~~Q18~~ Q8

$$m_0 = m_r + m_p + m_{payload}$$

$$1 = \frac{m_r}{m_0} + \frac{m_p}{m_0} + \frac{m_{payload}}{m_0}$$

$$1 = X_r + X_p + X_{payload}$$

where X_r is the structural fraction

X_p - propellant fraction

$X_{payload}$ - payload fraction

$\therefore$ ~~d = structural~~

define $d = \dfrac{m_{payload}}{m_r + m_p} = \dfrac{m_{payload}}{m_0 - m_{payload}}$

for single stage

$$X_{PL} = \frac{m_{PL}}{m_0}$$

so $d = \dfrac{m_{PL}}{m_0 - m_{PL}} = \dfrac{1}{\frac{m_0}{m_{PL}} - 1} = \dfrac{X_{PL}}{1 - X_{PL}}$

for double stage

let mass of stage 1 $\Rightarrow m_0 = m_{01}$

payload of stage 1 is equal to mass of stage 2

$\therefore d_1 = \dfrac{m_{02}}{m_{01} - m_{02}} = \dfrac{m_{02}}{m_0 - m_{02}}$

and $d_2 = \dfrac{m_{PL}}{m_{02} - m_{PL}}$

let two stages are similar $\boxed{d_1 = d_2}$

$$\frac{m_{02}}{m_0 - m_{02}} = \frac{m_{PL}}{m_{02} - m_{PL}} \Rightarrow \quad m_{02} = \sqrt{m_0}\,\sqrt{m_{PL}}$$

$$m_0 = \frac{m_{PL}}{x_{PL}}$$

$$m_{02} = \frac{m_{PL}}{\sqrt{x_{PL}}}$$

Now $\cancel{d_1 = d_2} = \dfrac{m_{02}}{\cancel{m_0} - m_{02}} = \dfrac{\dfrac{m_{PL}}{\sqrt{x_{PL}}}}{\quad}$

$$\Rightarrow \quad d_{2\,stage} = \frac{m_{PL}}{m_{02} - m_{PL}} = \frac{m_{PL}}{\dfrac{m_{PL}}{\sqrt{x_{PL}}} - m_{PL}} = \frac{\sqrt{x_{PL}}}{1 - \sqrt{x_{PL}}}$$

$$d_{2\text{-}stage} = \frac{\sqrt{x_{PL}}}{1 - \sqrt{x_{PL}}}$$

$$d_{1\text{-}stage} = \cancel{\sqrt{8}}\ \frac{x_{PL}}{1 - x_{PL}}$$

We can observe $d_{2\text{-}stage} > d_{1\text{-}stage} \Rightarrow$

payload capacity of 2 stage rocket is more that 1-stage rocket

ii) $\Delta V = I g_0 \ln\left(\dfrac{m_0}{m_f}\right)$

$$\frac{8}{4.5 \times \cancel{9.8}} = \ln \frac{m_0}{m_f} \qquad \frac{m_0}{m_f} = \frac{m_{PL} + m_{remp}}{m_{PL} + m_{s}}$$

$$= 5.87$$

Q2

- $F_{thrust} = -c\frac{dm}{dt}$

$F_{net} = -c\frac{dm}{dt} - mg = m\frac{dv}{dt}$

$-c\,dm - mg\,dt = m\,dv$

$-\frac{c}{m}dm - g\,dt = dv$

$\cancel{-cd}$

$-c\ln\left|\frac{m_f}{m_0}\right| - gt = v(t) - v_0$

$v(t) = v_0 + c\ln\left|\frac{m_0}{m_f}\right| - gt$

$$\boxed{v(t) = v_0 - c\ln|u| - gt}$$

Now let the burning rate of fuel is constant

$k = -\frac{dm}{dt}$

$\int_0^{t_b} k\,dt = -\int_{m_0}^{m'(t)} dm$

$\cancel{kt = -m(t) + m_0}$

$+kt_b = -m' + m_0$

$$\boxed{k = \frac{m_0 - m'}{t_b}}$$

where t_b is the fuel burning time or the time at which it is placed in the orbit

for $t < t_b$

$\int_0^t k\,dt = -\int_{m_0}^{m(t)} dm$

$\Rightarrow kt = -m(t) + m_0$

$(m_0 - m')\frac{t}{t_b} - m_0 = -m(t)$

m'

$$m(t) = m_0 - (m_0 - m')\frac{t}{t_b}$$

Now $$h = \int_0^t u\,dt$$

$$h = \int_0^t \left(-c \ln \frac{m(t)}{m_0} - gt\right) dt$$

$$h = -c \int_0^t \ln\left(\frac{m_0 - (m_0 - m')\frac{t}{t_b}}{m_0}\right) - g\int_0^t t\,dt$$

$$h = \left[-ct \ln\left(m_0 - (m_0 - m')\frac{t}{t_b}\right) + ct + \frac{c m_0 t_b}{m_0 - m'} \ln\left(m_0 - (m_0 - m')\frac{t}{t_b}\right) - \frac{gt^2}{2} \right]_0^t + ct \ln m_0$$

$$h = -ct \ln\left(m_0 - (m_0 - m')\frac{t}{t_b}\right) + ct - \frac{gt^2}{2} + ct \ln m_0$$
$$+ \frac{c m_0 t_b}{m_0 - m'} \ln\left|m_0 - (m_0 - m')\frac{t}{t_b}\right|$$
$$- \frac{c m_0 t_b}{m_0 - m'} \ln|m_0| + ct \ln m_0$$

$$h = -ct \ln[m(t)] + ct - \frac{gt^2}{2} + \frac{c m_0 t_b}{m_0 - m'} \ln|m(t)| - \frac{c m_0 t_b}{m_0 - m'} \ln|m_0|$$

$$h = -ct \ln|m(t)| + ct - \frac{gt^2}{2} + \frac{c m_0 t_b}{m_0 - m'} \ln\left|\frac{m(t)}{m_0}\right|$$

$$\boxed{h = -ct \ln|\mu m_0| + ct - \frac{gt^2}{2} + \frac{c m_0 t_b}{m_0 - m'} \ln|\mu| + ct \ln m_0}$$

$$h = -ct\ln|u| + ct - \frac{gt^2}{2} + \frac{c m_0 t_b}{m_0 - m'}\ln|u|$$

$$h = -c\ln|u|\left[t - \frac{m_0 t_b}{m_0 - m'}\right] + ct - \frac{gt^2}{2}$$

$$h = \frac{-c t_b \ln|u|}{m_0 - m'}\left[(m_0 - m')\frac{t}{t_b} - m_0\right] + ct - \frac{gt^2}{2}$$

$$h = \frac{-c t_b}{m_0 - m'}\ln|u|\left[m(t)\right] + ct - \frac{gt^2}{2}$$

$$h = \frac{-c t_b \ln|u|}{(m_0 - m')} \times u m_0 + ct - \frac{gt^2}{2}$$

$$\boxed{h = \frac{-c t_b \ln|u| \times u}{\left(1 - \frac{m'}{m_0}\right)} + ct - \frac{gt^2}{2}}$$

final expression

Now lets verify this for $t = t_b$

where $m(t) = m'$

i.e. $u = \frac{m'}{m_0}$

$$h_b = \frac{-c t_b \ln\left|\frac{m'}{m_0}\right|}{1 - \frac{m'}{m_0}} \times \frac{m'}{m_0} + c t_b - \frac{gt^2}{2}$$

$$h_b = \frac{-c t_b \ln\left|\frac{m'}{m_0}\right|}{\frac{m_0 - m'}{m'}} + c t_b - \frac{gt^2}{2}$$

$$\boxed{h_b = \frac{-ct_b \ln\left|\frac{m'}{m_0}\right|}{1-\frac{m'}{m_0}} + ct_b - \frac{gt_b^2}{2}}$$

which is the ~~sea~~ exact eqn for burnout altitude that we had calculated in the previous assignment

- total time of flight

= time to reach orbit and time to return back to earth

$\Rightarrow$

$$v(t) = v_0 - c\ln|\mu| - gt$$

for simplicity consider at $t=0 \Rightarrow v_0 = 0$

$$v(t) = -c\ln\left|\frac{m(t)}{m_0}\right| - gt$$

- $\eta = \frac{\text{Thrust}}{m_0 g}$ $\propto$ acceleration of the aircraft

so therefor if η is high then acceleration of the aircraft will be high and viceversa

for $\eta > 1$ the flight will be more easier because lift generated is very high

Q//

$$\Delta u = I_{sp}\, g \log\left(\frac{m_o}{m_f}\right)$$

$$7.6\times10^{3} = 400\times9.8 \log\frac{m_o}{m_f}$$

$$\log\frac{m_o}{m_f} = 1.938$$

$$\log\left(\frac{m_{prop} + m_{payload} + m_{rocket}}{m_{payload} + m_{rockets}}\right) = 1.98$$

$$\log\left(1 + \frac{m_{prop}}{m_{payload} + m_{rocket}}\right) = 1.938$$

$$1 + \frac{m_{prop}}{m_{payload} + m_{rocket}} = e^{1.938}$$

$$\frac{m_{prop}}{m_{payload} + m_{rocket}} = \frac{5.94}{1}$$

$$\frac{m_{prop}}{m_{payload} + m_{rocket} + m_{prop}} = \frac{5.94}{5.94+1} = \frac{5.94}{6.94}$$

$$= 0.85$$

Q4 $\Delta v = I g_0 \ln\left(\frac{m_0(1-d)}{m_{PL}} \right)$

$\Delta v = I g_0 \ln(m_0(1-d)) - I g_0 \ln m_{PL}$

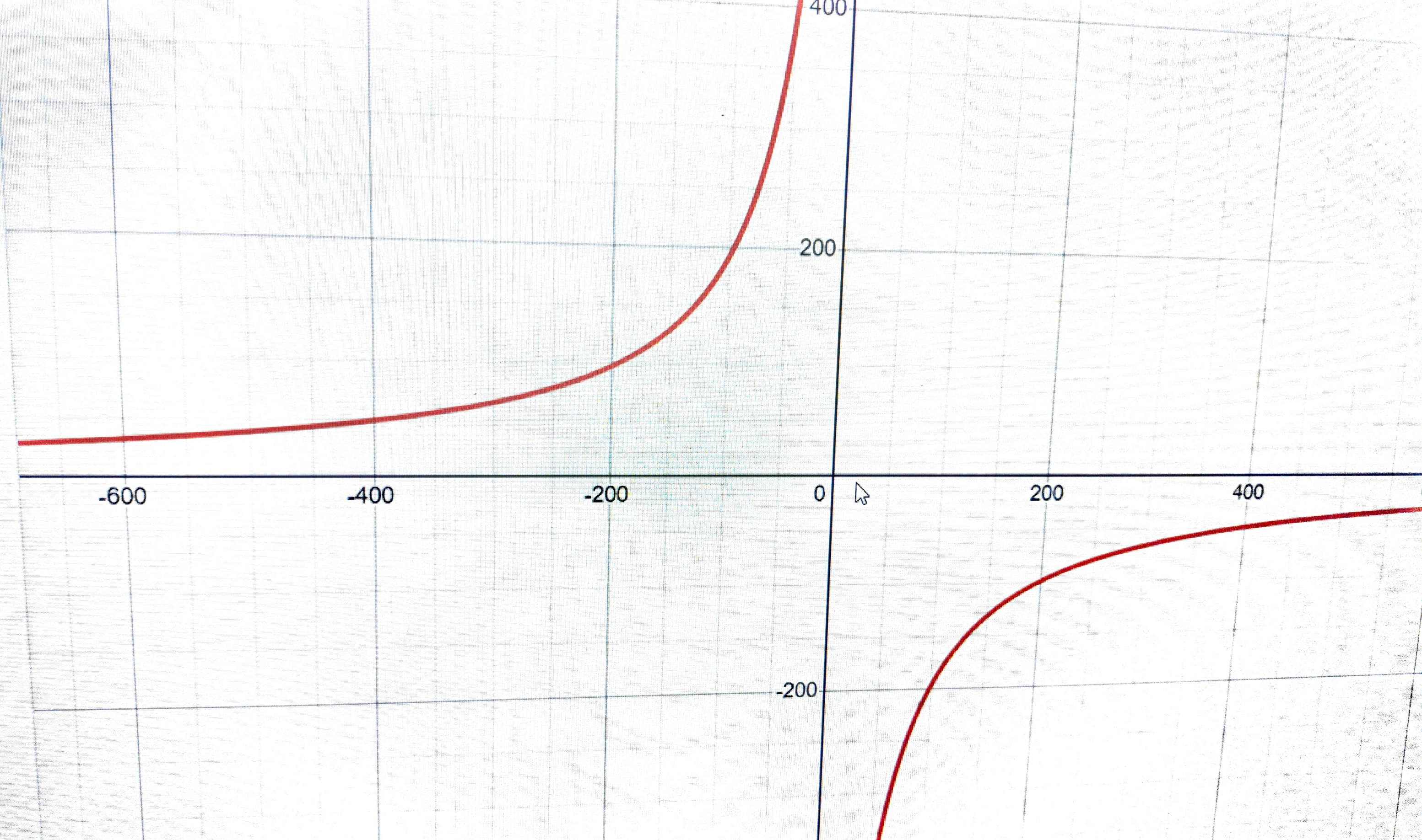
i) $\Delta v \Rightarrow 8000 = 350 \times 10 \left(\ln(m_0(1-d)) - \ln 2000 \right)$

$\frac{80}{35} + \ln 2000 = \ln(m_0(1-d))$

$2.28 + 7.6 = \ln(m_0(1-d))$

$9.88 = \ln(m_0(1-d))$

$m_0(1-d) = 19535.72$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-200
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