1) $\Delta v = I_{sp}\, g_0 \ln\left[1 + \frac{M_{prop}}{M_{pay} + M_{rocket}}\right]$.

$\Delta v = 7.6\,km/s$, $= 7600\ m/s$.
$I_{sp} = 400$ $\quad g_0 = 9.8\ m/s^2$.

$\ln\left[1 + \frac{M_{prop}}{M_{pay} + M_{rocket}}\right] = \frac{7600}{400 \times 9.8} = 1.9388$

$M_{prop} = \cancel{5.9504}\ 5.9504\,(M_{pay} + M_{rocket})$.

$\frac{M_{propellant}}{M_{initial}} = \frac{5.9504\,(M_{pay} + M_{rocket})}{\cancel{5.9504}\,(M_{pay} + M_{rocket} + M_{prop})}$

$= \frac{5.9504\,(M_{pay} + \cancel{M_{rocket}})}{6.9504\,(M_{pay} + \cancel{M_{rocket}})}$

$\boxed{\frac{M_{propellant}}{M_{initial}} = 0.85612}$

b) $\Delta v = I_{sp}\, g_0 \ln\left(\frac{M_i}{M_f}\right)$. $M_{pay} = 2000$ ~~kg~~ kg

$$e^{\Delta v / I_{sp} g_0} = \frac{M_i}{M_f}$$

$$\frac{M_i}{M_f} = \frac{M_{propellant} + M_{pay} + M_{rocket}}{M_{pay} + M_{rocket}}$$

$$M_{rocket} = \frac{2000\lambda}{(1-\lambda)}$$

$$e^{\Delta v / I_{sp} g_0} = \frac{M_i}{2000 + \frac{2000\lambda}{(1-\lambda)}}$$

$$e^{\Delta v / I_{sp} g_0} = \frac{M_i(1-\lambda)}{2000}$$

~~$\Delta v = 8$ km/s~~ $\Delta v = 8$ km/s $= 8000$ m/s.

$I_{sp} = [350, 400, 450)$

(a) $I_{sp} = 350$ sec

$$2000 \times e^{8000/350 \times 9.8} = M_i(1-\lambda)$$

$$M_i(1-\lambda) = 2.069 \times 10^4.$$

(b) $I_{sp} = 400$ sec.

$$M_i(1-\lambda) = 2000 \times e^{8000/400 \times 9.8}$$

$$= 1.5393 \times 10^4$$

(c) $I_{sp} = 450$

$$M_i(1-\lambda) = 2000\,\lambda\, e^{8000/450 \times 9.8} = 1.2270 \times 10^4.$$

$\therefore$ The graph is a hyperbola because as we increase the value of I_{sp}, the slope of the curve $\downarrow$ for the same M_i.

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~~It is so because~~

2. (i) $F_{thrust} = -c\frac{dm}{dt}$

$$-c\frac{dm}{dt} - mg = m\frac{dv}{dt}$$

$$-c\int\frac{dm}{m} - g\int dt = \int dv$$

Given

$$\mu = \frac{m(t)}{m_0}$$

$$-c(\ln m(t) - \ln m_0) - g(t-0) = v(t) - v_0$$

$$v(t) = v_0 - gt - c\ln\left(\frac{m(t)}{m_0}\right)$$

$$\boxed{v(t) = v_0 - gt - c\ln(\mu)}$$

(iii) total time of flight of the rocket

Let, $v_e \rightarrow$ orbital velocity of rocket.

$v_0 \rightarrow$ initial velocity.

$$v_e = v_0 - gt - c\ln(\mu)$$

$$\boxed{\frac{v_e - v_0 - c\ln(\mu)}{g} = t}$$

(iv) $n = \frac{F_{thrust}}{m_0 g}$ $\propto$ acceleration of rocket.

So, for $n > 1$, it is much easier to fly because thrust generated will be very high.

(iii) If the burnout rate is constant.

$$-\frac{dm}{dt} = p \Rightarrow -\int_{m_0}^{m'} dm = p\int_{t=0}^{t_0} dt$$

$$m' - m_0 = pt_0$$

$$\frac{m' - m_0}{t_0} = p.$$

$t_0 \rightarrow$ is the time to reach orbit.

For $t < t_0$, $m(t)$.

$$\int_0^t p\,dt = -\int_{m_0} dm$$

$$pt = -m(t) + m_0.$$

$$m_0 - \frac{m' - m_0}{t_0}t = m(t).$$

So, $z(t) = \int_0^t v\,dt$

$$= +\int_0^t \left(-c\ln\left(\frac{m(t)}{m_0}\right) - gt\right) dt$$

$$= -c\int_0^t \ln\left(\frac{m_0 - \frac{(m' - m_0)t}{t_0}}{m_0}\right) - g\int_0^t t\,dt.$$

Solving this eq^n we get the eq^n of z as a function of t, m.

Also, $t = t_0$, gives the burnout altitude.