Assignment 2

Q1) $\Delta V = 7.6\times10^3 \, m/s = I_{sp} g_0 \ln\left(\frac{M_0}{M_f}\right)$

$$\frac{7.6\times10^3}{400\times9.8} = \ln\left(\frac{M_0}{M_f}\right)$$

$$\frac{M_0}{M_f} = e^{19/9.8}$$

$$\frac{M_0 - M_{prop}}{M_0} = e^{-1.94}$$

$$1 - e^{-1.94} = \frac{M_{prop}}{M_0}$$

$$1 - 0.1437 = \frac{M_{prop}}{M_0}$$

$\therefore$ $$\boxed{0.856 = \frac{M_{prop}}{M_0}}$$

Q4) $\Delta V = I_{sp} g_0 \ln\left(\frac{M_0}{M_f}\right)$

$$e^{\delta/I_{sp}g_0} = \frac{M_0}{M_f} = \frac{M_0}{M_{rock}+M_{pay}} = \frac{M_{rock}}{M_{rock}+M_{pay}} \times \frac{M_0}{M_{rock}}$$

M_0

$$= \lambda \times \frac{M_0}{M_{rock}} = \frac{(1-\lambda)M_0}{2000}$$

$M_{pay} = 2000 \Rightarrow \frac{M_{rock}}{M_{rock}+M_{pay}} = \lambda$

$$M_{rock} = \frac{\lambda}{1-\lambda}(2000)$$

$$\boxed{2000\, e^{\delta/I_{sp}g_0} = (1-\lambda)M_0}$$

~~Q2) (i) $\frac{dv}{dt} = -\frac{c}{m}\frac{dm}{dt} - g_0$~~

Slope is very steep, which implies slight change λ leads to high change in M_0.

Q2 (i) $\frac{dv}{dt} = -\frac{c}{m}\frac{dm}{dt} - g_0$

$v = -\int \frac{c}{m} dm - \int g_0 \, dt$

$$\boxed{v_{(t)} = -c \ln\mu - g_0 t} \quad -①$$

(ii) $\frac{dz}{dt} = -c \ln\mu - g_0 t$

$z = -c\int \ln\mu \, dt - \int g_0 t \, dt$

$v = -v_{ex} \ln\mu$

$z = \frac{c}{v_{ex}} \int v \, dt - \frac{1}{2} g_0 t^2$

$z = \frac{c}{v_{ex}} z - \frac{1}{2} g_0 t^2$

$\frac{1}{2} g_0 t^2 = z\left(\frac{c - v_{ex}}{v_{ex}}\right)$

Also, $v = -v_{ex} \ln\mu$ & $v = -c\ln\mu - g_0 t$

$v_{ex} \ln\mu = c \ln\mu + g_0 t$

$\cancel{\frac{c - v_{ex}}{g_0 t} = \ln\mu}$ $\boxed{(c - v_{ex}) = -\frac{g_0 t}{\ln\mu}}$

$\frac{c\ln\mu + g_0 t}{\ln\mu} = v_{ex}$

$z = \left(\frac{v_{ex}}{c - v_{ex}}\right) \frac{1}{2} g_0 t^2$

$z = \frac{c\ln\mu + g_0 t}{\ln\mu} \times \left(-\frac{\ln\mu}{g_0 t}\right) \times \frac{1}{2} g_0 t^2$

$= \frac{1}{2}(c \ln\mu + g_0 t) t$

$$\boxed{z = \frac{c}{2} t \ln\left(\frac{1}{\mu}\right) - \frac{1}{2} g_0 t^2}$$

(iii) Fuel burnout time, t_b,

$$V(t_b) = -v_{ex} \ln(\mu_f) \qquad \mu_f = \frac{M_{rocket+payload}}{m_0}$$

$$= -c \ln \mu_f - g_0 t_b$$

$$(v_{ex} - c) \ln \mu_f = g_0 t_b$$

$$\frac{(v_{ex} - c) \ln \mu_f}{g_0} = t_b$$

$$Z = \frac{c(\ln \mu_f)^2 (c - v_{ex})}{2 g_0} - \frac{1}{2g_0}\left[(v_{ex} - c) \ln \mu_f\right]^2$$

$$= \frac{(\ln \mu_f)^2 (c - v_{ex})}{2g_0} \left(c - v_{ex} + c \right)$$

$$Z = \frac{(\ln(\mu_f))^2 (c - v_{ex})(2c - v_{ex})}{2g_0}$$

$$h = \frac{V^2(t_b)}{2g_0} = \frac{v_{ex}^2 (\ln \mu_f)^2}{2g_0}, \quad \frac{V(t_b)}{g_0} = t' \qquad \boxed{t_{1/2} = t' + t_b}$$

$$\sqrt{\frac{2(Z+h)}{g_0}} = t'_{1/2}$$

$$T = t'_{1/2} + t_{1/2}$$

Time of Flight $= T = \sqrt{\frac{(\ln \mu_f)^2}{g_0^2}\left[v_{ex}^2 + c(c - v_{ex})(2c - v_{ex})\right]} + \frac{(v_{ex} - c)\ln \mu_f}{g_0} - \frac{v_{ex} \ln(\mu_f)}{g_0}$

$$\boxed{T = \sqrt{\left(\frac{\ln \mu_f}{g_0}\right)^2 \left[v_{ex}^2 + c(c - v_{ex})(2c - v_{ex})\right]} - \frac{c \ln(\mu_f)}{g_0}}$$

iv) Ratio of Thrust & initial weight, if $n > 1 \Rightarrow$ Accelerating

$n < 1 \Rightarrow$ Weight > Thrust

$n \uparrow \Rightarrow$ High thrust to lift rocket

Q3) (i) $DV = V_{ex} \ln\left(\frac{M_0}{M_f}\right)$

$$\exp\left(\frac{8}{4.5}\right) = \frac{M_0}{M_f}$$

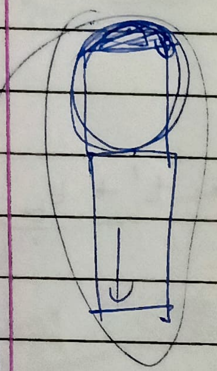
$$\exp\left(-\frac{8}{4.5}\right) = \frac{M_f}{M_0} = \frac{M_{pay}}{M_0(1-\lambda)}$$

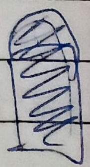
$$M_0(1-\lambda)\exp\left(-\frac{8}{4.5}\right) = M_{pay}$$

~~(ii)~~ $M_0\exp\left(-\frac{8}{4.5}\right) - M_{rocket} = M_{pay}$

(ii) For DSTO,

$\cancel{M_{pay}}$ $M_{o_1} \to$ Total initial Mass.





$$M_{o_2} = M_{o_1}(1-\lambda_1)\exp\left(-\frac{4}{4.5}\right)$$

$$\cancel{M_{pay} = M_{o_2}(1-\lambda_2)\exp\left(-\frac{4}{4.5}\right)}$$

$$\cancel{M_{pay} = M_{o_1}(1-\lambda_1)(1-\lambda_2)\exp\left(-\frac{8}{4.5}\right)}$$

$$\cancel{= M_{o_1}(1-M}$$

$$\cancel{M_{o_2} = (1-}$$

$$\cancel{M_{o_2} = M_o(M}$$

$$\cancel{M_o\exp\left(-\frac{4}{4.5}\right) - M_{rocket(1+2)}}$$

$$\cancel{+}$$

$$\cancel{M_o\exp\left(-\frac{4}{4.5}\right) - M_{rocket_2}}$$

$$M_{o_2} = M_{o(1+2)}\exp\left(-\frac{4}{4.5}\right) - M_{rocket(1)}$$

$$M_{pay} = M_{o_2}\exp\left(-\frac{4}{4.5}\right) - M_{rocket(2)}$$

$$M_{pay} = M_{o(1+2)}\exp\left(\frac{-8}{4.5}\right) - M_{rocket(1)}\, M_{o_2}\exp\left(\frac{-4}{4.5}\right) - M_{rocket(2)}$$

Untitled Graph

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1 $\frac{2000}{1-x}e^{\left(\frac{8000}{350\cdot 9.8}\right)}$

2 $\frac{2000}{1-x}e^{\left(\frac{8000}{400\cdot 9.8}\right)}$

3 $\frac{2000}{1-x}e^{\left(\frac{8000}{450\cdot 9.8}\right)}$

4

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