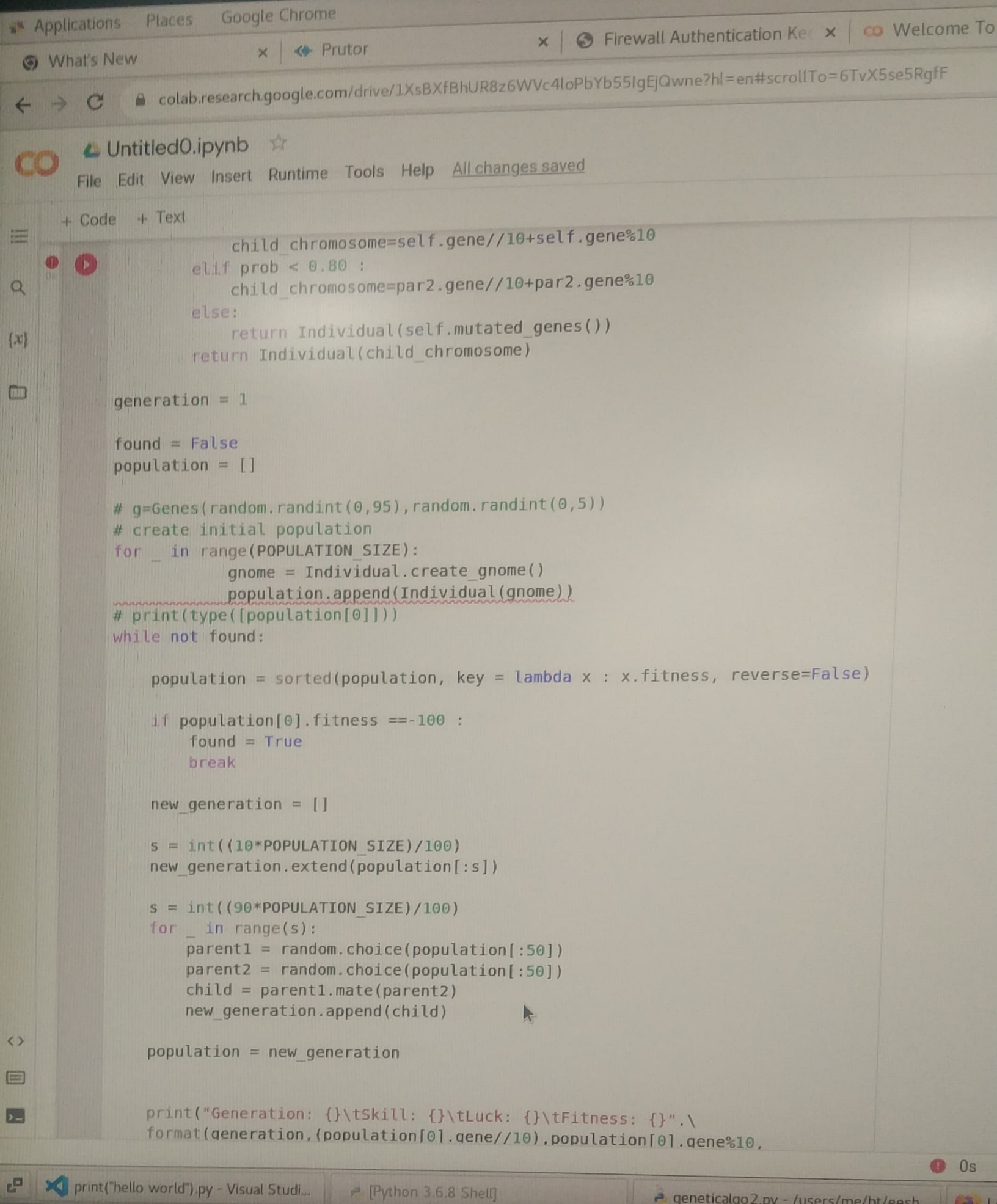


Initially $V_{P/S}$ was in ~~the~~ another direction but when it comes to range of Jupiter's gravity it changes its speed as well as direction.

So here we can apply momentum conservation. The satellite and Jupiter and also energy conservation since only gravity is acting here.

∴ Here $\vec{V}_{\text{new}} = \vec{V}_{P/S_{in}} + \vec{V}_{J/S}$

∴ $|\vec{V}_{\text{new}}| - |\vec{V}_0| = 16 \text{ km/s}$



for L_3

$$\frac{G M_2}{(r+r)^2} - \frac{h^2}{(2-r)^2} = \frac{G M_2}{(2-r)^2} \quad \text{--- (1)}$$

$$L_3 = -1.5000187 \times 10^{11} \quad L_3 = -R/1 + \frac{S}{L_2}$$

Substituting L_3 in eq (1)

$$eq (1) = 0$$

$\Rightarrow L_3$ satisfies eq (1)

$$= R/1 + \frac{S M_2}{L_2}$$

Putting
giving $L_1 = R \left(1 + \left(\frac{m_E m_S}{3} \right)^{1/3} \right)$

Putting L_1, m_E, m_S, R in eqⁿ L_1 and checking whether
 L_1 satisfies eqⁿ or not

$L_1 = 1.5 - 1.5 \times 10^{11} \text{ km}$

Putting it in eqⁿ

eqⁿ ① = 0

$\Rightarrow L_1$ satisfies eqⁿ ①

Similarly for L_2

eqⁿ ②

$$\frac{G M_S}{(x-r)^2} + \frac{G M_E}{(r-x)^2} - \frac{G(M_S + M_E)}{R^2} x = 0$$

$$\Rightarrow -M_S(x-r)^2 R^3 + G M_E (r-x)^2 R^3 = (M_S + M_E) x (x-r)^2 R^3$$

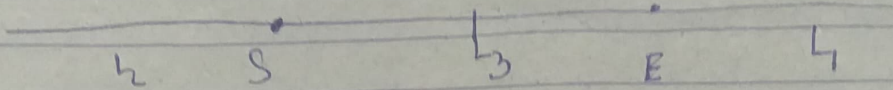
$$\Rightarrow \text{for } L_2 = 1.48 \times 10^{11} \text{ km} \quad L_2 = \left(1 - \left(\frac{m_E}{3m_S} \right)^{1/3} \right) R$$

Putting L_2 in eqⁿ ②

L_2 in eqⁿ ② = 0

$\Rightarrow L_2$ satisfies eqⁿ ②

Q2 For point L



∴ for an equi-potential region
 $V = \text{constant}$

$$\Rightarrow \frac{dV}{dr} = 0 \Rightarrow F = 0$$

$$\Rightarrow \cancel{G} (F_G)_S + (F_G)_E = F_{\text{centrifugal force}} = F_c$$

For L1 region

$$\frac{G M_S}{(x + r_1)^2} + \frac{G M_E}{(x - r_2)^2} = \frac{G (M_S + M_E)}{(\cancel{x} R_{SE})^3} x$$

$$r_1 = \cancel{MR}. \text{ Distance of sun from COM} = \frac{M_E}{M_S + M_E} R$$

$$r_2 = \text{Distance of earth from COM} = \frac{M_S}{M_S + M_E} R$$

Putting the value of M_E , M_S and R .

$$G M_S (x - r_1)^{-2} R^3 + G M_E (x + r_2)^{-2} R^3 = G (M_S + M_E) x$$

$$\Rightarrow M_S (x - r_1)^{-2} + M_E (x + r_2)^{-2} R^3 = (M_S + M_E) x$$

⓪

$$\Rightarrow \Delta v = \sqrt{\frac{GM}{r}} \left(1 - \sqrt{\frac{2r_2}{r_1+r_2}} \right)$$

$$\Delta v_{total} = \Delta v_1 + \Delta v_2$$

$$\Rightarrow \Delta v = \sqrt{\frac{GM}{r}} \left(\sqrt{\frac{2r_2}{r_1+r_2}} - 1 \right) + \sqrt{\frac{GM}{r}} \left(1 - \sqrt{\frac{2r_1}{r_1+r_2}} \right)$$

On putting ~~at~~ ~~in~~ value for transfer b/w geostationary orbit and graveyard orbit

$$r_1 = 36035800 \text{ km} = 42200 \text{ km}$$

$$r_2 = 36050000 \text{ km} = 42450 \text{ km}$$

$$\Rightarrow (\Delta v_{total})_{gy} = \Delta v_1 + \Delta v_2 = 0.07 \text{ m/s}$$

Similarly on putting values for transfer b/w geostationary orbit and Low earth orbit

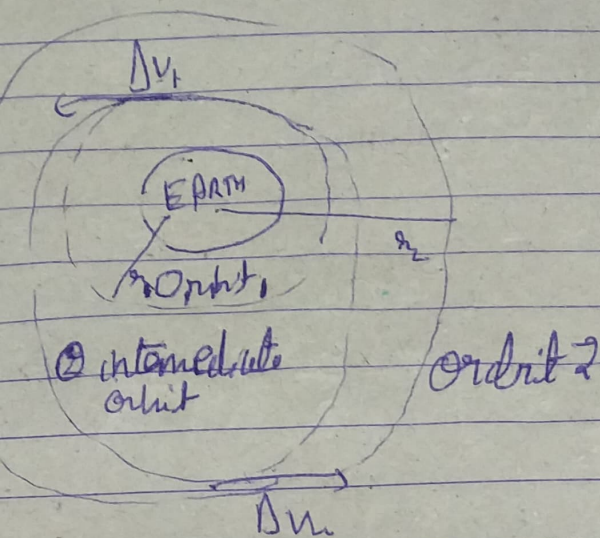
$$r_1 = 35800 \text{ km} = 42200$$

$$r_2 = 3400 \text{ km} = 6700$$

$$(\Delta v_{total})_{LEO} = \Delta v_1 + \Delta v_2 = -3856 \text{ m/s}$$

$$\therefore (\Delta v_{total})_{gy} > (\Delta v_{total})_{LEO}$$

\Rightarrow we would prefer transfer to graveyard orbit.



Transferring from orbit-1 to intermediate orbit

$$\frac{-GMEm}{2r_1} + \frac{1}{2}mv^2 = \frac{-GMEm}{r_1+r_2}$$

$$\Rightarrow 2 \left(\frac{GM_e}{2r_1} - \frac{GM_e}{r_1+r_2} \right) = v^2$$

$$\sqrt{\frac{GM_e}{r_1}} \cdot \sqrt{\frac{r_2}{r_1+r_2}} = v_1$$

$$\Rightarrow \Delta v_1 = \sqrt{\frac{GM_e}{r_1} \left(\frac{r_2}{r_1+r_2} - 1 \right)}$$

for transferring from orbit intermediate orbit to orbit

$$\frac{-GMEm}{2r_2} + \frac{1}{2}mv^2 = \frac{-GMEm}{r_1+r_2}$$

$$\Rightarrow v_2 = \sqrt{\frac{GM_e}{r_2} \left(\frac{r_1}{r_1+r_2} - 1 \right)}$$