Assignment - 2

1)

$$\Delta V = I_{sp} g_0 \ln\left[1 + \frac{M_{prop}}{M_{pay} + M_{rocket}}\right]$$

$\Delta V = 7.6\, km/s = 7600\, m/s$ $\quad I_{sp} = 400\, s \quad g_0 = 9.8\, m/s^2$.

$$\Rightarrow \ln\left[1 + \frac{M_{prop}}{M_{pay} + M_{rocket}}\right] = \frac{7600}{400 \times 9.8} = 1.9388$$

$$\Rightarrow 1 + \frac{M_{prop}}{M_{pay} + M_{rocket}} = e^{1.9388} = 6.95$$

$$\Rightarrow M_{prop} = (5.95)(M_{pay} + M_{rocket}).$$

Now

$$\frac{M_{prop}}{M_i} = \frac{5.95(M_{pay} + M_{rocket})}{M_{pay} + M_{rocket} + M_{prop}}$$

$$= \frac{5.95(M_{pay} + M_{rocket})}{6.95(M_{pay} + M_{rocket})} = 0.856$$

2) $F_{thrust} = -c\frac{dM}{dt}$

$$F_{net} = -c\frac{dm}{dt} - mg = m\frac{dv}{dt}$$

$$\Rightarrow -c\,dm - mg\,dt = m\,dv$$

$$\Rightarrow -\int_{m_0}^{m_f} \frac{c\,dm}{m} - \int g\,dt = \int_{v_0}^{v(t)} dv \quad \Rightarrow -c\ln\left|\frac{m_f}{m_0}\right| - gt = -u(t) - u_0$$

$$\Rightarrow v(t) = v_0 + c\ln\left|\frac{m_0}{m_f}\right| - gt$$

$$\Rightarrow \boxed{v(t) = v_0 - c\ln|\mu| - gt}$$

iv) $n = \frac{F_{thrust}}{m_0 g} \propto$ acceleration of rocket.

So, for $n > 1$ the flight will be more easier because lift generated will be very high.

ii) If the burning rate = constant.

$\Rightarrow \quad -\frac{dm}{dt} = k \quad \Rightarrow \int_0^{t_b} k\,dt = \int_{m_0}^{m'} dm \quad \Rightarrow k = \frac{m_0 - m'}{t_b}$

t_b = time to reach orbit.

For $t < t_b$,

$\int_0^{t} k\,dt = -\int_{m_0}^{m(t)} dm \quad \Rightarrow kt = -m(t) + m_0$

$\Rightarrow (m_0 - m')\frac{t}{t_b} - m_0 = -m(t)$

$\Rightarrow m(t) = m_0 - (m_0 - m')\frac{t}{t_b}$

Now,

$h = \int_0^t u\,dt \quad \Rightarrow h \int \left(-c \ln \frac{m(t)}{m_0} - gt\right) dt$

$\Rightarrow h = -c \int_0^t \ln\left(\frac{m_0 - (m_0 - m')\,t/t_b}{m_0}\right) - g\int_0^t t\,dt$

$\Rightarrow h = \Big[-ct \ln\left(m_0 - (m_0 - m')\frac{t}{t_b}\right) + ct$

$+ \frac{c m_0 t_b}{m_0 - m'} \ln\left[m_0 - (m_0 - m')\frac{t}{t_b}\right) - \frac{g t^2}{2}\Big]_0^t$

$+ ct \ln m_0$

$\Rightarrow h = -ct \ln\left(m_0 - (m_0 - m')\frac{t}{t_b}\right) + ct - \frac{g t^2}{2} + ct \ln m_0$

$+ \frac{c m_0 t_b}{m_0 - m'} \ln\left[m_0 - (m_0 - m')\frac{t}{t_b}\right]$

$- \frac{c m_0 t_b}{m_0 - m'} \ln(m_0) - c t \ln m_0$

$$\Rightarrow h = -ct\ln|m(t)| + ct - \frac{gt^2}{2} + \frac{cm_0t_b}{m_0-m'}\ln|m(t)|$$

$$- \frac{cm_0t_b}{m_0-m'}\ln|m_0|$$

$$\Rightarrow h = -ct\ln|m(t)| + ct - \frac{gt^2}{2} + \frac{cm_0t_b}{m_0-m'}\ln\left|\frac{m(t)}{m_0}\right|$$

$$\Rightarrow h = -ct\ln|\mu m_0| + ct - \frac{gt^2}{2} + \frac{cm_0t_b}{m_0-m'}\ln|\mu| + ct\ln m_0$$

$$\Rightarrow h = -ct\ln|\mu| + ct - \frac{gt^2}{2} + \frac{cm_0t_b}{m_0-m'}\ln|\mu|$$

$$\Rightarrow h = -c\ln|\mu|\left[t - \frac{m_0t_b}{m_0-m'}\right] + ct - \frac{gt^2}{2}$$

$$\Rightarrow h = \frac{-ct_b\ln|\mu|}{m_0-m'}\left[(m_0-m')\frac{t}{t_b} - m_0\right] + ct - \frac{gt^2}{2}$$

$$\Rightarrow h = \frac{-ct_b}{m_0-m'}\ln|\mu|[m(t)] + ct - \frac{gt^2}{2}$$

$$\Rightarrow h = \frac{-ct_b\ln|\mu|}{(m_0-m')} \times \mu m_0 + ct - \frac{gt^2}{2}$$

$$\Rightarrow \boxed{h = -\frac{ct_b\ln|\mu|}{\left(1-\frac{m'}{m_0}\right)} \times \mu + ct - \frac{gt^2}{2}}$$

for $t = t_b$ then $m(t) = m'$ i.e. $\mu = \frac{m'}{m_0}$

$$h_b = \frac{-ct_b\ln\left|\frac{m'}{m_0}\right|}{1-\frac{m'}{m_0}} \cdot \frac{m'}{m_0} + ct_b - \frac{gt^2}{2}$$

$$\Rightarrow h_b = \frac{-ct_b\ln\left|\frac{m'}{m_0}\right|}{\frac{m_0-m'}{m'}} + ct_b - \frac{gt^2}{2}$$

$$\boxed{h_{b_t} = \frac{-ct_b \ln\left|\frac{m'}{m_0}\right|}{1-\frac{m'}{m_0}} + ct_b - \frac{g t_b^2}{2}}$$ — burnout altitude

4) $\Delta v = I_{sp}\, g_0 \ln\left(\frac{M_i}{M_f}\right)$ $\quad M_{pay} = 2000\ kg$

$$\Rightarrow e^{\frac{\Delta v}{I_{sp} g_0}} = \frac{M_i}{M_f}$$

$$\Rightarrow e^{\Delta v / I_{sp} g_0} = \frac{M_{prop} + M_{pay} + M_{rocket}}{M_{pay} + M_{rocket}}$$

$$\Rightarrow e^{\Delta v / I_{sp} g_0} = \frac{M_i}{2000 + \frac{2000\lambda}{1-\lambda}}$$

$$\Rightarrow e^{\Delta v / I_{sp} g_0} = \frac{M_i(1-\lambda)}{2000\lambda}$$

$\Delta v = 8\ kmps = 8000\ m/s$ $\quad I_{sp} = [350, 400, 450]$

i) $I_{sp} = 350\ s$

$$2000 \times e^{\frac{8000}{350 \times 9.8}} = M_i(1-\lambda)$$

$$\Rightarrow M_i(1-\lambda) = 2.06 \times 10^4$$

ii) $I_{sp} = 400\ s$

$$M_i(1-\lambda) = e^{\frac{8000}{400 \times 9.8}} \times 2000 = 1.54 \times 10^4$$

iii) $I_{sp} = 450$

$$M_i(1-\lambda) = e^{\frac{8000}{450 \times 9.8}} \times 2000 = 1.227 \times 10^4$$

The graph is rectangular hyperbola.