Stock Price Simulation using Markov Chains and GARCH Models

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Abstract:

A brief overview summarizing the purpose of the project (stock price modeling), the techniques used (Markov Chains, Monte Carlo simulations, and GARCH models), and the key results, including risk analysis metrics like Value at Risk (VaR) and Conditional VaR (CVaR).

Introduction:

Stock price simulation is a critical task in financial markets to evaluate future asset movements and manage risk. In this project, we model stock prices using Markov chains to account for transitions between discrete states (e.g., price increase, decrease, or stable). Additionally, Monte Carlo methods allow for the generation of multiple potential price paths, providing insight into a wide range of possible outcomes. The incorporation of GARCH models further improves accuracy by modeling time-varying volatility, a key characteristic of real-world stock prices.

Methodology:

1-Data Collection:

Historical stock data (e.g., Apple Inc. stock prices) was collected using the quantmod package. Closing prices from 2015 to 2023 were used for analysis, and daily returns were calculated to categorize stock movements.

2-Markov Chain:

We categorized daily price changes into three states: **Up**, **Down**, and **Stable**, based on a threshold of ±1%. A Markov chain model was built by calculating the transition probabilities between

these states using historical data. This yielded a transition matrix that guides the stock price transitions.

Example Transition Matrix:

	Down	Stable	Up
Down	0.2325581	0.4697674	0.2976744
Stable	0.1536424	0.6529801	0.1933775
Up	0.1743772	0.5729537	0.2526690

3-Monte Carlo Simulation:

Monte Carlo simulations were run using the transition matrix, simulating stock prices over 365 days (1 year). We conducted multiple simulations (1000 paths) to capture a range of possible future stock price paths.

4-Garch Modeling:

To incorporate time-varying volatility, a GARCH(1,1) model was applied to the historical returns. The GARCH model allows for clustering volatility, which better reflects the behavior of financial markets. Future price paths were adjusted accordingly based on simulated volatility.

Results:

1-Monte Carlo Simulated Paths:

The Monte Carlo simulation produced multiple possible stock price paths. The majority of simulated paths exhibited moderate growth, while some paths showed larger deviations due to increased volatility.

2-Risk Analysis:

We calculated two key risk metrics: **Value at Risk (VaR)** and **Conditional Value at Risk (CVaR)**, both at a 95% confidence level.

- **VaR (95%)**: Indicates the worst expected loss over 1 year, with a 5% chance of exceeding this loss.
- **CVaR (95%)**: The average loss expected in the worst 5% of cases.

Example Risk Metrics:

```
      5%
68.17928
[1] 65.52618
```

Additionally, the **maximum drawdown** (the peak-to-trough decline) was calculated, revealing the worst-case scenario of loss in a single path.

References:

1-Packages:

quantmod, markovchain, rugarch, PerformanceAnalytics.

2-Research literature on Monte Carlo methods and GARCH models.