

Assignment 1

2) Weight = mg

$$F_{\text{Total}} = F_{\text{Thrust}} - mg$$

$$\Rightarrow m \frac{dv}{dt} = -u \frac{dm}{dt} - mg$$

$$\frac{dv}{dt} = -\frac{u}{m} \frac{dm}{dt} - g$$

$$\int_u^v dv = -u \int_{m_0}^m \frac{dm}{m} - g \int_0^t dt$$

$$\Rightarrow v - u = -u \ln \frac{m}{m_0} - gt = \Delta u$$

$$\Rightarrow u(t) = -u \ln \frac{m}{m_0} - gt = \frac{dv}{dt}$$

$$\Rightarrow \int_0^y dy = u(t) dt = -u \int_0^{t_b} \ln \frac{m}{m_0} dt - g \int_0^{t_b} dt$$

$t_b = \text{burnout time}$

$$\Rightarrow \boxed{y = -u \ln \frac{m}{m_0} t_b - g t_b^2} \quad \text{Altitude of rocket at } t_b$$

At burnout $v=0$

$$\text{So, } 0 - u^2(t) = -2gh$$

$$\Rightarrow h = \frac{u^2(t)}{2g}$$

$$\Rightarrow h = \frac{(-gt_b - u \ln \frac{m}{m_0})^2}{2g}$$

• Max altitude = $y + h$

$$= -u \ln \frac{m}{m_0} t_b - g t_b^2 + \frac{(-gt_b - u \ln \frac{m}{m_0})^2}{2g}$$