1) $\Delta v = I_{sp}\, g_0 \ln\left(\frac{M_i}{M_f}\right)$

$= -I_{sp}\, g_0 \ln\left(\frac{M_f}{M_i}\right)$

$\Rightarrow 7.6\times10^3 = -I_{sp}\, g_0 \ln\left(1-\frac{M_{propellent}}{M_{initial}}\right)$

$\Rightarrow 7.6\times10^3 = -400\times9.8 \ln\left(1-\frac{M_{propellent}}{M_{initial}}\right)$

$\Rightarrow \frac{-19}{9.8} = \ln\left(1-\frac{M_{propellent}}{M_{initial}}\right)$

$\Rightarrow 0.144 = 1-\frac{M_{propellent}}{M_{initial}}$

$\Rightarrow \frac{M_{propellent}}{M_{initial}} = 0.856$

4) $\Delta v = I_{sp}\, g_0 \ln\left(\frac{M_i}{M_f}\right)$

$\Rightarrow e^{\frac{\Delta v}{I_{sp} g_0}} = \left(\frac{M_i}{M_f}\right)$

$\Rightarrow e^{\frac{\Delta v}{I_{sp} g_0}} = \frac{M_{propellent}+M_{pay}+M_{rocket}}{M_{pay}+M_{rocket}}$

$\Rightarrow e^{\frac{\Delta v}{I_{sp} g_0}} = \frac{M_i}{2000+\frac{2000d}{1-d}}$

$\Rightarrow e^{\frac{\Delta v}{I_{sp} g_0}} = \frac{M_i(1-d)}{2000}$

$\Rightarrow 2000\, e^{\frac{\Delta v}{I_{sp} g_0}} = M_i(1-d)$

$\Rightarrow \frac{-2000\, e^{\frac{\Delta v}{I_{sp} g_0}}}{(1-d)^2}\frac{dd}{dm_i} = 1$

$\Rightarrow \frac{dd}{dm_i} = \frac{-(1-d)^2 e^{-\frac{\Delta v}{I_{sp} g_0}}}{2000}$

1) $I_{sp} \rightarrow 350$

$e^{\frac{8000}{350\times9.8}} = \frac{M_i(1-d)}{2000}$

$\Rightarrow 2.06\times10^3 = M_i(1-d)$

2) $I_{sp} \rightarrow 400$

$e^{\frac{8000}{400\times9.8}}\times 2000 = M_i(1-d)$

$\Rightarrow 15393.78 = M_i(1-d)$

③ $e^{\frac{8000}{450 \times 9.8}} \times 2000 = M_i(1-d)$

$\Rightarrow 12270.60 = M_i(1-d)$

~~As we increase Isp, the slope of the graph decreases for different same initial mass and slope of the graph between initial mass~~

The graph obtained is rectangular hyperbola and as we increase Isp, the slope of the graph decreases for same initial mass.

2) a) $F_{total} = F_{thrust} - mg = -C\frac{dm}{dt} - mg = m\frac{dv}{dt}$

$\Rightarrow m\,dv = -C\,dm - mg\,dt$

$\Rightarrow dv = -\frac{C}{m}dm - g\,dt$

$\Rightarrow v(t) - v_0 = -C\ln\left|\frac{m(t)}{M_i}\right| - gt$

$\Rightarrow \boxed{v(t) = C\ln|\mu| - gt}$ $\left[\begin{array}{c}\text{Taking} \\ v_0 \to 0\end{array}\right]$

b) Let $p = -\frac{dm}{dt} = \text{const.}$

$\Rightarrow \int_0^{t_b} p\,dt = -\int_{m_0}^{m'} dm$ $\left[\begin{array}{c}\text{Take} \\ m' = \text{final mass}\end{array}\right]$

$\Rightarrow p\,t_b = m_0 - m'$

$\Rightarrow p = \frac{m_0 - m'}{t_b}$ t_b is the time when rocket is placed in the orbit.

$\Rightarrow \int_0^t p\,dt = -\int_{m_0}^{m(t)} dm$

$\Rightarrow pt = m_0 - m(t)$

$\Rightarrow m(t) = m_0 - \frac{(m_0 - m')t}{t_b}$

$$\cancel{h} = \int_0^{t} v\, dt \qquad z = \int_0^{t} v\, dt$$

$$\Rightarrow z = \int_0^{t} \left(-c \ln \frac{m(t)}{m_0} - gt\right) dt$$

$$= -c \int_0^{t} 1 \cdot \ln\left(\frac{m_0 - (m_0 - m')\frac{t}{t_b}}{m_0}\right) - \frac{gt^2}{2}$$

$$= -ct \ln|m(t)| + ct - \frac{gt^2}{2} + \frac{c m_0 t_b}{m_0 - m'} \ln|m(t)| - \frac{c m_0 t_b \ln|m_0|}{m_0 - m'}$$

$$\Rightarrow h = -ct \ln|\mu m_0| + ct - \frac{gt^2}{2} + \frac{c m_0 t_b}{m_0 - m'} \ln|\mu| + ct \ln m_0$$

$$\Rightarrow h = -ct \ln|\mu| + ct - \frac{gt^2}{2} + \frac{c m_0 t_b}{m_0 - m'} \ln|\mu|$$

$$\Rightarrow h = -ct \ln\mu \left[t - \frac{m_0 t_b}{m_0 - m'}\right] + ct - \frac{gt^2}{2}$$

$$\Rightarrow h(t) = \frac{-c t_b \ln|\mu|}{m_0 - m'} \times \mu(m_0) + ct - \frac{gt^2}{2}$$

3) Time of flight of the rocket —

$$\cancel{\int_{m_0}^{m'} m(t) = \int_0^{t} m_0 - \frac{(m_0 - m')\, t}{t_b}}$$

$$\cancel{\Rightarrow m' - m_0 = m_0 t - \frac{(m_0 - m')\, t^2}{2 t_b}}$$

Let the final orbital velocity be V.

$$\therefore \quad V = c \ln|\mu| - g t'$$

$$\Rightarrow t' = \frac{c \ln|\mu| - V}{g}$$ is the time of flight