EEE321 Lab Work 4

(Clearly justify all answers.)

(Due 11 November 2022)

Work on these questions as a homework first: answer the analytical parts of the questions and write the answers on a paper, write a MATLAB program (or many such programs) to perform the tasks that need computation, print your MATLAB code(s), print your computer outputs (numerical and graphic) whenever needed; the collection of all those will be your lab report. Be ready to run your computer codes during the face-to-face lab session. During your lab session, your TA will ask you questions, observe your results as you run them while sharing your screen, and may go over your codes as needed. Based on your results, you may be asked to modify your lab report, including any modifications needed in your MATLAB codes, during the lab hours. Your face-to-face lab session will continue until your TA is satisfied with your results, during the allocated lab hours. You may upload your entire lab report during or right after your lab session to MOODLE. If you need some more time to modify your report, you may do so, until the end of the day. Your report will get a grade based on your preparedness when you come to the lab session, performance of your codes during the lab session, (including any modifications done during the lab session) your answers to the oral questions during your demo(s), and the entire content of the submitted report.

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In this work, you will learn the Fourier series expansion and some related approximations.

1- $y_a(t)$ is a rectangular waveform defined as:

$$y_a(t) = \begin{cases} 0 & t \in [0,6)\text{s.} \\ 4 & t \in [6,10)\text{s.} \\ 0 & t \in [10,16)\text{s.} \end{cases}$$

and $y_a(t)$ is periodic with a period of 16 seconds; t is in seconds.

Any signal processing in a digital environment involves sampling; that includes your MATLAB environement, as well.

a) Discretize $y_a(t)$, using a sampling period $T_s = 1/8$ s. Plot (using MATLAB) $y[n] = y_a(nT_s)$, for $n \in [-30, 225]$.

b) Find the Fourier series expansion of $y_a(t)$.

c) The spectrum of a periodic signal is just a table of frequencies and the corresponding complex amplitudes of the complex sinusoidal components of a signal. A plot of the spectrum is, therefore, a *representative* plot of those complex amplitudes with respect to a horizontal frequency axis. The *representative* plot means, any style that you adopt by making sure that anyone looks at it easily understands the relative amplitudes of the signal components. (Remark: sometimes the *spectrum* is interpreted as the magnitude of those complex amplitudes. We will stick to the complex amplitude interpretation.)

Plot (by hand) the spectrum of $y_a(t)$.

d) Write a MATLAB code that computes the discrete function $z_N[n]$, where,

$$z_N[n] = \sum_{k=-N}^{N} a_k e^{j\omega_0 k n T_s}$$

where a_k's are the FSE coefficients that you found in part (b). Plot the result for $n \in [-30, 225]$, and for $N = 120$. (Adopt a proper graph style; note that your computed functions, in this homework, represent continuous-time functions, even if they are computed by digital means.)

Comment on the results: does the plot look like $y_a(t)$?

e) Repeat (e) for $N = 60$.

e) Repeat (e) for $N = 30$

f) Repeat (e) for $N = 5$.

g) Repeat (e) for $N = 3$.

h) Repeat (e) for $N = 1$.

Consider your plots in (d) through (h) and comment on the quality of approximations $z_N[n]$, of $y_a(t)$, by taking only some of the FSE components during the synthesis; pay attention also to the Gibbs phenomenon.

i) Plot the zeroth, first, second, and third harmonics of $y_a(t)$, using the same scale for all these plots. (Hint: zeroth harmonic is also the DC component of $y_a(t)$ that is a constant function whose amplitude (value) is equal to a_0. $k'th$ harmonic of this real valued $y_a(t)$ is $a_{-k}e^{-j\omega_o kt} + a_k e^{j\omega_o kt}$ which is a real valued sinusoidal. Hint: Though you cannot plot a continuous function using MATLAB, you can get a very good approximation if the sampling rate is high enough.)

2- Replace $y_a(t)$ in (1), as,

$$y_a(t) = \left| 3\cos\left(\frac{\pi}{8}t\right) \right| \quad .$$

Find the fundamental period of this function first; and then repeat (1) using that fundamental period. (Full-wave rectifier.)

3- Replace $y_a(t)$ in (1), as,

$$y_a(t) = \begin{cases} \left|3\cos\left(\frac{\pi}{8}t\right)\right| & t \in [-4, 4)\,\mathrm{s}\,. \\ 0 \quad t \in [4, 12)\mathrm{s}. \end{cases}$$

and it is periodic with period $T = 16s$. Repeat (1). (Half-wave rectifier.)