CS 302 Automata Theory Fall 2024

Text :

Introduction to Automata Theory, Languages and Computation

Third edition, Pearson 2006

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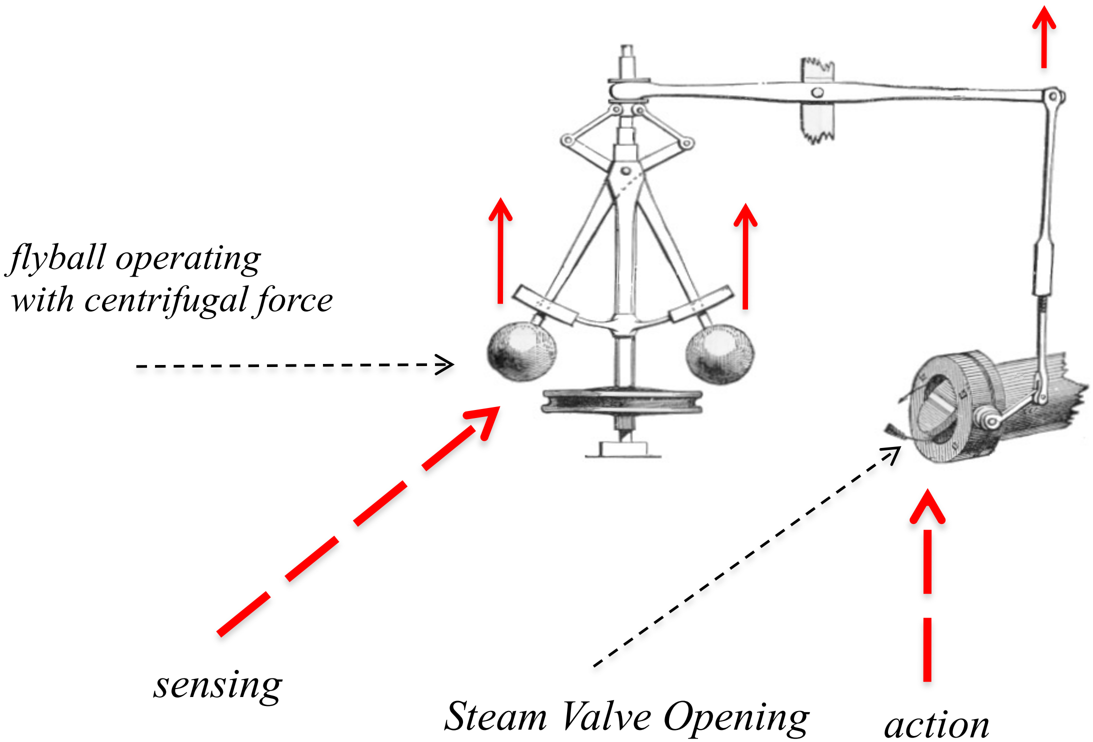
PREMODERN AUTOMATION

James Watt's Governor (Speed regulator) 1788



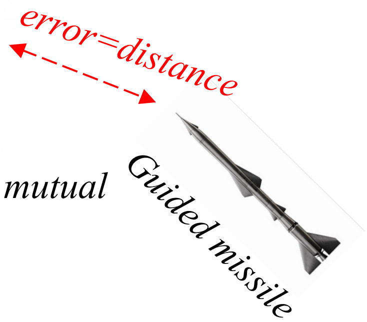
Almost a century later : ***James Clerk Maxwell's famous paper (1868) : On Governors***

First mathematical treatment of stability

Classical Example of Negative Feedback

Automation examples using arithmetic error

1- fast : seconds ; game theoretic



radars to monitor ; mutual cruise control to manipulate error to win the game

2- slow : minutes; temperature control

error = desired – actual (room temperature)

thermostat to monitor ; heater to control

3- super slow : months ; inflation control

error = desired – actual (price levels)

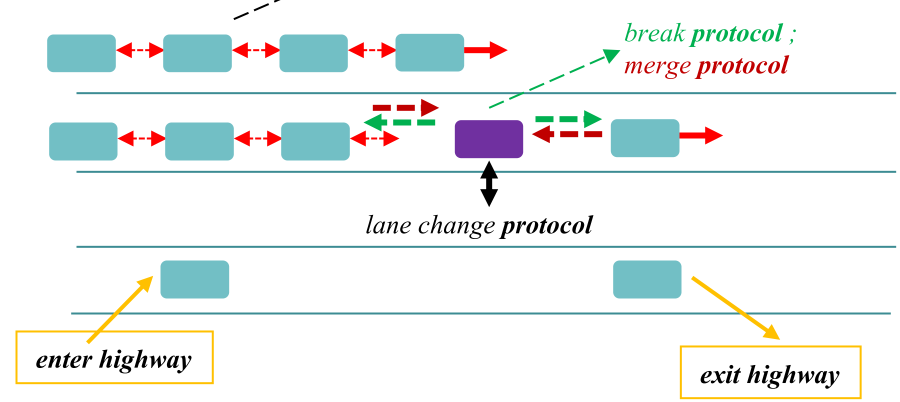
sampling prices to monitor ;

monetary or fiscal policy to control

Automation examples using linguistic techniques

***platoons** of vehicles*

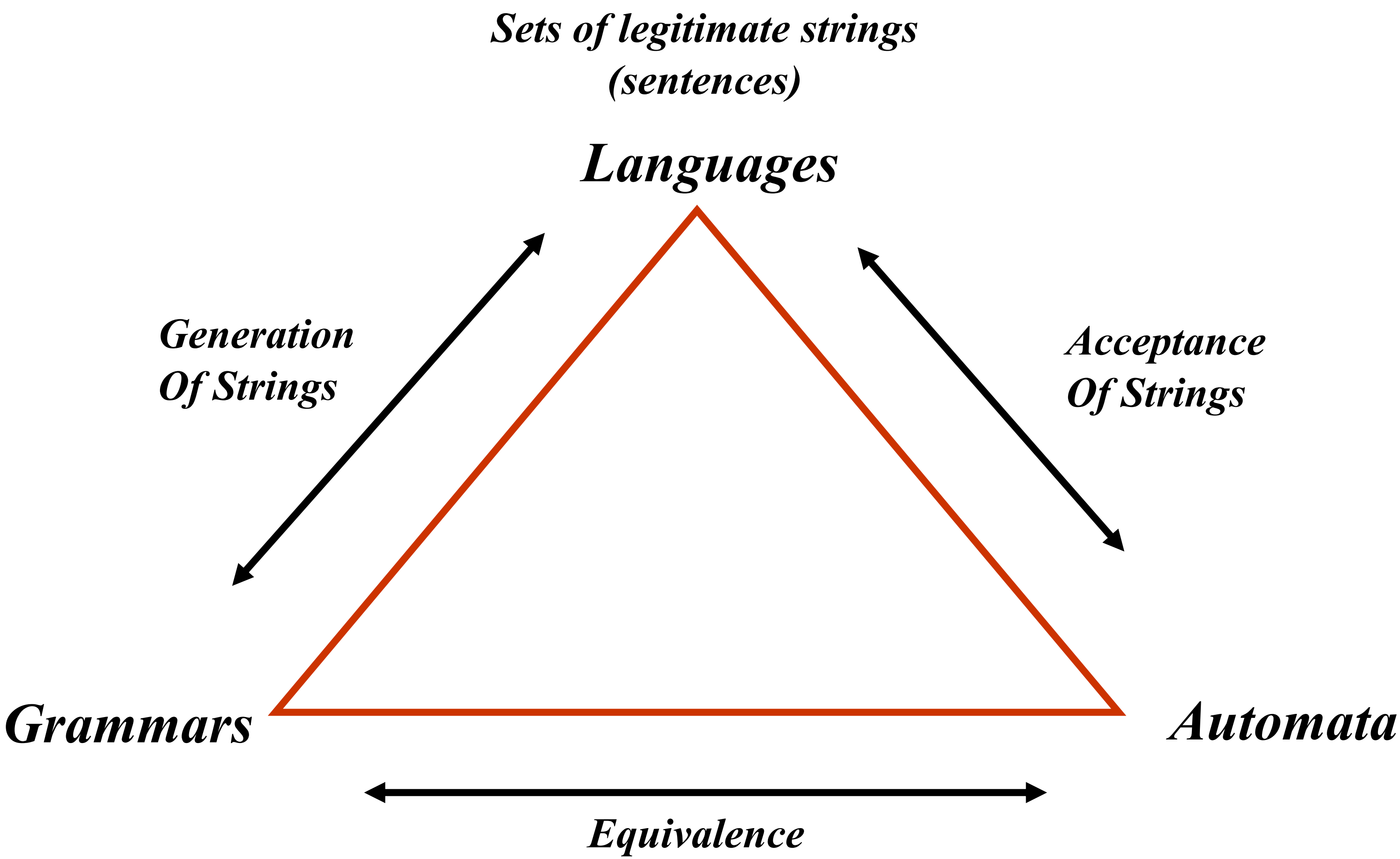
1- automated highways (PATH project)



2 – manouvering in heteregenous environments

monitor shelves in a store and automatically renew commercial items of varying shape and size bringing them from the storage space meanwhile keeping the statistics of both the shelves and the storage area

Topic of the Course → MODERN : LINGUISTIC BASED AUTOMATION



Definition of a Language

(1) A finite set $\boldsymbol{\Sigma}$**,** *called the* ***alphabet*** *set.*

(2) A ***set*** *of strings with elements in* $\boldsymbol{\Sigma}$ *is called a* ***language*** *over* $\boldsymbol{\Sigma}$

Formal Definition of a Language $L \subseteq \Sigma^*$ *where :*

emptystring

$$\Sigma^* := e \cup \Sigma^+$$

$$\Sigma^+ := \Sigma \cup \Sigma^2 \cup \ldots \cup \Sigma^n \cup \ldots = \cup_{k=1,+\infty} \Sigma^k$$

where *n times*

$$\Sigma^n := \Sigma . \Sigma . \ldots . \Sigma := \{\sigma_1 \sigma_2 \ldots \sigma_n \mid \sigma_j \in \Sigma,\ j=1,2,\ldots,n\}$$

$=$ *set of all strings with elements in* Σ *of length* $\boldsymbol{n}$

String Operations and Terminology

String concatenation notation : $\boldsymbol{u.v} \in \Sigma^*$ *where* $\boldsymbol{u} \in \Sigma^*, \boldsymbol{v} \in \Sigma^*$

A nonempty string $\boldsymbol{v} \in \Sigma^+$ *is called :*

a substring of $\boldsymbol{s}$ *if* $\boldsymbol{s} = \boldsymbol{u.v.w}$ *, where* $\boldsymbol{u,w} \in \Sigma^*$

a prefix of $\boldsymbol{s}$ *if* $\boldsymbol{s} = \boldsymbol{v.w}$ *, where* $\boldsymbol{w} \in \Sigma^*$

a postfix of $\boldsymbol{s}$ *if* $\boldsymbol{s} = \boldsymbol{u.v}$ *where* $\boldsymbol{u} \in \Sigma^*$

$\boldsymbol{s^n}$ *denotes a string* $\boldsymbol{s}$ *concatenated with itself* $\boldsymbol{n}$ *times*

length($\boldsymbol{s}$*)= # characters in* $\boldsymbol{s} = |\boldsymbol{s}|$

How can we define a language ***L*** *?*

$$L := (s \in \Sigma^* \mid F(s))$$

A logical condition on **s** *;* ***F*** *is a truth valued function*

There is a problem in this definition :

Is ***F*** *computable ?*

What does ***computable*** *mean ?*

Two ***possibly computable*** *tools are introduced :*

(1) Grammars ; (2) Automata

Examples of languages :

(1) Natural Languages *; strings of characters from a keyboard that are* ***syntactically*** *correct in* ***English*** *e.g.* ***The chair ate the elephant*** *is a syntactically correct string (sentence) in the English language ;* ***The ate elephant chair the*** *is not syntactically correct !*

(2) Formal *(Computer)* ***Languages :*** *i.e. strings of symbols that are syntactically correct ; such as a C++ program for which the compiler does not give a syntax error*

Simple examples of formal languages

(3) *Well-defined expressions. eg. arithmetic expressions using the operators + and × and nonnegative integers*

Operation not specified

$(32+560)\times(3+54\times 7)$ *is correct whereas* $32(+056\times 7(3))$ *is not correct*

integer cannot start with a 0

(4) Problems : ***encoding ?*** *of* ***decision ?*** *problems*

Examples :

(i) Decision problem :

$$E = \{ (n, m, k) \in Z \times Z \times Z \mid n+m = k ? \} ; E \subseteq Z^3$$

(ii) Encoding of a graph **G** *that solves the decision problem of connectedness !*

Context Free Grammars

$$G = (V, T, P, S)$$

Terminal symbols $T \subseteq V$

Start symbol $S \in V\text{-}T$

Finite set of symbols

Finite set of production rules
$P \subseteq (V\text{-}T) \times V^*$

Example : generation of integers in decimal notation

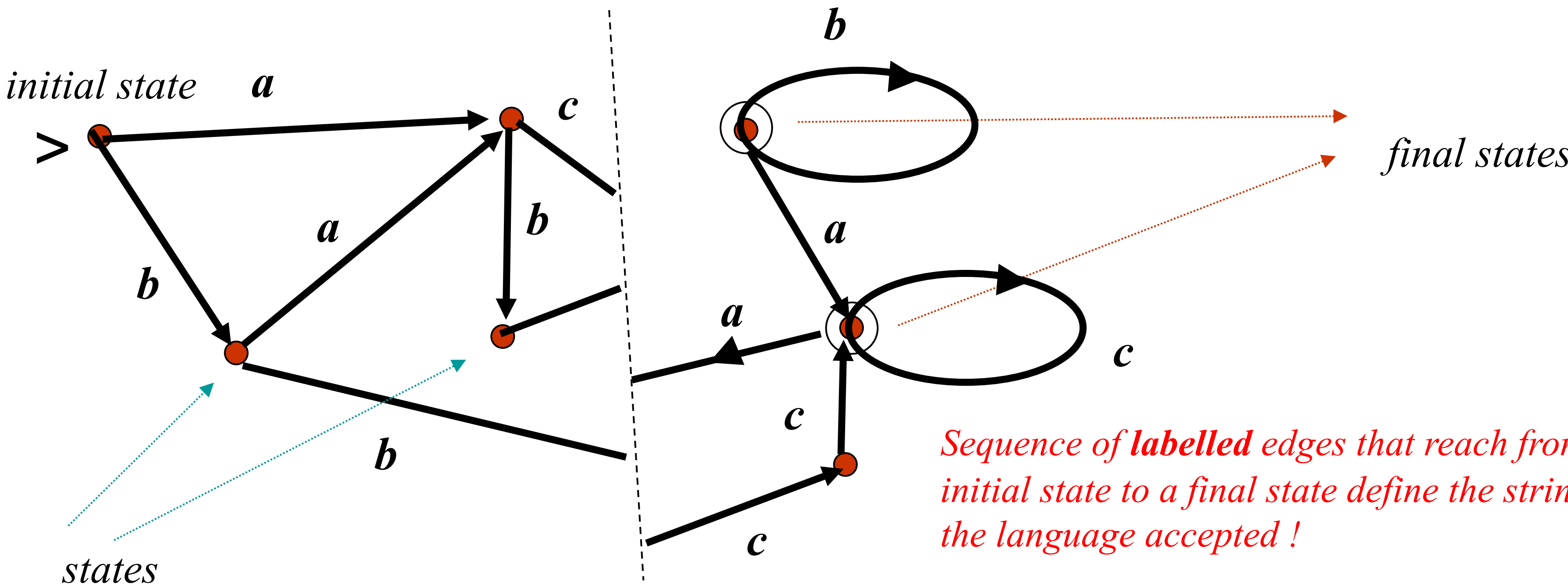
-108970 and +67 and 564 are legitimate strings but 034

and 1-3 and 90+1 are not!

Find a grammar that generates integers in decimal notation !

(Deterministic Finite) Automata over a set $\Sigma = \{a, b, c, \ldots\}$

Simple way to define is by directed graphs where edges are labeled by symbols in Σ

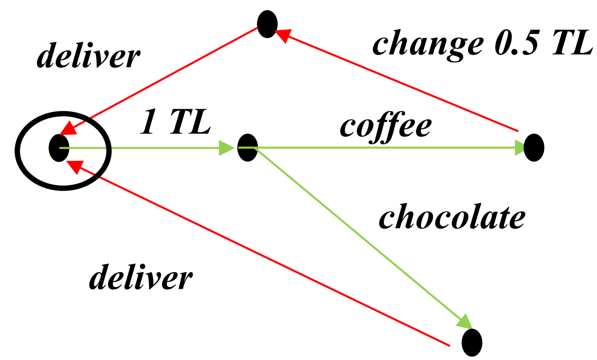


*Sequence of **labelled** edges that reach from the initial state to a final state define the strings of the language accepted !*

*In **CS 302** we use **Automata** as a language acceptor (or generator)*

But there are other uses in modeling real systems :

(1) Coffee & Chocolate Machine



(2) Digital Integrated Circuits with input and output