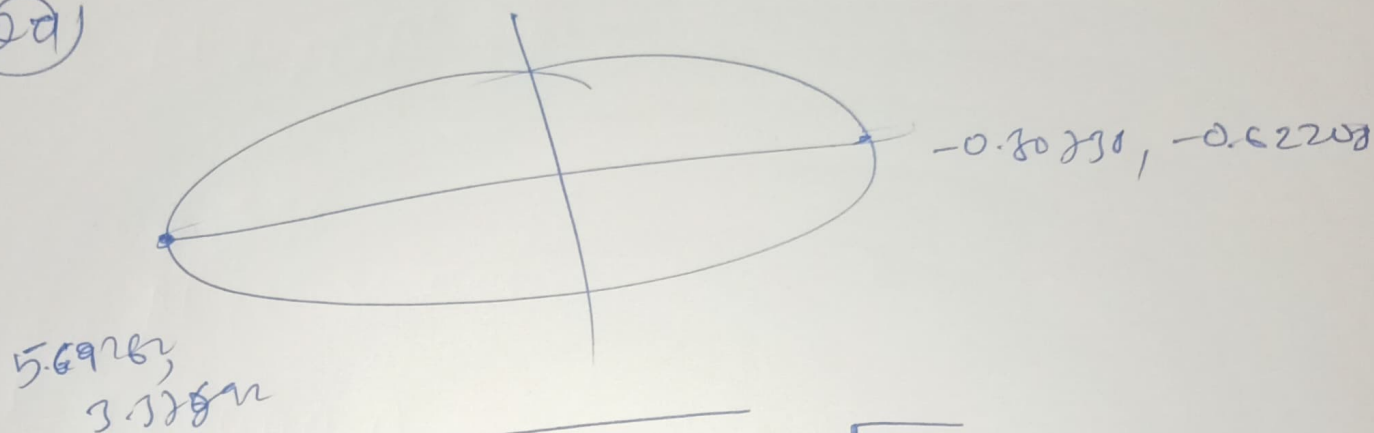


Chart Title



Q9)



Length of semi major axis $2a = \sqrt{6^2 + 4^2} = \sqrt{52}$

$a = \frac{\sqrt{52}}{2} = \sqrt{13}$

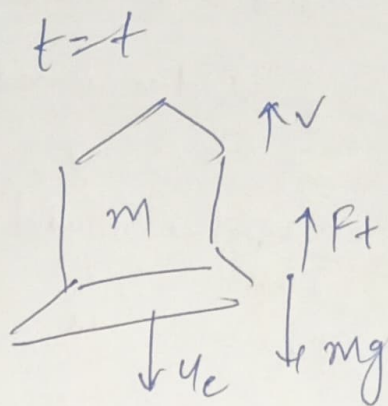
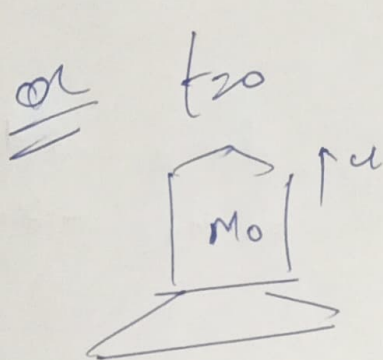
centre $x = \frac{5.69262 - 0.30738}{2} = 2.69262$

centre $y = \frac{3.37892 - 0.62208}{2} = 1.37892$

we got centre as

$$\frac{(x - x_1)^2}{a^2} + \frac{(y - y_1)^2}{b^2} = 1$$

but x_1, y_1 and other points we can get eqn of trajectory



$$F_{thrust} = -u_e \frac{dm}{dt}$$

weight = mg

$$F_{net} = F_{thrust} - mg$$

$$m \frac{dv}{dt} = -u_e \frac{dm}{dt} - mg$$

$$\frac{dv}{dt} = -\frac{u_e}{m} \frac{dm}{dt} - g \quad \text{--- (1)}$$

$$\int_u^v dv = \int_{m_0}^m -\frac{u_e}{m} dm - \int_0^t g dt$$

$$v - u = -\frac{u_e}{m} \ln \frac{m}{m_0} - gt$$

$$v - u = \frac{u_e}{m} \ln \frac{m_0}{m} - gt$$

$$\Delta u = \frac{u_e}{m} \ln \frac{m_0}{m} - gt$$

Now $\frac{dm}{dt} = \text{constant} = -k$

$$dm = -k dt \quad \text{or} \quad dt = -\frac{dm}{k}$$

By eqⁿ (1)

$$m \frac{dv}{dt} = -u_e \frac{dm}{dt} - mg$$

$$m dv = -u_e dm - mg dt$$

$$-mg dt = u_e dm + m dv$$

$$-mg \left(\frac{-dm}{k} \right) = u_e dm + m dv$$

$$\int_{m_0}^m \frac{mg}{k} dm = \int_{m_0}^m \frac{u_e dm}{m} + \int_u^v m dv$$

$$dm = -k dt$$

$$m - m_0 = -kt$$

$$\frac{g}{k} (m - m_0) = u_e \ln \frac{m}{m_0} + u - u$$

$$v(t) = u - u_e \ln \frac{m}{m_0} + \frac{g}{k} (m - m_0)$$

$$u(t) = u - u_e \ln \frac{m}{m_0} - gt$$

consider $u=0$ a special case for easy calculation

$$u(t) = -u_e \ln \frac{m}{m_0} - gt$$

$$dr = \frac{u(t)}{t_b} dt$$

$$y = -\int_0^{t_b} gt dt - \int_0^{t_b} u_e \ln \frac{m}{m_0} dt$$

$$y = -g t_b^2 - u_e \ln \frac{m}{m_0} t_b \quad \text{--- (2)}$$

burnout altitude

Now after burnout altitude thrust = 0

$$a = -g$$

$$\frac{dv}{dt} = -g$$

$$\int_{u(t)}^0 v dv = \int_0^h -g dh$$

$$0 - \frac{v^2(t)}{2} = -gh$$

$$h = \frac{v^2(t)}{2g}$$

$$-mg dt = u_e dm + m dv$$

$$h = \frac{(-gt_b - u_e \ln \frac{m}{m_0})^2}{2g} \quad \text{--- (3)}$$

hence max altitude

$$\Rightarrow \boxed{y + h}$$

$$-k dt$$

$$-k t$$

