Q1 $\Delta v = I_{sp}\, g_0 \ln\left(\frac{M_0}{M_f}\right)$

$$7.6\times10^{3} = 400\times9.8 \ln\left(\frac{M_0}{M_f}\right)$$

$$\frac{7.6\times10^{3}}{400\times9.8} = \frac{7600}{9.8\times400} = \boxed{1.939 = \ln\left(\frac{M_0}{M_f}\right)}$$

$$\left(\frac{M_0}{M_f}\right) = e^{1.939}$$

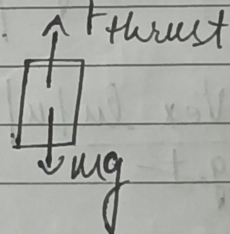
$M_0 \to M_p + M_s$

$M_f \to M_s$

$$\frac{M_p}{M_s} + 1 = e^{1.939}$$

$$\frac{M_p}{M_0} = \frac{\cancel{M_s}\,(e^{1.939}-1)}{\cancel{M_s}\,(e^{1.939}-1)+\cancel{M_s}} = \frac{e^{1.939}-1}{e^{1.939}} = 0.856$$

Q2 $F_{thrust} = -C\frac{dm}{dt}$



$$ma = F_{thrust} - mg$$

$$ma = -C\frac{dm}{dt} - mg$$

$$m\frac{dv}{dt} = -C\frac{dm}{dt} - mg$$

$$\mu \cancel{m_0}\frac{dv}{dt} = -C\cancel{m_0}\frac{d\mu}{dt} - \mu\cancel{m_0}\, g$$

$$\mu\, dv = -C\, d\mu - \mu g\, dt$$

$$\int_0^v dv = \int_1^{\mu} -C\frac{d\mu}{\mu} - \int_0^t g\, dt$$

$$v = -C\ln|\mu| - gt$$

$\underline{v = -V_{ex} \ln(\mu)}$ $\qquad v = -V_{ex} \ln\left(\frac{M_f}{M_0}\right)$

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$$v = \frac{dz}{dt} = -c\ln\mu - g_0 t$$

$$v = \frac{cv}{V_{ex}} - g_0 t$$

$$g_0 t = \left(\frac{c}{V_{ex}} - 1\right) v = \left(\frac{c}{V_{ex}} - 1\right)\frac{dz}{dt}$$

$$\frac{1}{2} g_0 t^2 = z\left(\frac{c - V_{ex}}{V_{ex}}\right)$$

$$z = \left(\frac{V_{ex}}{c - V_{ex}}\right)\frac{1}{2} g_0 t^2$$

$$-c\ln|\mu| - g_0 t = -V_{ex}\ln|\mu|$$

$$(V_{ex} - c)\ln|\mu| = g_0 t$$

$$-(c - V_{ex}) = \frac{-g_0 t}{\ln|\mu|}$$

$$V_{ex} = c + g_0 t / \ln|\mu|$$

$$z = \frac{V_{ex}\ln|\mu|}{\cancel{-g_0 t}} \frac{1}{2} g_0 t^2 t$$

$$= -\frac{t}{2}\ln|\mu|\left(c + \frac{g_0 t}{\ln|\mu|}\right)$$

$$z = \frac{-ct\ln|\mu|}{2} - \frac{1}{2} g_0 t^2$$

for time of flight t_f

when $v = 0$, $\mu_f = \frac{m_{structure}}{m_0}$ → all the fuel has burnt

so using the relⁿ,

$$0 \leftarrow v = -c\ln|\mu_f| - g t_f$$

$$t_f = \frac{-c\ln|\mu_f|}{g}$$

(4) $\Delta V = I_{sp} g_0 \ln\left(\frac{M_{pay} + M_{rock} + M_{prop}}{M_{pay} + M_{rckt}}\right)$

$$= I_{sp} g_0 \ln\left(\frac{M_0}{M_{pay} + M_{rckt}}\right)$$

$$\alpha = \frac{M_{rckt}}{M_{rckt} + M_{pay}}$$

$$\frac{1}{\alpha} = 1 + \frac{M_{pay}}{M_{rckt}}$$

$$\left(\frac{1}{\alpha} - 1\right) = \frac{M_p}{M_r}$$

$$\left(\frac{1-\alpha}{\alpha}\right) M_r = M_p \Rightarrow M_r = \left(\frac{\alpha}{1-\alpha}\right) M_p$$

$$\frac{M_0}{M_p + \left(\frac{\alpha}{1-\alpha}\right) M_p} = \frac{M_0}{M_p\left[\frac{1-\cancel{\alpha}+\cancel{\alpha}}{1-\alpha}\right]} = \frac{(1-\alpha) M_0}{M_p}$$

$$\Delta V = I_{sp} g_0 \ln\left(\frac{(1-\alpha) M_0}{M_p}\right)$$

$$e^{\left(\frac{\Delta V}{I_{sp} g_0}\right)} = \frac{(1-\alpha) M_0}{M_p}$$

$$\left(\frac{M_p}{1-\alpha}\right) e^{\left(\frac{\Delta V}{I_{sp} g_0}\right)} = M_0$$

$$\left(\frac{2000}{1-\alpha}\right) e^{\left(\frac{8000}{I_{sp} \times 9.8}\right)} = M_0$$


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1
$\frac{2000}{1-x}e^{\left(\frac{8000}{350\cdot 9.8}\right)}$
2
$\frac{2000}{1-x}e^{\left(\frac{8000}{400\cdot 9.8}\right)}$
3
$\frac{2000}{1-x}e^{\left(\frac{8000}{450\cdot 9.8}\right)}$
4
400
200
-400
-200
0
200
400
600
-200
-400
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