classmate

Date
Page

Assignment 2

1. $MR = \frac{m_f}{m_e} = \frac{m_p + m_e}{m_e} = \left(1 - r\right)^{-1}$

where r is the ~~ratio~~ fraction we're supposed to find out.

Ideal rocket eqⁿ:

$$\Delta V = I_{sp} \cdot g_0 \cdot \ln(MR)$$

$$\therefore \quad \Delta v = I_{sp}\, g_0 \ln\left(\frac{1}{1-r}\right)$$

$$\therefore \quad \ln\left(\frac{1}{1-r}\right) = \frac{\Delta v}{I_{sp}\, g_0}$$

$$= \frac{7600}{400 \times 9.8}$$

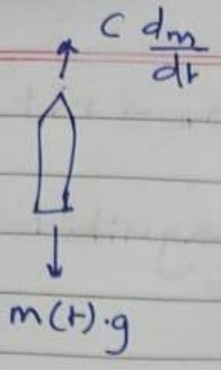
$$= 1.94$$

$$\therefore \quad \frac{1}{1-r} = 6.95$$

$$\therefore \quad r = 1 - \frac{1}{6.95}$$

$$= \boxed{0.856}$$

2.



The net force acting on the rocket in +z dir^n would be

$$\left(c\frac{dm}{dt} - mg\right)$$

The acc. would be

$$\frac{c}{m}\frac{dm}{dt} - g, \quad \text{thus}$$

$$v(t) = \int\left(\frac{c}{m}\frac{dm}{dt} - g\right)dt$$

$$= c\ln\left(\frac{m(t)}{m_0}\right) - gt + k$$

Putting $t=0$, we get $v=0$, thus $k=0$.

$$\therefore v(t) = c\ln\left(\frac{m(t)}{m_0}\right) - gt$$

$$= \boxed{c(\ln\mu) - gt}$$

$$z(t) = \int v(t)\,dt$$

$$= \int \left(c\ln(m(t)) - c\ln m_0 - gt\right)dt$$

We assume $\frac{dm}{dt}$ to be constant, which is the case in solid (and liquid) propellant rockets.

PTO

Thus $m(t) = m_0 - \dot{m}t$

(Here $\dot{m}$ is the magnitude of the flow rate)

Note that

$$\int \ln(a-bx)\,dx = -\frac{a}{b}\ln(a-bx) + x\ln(a-bx) - x + d.$$

Thus

$$z(t) = c\left(-\frac{m_0}{\dot{m}}\ln(m_0-\dot{m}t) + t\ln(m_0-\dot{m}t) - t\right) - c(\ln m_0)t - \frac{gt^2}{2} + \frac{cm_0\ln m_0}{\dot{m}}$$

Now note that

$$m_0 - \dot{m}t = m(t) = \mu m_0$$

and

$$\dot{m} = \cancel{m(t) - m_0} \frac{m_0 - m(t)}{t}$$

$$= \frac{m_0 - \mu m_0}{t}$$

$$= \frac{m_0}{t}(1-\mu).$$

Thus $$\frac{m_0}{\dot{m}} = \frac{m_0}{\frac{m_0(1-\mu)}{t}}$$

$$= \frac{t}{1-\mu}.$$

Putting these expressions in one eq^n for $z(t)$,

$$z(t) = \boxed{c\left(t\ln(\mu m_0) - t - \frac{t}{1-\mu}\ln(\mu m_0)\right) - ct\ln m_0 - \frac{gt^2}{2} + \frac{ct\ln m_0}{1-\mu}}$$

We haven't been given the ratio of ~~masses~~ of propellant and dry mass, thus the total 'powered-flight' time would ~~be~~ have an upper limit, given by

$$t_b = \frac{m_0}{\dot{m}} = \frac{m_0 c}{F_{thrust}} = \boxed{\frac{c}{gn}}$$

Note that $$I_{sp} = \frac{F_{thrust}}{\dot{m} g}$$

Thus $$n = \frac{F_{thrust}}{m_0 g} = \frac{F_{thrust}}{\dot{m} g t_b} = \frac{I_{sp}}{t_b}.$$

i.e. the ratio of specific impulse and burning time.