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ASSIGNMENT-1

Q1) (x_1, y_1) $P_1 \equiv (-0.30738, -0.62208)$
(x_2, y_2) $P_2 \equiv (3.69262, 3.37792)$
(x_3, y_3) $P_3 \equiv (4.69262, 0.37792)$
(x_4, y_4) $P_4 \equiv (3.69262, -0.62208)$
(x_5, y_5) $P_5 \equiv (5.69262, 3.37792)$
(x_6, y_6) $P_6 \equiv (0.69262, 2.37792)$

(a) General 2nd order eq^n for a conic section :-

$$ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$$

On putting values of x and y $((x_1, y_1), (x_2, y_2), \dots, (x_6, y_6))$ we get a system of 6 eq^{n}s and 6 unknowns (a, b, c, h, g, f).

On solving those, we get the desired eq^n of ellipse.

eccentricity, $e = \sqrt{1 - \frac{b'^2}{a'^2}}$. (where b', a' are lengths of semi-minor and semi-major axis of the ellipse.)

(b)

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Q2

(a) $\cancel{u = u_e \ln}$

Initial mass of rocket $= m_0$

$$I_{sp} = \frac{F_{Th}}{\dot{m} g_0} \quad (F_{Th} \rightarrow \text{thrust})$$

$$F_{Th} = I_{sp}\, \dot{m}\, g_0 \quad \text{—— (1)}$$

$$(F_{Th} - mg) = ma$$

$$\frac{F_{Th}}{m} - g = \frac{dv}{dt}$$

$$\int_0^t \frac{F_{Th}}{m_0 - \dot{m}t}\, dt - gt = v$$

$$\int_0^t \frac{I_{sp}\, \dot{m}\, g_0\, dt}{m_0 - \dot{m}t} - gt = v \quad (\text{from } 1)$$

$$- I_{sp}\, g_0 \int_0^t \frac{-\dot{m}}{m_0 - \dot{m}t}\, dt - gt = v$$

$$\Rightarrow \quad v = I_{sp} g_0 \ln \left(\frac{m_0}{m_0 - \dot{m}t}\right) - gt$$

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(8) We have,

$$V = I_{sp} g_0 \ln\left(\frac{m_0}{m_0 - \dot{m} t}\right) - g_0 t$$

$t_b \rightarrow$ burnout time

Velocity at $t_b \Rightarrow v_b = I_{sp} g_0 \ln\left(\frac{m_0}{m_0 - \dot{m} t_b}\right) - g t_b \quad \text{—— (1)}$

Altitude at t_b = H (say)

$$\frac{dH}{dt} = I_{sp} g_0 \ln\left(\frac{m_0}{m_0 - \dot{m} t}\right) - gt$$

$$\int_0^H dH = \int_0^{t_b} I_{sp} g_0 \ln m_0 \, dt - \int_0^{t_b} I_{sp} g_0 \ln(m_0 - \dot{m} t)\, dt - \int_0^{t_b} gt \, dt$$

$$\Rightarrow H = I_{sp} g_0 \ln(m_0)\, t_b + \frac{I_{sp} g_0}{\dot{m}} (m_0 - \dot{m} t_b)\left[\ln(m_0 - \dot{m} t_b) - 1\right] - \frac{g t_b^2}{2} \quad \text{—— (2)}$$

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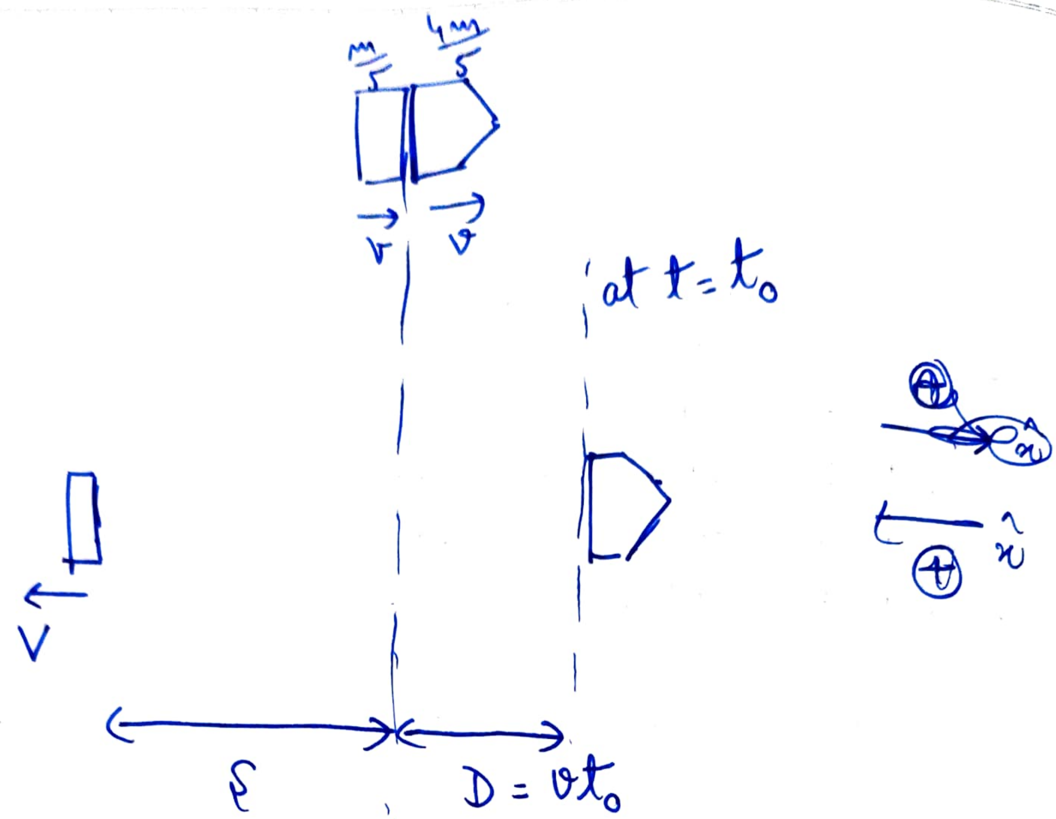
$$\text{Max height} = H + \frac{v_0^2}{2g}$$

(H, v_0 are calculated above)

Q3



$$\text{Blast radius} = \frac{16}{v}\sqrt{GM} = \frac{16}{v}\sqrt{G \times \frac{4m}{5}}$$

$$= \frac{32}{v}\sqrt{\frac{Gm}{5}}$$

Necessary condition :-

$$\xi + D \geq \frac{32}{v}\sqrt{\frac{Gm}{5}}$$

~~$V_g = -V\hat{x}$~~

~~$V_l = v\hat{x}$~~

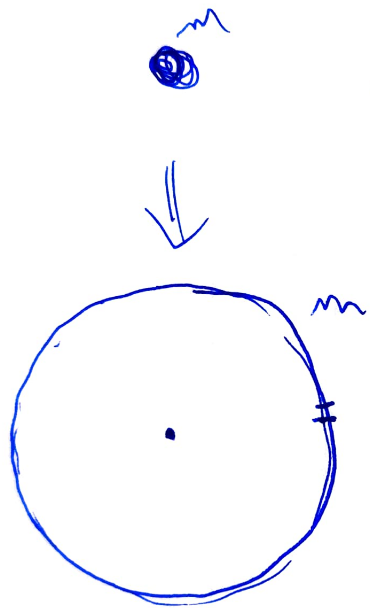
~~$-V - v = I_{sp}\, g_0 \ln 5$~~

~~$V = -(v + I_{sp}\, g_0 \ln 5)\hat{x}$~~

~~For dir^n~~

$$v = u_e \ln\left(\frac{\frac{m}{5}}{\frac{m}{5} - \left(\frac{dm}{dt}\right)t_0}\right)$$

$$\frac{G\,m\,dm}{R^2}$$



$$\boxed{R = \sqrt{\frac{16\,GM}{v^2}}} \quad \boxed{= \frac{16}{v}\sqrt{GM}}$$

$$\frac{1}{2}(dm)\left(\frac{v}{2}\right)^2$$

$$\frac{1}{2}(\cancel{dm})\left(\frac{v}{2}\right)^2 = \frac{G\,m\,\cancel{dm}}{R^2}$$

$$\frac{v^2}{16} = \frac{G\,m}{R^2}$$

$$v_g = -V\hat{x}$$
$$v_i = v\hat{x}$$

$$\cancel{\Delta v = -V - v = -u_e \ln\left(\frac{\frac{m}{5} - \dot{m}t_0}{\frac{m}{5}}\right)}$$

$$\cancel{v = \left[u_e \ln\left(\frac{m - 5\dot{m}t_0}{m}\right) - v\right]\hat{x}}$$

$F_{Th} = I_{sp}\,\dot{m}\,g_0$

$$\cancel{V = \left[v - u_e \ln\left(\frac{m - 5\dot{m}t_0}{m}\right)\right](-\hat{x})}$$

$$S = -vt_0 + \frac{1}{2}\left(I_{sp}\,\dot{m}\,g_0\right)t_0^2$$

$$D = vt_0$$

$$S + D = -2vt_0 + \frac{I_{sp}\dot{m}\,t_0^2 g_0}{2} \geq \frac{32}{v}\sqrt{\frac{Gm}{5}}$$

$$\boxed{\dot{m} \geq \frac{2\left(\frac{32}{v}\sqrt{\frac{Gm}{5}} + 2vt_0\right)}{I_{sp}\,t_0^2\,g_0}}$$