t_b}{m_0 - m'} \ln\left(m_0 - (m_0 - m')\,t/t_b\right) - \frac{c m_0 t_b}{m_0 - m'} \ln(m_0) + ct \ln m_0$$

$$h = -ct\ln|m(t)| + ct - \frac{gt^2}{2} + \frac{cm_0t_b}{m_0-m'}\ln|m(t)| - \frac{cm_0t_b}{m_0-m'}\ln|m_0|.$$

$$h = -ct\ln|m(t)| + ct - gt^2/_2 + \frac{cm_0t_b}{m_0-m'}\ln|m(t)| - \frac{cm_0t_b\ln|m_0|}{m_0-m'}.$$

$$h = -ct\ln|mt| + ct - gt^2/_2 + \frac{cm_0t_b}{m_0-m'}\ln\left|\frac{m(t)}{m_0}\right|.$$

$$\boxed{h = -ct\ln|u m_0| + ct - \frac{gt^2}{2} + \frac{cm_0t_b}{m_0-m'}\ln|u| + ct\ln m_0}$$

$$h = -ct\ln|u| + ct - \frac{gt^2}{2} + \frac{cm_0t_b}{m_0-m'}\ln|u|$$

$$h = -c\ln|u|\left[t - \frac{t_b}{m_0-m'}\right] + ct - gt^2/_2.$$

$$h = \frac{-ct_b}{m_0-m'}\ln|u|\left[m(t)\right] + ct - gt^2/_2$$

$$h = \frac{-ct_b}{m_0-m'}\ln|u|\left[m(t)\right] + ct - gt^2/_2$$

$$h = \frac{-ct_b\ln|u|}{(m_0-m')} \times u m_0 + ct - \frac{gt^2}{2}$$

$$\boxed{h = \frac{-c t_b \ln|\mu|}{(1 - M'/M_0)} \times \mu + ct - gt^2/2}$$ fuel expression

Q.2

~~Q7 $\Delta v = I_{sp} g_0 \ln$~~

Now, lets verify this for $t = t_b$.

where, $m(t) = M'$ i.e. $\mu = M'/M_0$.

$$h_b = \frac{-c t_b \ln\left|\frac{M'}{M_0}\right|}{1 - M'/M_0} \times \frac{M'}{M_0} + c t_b - gt^2/2$$

$$h_b = \frac{-c t_b \ln|M'/M_0|}{\frac{M_0 - M'}{M'}} + c t_b - gt^2/2$$

$$n = \frac{F_{thrust}}{M_0 g} \propto acc^n \text{ of the aircraft.}$$

So, therefore of n is high then, acc^n of the aircraft will be high and ~~viceversa.~~

for, $n>1$ the ~~furnt~~ will be more ~~eagiel~~. because left generated is very high.

$$\boxed{h_b = \frac{-c t_b \ln\left|\frac{M'}{M_0}\right|}{1 - M'/M_0} + c t_b - g t_b^2/2.}$$

which is the exact eq^n for burnout altitude that we had calculated on the precious assignment.

- total time of flight
  = time to reach orbit and time to return back to earth.

$\Rightarrow$

$$v(t) = v_0 - c \ln |u| - gt$$

for simplicity conclude at $t = 0 \Rightarrow v_0 = 0$.

$$v(t) = -c \ln \left| \frac{m(t)}{m_0} \right| - gt.$$