Date
Page

Assignment-1.

(a) we know, eq^n of ellipse,

$$\frac{x^2}{a^2} + y^2/b^2 = 1.$$

let $1/a^2 = k$ $1/b^2 = \ell$.

$\therefore kx^2 + \ell y^2 = 1.$

Now, using observed coordinate provided in AU, we form a matrix

i.e.

$$\begin{bmatrix} (-0.30738)^2 & (-0.62208)^2 \\ \vdots & \\ (0.69262)^2 & (2.37792)^2 \end{bmatrix} \begin{bmatrix} k \\ \ell \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

solve using least square method.

$A^T A X = A^T b$. where, $X = \begin{bmatrix} k \\ \ell \end{bmatrix}$

A → cofficient matrix, and b = unit matrix.

$$A = \begin{bmatrix} 0.09 & 0.38 \\ 13.61 & 11.42 \\ 21.99 & 0.14 \\ 13.62 & 0.38 \\ 32.38 & 11.42 \\ 0.48 & 5.66 \end{bmatrix}_{6\times 2} \Rightarrow A^T = \begin{bmatrix} \cancel{0.09 \quad 13.61} \\ \\ \\ \end{bmatrix}$$

Now,

$$A^T = \begin{bmatrix} 0.09 & 13.61 & 21.99 & 13.62 & 32.38 & 0.48 \\ 0.38 & 11.42 & 0.14 & 0.38 & 11.42 & 5.66 \end{bmatrix}_{2\times 6}.$$

Now, $[A^T A x]_{2\times 1} = [A^T b]_{2\times 1}$

or,

$$\begin{bmatrix} 0.324 \\ 293.19 \end{bmatrix} \begin{bmatrix} k \\ t \end{bmatrix} = \begin{bmatrix} 82.17 \\ 29.4 \end{bmatrix}$$

$0.324k = 82.17 \Rightarrow k = \frac{82.17}{0.324} = 253.6$

$293.19t = 29.4$

$t = \frac{29.4}{293.19} = 0.100$

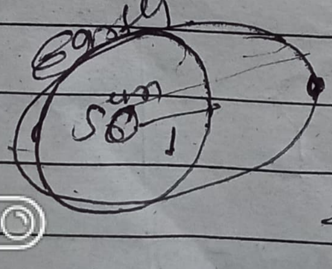
Now, $\frac{1}{a^2} = 253.6$ $\qquad \frac{1}{b^2} = 0.100$

$\boxed{a = 0.06}$ $\qquad \boxed{b = 3.125}$

Now, required, eq^n of trajectory,

$$\boxed{\frac{x^2}{3.6\times 10^{-3}} + \frac{y^2}{9.76} = 1.}$$

(b)



Minimum distance

$= 9.76 - 1 = 8.76.$

2.) a.) $\Delta V = I_{sp}\, g_0 \ln\left(\frac{M_0}{M_0 - \dot{m}t}\right)$ $\left[\begin{array}{l} M_0 \equiv \text{initial} \\ \text{mass of rocket} \end{array}\right]$

$$v = I_{sp}\, g_0 \ln\left(\frac{M_0}{M_0 - \dot{m}t}\right)$$

b.) $t_b \longrightarrow$ burnout time.

~~Distance travelled~~ before burnout time

$$\int_0^{t_b} v\,dt = I_{sp}\, g_0 \int_0^{t_b} \ln\left(\frac{M_0}{M_0 - \dot{m}t}\right) dt$$

$$\Rightarrow x = I_{sp}\, g_0 \int_0^{t_b} \ln\left(\frac{M_0}{M_0 - \dot{m}t}\right) dt$$

take $\ln\left(\frac{M_0}{M_0 - \dot{m}t}\right)$ as V.

$$\Rightarrow x = \frac{I_{sp}\, g_0\, M_0}{\dot{m}}\left[1 - \ln\left(\frac{M_0}{M_0 - \dot{m}t}\right)\left(\frac{M_0 - \dot{m}t_b}{M_0}\right)\left(\frac{M_0 - \dot{m}t_b}{M_0}\right)\right. \quad -(1)$$

After t_b: distance travelled

$$v^2 - u^2 = 2gS$$

$$S = \frac{I_{sp}^2 \left(\ln\left(\frac{M_0}{M_0 - \dot{m}t}\right)\right)^2 g_0}{2} \quad -(2)$$

total distance $= (1) + (2)$

$$\frac{I_{sp}\, g_0\, M_0}{\dot{m}}\left[1 - \ln\left(\frac{M_0}{M_0 - \dot{m}t}\right)\left(\frac{M_0 - \dot{m}t_b}{M_0}\right) - \left(\frac{M_0 - \dot{m}t_b}{M_0}\right)\right] + \frac{I_{sp}^2 \left[\ln\left(\frac{M_0}{M_0 + \dot{m}t}\right)\right]^2 g_0}{2}$$