Assignment-2

Q.1) Total initial wet mass $\rightarrow M_i = M_{prop} + M_{rocket} + M_{pay}$

$$M_{prop} = M_{fuel} + M_{oxi}$$

$$M_{prop} = x M_i$$

$$\Delta V = I_{sp} g_0 \cdot \ln\left[\frac{M_i}{M_i - M_{prop}}\right] = I_{sp} \cdot g_0 \cdot \ln\left[\frac{M_i}{M_i - x M_i}\right]$$

$$\Delta V = I_{sp} \cdot g_0 \cdot \ln\left[\frac{1}{1-x}\right]$$

$$\Rightarrow \quad 1 - x = e^{-(\Delta V / I_{sp} \cdot g_0)}$$

$$\Rightarrow \quad x = 1 - e^{-(\Delta V / I_{sp} \cdot g_0)}$$

$\Delta V = 7.6\ km/s = 7600\ m/s$ $\qquad I_{sp} = 400 s$

$g_0 = 9.8\ m/s^2$

$$\Rightarrow \frac{\Delta V}{I_{sp} \cdot g_0} = \frac{760\cancel{0}}{4\cancel{0} \times 98} = \frac{190}{98} = 1.938$$

$$\Rightarrow x = 1 - e^{-1.938} = 1 - 0.143 = 0.857$$

$\Rightarrow$ $\boxed{x = 0.857}$ $\rightarrow$ fraction of M_i, M_{prop} should be.

Q.2) $F_{thrust} = -c\frac{dm}{dt}$ $\qquad n = \frac{F_{thrust}}{m_0 g}$ $\qquad \mu = \frac{m(t)}{m_0}$

$$I_{sp} = \frac{F_{thrust}}{g_0 \frac{dm}{dt}} = \frac{-c}{g_0} \Rightarrow \boxed{I_{sp} \cdot g_0 = -c}$$

$$\Delta V = I_{sp} \cdot g_0 \ln\left(\frac{M_0}{M_f}\right) = -c \ln\left(\frac{M_0}{M_f}\right)$$

$$-c\frac{dm}{dt} = n m_0 g \rightarrow -c\int dm = m_0 g \int n\, dt$$

$$F_{thrust} = m\frac{dv}{dt} = -c\frac{dm}{dt} \Rightarrow \frac{dv}{dt} = \frac{c\, dm}{m(t)\, dt}$$

$$\Rightarrow \int_0^{\Delta V} dv = c\int_{m_0}^{m} \frac{dm}{m} \Rightarrow \Delta V = c\ln\left(\frac{m(t)}{m_0}\right) = c\ln(\mu)$$

$\Rightarrow$ $\boxed{V = c\ln(\mu)}$ $\rightarrow$ Velocity of rocket
{Initial velocity is 0}

$$V = \frac{dz}{dt} = c\ln\left(\frac{m(t)}{m_0}\right) \Rightarrow \int dz = c\int \ln\left(\frac{m(t)}{m_0}\right) dt$$

$$\Rightarrow z(t) = c\int \ln(m(t))\,dt - c\ln(m_0)\int dt$$

$$\Delta V = -V_{ex}\ln\left(\frac{m(t)}{m_0}\right)$$

$$\Rightarrow \ln\left(\frac{m(t)}{m_0}\right) = \frac{-V}{V_{ex}}$$

$$\Rightarrow \ln(\mu) = \frac{-V}{V_{ex}} \quad \& \quad \ln(\mu) = \frac{V}{c}$$

$$\Rightarrow \boxed{c = -V_{ex}}$$

$\cancel{m(t)\ \Delta z}$

$\cancel{\Rightarrow \frac{dz}{dt} = \frac{dm}{dt}}$

$\cancel{\frac{dm}{dt} = -\frac{m}{V}\frac{dv}{dt}}$

$\cancel{\Rightarrow dm = -m\frac{dv}{V}}$

$\cancel{\Rightarrow dz = -\frac{m\,dv}{V}}$

Final mass of rocket is m_f at $t = t_0$

$$\frac{dv}{dt} = \frac{c}{\mu m_0}\left(\frac{dm}{dt}\right) \quad \& \quad \frac{dv}{dt} = -\frac{V_{ex}}{m_0}\frac{dm}{dt}$$

$$\Rightarrow m(t) = \mu m_0$$

$$\boxed{z(t) = c\int \ln(\mu)\,dt}$$

$$V = \frac{dz}{dt} \Rightarrow dt = \frac{dz}{c\ln(\mu)} \Rightarrow \int \ln(\mu)\,dt = \int \frac{dz}{c}$$

$\Rightarrow$

$$\eta = \frac{F_{thrust}}{m_0 g}$$ $\rightarrow$ [it is the ratio of the rocket's acceleration and acceleration due to gravity]

Q3) Initial mass $\rightarrow M_0 = (M_{prop1} + M_{prop2}) + M_{pay1} + (M_{rock1} + M_{rock2})$

$\boxed{\text{For DSTO}}$

$$M_{f1} = M_{prop2} + M_{pay1} + M_{rock2}$$

$$M_{f2} = M_{pay} + M_{rock2}$$

$$\Delta V = I_{sp} \cdot g_0 \cdot \ln\left(\frac{M_{in}}{M_{fin}}\right)$$

$$\Delta V_1 = I_{sp} \cdot g_0 \cdot \ln\left(\frac{M_{prop1} + M_{prop2} + M_{pay1} + M_{rock1} + M_{rock2}}{M_{prop2} + M_{pay1} + M_{rock2}}\right)$$

$$\Delta V_2 = I_{sp} \cdot g_0 \cdot \ln\left(\frac{M_{prop2} + M_{pay1} + M_{rock2}}{M_{pay1} + M_{rock2}}\right)$$

$$\Delta V_1 = I_{sp} \cdot g_0 \cdot \ln\left[1 + \frac{M_{prop1} + M_{rock1}}{M_{prop2} + M_{pay1} + M_{rock2}}\right]$$

$$\Delta V_2 = I_{sp} \cdot g_0 \cdot \ln\left[1 + \frac{M_{prop2}}{M_{pay1} + M_{rock2}}\right]$$

$$\Delta V = \Delta V_1 + \Delta V_2 = I_{sp} \cdot g_0 \ln\left[\frac{M_0}{M_{f2}}\right]$$

$$= I_{sp} g_0 \ln\left[1 + \frac{M_{prop1} + M_{prop2} + M_{rock1}}{M_{pay1} + M_{rock2}}\right] \quad \text{(i)}$$

$\Delta V_1 = \Delta V_2 = 4 \text{ kmps}$

$\Delta V = 8 \text{ kmps}$

$$\frac{M_{prop1} + M_{rock1}}{M_{prop2} + M_{pay1} + M_{rock2}} = \frac{M_{prop2}}{M_{pay1} + M_{rock2}}$$

$\boxed{\text{For SSTO}}$

$$\Delta V = I_{sp} \cdot g_0 \cdot \ln\left[\frac{M_{prop} + M_{pay2} + M_{rock}}{M_{pay2} + M_{rock}}\right]$$

$$= I_{sp} \cdot g_0 \cdot \ln\left[1 + \frac{M_{prop}}{M_{pay2} + M_{rock}}\right] = 8 \text{ kmps} \quad \text{(ii)}$$

Now (i) = (ii) as $\Delta V = 8$ kmph

$$\Rightarrow \frac{M_{prop1} + M_{prop2} + M_{rock1}}{M_{pay1} + M_{rock2}} = \frac{M_{prop}}{M_{pay2} + M_{rock}}$$

Now take $M_{prop1} + M_{prop2} = M_{prop}$
$M_{rock1} + M_{rock2} = M_{rock}$

$$\Rightarrow \frac{M_{prop} + M_{rock1}}{M_{pay1} + M_{rock2}} = \frac{M_{prop}}{M_{pay2} + M_{rock}}$$

$$\Rightarrow \left[M_{pay1} + M_{rock2}\right] = \left[1 + \frac{M_{rock1}}{M_{prop}}\right]\left[M_{pay2} + M_{rock}\right]$$

$$\Rightarrow \left[M_{pay1} + \cancel{M_{rock}}\right] - M_{rock1} = \left[M_{pay2} + \cancel{M_{rock}}\right] + \left[M_{pay2} + M_{rock}\right]\left[\frac{M_{rock1}}{M_{prop}}\right]$$

$$\Rightarrow M_{pay1} = \left[M_{pay2} + M_{rock}\right]\left[\frac{M_{rock1}}{M_{prop}}\right] + M_{rock1}$$

$M_{pay1} > M_{pay2}$

↳ DSTO can carry more payload ~~that~~ than SSTO

Q.4 $\Delta V = 8$ kmps $\quad I_{sp} \to [350, 400, 450]$

$$\Delta V = I_{sp} \cdot g_0 \cdot \ln\left[\frac{M_i}{M_f}\right] \qquad \lambda = \frac{M_{rocket}}{M_{rocket} + M_{pay}}$$

$$= I_{sp} \cdot g_0 \cdot \ln\left[1 + \frac{M_{prop}}{M_{pay} + M_{rocket}}\right]$$

$M_{pay} = 2000$ kg. $\quad M_0 \to$ initial rod mass
$\lambda \to$ structural coeff.

(a) $M_i = 2000 + M_{rocket} + M_{prop}$

$$\frac{1}{\lambda} = 1 + \frac{M_{pay}}{M_{rocket}}$$

$\zeta \Rightarrow$ Propellant mass fraction

$$\zeta = \frac{M_{prop}}{M_0}$$ Assume $M_{prop} \rightarrow$ constant,

$$\Rightarrow M_i = \frac{M_{prop}}{\left[1 - \left(\frac{\zeta}{1-\lambda}\right)\right]}$$

$$e^{\left(\frac{\Delta V}{I_{sp} \cdot g_0}\right)} = \frac{M_i}{M_i - M_{prop}} = \frac{1/\left[1-\left(\frac{\zeta}{1-\lambda}\right)\right]}{1/\left[1-\left(\frac{\zeta}{1-\lambda}\right)\right] - 1}$$

$$\Rightarrow \frac{1}{1-1+\left(\frac{\zeta}{1-\lambda}\right)} = \left(\frac{1-\lambda}{\zeta}\right) = e^{\left(\frac{\Delta V}{I_{sp} \cdot g_0}\right)}$$

$$\Rightarrow \left[\frac{1-\lambda}{M_{prop}}\right] = M_i e^{(\Delta V / I_{sp} \cdot g_0)}$$

$$\Rightarrow \boxed{\lambda = 1 - M_i \left[M_{prop} e^{\left(\Delta V / I_{sp} \cdot g_0\right)}\right]}$$

$\Delta V = 8000 \, m/s$ $\quad I_{sp} = [350, 400, 450]$

$g_0 = 10 \, m/s$ $\quad M_{pay} = 2000 \, kg$

$$\lambda = \frac{M_{rocket}}{M_i - M_{prop}} \qquad \lambda + 1 = \frac{2M_{rocket} + M_{pay}}{M_i - M_{prop}}$$

$$\lambda + 1 = \frac{M_{rocket} + M_i - M_{pay}}{M_i - M_{prop}}$$

$$M_i = M_{prop} + \left[M_{pay} + M_{rocket} \right] = M_{prop} + \frac{M_{rocket}}{\lambda}$$