Date ___ / ___ / ______

Assignment-1

Q2) $ma = F_{thrust} - mg$

$$\frac{du}{dt} = \frac{I_{sp} g_0 \dot{m}}{m} - g.$$

$$\frac{du}{dt} = g_0\left(\frac{I_{sp}\,\dot{m}}{m} - 1\right)$$

$$\frac{du}{dt} = g_0\left(\frac{I_{sp}\,\dot{m}}{\dot{m}t + M_0} - 1\right)$$

$\dot{m}(t) = k$

$\Rightarrow \boxed{m(t) = \dot{m}t + M_0}$

$$\boxed{u = g_0 I_{sp} \ln\left(\frac{\dot{m}t + M_0}{M_0}\right) - g_0 t}$$

$M_0 e^{v/v_{ex}}$

(b) $r_{(t)} = g_0 I_{sp} \int \ln\left(\frac{\dot{m}t + M_0}{M_0}\right) dt - \frac{1}{2} g t^2$

$$r(t) = g_0 I_{sp} \frac{M_0}{\dot{m}}\left(\frac{\dot{m}t}{M_0}+1\right)\left[\ln\left(\frac{\dot{m}t}{M_0}+1\right) - 1\right]\Bigg|^{t} - \frac{1}{2} g t^2$$

$$\boxed{r(t) = g_0 I_{sp} \frac{M_0}{\dot{m}}\left\{\left(\frac{\dot{m}}{M_0}t+1\right)\left[\ln\left(\frac{\dot{m}}{M_0}t+1\right) - 1\right] + 1\right\} - \frac{1}{2} g t^2}$$

$\cancel{g_0 I_{sp} \frac{M_0}{\dot{m}}}$

$$\boxed{r(t_b) = g_0 I_{sp} \frac{M_0}{\dot{m}}\left\{\left(\frac{M_{payload}+M_{rocket}}{M_0}\right)\left[\ln\left(\frac{M_{payload}+M_{rocket}}{M_0}\right) - 1\right] + 1\right\} - \frac{1}{2} g t_b^2}$$

$u(t) = 0$

$I_{sp} \ln\left(\frac{\dot{m}t + M_0}{M_0}\right) = t$

$\cancel{r(t)}\ r_{max} = g_0 I_{sp} \frac{M_0}{\dot{m}} \{$

Date __/__/__

~~Q1) a)~~

Q2) b) After t_b it is just moving under gravity

$$v^2-u^2=-2gh$$

$$\left(g_0 I_{sp}\ln\left(\frac{\dot m t_b+M_0}{M_0}\right)-g_0t_b\right)^2=2g_0h$$

$$\boxed{\frac{g_0}{2}\left(I_{sp}\ln\left(\frac{\dot m t_b+M_0}{M_0}\right)-t_b\right)^2=h}$$

So Max. Altitude $= r(t_b)+h$

$$= g_0 I_{sp}\frac{M_0}{\dot m}\left\{\left(\frac{M_{payload+round}}{M_0}\right)\left[\ln\left(\frac{M_{payload+round}}{M_0}\right)-1\right]+1\right\}$$

$$-\frac{1}{2}g_0t_b^2+\frac{g_0}{2}\left[I_{sp}\ln\left(\frac{M_{payload+round}}{M_0}\right)-t_b\right]^2$$

Ans

$$= g_0 I_{sp}\frac{M_0}{\dot m}\left\{\left(\frac{M_{payload+round}}{M_0}\right)\left[\ln\left(\frac{M_{payload+round}}{M_0}\right)-1\right]+1\right\}$$

$$-I_{sp}g_0t_b\ln\left(\frac{M_{payload+round}}{M_0}\right)+\frac{1}{2}g_0I_{sp}^2\left[\ln\left(\frac{M_{payload+round}}{M_0}\right)\right]^2$$

Q1) a) Shift the center to $(-0.69262, 0.37792)$

Now put values of (x,y) in

$$\boxed{x^2+ay^2+bxy+cx+dy+e=0}$$

We get 6 equation which have the following matrix form,

$$\begin{pmatrix} 1 & 1 & -1 & -1 & 1 \\ 9 & 9 & 3 & 3 & 1 \\ 0 & 0 & 4 & 0 & 1 \\ 1 & -3 & 3 & -1 & 1 \\ 9 & 15 & 5 & 3 & 1 \\ 4 & 0 & 0 & 2 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \\ d \\ e \end{pmatrix}=\begin{pmatrix} -1 \\ -9 \\ -16 \\ -9 \\ -25 \\ 0 \end{pmatrix}$$

Date ___/___/___

This can be represented as

$Ax = b$

The least squares solution to this, is

$$\boxed{C = (A^T A)^{-1} A^T b}$$

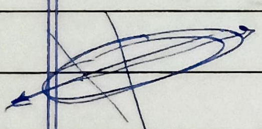
On solving,

$$C = \begin{pmatrix} 1.733 \\ -1.224 \\ -2.934 \\ -0.404 \\ -5.195 \end{pmatrix}$$

So, the equation is,

$$\boxed{\begin{aligned}(x-0.69262)^2 + 1.733(y-0.37792)^2 \\ -1.224(x-0.69262)(y-0.37792) - 2.934(x-0.69262) \\ - 0.404(y-0.37792) - 5.195 = 0\end{aligned}}$$

$y = mx$ $\quad 0.69262 = h, \; 0.37792 = k$



$$\frac{\partial f}{\partial x} = 2(x-h) - 1.224(y-k) - 2.934 = 0$$

$$\frac{\partial f}{\partial y} = 3.466(y-k) - 1.224(x-h) - 0.404 = 0$$

$2X - 1.224Y - 2.934 = 0$

$3.466Y - 1.224X - 0.404 = 0$

$$x = h + \frac{\begin{vmatrix} 2.934 & -1.224 \\ 0.404 & 3.466 \end{vmatrix}}{\begin{vmatrix} 2 & -1.224 \\ -1.224 & 3.466 \end{vmatrix}} = h + 1.962$$

$$= 2.655 = x$$

Page No.

Date ___/___/___

$$y = k + \frac{\begin{vmatrix} 2 & 2.934 \\ -1.224 & 0.404 \end{vmatrix}}{\begin{vmatrix} 2 & -1.224 \\ -1.224 & 3.466 \end{vmatrix}}$$

$$= k + \frac{4.399}{5.433824} = 0.81 + 0.37792$$

$$y = 1.18792$$

Center of ~~the~~ ellipse $= (2.655, 1.18792)$

Sun lies at focus of ellipse,
So equation of major axis,

$y = mx$

$m = 0.447$

$$\boxed{y = 0.447x}$$

So, ~~they~~ this line intersects ellipse at,

$(-0.555, -0.248)$ and $(5.866, 2.622)$

endpoints of major axis.

$$2a = \sqrt{6.421^2 + 2.87^2}$$

$$a = \frac{1}{2}\sqrt{49.466}$$

$$= \frac{7.054}{2}$$

~~$a = 3.52$~~

$$\boxed{a = 3.527}$$ ans

$(0,0)$ is focus,

So $ae = \sqrt{2.655^2 + 1.18792^2}$ = distance from center of ellipse

$= \sqrt{8.4601}$

$ae = 2.909$

$$\boxed{e = \frac{2.909}{3.527} = 0.82}$$ ans