2. (a) From the rocket eq^n, we get

$$u = u_e \ln\left(\frac{M/\dot{m}}{M/\dot{m} - t}\right) - g_0 t$$

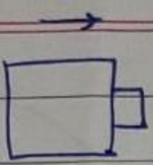
Now ~~$I_{sp} = F_{thrust}$~~

note that $\dot{m} t_b = M$

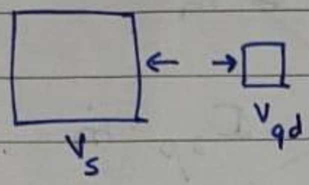
where M is total mass.

$$\therefore \boxed{u = u_e \ln\left(\frac{t_b}{t_b - t}\right) - g_0 t}$$

Q 3.



By cons. of momentum,



(assuming separation by explosive bolts)

$$m\vec{v} = \frac{4m}{5}\cdot\vec{v_s} + \frac{m}{5}\cdot\vec{V_{qd}}$$

We assume $|v-v_s|$ and $|v-v_{qd}|$ to be very small.
Now in the frame of ref. of the remaining ship,

$$v_{debris} = \frac{v}{2}.$$

$$\therefore \quad s_{debris} = \frac{v}{2}(t-t_0), \quad t > t_0.$$

~~v_{qd} =~~ And by the rocket eq^n (for free space)

$$\frac{v_{qd}}{u_e} = -\ln\left(\frac{m_f}{m_0}\right)$$

$$\therefore \quad ① \quad v_{qd} = -u_e \ln\left(\frac{m_f}{m_0}\right).$$

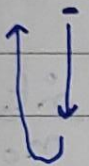
$$\cancel{② \quad I_{sp} = \frac{F_{thrust}}{\dot{m}} = \frac{u_e\,\dot{m}}{\dot{m}} = u_e.}$$

Equating s_{qd} with s_{debris} we get

$$② \quad m_f = m_0 - \dot{m} t .$$

Now consider the frame ~~of debr~~ of reference fixed to the debris.

- quadrant (at time $t = t_0$)



- debris

We want velocity of quadrant w.r.t. to debris to be zero when the quadrant almost touches it.

Let v'_{qd} be the vel. of qd. in that ref. frame. We get

$$v'_{qd} = V_{qd} - \frac{v}{2}.$$ In the end, $v'_{qd} = 0$, so $\Delta v = -V_{qd} + \frac{v}{2}$.

$$\therefore \Delta v'_{qd} \cancel{\therefore\ v'_{qd} + \frac{v}{2}} = -u_e \ln\left(1 - \frac{\dot{m}t}{m'_0}\right)$$

$$\therefore -V_{qd} + \frac{v}{2} = -u_e \ln\left(1 - \frac{\dot{m}t}{m'_0}\right) \text{ --- } ⑨$$

~~Now since~~ $\cancel{v = \frac{ds}{dt}}$,

It suffices to find t. It is the time required for the quadrant to decelarate to zero velocity w.r.t. debris.

We will need to know its initial velocity i.e. at 't_0'.

$$V_{qd} = -u_e \ln\left(\frac{m_0 - \dot{m}t_0}{m_0}\right)$$

$$= -u_e \ln(m_0 - \dot{m}t_0) + u_e \ln(m_0)$$

~~V_{qd}~~ Now

$$\Delta v = \frac{v}{2} - V_{qd}$$

$$= \frac{v}{2} + u_e \ln(m_0 - \dot{m} t_0) - u_e \ln(m_0)$$

$m_0' = m_0 - \dot{m} t_0$, So

$$\frac{v}{2} + u_e \ln(m_0 - \dot{m} t_0) - u_e \ln(m_0)$$

$$= -u_e \ln\left(1 - \frac{\dot{m} t}{m_0 - \dot{m} t_0}\right)$$

$$\therefore \quad 1 - \frac{\dot{m} t}{m_0 - \dot{m} t_0} = \frac{e^{-\frac{v}{2u_e}}}{m_0 - \dot{m} t_0} \cdot m_0$$

$$\therefore \quad m_0 - \dot{m}(t + t_0) = m_0 e^{-\frac{v}{2u_e}}$$

$$\therefore \quad \boxed{\dot{m} = \frac{m}{5t_0}\left(1 - e^{-\frac{v}{2u_e}}\right)}$$