Mining Investments Analysis

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1 Introduction

In this practical assignment, we analyze the net present value (NPV) for a multimillion mining business using three different cases. Case 1 is 11 periods (years) long and the mill produces approximately 8 million tons (Mt), case 2 is 14 periods long with 6Mt, and case 3 is 20 periods long with 4Mt. The NPV is calculated using a dynamic system, and since the analysis is long-term, Monte Carlo simulation is used to model uncertainty in future gold prices.

2 Model

For the gold price, Geometric Brownian Motion (GBM) is used. The GBM is defined as

$$dS_t = \mu S_t dt + \sigma S_t dW_t, \tag{1}$$

where μ is the trend, σ is the volatility, dW_t is the Wiener process, and S_t is the gold price at time t. Markov chain Monte Carlo (MCMC) is a method used to estimate posterior distributions (Härkönen, 2024). MCMC is used to estimate parameters μ and σ in the GBM. When using GBMs we are interested in log-returns, so using Equation (1) the GBM can be written as

$$d(lnS_t) = (\mu - \frac{1}{2}\sigma^2)dt + \sigma dW_t$$

(Ibe, 2013). After integration on both sides from t to $t + \Delta t$ the log-returns r_t becomes

$$r_t = ln(\frac{S_t}{S_{t-1}}) = (\mu - \frac{1}{2}\sigma^2)\Delta t + \sigma\epsilon_t, \tag{2}$$

where $\epsilon_t \sim \mathcal{N}(0, 1)$ is normally distributed noise. For the prior distribution of the parameters μ and σ, we use uniform distributions $\mu \sim \mathcal{U}(-1, 1)$ and $\sigma \sim \mathcal{U}(0, 10)$. Thus, we only assume the μ to be between -1 and 1, and the σ between 0 and 10. From the posterior probability density function (PDF)

$$p(r_t|\mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} exp(-\frac{(r_t - (\mu - \frac{1}{2}\sigma^2))^2}{2\sigma^2}),$$

can be calculated the negative log-likelihood of the posterior distribution (Nkemnole and Abass, 2019). The negative log-likelihood $\mathcal{L}(\mu, \sigma)$ becomes,

$$\mathcal{L}(\mu, \sigma) = \frac{N}{2} log(2\pi\sigma^2) + \frac{1}{2\sigma^2} \sum_{t=1}^{N} (r_t - (\mu - \frac{1}{2}\sigma^2))^2, \tag{3}$$

where N is the total number of log-returns. Since the GBM is a yearly process, we need to calculate the parameters μ and σ yearly estimates

$$\begin{aligned} \hat{\mu} &= \mu_{posterior} \times 12 \\ \hat{\sigma} &= \sqrt{12} \times \sigma_{posterior} \end{aligned} \tag{4}$$

The MCMC sampling process is done using Delayed Rejection Adaptive Metropolis from MCMCSTAT Matlab package provided by Laine (2018). We also use a 10% burn-in period to eliminate the least accurate estimates at the start of the chain.

In Figure (1) shows the dynamic system used to calculate the NPVs.

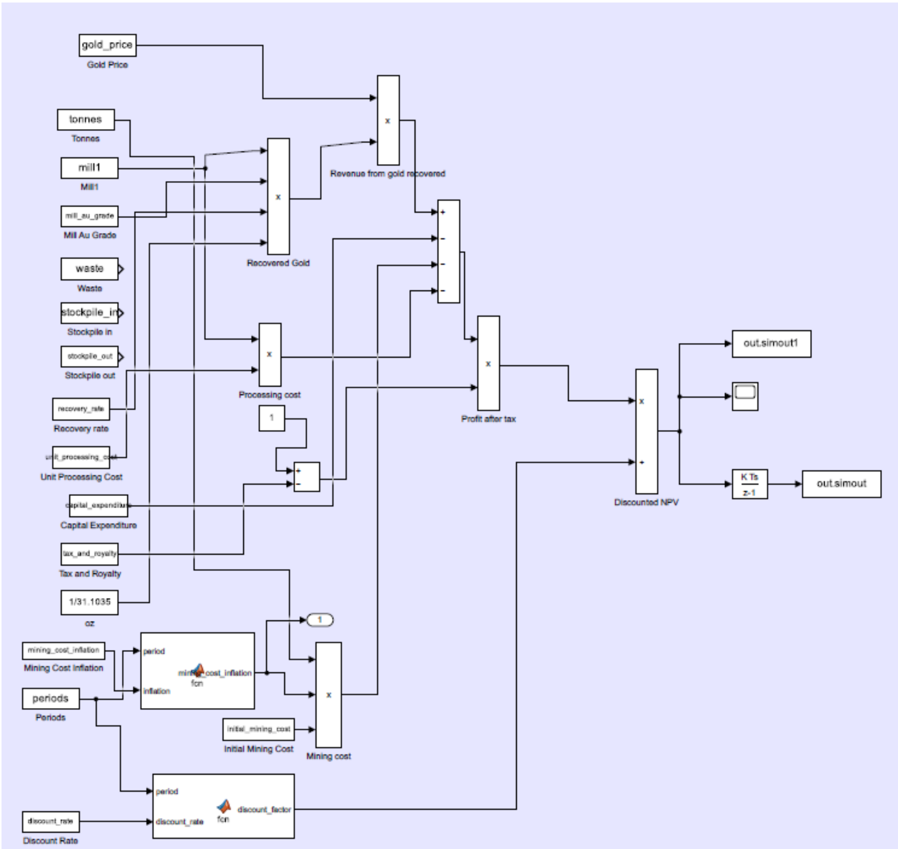

Figure 1: Dynamic system for NPV calculation

The discount factor is calculated as

```
function discount_factor = fcn(period, discount_rate)
    discount_factor = (1+(discount_rate/100))^(period-1);
```

, and the mining cost inflation is calculated as

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function mining_cost_inflation = fcn(period, inflation)
    mining_cost_inflation = (1+(inflation/100))^(period-1);
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3 Results

From Figure (2), it can be seen that both estimates converge, and we get estimated values $\hat{\mu} = 0.1062$, and $\hat{\sigma} = 0.1302$.

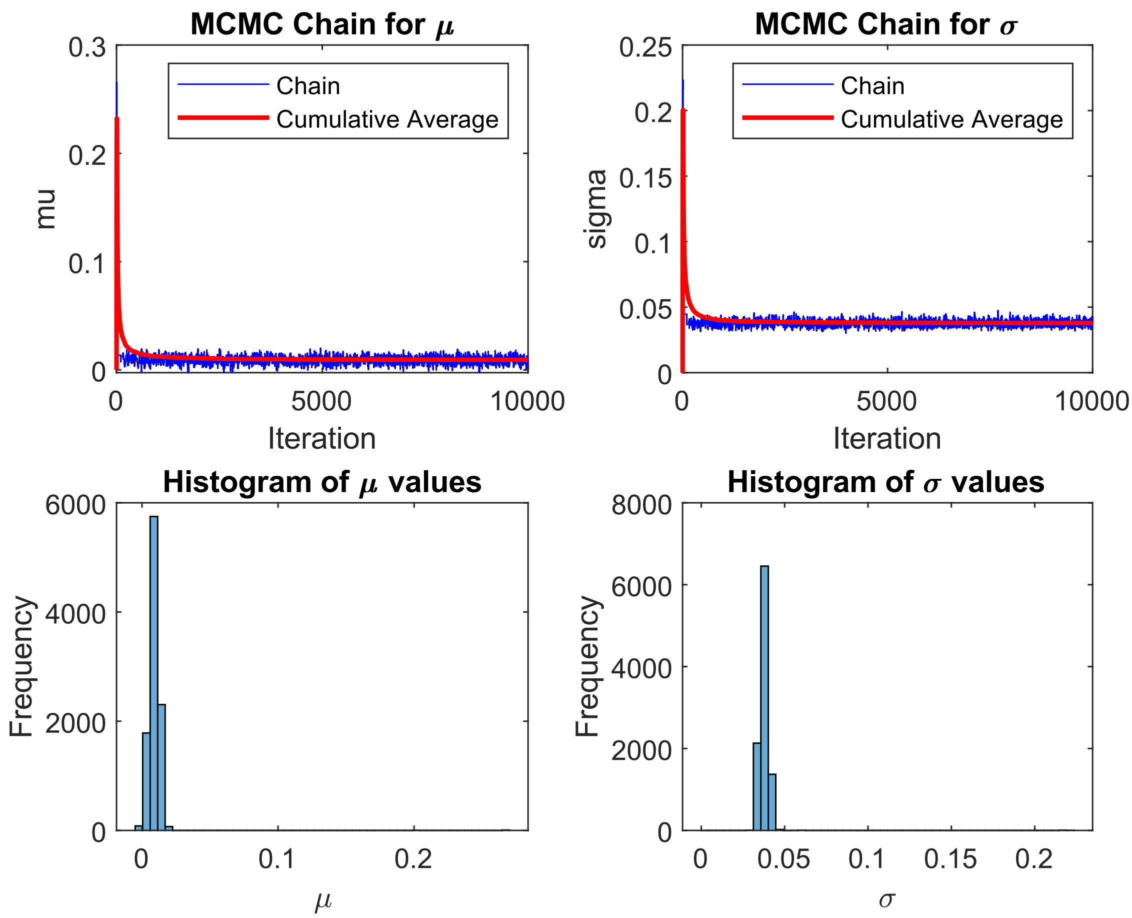

Figure 2: From the top row are the MCMC chains for the estimated parameters and the histograms of the estimated parameter values.

In Figures (3), (4), and (5) are the simulation results from case 1, case 2, and case 3, respectively. The mean value of the NPV is used to tackle the uncertainty in the gold price. The cumulative mean shows how the total NPV has grown during the periods.

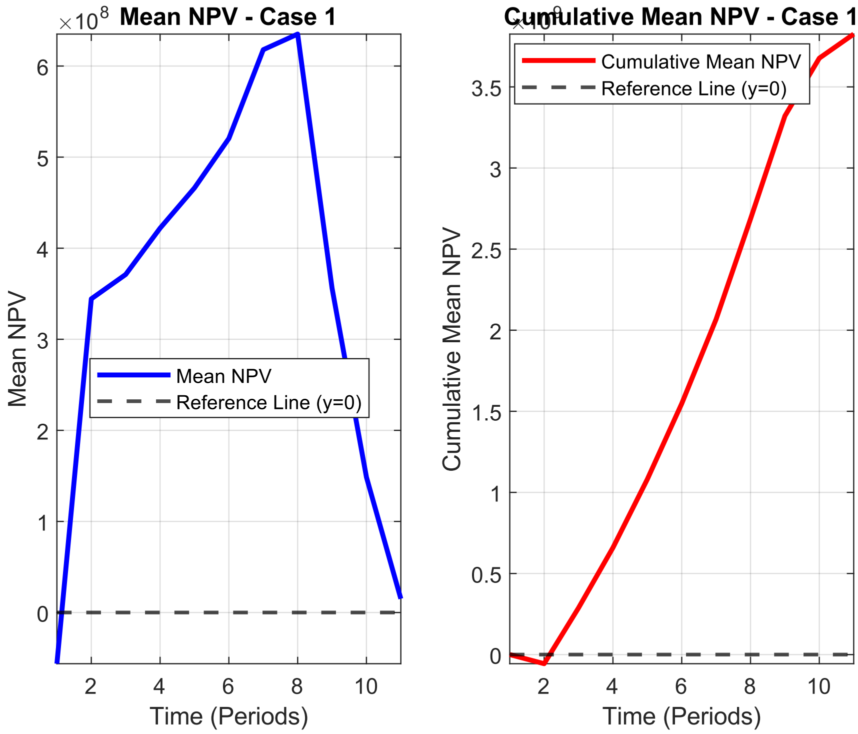

Figure 3: From left to right are the mean NPV values and the cumulative mean NPV values of case 1 simulation.

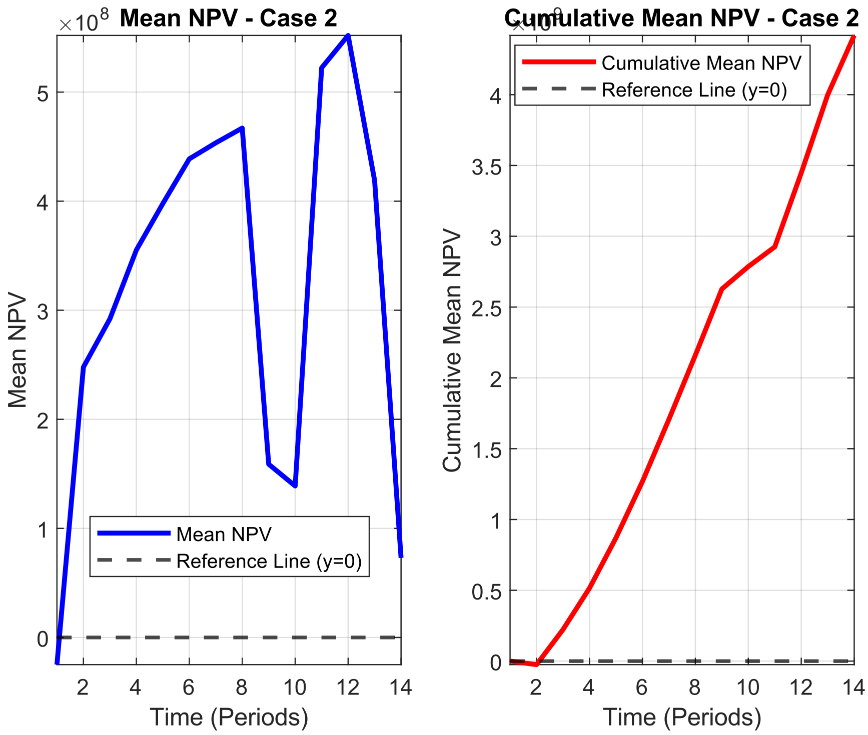

Figure 4: From left to right are the mean NPV values and the cumulative mean NPV values of case 2 simulation.

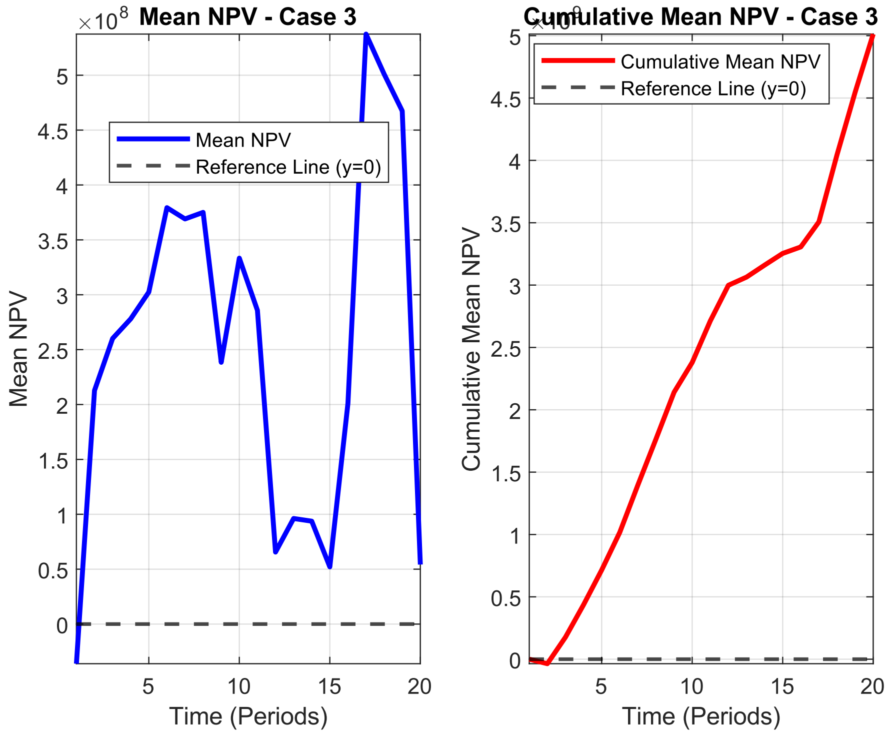

Figure 5: From left to right are the mean NPV values and the cumulative mean NPV values of case 3 simulation.

We can calculate the percentage of negative simulation results to evaluate if the mine should be closed early. After the first year, the case 1 mine is under a 0.1 % probability of a negative NPV result each year until the last year, when the probability is 0.2 %. Case 2 mine has 0.76 % and 1.26 % probability of negative NPV result on the 9th and 10th years, respectively. Otherwise, the probability is under 0.03%. After 11 years, the case 3 mine's probability of negative NPV result increases and stays between 1.4% and 7.4% for 4 years.

Years	Case 1 ($)	Case 2 ($)	Case 3 ($)
11	3,825,754,585	2,925,019,110	2,712,538,096
14	-	4,418,213,082	3,160,081,331
20	-	-	5,012,854,957

Table 1: In the table are the cumulative NPV values after a certain number of years.

The worst results come from case 3, where we add 0.05 to the recovery rate each year, and the mine's probability of a negative NPV result decreases to between 1.0% and 5.9% for 4 years. Even after changing the recovery rate to 1 for each year, the probability stays between 0.7 % and 4.4 %. In Figure (6) shows the simulation results with a 0.05 increase to the original value of the recovery rate each year.

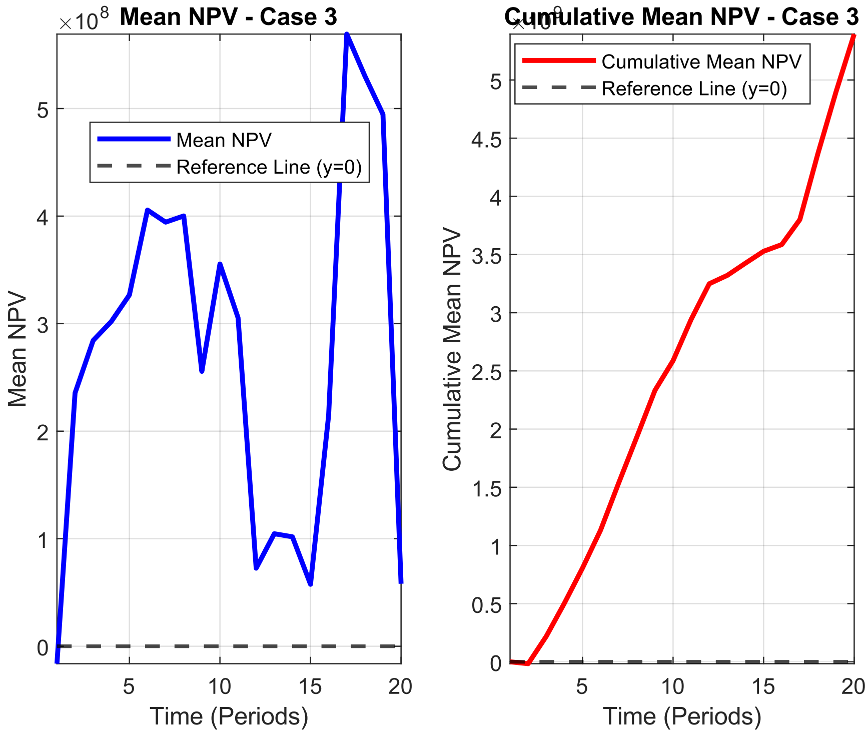

Figure 6: From left to right are the mean NPV values and the cumulative mean NPV values of case 3 simulation with the increased recovery rate.

4 Conclusions

From Table (1), it can be seen that case 1 produces the most money in 11 years. In 14 years, case 2 produces even more money, and in 20 years, case 3 produces the most money of the mines. However, the probability of a negative NPV result is the highest in case 3. Thus, case 1 is the safest mine plan and produces the most money in the 11-year window. In the long term, case 3 mine plan produces the most money, but also has the biggest risk factor. Since, even with a 100% recovery rate, the risk of negative NPV is the biggest among the mine plans.

References

Härkönen, T. (2024). Sequential monte carlo and stochastic processes in spectroscopy.

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