Q1 a Shifting the center to $(0.69262, 0.37792)$
now put values of (x, y) in
$$x^2 + ay^2 + bxy + cx + dy + e = 0$$
we get 6 eqn which have the following matrix form,

$$\begin{pmatrix} 1 & 1 & -1 & -1 & 1 \\ 9 & 9 & 3 & 3 & 1 \\ 0 & 0 & 4 & 0 & 1 \\ 1 & -3 & 3 & -1 & 1 \\ 9 & 15 & 5 & 3 & 1 \\ 4 & 0 & 0 & 2 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \\ e \end{pmatrix} = \begin{pmatrix} -1 \\ -9 \\ -16 \\ -9 \\ -25 \\ 0 \end{pmatrix}$$

This can be represented as,
$$AX = B$$
The least sq. soln to this is,
$$C \Rightarrow (A^T A)^{-1} A^T B$$

on solving,

$$C \Rightarrow \begin{pmatrix} 1.733 \\ -1.224 \\ -2.934 \\ -0.404 \\ -5.195 \end{pmatrix}$$

so eq^n of the ellipse is,

$$(X-0.69262)^2 + 1.733(y-0.37792)^2 - 1.224(X-0.69262).(y-0.37792)$$
$$-2.934(X-0.69262)$$
$$-0.404(y-0.37792) = 5.195$$

$y \Rightarrow mx$ will be the major axis of this ellipse,
for finding center,

$$\frac{\partial f}{\partial X} = 0, \quad \frac{\partial f}{\partial y} = 0$$

$h \Rightarrow 0.69262$
$k \Rightarrow 0.37792$

$$2(X-h) - 1.224(y-k) - 2.934 = 0$$
$$3.466(y-k) - 1.224(X-h) - 0.404 = 0$$

solving this we get
center of ellipes as $\rightarrow (2.655, 1.18792)$

$$m \Rightarrow \frac{1.18792}{2.655} \Rightarrow 0.447$$

$\boxed{y \Rightarrow 0.447X}$ is the major axis of ellipse intersecting the ellipse at $(-0.555, -0.248)$ and $(5.866, 2.622)$

end pts of major axis

$$2a \Rightarrow \sqrt{6.421^2 + 2.87^2} \Rightarrow \boxed{a \Rightarrow 3.527}$$

(0,0) is focus of ellipse so,

$$ae \Rightarrow \sqrt{2.655^2 + 1.1879^2} \Rightarrow 2.909$$

$$e \Rightarrow \frac{2.909}{a} \Rightarrow \frac{2.909}{3.527} \Rightarrow 0.82$$

$$\boxed{e \Rightarrow 0.82}$$

(2) $F_{thrust} = I_{sp}\dot{m}g_0$

$F_{net} = I_{sp}\dot{m}g_0 - mg_0$

$a = \frac{I_{sp}\dot{m}g_0}{m} - g_0$

$\frac{dv}{dt} = \frac{I_{sp}\dot{m}g_0}{m} - g_0$ taking M_0 as the total mass of the rocket

$m = M_0 - \dot{m}t$

$\frac{dv}{dt} = \frac{I_{sp}\dot{m}g_0}{M_0 - \dot{m}t} - g_0$

$\int_0^v dv = \int_0^t \frac{I_{sp}\dot{m}g_0\,dt}{M_0 - \dot{m}t} - \int_0^t g_0\,dt$

$= \frac{-1}{\cancel{\dot{m}}} \ln(M_0 - \dot{m}t)\Big|_0^t I_{sp}\cancel{\dot{m}}g_0 - g_0 t$

$v = +I_{sp}g_0 \ln\left(\frac{M_0}{M_0 - \dot{m}t}\right) - g_0 t$

$$\boxed{v = I_{sp}g_0 \ln\left(\frac{M_0}{M_0 - \dot{m}t}\right) - g_0 t}$$

$v = \frac{dh}{dt} = I_{sp}g_0 \ln\left(\frac{M_0}{M_0 - \dot{m}t}\right) - g_0 t$

$\int_0^h h = \int_0^{t_b} I_{sp}g_0 \ln\left(\frac{M_0}{M_0 - \dot{m}t}\right) dt - \frac{1}{2}g t_b^2$

$h_b = I_{sp}g_0\left[\ln M_0 \left(t_b - \frac{M_0}{\dot{m}}\right) + \left(\frac{M_0}{\dot{m}} - t_b\right)\ln\left(M_0 - \dot{m}t_b\right) + t_b\right]$

$-\frac{1}{2}g t_b^2$ ⟶ Altitude of rocket at burnout time t_b

at t_b,

$v = I_{sp}g_0 \ln\left(\frac{M_0}{M_0 - \dot{m}t_b}\right) - g_0 t_b$

now after t_b the rocket moves under the force of gravity only so max height attained after burnout time t_b,

$$\Rightarrow \frac{v^2}{2g_0} \Rightarrow \frac{1}{2g_0}\left[I_{sp}\, g_0 \ln\left(\frac{M_0}{M_0 - \dot{m} t_b}\right) - g_0 t_b\right]^2$$

$$\Rightarrow \frac{g_0}{2}\left[I_{sp} \ln\left(\frac{M_0}{M_0 - \dot{m} t_b}\right) - t_b\right]^2$$

so max altitude attained by rocket,

$$h_b + \frac{g_0}{2}\left[I_{sp} \ln\left(\frac{M_0}{M_0 - \dot{m} t_b}\right) - t_b\right]^2$$

Altitude at burnout time (t_b)