1 ✓

(a) General equation of conic.

$$Ax^2 + By^2 + Cxy + Dx + Ey + F = 0.$$

① $0.0944A + (0.3869)B + C(0.1912) - D(0.30738) - E(0.62208) + F = 0$

② $(13.635)A + B(11.410) + C(12.4733) + D(3.6926) + E(3.377) + F = 0$

③ $(22.020)A + B(0.14202) + C(1.7734) + D(4.6926) + E(0.377) + F = 0$

④ $(3.635)A + B(0.3869) + C(2.0297) + D(3.6926) + E(-0.6220) + F = 0$

⑤ $(32.405)A + B(11.4103) + C(19.229) + D(5.6926) + E(3.37792) + F = 0$

⑥ $(0.4797)A + (5.6545)B + C(1.6469) + D(0.6926) + E(2.3778) + F = 0$

~~Solve eqn ①, ②, 3, 4, 5, & 6.~~

$$A^T A X = A^T b$$

Page

(2) (a) $\Delta v = I_{sp}\, g_0 \ln\left(\frac{M_0}{M_f}\right)$ Total mass of rocket $= M_0$.

$v = I_{sp}\, g_0 \ln\left(\frac{M_0}{M_0 - \dot{m}t}\right)$

Initial velocity $= 0$.

(b) Altitude of rocket at burnout time t_b.

$$\frac{dx}{dt} = I_{sp}\, g_0 \ln\left(\frac{M_0}{M_0 - \dot{m}t}\right)$$

$$x = \int_0^{t_b} I_{sp}\, g_0 \ln\left(\frac{M_0}{M_0 - \dot{m}t}\right) dt$$

$$x = \frac{I_{sp}\, g_0 M_0}{\dot{m}}\left[1 - \left(\frac{M_0 - \dot{m}t_b}{M_0}\right)\ln\left(\frac{M_0}{M_0 - \dot{m}t_b}\right) - \left(\frac{M_0 - \dot{m}t_b}{M_0}\right)\right]$$

velocity at $t = t_b$ $= I_{sp}\, g_0 \ln\left(\frac{M_0}{M_0 - \dot{m}t_b}\right)$

After $t = t_b$ distance travelled $= \frac{v_b^2}{2g_0}$

$$= \frac{I_{sp}^2\, g_0}{2}\left[\ln\left(\frac{M_0}{M_0 - \dot{m}t_b}\right)\right]^2$$

Maximum altitude $= \frac{I_{sp}^2\, g_0}{2}\left[\ln\left(\frac{M_0}{M_0 - \dot{m}t_b}\right)\right]^2 +$

$$\frac{I_{sp}\, g_0 M_0}{\dot{m}}\left[1 - \left(\frac{M_0 - \dot{m}t_b}{M_0}\right)\ln\left(\frac{M_0}{M_0 - \dot{m}t_b}\right) - \left(\frac{M_0 - \dot{m}t_b}{M_0}\right)\right]$$