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Q.2) (a) Let the initial mass be M.

and initial velocity be ~~Δv~~ V

After time t

Final mass $= -\dot{m}t + M$.

Final velocity $u = v + \Delta v$.

Conserving momentum,

$$Mv = \cancel{M(v+\ldots} \; (M - \dot{m}t)(v + \Delta v) - (\dot{m}t)(v - v_{exhaust})$$

$$\cancel{Mv} = \cancel{Mv} + M\Delta v - \dot{m}tv - \dot{m}t\Delta v - \dot{m}tv + \dot{m}tv_{ex}.$$

$$2\dot{m}tv - \dot{m}t\, v_{exhaust} = (M - \dot{m}t)\,\Delta v.$$

At initial $v = v_{exhaust}$.

$$\frac{\dot{m}t\, v_{exhaust}}{(M - \dot{m}t)} = \Delta v.$$

So, u (speed) $= v + \Delta v$.

$$= v_{ex} + \frac{\dot{m}t\, v_{ex}}{(M - \dot{m}t)}$$

$$\boxed{u = \frac{M v_{ex}}{(M - \dot{m}t)}}$$

(b) $u = \frac{dH}{dt}$ $\quad H =$ height

$$\frac{dH}{dt} = \frac{M v_{ex}}{(M - \dot{m}t)}$$

$$\int_0^H dH = M v_{ex} \int_0^t \frac{dt}{(M - \dot{m}t)}.$$

$$\cancel{H = M v_{ex} \ln(M - \dot{m}t)}$$

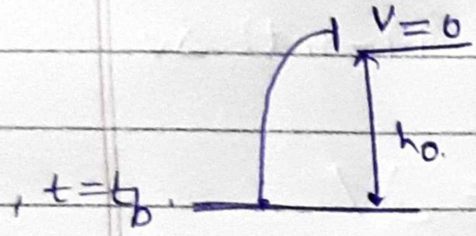
$$H = -\frac{M v_{ex}}{\dot{m}} \ln\left(1 - \frac{\dot{m}}{M} t\right).$$

$$H = \frac{M v_{ex}}{\dot{m}} \ln\left(\frac{M}{M - \dot{m} t}\right).$$

height at t_b = burnout time,

$$\boxed{H_b = \frac{M v_{ex}}{\dot{m}} \ln\left(\frac{M}{M - \dot{m} t_b}\right)}$$

For max. height, only g_0 is considered :-
so eqns of motion can be applied.



$$\overset{0}{\cancel{v^2}} - u^2 = 2 g_0 S$$

$$+u^2 = +2 g_0 h_0$$

$$\frac{1}{2 g_0}\left(\frac{M v_{ex}}{(M - \dot{m} t_b)}\right)^2 = h_0$$

$$\boxed{\text{Max. height} = H_b + h_0 = \frac{M v_{ex}}{m} \ln\left(\frac{M}{M - \dot{m} t_b}\right) + \frac{1}{2 g_0}\left(\frac{M v_{ex}}{(M - \dot{m} t_b)}\right)^2}$$