(1)

$$\Delta v = I_{sp}\, g_0 \ln\left[1 + \frac{M_{prop}}{M_{pay} + M_{rocket}}\right]$$

$M_i = M_{prop} + M_{pay} + M_{rocket}$

$M_f = M_{pay} + M_{rocket}$.

$$7.6 \times 10^3 = -400 \times 9.8 \times \ln\left(\frac{M_i - M_p}{M_i}\right)$$

$$1 - \frac{M_p}{M_0} = 0.144$$

$M_p = M_{propellant}$

$$\frac{M_p}{M_0} = 0.856 \checkmark$$

(2)

$$F_{thrust} = -c\frac{dm}{dt}.$$

$$-c\frac{dm}{dt} - mg = m\frac{dv}{dt}.$$

$$-c\,dm - mg\,dt = m\,dv$$

$$-\frac{c}{m}\,dm - g\,dt = dv.$$

$$-c\ln m\Big|_{m_0}^{m_f} - gt = v(t) - v_0.$$

$\mu = \frac{m(t)}{m_0}$

$$\boxed{v(t) = v_0 - c\ln|\mu| - gt}$$ *

~~v_0=~~

Now let burning fuel rate is constant.

$$a = -\frac{dm}{dt}$$

$$\int a\,dt = -\int dm$$

$$a = \frac{m_0 - m^f}{t_b}$$

where $t_b \rightarrow$ fuel burning time

$$\int_0^t \alpha\, dt = -\int_{m_0}^{m(t)} dm \qquad m' = \text{final mass.}$$

$$\alpha t = -m(t) + m_0$$

$$-m(t) = (m_0 - m')\frac{t}{t_b} - m_0$$

$$m(t) = m_0 - (m_0 - m')\frac{t}{t_b}$$

displacement of rocket $z(t)$

$$z(t) = \int_0^t v\, dt$$

$$= \int_0^t \left(-c \ln\frac{(m(t))}{m_0} - gt\right) dt$$

$$z(t) = -c\int_0^t \ln\left(\frac{m_0 - (m_0 - m')\frac{t}{t_b}}{m_0}\right) - g\int_0^t t\, dt$$

after solving this eq^n

$$z(t) = -ct\ln(\mu m_0) + ct - \frac{gt^2}{2} + \frac{cm_0 t_b}{m_0 - m'}\ln|\mu| + ct\ln m_0$$

$$z(t) = -ct\ln|\mu| + ct - \frac{gt^2}{2} + \frac{cm_0 t_b}{m_0 - m'}\ln|\mu|$$

$$\left\{ z(t) = -\frac{ct_b \ln|\mu| \times \mu}{\left(1 - \frac{m'}{m_0}\right)} + ct - \frac{gt^2}{2} \right\} **$$

- total time of flight of the rocket.

let $v_e \rightarrow$ orbital velocity of rocket

$$v_e = v_0 - c \ln|\mu| - gt$$

$$\left[t = \frac{v_0 - v_e - c\ln|\mu|}{g} \right]$$

$v_0 \rightarrow$ launced velocity

4) $\lambda = \dfrac{M_{rocket}}{M_{rocket} + M_{pay}}$ $\rightarrow$ 2000 Kg.

$M_0 \rightarrow$ Initial wet mass

$$M_{rocket} = \frac{2000\lambda}{1-\lambda}$$

$$\Delta V = I_{sp}\, g_0 \ln\left(\frac{M_{prop} + M_{pay} + M_{rocket}}{M_{pay} + M_{rocket}}\right)$$

$$e^{\Delta V / I_{sp} g_0} = \frac{M_0}{M_{pay} + M_{rocket}}$$

$$e^{\Delta V / I_{sp} g_0} = \frac{M_0}{2000 + \frac{2000\lambda}{1-\lambda}}$$

$$e^{\Delta V / I_{sp} g_0} = \frac{M_0(1-\lambda)}{2000}$$

1st) $I_{sp} \rightarrow 350$

$$2000\, e^{\frac{8000}{350 \times 9.8}} = M_0(1-\lambda)$$

$$M_0(1-\lambda) = 20604.4$$

2nd $I_{sp} \rightarrow 400$

$$2000\, e^{\frac{8000}{400 \times 9.8}} = M_0(1-\lambda)$$

$$M_0(1-\lambda) = 15393.77$$

3rd $I_{sp} \rightarrow 450$

$$2000\, e^{\frac{8000}{450 \times 9.8}} = M_0(1-\lambda)$$

$$M_0(1-\lambda) = 12270.599$$

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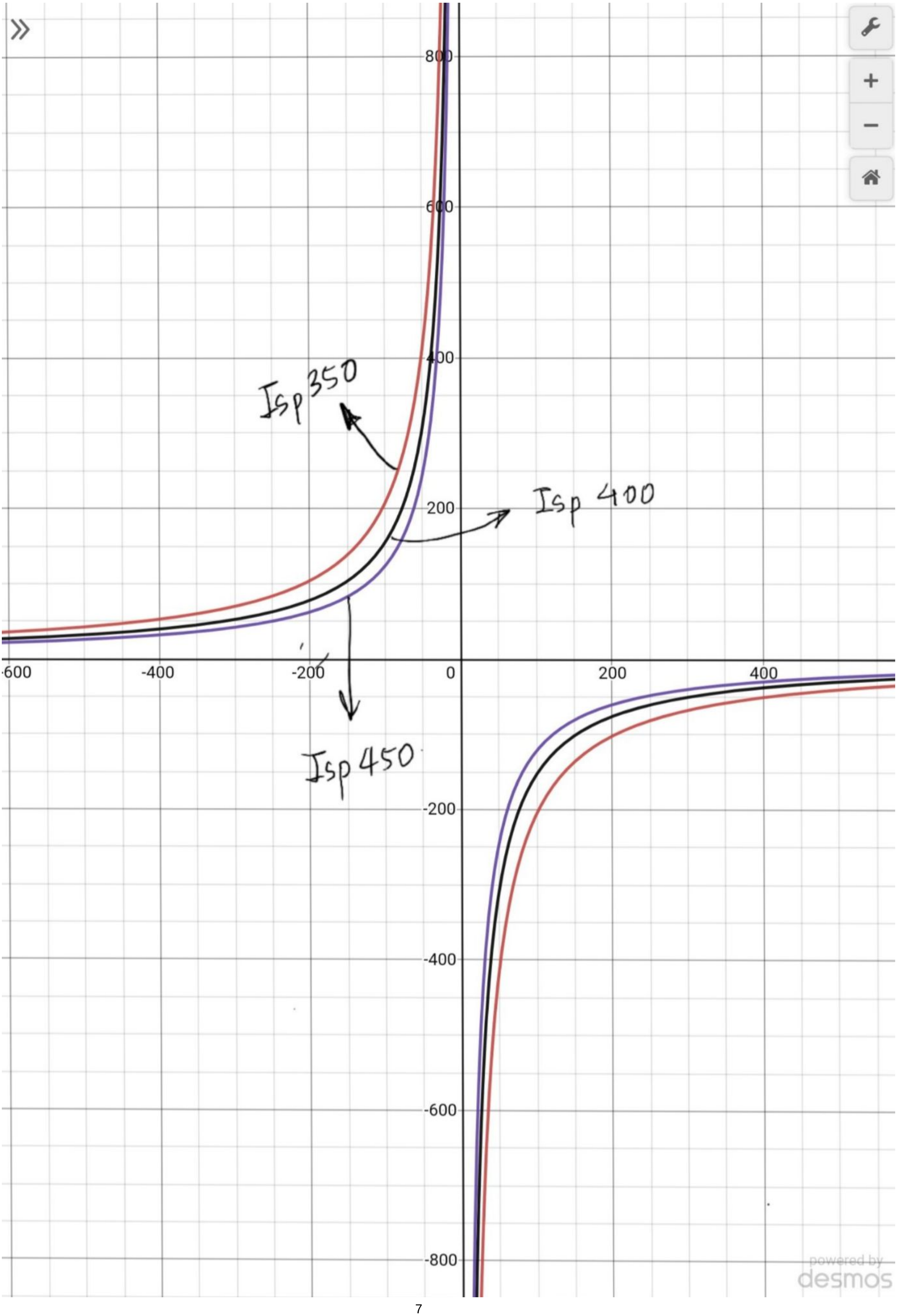
As we increase the value of Isp (Specific Impulse) the slope of the curve decreases for same initial mass.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