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EEE321-02

EEE321 LAB WORK 5

1)

My school number is 22002840. Therefore $N_1 = 4$ and $N_2 = 8$. This implies $M_1 = 6$ and $M_2 = 10$. According to the specifications of the lab the passband of the signal is between $\left(\frac{\pi}{10}, \frac{\pi}{6}\right)$ and $\left(-\frac{\pi}{6}, -\frac{\pi}{10}\right)$. Reason of this is because of the following property of the z-transform.

$$X\left(e^{jw}\right) = \sum_{k=-\infty}^{\infty} h[k] \cdot e^{-jwk}$$

$$X^*\left(e^{jw}\right) = \sum_{k=-\infty}^{\infty} h^*[k] \cdot e^{jwk}$$

$$X^*\left(e^{-jw}\right) = \sum_{k=-\infty}^{\infty} h^*[k] \cdot e^{-jwk} = X^*\left(e^{-jw}\right) = \sum_{k=-\infty}^{\infty} h[k] \cdot e^{-jwk}$$

$$X\left(e^{-jw}\right) = X^*\left(e^{-jw}\right)$$

This means $X\left(e^{jw}\right)$ is conjugate symmetric and therefore all the zeros and the pass-band must be symmetric for x axis. After finding this result another specification of the system is satisfied which is to have $L = N_1 + 11 = 15$. This shows that there must be 15 zeros of the z-transform and they must be on the unit circle for maximum suppression of the areas rather than passband. Zeros are distributed uniformly to the stopband areas of the discrete time Fourier transform plot. After that h[n] is found by using the digital calculations of MATLAB. Following is the method used for finding the impulse response of the system where $b_k's$ are zeros of the system.

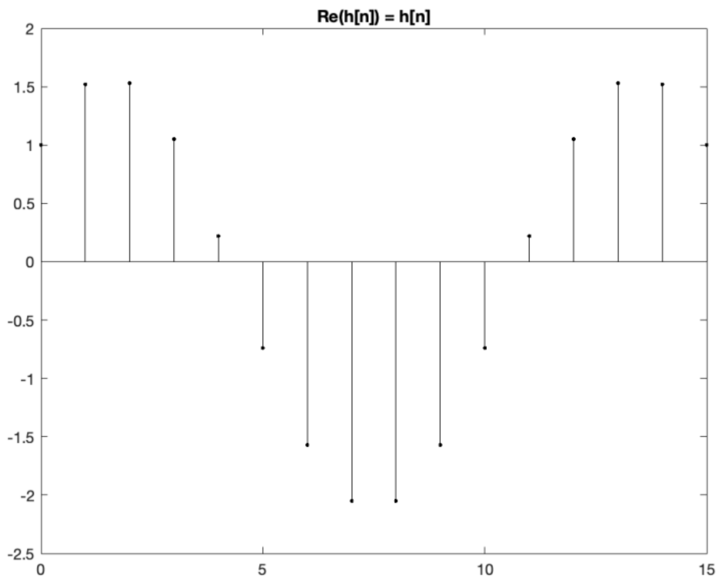
$$\frac{\prod_{k=0}^{k=14}(z - b_k)}{z^{15}} = H(z) = \sum_{k=-\infty}^{\infty} h[k] \cdot z^{-k} = \sum_{k=0}^{14} h[k] \cdot z^{-k}$$

Therefore h[n] of this signal can be found by using the coefficients of the Z-transform of the impulse response of the designed system. h[n] is found as a number array which can be inspected from Fig.1.

```
h =

  Columns 1 through 9

    1.0000    1.5195    1.5297    1.0513    0.2199   -0.7398   -1.5705   -2.0499   -2.0499

  Columns 10 through 16

   -1.5705   -0.7398    0.2199    1.0513    1.5297    1.5195    1.0000
```

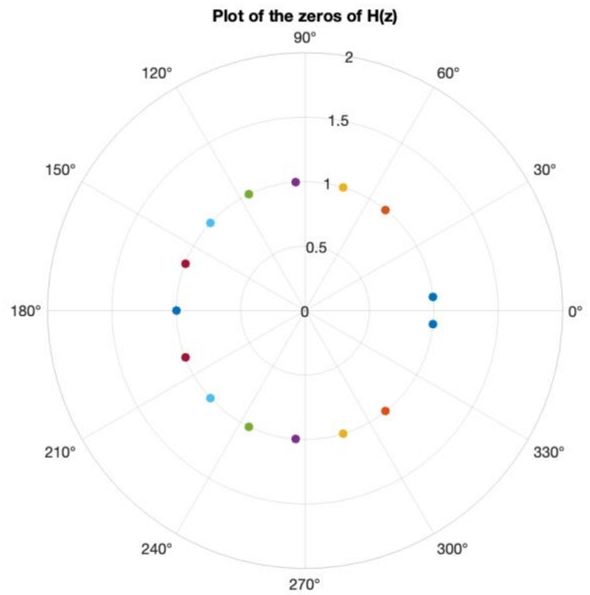
(Fig.1 Array values of h[n] by using MATLAB)

The h[n] is purely real as specified in the lab manual. Plot of h[n] can also be seen below in Fig. 2.



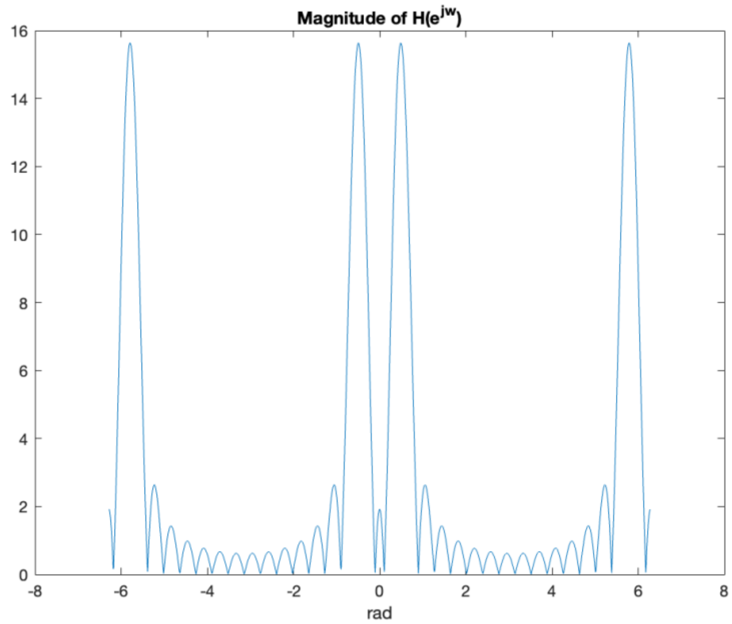
(Fig.2 Plot of h[n])

Zeros found to satisfy the specification of the bandpass filter can be seen in Fig.3.

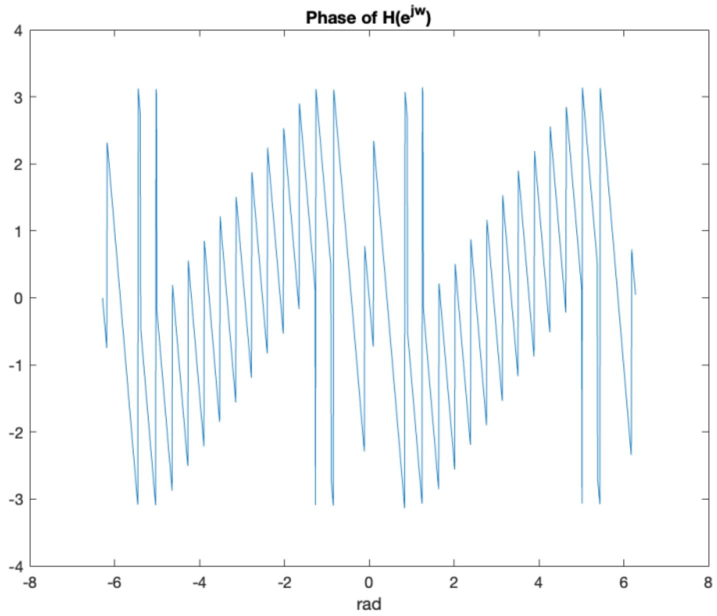


(Fig.3 Zeros of H(z) chosen for the best suppression of stopband region)

Also plot of $\left|H(e^{jw})\right|$ and $\sphericalangle H(e^{jw})$ can be found in Fig.4 and Fig.5 respectively.



(Fig.4 Magnitude of Discrete Time Fourier Transform of h[n])



(Fig.5 Phase of Discrete Time Fourier Transform of h[n])

2)

Let,

$$x_a(t) = \cos(\alpha t^2) \ \ for\ t \in (-\infty, \infty)\ and\ \alpha = 1$$

where $x_a(t)$ becomes.

$$x_a(t) = \cos(t^2) \ \ for\ t \in (-\infty, \infty)$$

Then first it will be examined if this signal is periodic or not. Let,

$$x_a(t) = x_a(t+T) = \cos((t+T)^2) = \cos(t^2)$$
$$\cos(t^2 + 2tT + T^2) = \cos(t^2 + 2\pi k)$$
$$T^2 + 2tT = 2\pi k\ where\ k\ is\ an\ integer$$

Solutions to this equality are the following:

$$T = \frac{-2t \pm \sqrt{4t^2 - 8\pi k}}{2}$$

This result shows that T is a function of time, since there is not a constant fundamental frequency of the signal and there is not an integer k that holds this equation for some non-integer time values therefore this signal cannot be considered as periodic in its analog form. The instantaneous frequency of this signal can be found by using the following equation:

$$\frac{d(t^2)}{dt} = 2t.$$

is the instantaneous frequency of $x_a(t)$.

This signal will be sampled with sampling period $T_s = \sqrt{\frac{\pi}{512}} \cong 0.078\ s$. The sampled signal $x_1[n]$ will be found with the following:

$$x_1[n] = x_a(n \cdot T_s) = \cos\left[\frac{n^2\pi}{512}\right].$$

Periodicity check for this function can be done by examining the validity of the following equality:

$$x_a(n \cdot T_s) = \cos\left[\frac{n^2\pi}{512} + 2\pi k\right] = x_a\big((n+N) \cdot T_s\big) = \cos\left[\frac{(n+N)^2\pi}{512}\right] \;\; for\ k \in integers.$$

$$\frac{\pi(2nN + N^2)}{512} = 2\pi k$$

$$N^2 + 2nN = 1024k$$

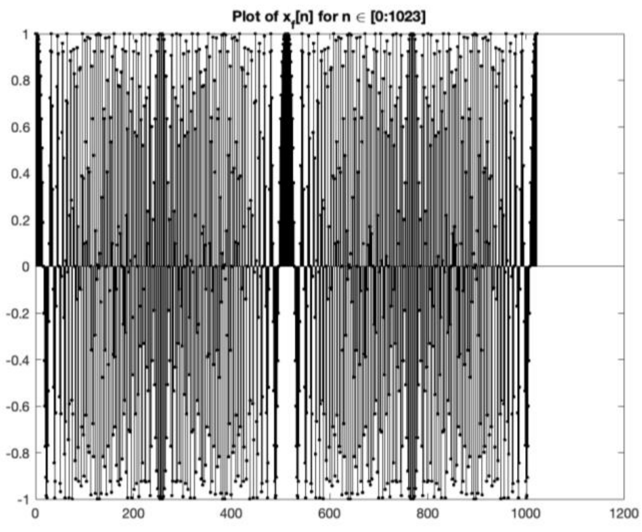
$$N + 2n = \frac{1024k}{N}$$

$$n = \frac{512k}{N} - \frac{N}{2}$$

There must be an integer k for any n for periodicity so, N must be at least 512. Then

$$n = k - 256$$

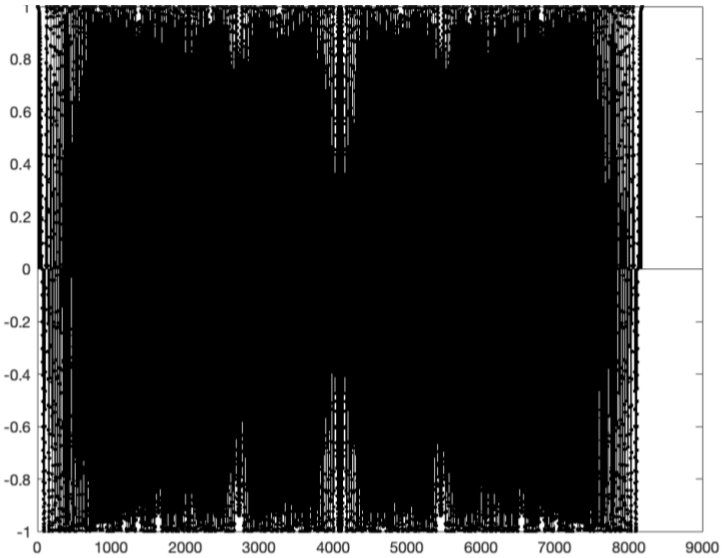
Therefore, there is always an integer k that satisfies this equation for any selected n. Hence it can be stated that the signal is periodic with fundamental period N = 512. Plot of $x_f[n]$ for $n \in [0{:}1023]$ can be seen in Fig.6.



(Fig.6 Plot of $x_f[n] = \cos\left(\frac{\pi n^2}{512}\right)$ for n∈ $[0\!:1023]$)

3)

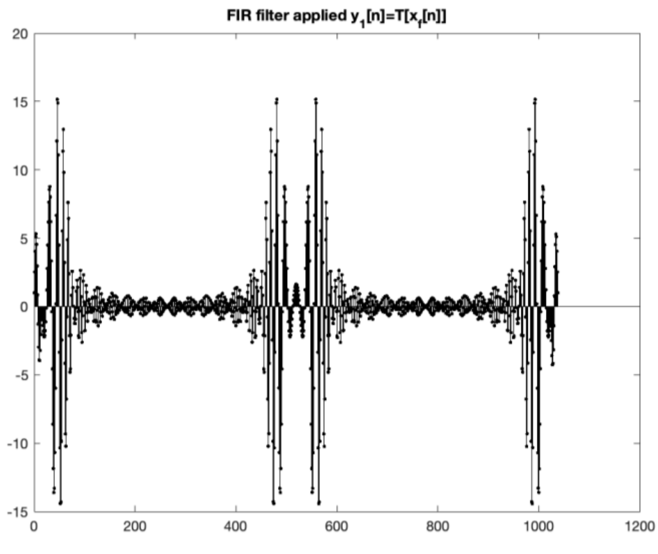
$x_g[n]$ is the discrete time signal where $x_g[n] = \cos(\alpha(n \cdot T_s)^2)$ $where\ n \in [0\!:8192]$ and $T_s = 1000\ rad^{-2}$. Plot of this signal can be seen in Fig.7.



(Fig.7 Plot of $x_g[n] = \cos\left(\frac{\pi n^2}{8192}\right)$ for n∈ $[0\!:8192]$)

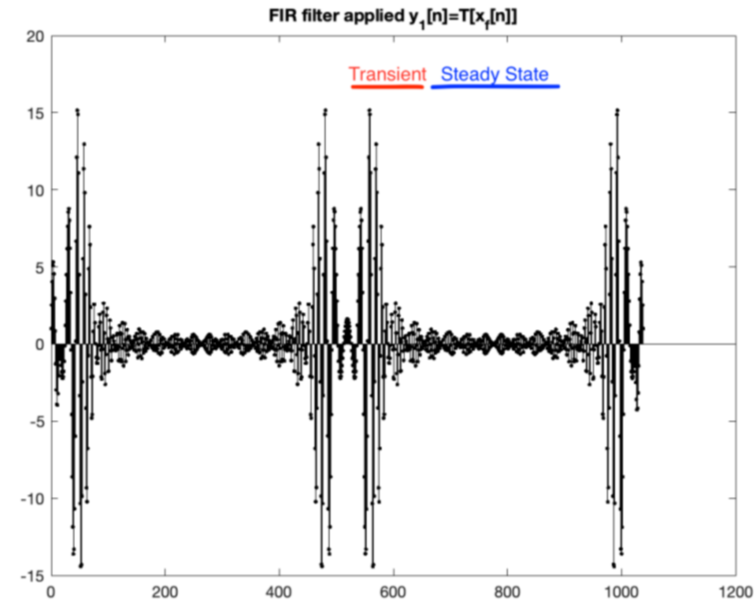
4)

Then the generated $x_f[n]$ signal is convolved with the coefficients of the Z-transform polynomial which is the impulse response of the designed FIR filter. Output of the filter can be seen in Fig.8.



(Fig.8 Output of the FIR filter which has input as $x_f[n]$)

Transient and steady state responses of the output are shown in Fig. 9



(Fig.9 Transient and Steady State intervals of $y_1[n]$)

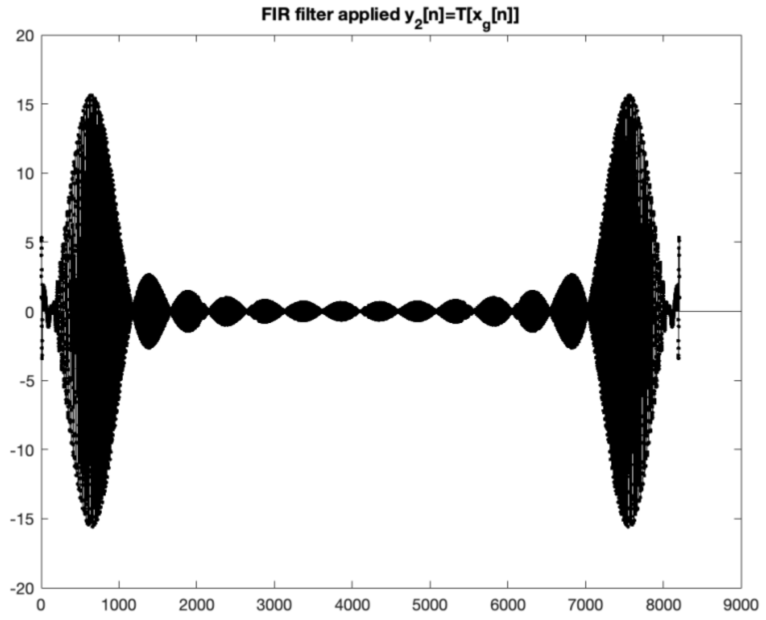
Transient part of the system output is the finite duration where the signal approaches to a bounded value and steady state duration is where the signal has approached a stable value interval.

Output of this system represents the response of the system to the given frequency values because of the nature of the input signal. The chirp signal $cos(\alpha t^2)$ has instantaneous frequency of $2\alpha t$ which is the value of the time instance where the frequency is examined. Therefore, instantaneous frequency of the chirp signal can be exactly the point it is examined for $\alpha = \frac{1}{2}$. The output of the filter by using the chirp signal as input basically gives the behavior of the filter for every time instance t. Therefore, $y_1[n]$ shows the frequency response of the filter.

Resolution of the frequency response of the system is determined by the sampling rate used in the chirp signal. The sampling rate must be chosen as a high value in order to ensure that the frequency response of the filter represented accurately.

Chirp signal is not a stable signal. Therefore, convolving it with another signal may result in divergence, so one must be careful when using chirp signal as the input to the system.

After that the $x_g[n]$ signal is used as input to the system and the output of the filter can be seen in Fig.10.



(Fig.10 Output of the FIR filter which has input as $x_g[n]$)

5)

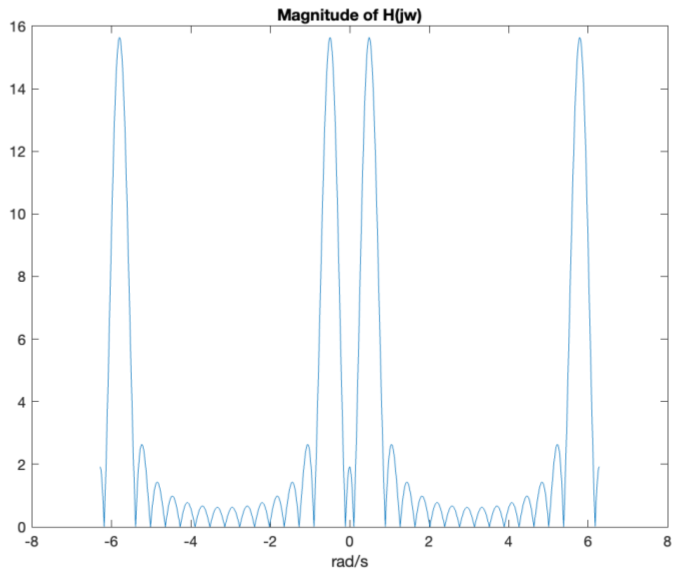
$x_a(t)$ is not a periodic signal but $x_r(t)$ is generated by using a periodic sampling of the signal $x_a(t)$ so, the sounds that are being heard are not the same since one is periodic and the other is not. This can also be understood when listening the audio format of both signals. The original chirp signal has a constant increasing frequency therefore the generated sound gets higher toned through time. But the signal generated by sampling the original signal is periodic therefore the pitch of the sound generated by that signal increases and decreases periodically.

6)

$y_r(t)$ and $y_2[n]$ are similar signals since both are periodic. $y_r(t)$ is generated by using the discrete version of the signal and using a sample rate. Each instance of the discrete signal lasts for the sampling rate and therefore a continuous signal estimation can be generated by this procedure which is also called as interpolation. Both $y_r(t)$ and $y_2[n]$ are signals that has repeated sounds which makes them similar.

7)

Cut-off frequencies of the impulse response of the system that produces $y_r(t)$ from $x_a(t)$ are the same with the designed filter at the first. So, cut-off frequencies are $\frac{\pi}{10}$ and $\frac{\pi}{6}$ and the symmetric of them according to the y axis that are $-\frac{\pi}{10}$ and $-\frac{\pi}{6}$. Those cutoff frequencies can also be seen from the frequency response plot of the analog system which is available as Fig. 11.



(Fig.11 Plot of $|H(jw)|$ for the signal $y_r(t)$)

8)

This filter allows the pass of components of a signal that lay in the bandpass domain of the filter. Therefore, only sounds within a small range of frequency band can be heard from the generated output sound file. This makes the output sound sharper and cleaner since many frequencies are removed from the recording by suppression effect of the filter on the stopbands. Therefore most of the information that the input signal carries remains in the output but there are slight changes in the sound of the signal which result from the properties of the designed system.

MATLAB CODES:

PARTS 1-2-3-4-7

```
%PART 1

% School number is 22002840
% N1 = 4   M1 = 6
% N2 = 8   M2 = 10
% L = 11 + N1 = 15
% Cutoff: pi/10, pi/6
% Causal

z0 = pi/30;
z1 = pi/6 + 1 * ((10*pi/6)/14);
z2 = pi/6 + 2 * ((10*pi/6)/14);
z3 = pi/6 + 3 * ((10*pi/6)/14);
z4 = pi/6 + 4 * ((10*pi/6)/14);
z5 = pi/6 + 5 * ((10*pi/6)/14);
z6 = pi/6 + 6 * ((10*pi/6)/14);
z7 = pi/6 + 7 * ((10*pi/6)/14);
z8 = -pi/6 -1 * ((10*pi/6)/14);
z9 = -pi/6 -2 * ((10*pi/6)/14);
z10 = -pi/6 -3 * ((10*pi/6)/14);
z11 = -pi/6 -4 * ((10*pi/6)/14);
z12 = -pi/6 -5 * ((10*pi/6)/14);
z13 = -pi/6 -6 * ((10*pi/6)/14);
z14 = -pi/30;

Hzeros = [z0,z1,z2,z3,z4,z5,z6,z7,z8,z9,z10,z11,z12,z13,z14];

syms z
H = (z-exp(1i*Hzeros(1)))*(z-exp(1i*Hzeros(2)))*(z-exp(1i*Hzeros(3)))*(z-exp(1i*Hzeros(4)))
w = -2*pi:0.01:2*pi;
H1(z) = H/z^15;

figure(1)
h = round(double(coeffs(H)),8);
```

```
figure(1)
h = round(double(coeffs(H)),8);
save('h.mat','h');
stem(0:15,h,'filled','.k');
title('Re(h[n]) = h[n]')

figure(2)
polarscatter(Hzeros,1,'filled');
title('Plot of the zeros of H(z)')

figure(3)
plot(w,abs(H1(exp(1j.*w))));
title('Magnitude of H(e^{jw})')
xlabel('rad')

figure(4)
plot(w,angle(H1(exp(1j.*w))));
title('Phase of H(e^{jw})')
xlabel('rad')


%PART 2
n = 0:1023;
xf = cos(pi*n.^2/512);
figure(5)
stem(n,xf,'filled','.k')
title('Plot of x_f[n] for n \in [0:1023]')

save('xf.mat','xf')


n = 0:8192;
xg = cos(pi*n.^2/8192);
figure(6)
```

```
save('xf.mat','xf')


n = 0:8192;
xg = cos(pi*n.^2/8192);
figure(6)
stem(n,xg,'filled','.k')
title('Plot of x_g[n] for n \in [0:8192]')

save('xg.mat','xg')

yf = conv(h,xf);
save('yf.mat','yf');
figure(7)
stem(0:1038,yf,'filled','.k');
title('FIR filter applied y_1[n]=T[x_f[n]]')

yg = conv(h,xg);
save('yg.mat','yg');
figure(8)
stem(0:8207,yg,'filled','.k');
title('FIR filter applied y_2[n]=T[x_g[n]]')

Ts=sqrt(pi/(1000*8207));
w = -2*pi:Ts:2*pi;
figure(9)
plot(w,abs(H1(exp(1j.*w))));
title('Magnitude of H(jw)')
xlabel('rad/s')
```

PART 5-6

```
load("yg.mat","yg");

a = 1000;
Ts=sqrt(pi/(a*8207));
samplerate = 1/Ts;

player = audioplayer((yg), samplerate);
period = Ts.*length(yg);
while(1)
    play(player);
    pause(period);
    stop(player);
end
```

PART 8-9

```
load('h.mat','h');
[self,fs1] = audioread('selfaudio.m4a');
[synph,fs2] = audioread('Symphony.m4a');

FIRself = conv(h,self);
FIRsynph = conv(h,synph(:,1));

audiowrite('FIRself.m4a',FIRself,fs1);
audiowrite('FIRsynph.m4a',FIRsynph,fs2);
audiowrite('Symphony.m4a',synph(:,1),fs2);
```