Uppsala University

Optimization

Assignment 2 - Basic Option Portfolio Optimization

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1 Short Description

In this assignment, the Markowitz framework for portfolio optimization is implemented. The performance of investments shared among five different assets is examined. The expected rate of return and variance of portfolio consist the criteria for measuring its performance. More precisely, the variance represents the involved risk. It is important to state that there exists a *trade off* between return and risk; the higher the return, the higher the risk.

Before answering the questions, I would like to rewrite the given formulae regarding the portfolio of the five given assets, using matrices that simplify computations and facilitate Matlab implementation.

First and foremost, the rates of returns and covariances can be expressed respectively using matrix notation as,

$$\mathbf{r} = \begin{pmatrix} \bar{r}_1 \\ \bar{r}_2 \\ \bar{r}_3 \\ \bar{r}_4 \\ \bar{r}_5 \end{pmatrix} = \begin{pmatrix} 0.1300 \\ 0.0530 \\ 0.1050 \\ 0.0500 \\ 0.1260 \end{pmatrix} \tag{1}$$

$$\mathbf{Q} = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} & \sigma_{14} & \sigma_{15} \\ \sigma_{21} & \sigma_2^2 & \sigma_{23} & \sigma_{24} & \sigma_{25} \\ \sigma_{31} & \sigma_{32} & \sigma_3^2 & \sigma_{34} & \sigma_{35} \\ \sigma_{41} & \sigma_{42} & \sigma_{43} & \sigma_4^2 & \sigma_{45} \\ \sigma_{51} & \sigma_{52} & \sigma_{53} & \sigma_{54} & \sigma_5^2 \end{pmatrix} = \begin{pmatrix} 0.0401 & -0.0119 & 0.0060 & 0.0074 & -0.0021 \\ -0.0119 & 0.0112 & 0.0021 & 0.0054 & 0.0055 \\ 0.0060 & 0.0021 & 0.0304 & 0.0077 & 0.0029 \\ 0.0074 & 0.0054 & 0.0077 & 0.0374 & -0.0104 \\ -0.0021 & 0.0055 & 0.0029 & -0.0104 & 0.0380 \end{pmatrix} \tag{2}$$

Based on (1) and (2), the expected rate of return (μ) and variance(σ^2) of the portfolio are defined as:

$$\mu = \sum_{i=1}^{5} \bar{r}_i w_i \overset{(1)}{=} \mathbf{w}^\intercal \mathbf{r} \quad = \begin{pmatrix} w_1 & w_2 & w_3 & w_4 & w_5 \end{pmatrix} \cdot \begin{pmatrix} 0.1300 \\ 0.0530 \\ 0.1050 \\ 0.0500 \\ 0.1260 \end{pmatrix} \tag{3}$$

Similarly,

$$\sigma^2 = \sum_{i=1}^{5} \sum_{j=1}^{5} w_i w_i \sigma_{ij} = \mathbf{w}^\intercal \mathbf{Q} \mathbf{w} \overset{(2)}{=} \tag{4}$$

$$= \begin{pmatrix} w_1 & w_2 & w_3 & w_4 & w_5 \end{pmatrix} \cdot \begin{pmatrix} 0.0401 & -0.0119 & 0.0060 & 0.0074 & -0.0021 \\ -0.0119 & 0.0112 & 0.0021 & 0.0054 & 0.0055 \\ 0.0060 & 0.0021 & 0.0304 & 0.0077 & 0.0029 \\ 0.0074 & 0.0054 & 0.0077 & 0.0374 & -0.0104 \\ -0.0021 & 0.0055 & 0.0029 & -0.0104 & 0.0380 \end{pmatrix} \cdot \begin{pmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \end{pmatrix}$$

In addition, there are constraints in the given problem. Being more specific,

$$\sum_{i=1}^{5} w_i = 1 \Leftrightarrow \mathbf{e}^\mathsf{T}\mathbf{w} = 1, \text{where} \quad \mathbf{e}^\mathsf{T} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \end{pmatrix} \tag{5}$$

As far as short selling is concerned, when it is allowed, there are no constraints on the weights of the five assets. On the contrary, when it is forbidden, the weights are required to be non-negative; $w_i \geq 0, \quad i = 1, \ldots, 5$.

The required Matlab code for this assignment as well as the detailed print out of results are included in the appendix section.

2 Tasks

2.1 Task 1

The linear program for maximizing the expected return of the portfolio, when short selling is not allowed is,

$$\begin{aligned} \underset{\mathbf{w}}{\text{maximize}} \quad & \mu = \mathbf{w}^\mathsf{T}\,\mathbf{r} \\ \text{subject to} \quad & \mathbf{e}^\mathsf{T}\mathbf{w} = 1, \\ & w_i \geq 0,\ i = 1, \ldots, 5. \end{aligned}$$

The expected rate of return; μ, is the weighted average of $\bar{r}_i$ with w_i being non-negative, since short selling is not allowed. To maximize μ, we let $w_i = 1$ for i with maximum $\bar{r}_i$. This is $i = 1$ and $\bar{r}_1 = 0.13$. Consequently, the solution of the maximization problem and the value of the objective function at the solution are respectively (equation (3)):

$$\mathbf{w} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \qquad \mu = w_1 \cdot r_1 + w_2 \cdot r_2 + w_3 \cdot r_3 + w_4 \cdot r_4 + w_5 \cdot r_5 = 1 \cdot 0.13 = 0.13$$

The variance of the portfolio is equal to the variance of asset 1, since the other weights are zero (equation (4)). As a result,

$$\sigma^2 = \sigma_1^2 = 0.0401$$

2.2 Task 2

The quadratic program of minimizing the variance of the portfolio, when short selling is allowed, is,

$$\begin{aligned} \underset{\mathbf{w}}{\text{minimize}} \quad & \sigma^2 = \frac{1}{2}\mathbf{w}^\mathsf{T}\mathbf{Q}\mathbf{w} \\ \text{subject to} \quad & \mathbf{e}^\mathsf{T}\mathbf{w} = 1. \end{aligned}$$

The constraint of this minimization problem is a linear equality constraint, which indicates that a part of the KKT conditions are required for its solution. Being more specific, the solution to this problem originates from the satisfaction of the first and second order optimality conditions, which in terms of Lagrangian function can be stated as:

- $\nabla \mathcal{L}(w_*, \lambda_*) = 0$
- $z^\intercal \nabla^2 \mathcal{L}(w_*, \lambda_*) z > 0$

The Lagrangian function is,

$$\mathcal{L}(\mathbf{w}, \lambda) = \frac{1}{2}\mathbf{w}^\intercal \mathbf{Q} \mathbf{w} - \lambda^\intercal \mathbf{e}^\intercal \mathbf{w}$$

The gradient of the aforementioned Lagrangian function is,

$$\nabla \mathcal{L}(\mathbf{w}, \lambda) = \frac{1}{2}(\mathbf{Q} + \mathbf{Q}^\intercal)\mathbf{w} - \lambda \mathbf{e} = \mathbf{Q}\mathbf{w} - \lambda \mathbf{e}$$

As a result, the first order optimality condition becomes,

$$\mathbf{Q}\mathbf{w} - \lambda \mathbf{e} = 0 \tag{6}$$

This can be expressed in terms of the following system, as

$$\begin{pmatrix} \mathbf{Q} & -\mathbf{e} \\ \mathbf{e}^\intercal & 0 \end{pmatrix} \cdot \begin{pmatrix} \mathbf{w} \\ \lambda \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ 1 \end{pmatrix} \tag{7}$$

The system described by equation (7) is going to be inserted in Matlab in the form of block matrices, in order to be solved using the *backslash* operator.

As far as the second order optimality condition is concerned, it is derived that

$$\nabla^2 \mathcal{L}(\mathbf{w}, \lambda) = \mathbf{Q}^\intercal = \mathbf{Q}, \quad \text{since } \mathbf{Q} \text{ is symmetric} \tag{8}$$

It is also given that $\mathbf{Q}$ is positive semidefinite, which means that $\mathbf{Q} \geq 0$. If $\mathbf{Q}$ is multiplied from left and right by $\mathbf{z}^\intercal$ and $\mathbf{z}$ respectively, it still holds that

$$\mathbf{z}^\intercal \nabla^2 \mathcal{L}(\mathbf{w}_*, \lambda_*) \mathbf{z} = \mathbf{z}^\intercal \mathbf{Q} \mathbf{z} \geq 0 \tag{9}$$

Therefore, the second order necessary optimality condition is satisfied, as indicated by formula (9).

By solving the linear system described by formula (7), the following results are obtained

$$\begin{pmatrix} \mathbf{w} \\ \lambda \end{pmatrix} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ \lambda \end{pmatrix} = \begin{pmatrix} 0.3198 \\ 0.7213 \\ 0.0350 \\ -0.0708 \\ -0.0052 \\ 0.0039 \end{pmatrix}, \quad \sigma^2 = 0.0020$$

It is true that the obtained variance of the portfolio; $\sigma^2 = 0.0020$, is remarkably smaller compared to this one of question 1, where $\sigma^2 = 0.0401$. Therefore, the portfolio with the maximum expected return is not the one with the minimum variance. Actually, the higher the expected return, the higher the variance. Furthermore, it is indicated that the diversification of fractions w_i among different assets reduces the variance of the portfolio.

2.3 Task 3

(a) *When short selling is allowed,*

$$\begin{aligned} \underset{\mathbf{w}}{\text{minimize}} \quad & \frac{1}{2}\mathbf{w}^\mathsf{T}\mathbf{Q}\mathbf{w} \\ \text{subject to} \quad & \mathbf{w}^\mathsf{T}\mathbf{r} = \rho, \\ & \mathbf{e}^\mathsf{T}\mathbf{w} = 1. \end{aligned}$$

When short selling is not allowed,

$$\begin{aligned} \underset{\mathbf{w}}{\text{minimize}} \quad & \frac{1}{2}\mathbf{w}^\mathsf{T}\mathbf{Q}\mathbf{w} \\ \text{subject to} \quad & \mathbf{w}^\mathsf{T}\mathbf{r} = \rho, \\ & \mathbf{e}^\mathsf{T}\mathbf{w} = 1, \\ & w_i \geq 0, \quad i = 1, \ldots, 5 \end{aligned}$$

(b) *When short selling is allowed,* the weights and variance found by using *quadprog* are

$$\mathbf{w} = \begin{pmatrix} 0.3738 \\ 0.4444 \\ 0.1266 \\ -0.1000 \\ 0.1552 \end{pmatrix}, \quad \sigma^2 = 0.0031$$

When short selling is not allowed, the weights and variance found by using *quadprog* are

$$\mathbf{w} = \begin{pmatrix} 0.3362 \\ 0.3403 \\ 0.1192 \\ 0.0000 \\ 0.2043 \end{pmatrix}, \quad \sigma^2 = 0.0032$$

(c) *When short selling is allowed,* the weights and variance found by using *quadprog* are

$$\mathbf{w} = \begin{pmatrix} 0.6346 \\ -0.8917 \\ 0.5688 \\ -0.2410 \\ 0.9293 \end{pmatrix}, \quad \sigma^2 = 0.0398$$

When short selling is not allowed, there is

No feasible solution found.

In this case, the algorithm cannot find a feasible solution. This fact can be explained by looking carefully at the value of ρ, which has been set equal to 0.2. Looking back at Task 1, the maximum value of the expected return, provided that short selling is not allowed, has been found to be **0.13**. However, in this question, $\rho = 0.2 > maximum$, meaning that problem is infeasible and the constraints cannot be satisfied

Moreover, comparing the two different cases; $\rho = 0.1$ and $\rho = 0.2$, when short selling is allowed, it is observed that the higher the expected return, the higher the risk; variance.

2.4 Task 4

The given optimization problem could be restated as,

$$\begin{aligned} \underset{\mathbf{w}}{\text{minimize}} \quad & \alpha \frac{1}{2}\mathbf{w}^\mathsf{T}\mathbf{Q}\mathbf{w} - (1-\alpha)\,\mathbf{w}^\mathsf{T}\mathbf{r} \\ \text{subject to} \quad & \mathbf{e}^\mathsf{T}\mathbf{w} = 1, \quad \text{and when short selling is not allowed} \\ & w_i \geq 0, \quad i = 1, \dots, 5 \end{aligned} \tag{10}$$

(a) When $\alpha = 0$, the restated equation (10) is in the form,

$$\begin{aligned} \underset{\mathbf{w}}{\text{minimize}} \quad & -\mathbf{w}^\mathsf{T}\mathbf{r} \equiv \underset{\mathbf{w}}{\text{maximize}} \quad \mathbf{w}^\mathsf{T}\mathbf{r} \\ \text{subject to} \quad & \mathbf{e}^\mathsf{T}\mathbf{w} = 1, \quad \text{and when short selling is not allowed} \\ & w_i \geq 0, \quad i = 1, \dots, 5 \end{aligned} \tag{11}$$

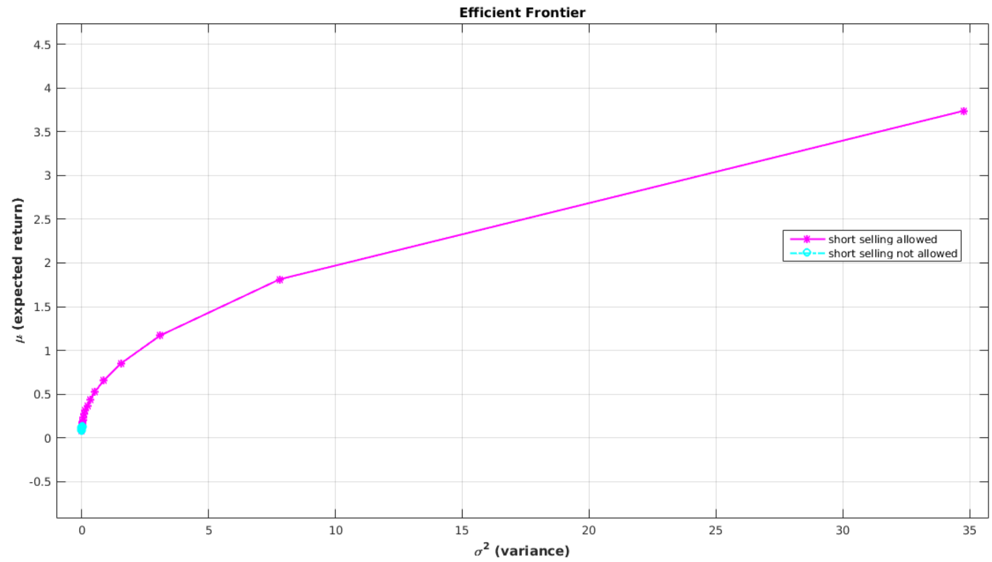
This is actually, the maximization problem that has been stated and solved in Task 1. The expected return of the portfolio is maximized under the constraints that the sum of the found weights is must be equal to 1 and if short selling is not allowed, the non-negativity of the weights is also required.

When $\alpha = 1$, the restated equation (10) is in the form,

$$\begin{aligned} \underset{\mathbf{w}}{\text{minimize}} \quad & \frac{1}{2}\mathbf{w}^\mathsf{T}\mathbf{Q}\mathbf{w} \\ \text{subject to} \quad & \mathbf{e}^\mathsf{T}\mathbf{w} = 1, \quad \text{and when short selling is not allowed} \\ & w_i \geq 0, \quad i = 1, \dots, 5 \end{aligned} \tag{12}$$

In this case, the problem is simplified to that one of Task 2. In other words, the variance of the portfolio needs to be minimized, provided that the sum of the weights is equal to 1. In addition, if short selling is not allowed, weights must be non-negative, as well.

(b) The efficient frontiers described by formula (10), are calculated and depicted in figure 1.



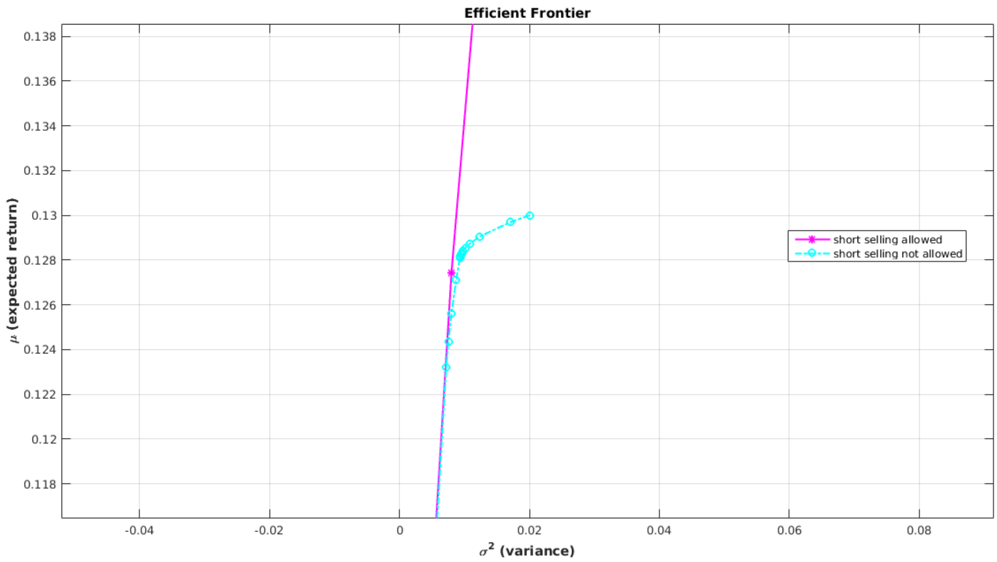


Figure 1: *Efficient Frontiers*

As indicated by figure 1, when short selling is allowed, the investor is able to achieve larger expected returns, but this large increase in the expected return is accompanied with a very large increase in risk. On the other hand, when short selling is not allowed the variance is remarkably reduced, and this fact affects similarly the behavior of the expected return. As a result, there is a trade off between return and risk. The shape between these two different efficient frontiers appears to be similar. However, in the case of unlimited short selling, there is not finite upper bound.

Appendices

A Matlab Code

Version used: MATLAB R2014b.

```
%% Chantzi Efthymia - Optimization - Assignment 2 %%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% PORTFOLIO OPTIMIZATION                                                       %
% This is the .m file that implements all the questions requested in          %
% assignment 2. If run, the results are saved in a .txt file named            %
% output.txt. The figure of efficient frontiers is also saved as a .png file %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

clc;
clear all;
close all;

diary on
diary('output.txt'); %save output displayed results in output.txt

fprintf('%s\n', date);
fprintf('---- Portfolio Optimization ----\n');
fprintf('\n running...\n');

%%%%%%% general global parameters needed for all questions %%%%%%%

%number of assets
numOfAssets = 5;

%vector of yearly expected rates of returns of assets according to Table 1
annualExpectedRate = 10^(-2)*[13 5.3 10.5 5 12.6];

%symmetric matrix of covariances of assets according to Table 1
covarianceMatrix = 10^(-2)*[4.01 -1.19 0.6 0.74 -0.21;
                            -1.19 1.12 0.21 0.54 0.55;
                            0.6 0.21 3.04 0.77 0.29;
                            0.74 0.54 0.77 3.74 -1.04;
                            -0.21 0.55 0.29 -1.04 3.8];

%change default options, so as to use a medium-scale active set algorithm
%as requested
oldOptions = optimset('quadprog');
options = optimset(oldOptions, 'LargeScale', 'off');


%% Question 2 %%

fprintf('---- Question 2, Results ----');
fprintf('\n\n');

%vector e for the definition of the constraint sum{i=1..n}w_i = 1
```

```matlab
e = ones(numOfAssets, 1);

%parts of the reformulated system
LagragianRightSide = zeros(size(e));
constraintRightSide = 1;
b = [LagragianRightSide ; constraintRightSide];

%formulated system by the KKT conditiions
A = [covarianceMatrix -e ; e' 0];

%solving by backslash
w_lambda_sol = A\b

%variance
variance_question2 = (1/2)*(w_lambda_sol((1 : 5), 1)'*covarianceMatrix*w_lambda_sol((1 : 5), 1))

%checks if the constraint is satisfied
constraintSatisfied = (w_lambda_sol(1 : 5, 1)'*e);

if((constraintSatisfied - 1) == 0)

    fprintf('The linear constraint is satisfied.\n');

end

%in order to validate the result, solve the same problem with quadprog
%not asked in the assignment, but it is done just for validation of the
%result
Aeq = [1 1 1 1 1];
beq = 1;
[w_quadprog, variance_quadprog] = quadprog(covarianceMatrix, [], [], [], Aeq, beq, [], [], [], options)



%% Question 3 %%
fprintf('\n\n');
fprintf('---- Question 3, Results ----');
fprintf('\n\n');


%% question 3b
p = 0.1;
fprintf('--------- p = %.2f ---------\n', p);

%short selling allowed
Aeq = [annualExpectedRate ; e'];
beq = [p  1];

fprintf('---short selling---\n');
fprintf('weights, variance, expected rate\n');
[w, variance, exitflag] = quadprog(covarianceMatrix, [], [], [], Aeq, beq, [], [], [], options)
expectedReturn = annualExpectedRate*w


%short selling not allowed
```

```
lb = zeros(numOfAssets, 1); %lower bound, weights non-negative
fprintf('---no short selling---\n');
fprintf('weights, variance, expected rate\n');
[w, variance, exitflag] = quadprog(covarianceMatrix, [], [], [], Aeq, beq, lb, [], [], options)
expectedReturn = annualExpectedRate*w


%% question 3c
p = 0.2;
beq = [p 1];
fprintf('--------- p = %.2f ---------\n', p);

%short selling allowed
fprintf('---short selling---\n');
fprintf('weights, variance, expected rate\n');
[w, variance, exitflag] = quadprog(covarianceMatrix, [], [], [], Aeq, beq, [], [], [], options)
expectedReturn = annualExpectedRate*w

%short selling not allowed
fprintf('---no short selling---\n');
fprintf('weights, variance, expected rate\n');
[w, variance, exitflag] = quadprog(covarianceMatrix, [], [], [], Aeq, beq, lb, [], [], options)
expectedReturn = annualExpectedRate*w


%% Question 4 %%
fprintf('\n\n');
fprintf('---- Question 4, Results ----');
fprintf('\n\n');

a = [0.05 : 0.05 : 1];

%equality constraint
Aeq = e';
beq = 1;

%weights for short selling allowed
weightsS = zeros(numOfAssets, length(a));

%matrix for storing variance and expected rate of return
%of efficient frontier for plotting
efficientPairsS = zeros(2, length(a));

fprintf('Short selling allowed.\n');
fprintf('\n');
for i = 1 : length(a)

    %inserting parameter a in the problem
    newCov = a(i)*covarianceMatrix;
    linearTerm = -(1 - a(i))*annualExpectedRate;

    [w, fval] = quadprog(newCov, linearTerm, [], [], Aeq, beq, [], [], [], options);

    %evaluation of variance because fval from above contains the value of
    %the reformulated problem described in question 4
```

```
    variancePort = (1/2)*w'*covarianceMatrix*w;

    %expected return of portfolio
    rPort = annualExpectedRate*w;

    %weights
    weightsS(:, i) = w;

    %efficient pairs
    efficientPairsS(1, i) = variancePort;
    efficientPairsS(2, i) = rPort;

end


%the same procedure as above applied for short selling not allowed

%weights for short selling not allowed
weights = zeros(numOfAssets, length(a));

%matrix for storing variance and expected rate of return
%of efficient frontier for plotting
efficientPairs = zeros(2, length(a));

fprintf('Short selling not allowed.\n');
fprintf('\n');
for i = 1 : length(a)

    %inserting parameter a in the problem
    newCov = a(i)*covarianceMatrix;
    linearTerm = -(1 - a(i))*annualExpectedRate;

    [w, fval] = quadprog(newCov, linearTerm, [], [], Aeq, beq, lb, [], [], options);

    %evaluation of variance because fval from above contains the value of
    %the reformulated problem described in question 4
    variancePort = (1/2)*w'*covarianceMatrix*w;

    %expected return of portfolio
    rPort = annualExpectedRate*w;

    %weights
    weights(:, i) = w;

    %efficient pairs
    efficientPairs(1, i) = variancePort;
    efficientPairs(2, i) = rPort;

end

%plot efficient pairs when short selling allowed and not allowed
figure();
plot(efficientPairsS(1, :), efficientPairsS(2, :), 'm*-', 'linewidth', 1.5);
hold on;
plot(efficientPairs(1, :), efficientPairs(2, :), '-.oc', 'linewidth', 1.5);
```

```
xlim([(min(efficientPairsS(1, :)) - 1) (max(efficientPairsS(1, :)) + 1)]);
ylim([(min(efficientPairsS(2, :)) - 1) (max(efficientPairsS(2, :)) + 1)])
xlabel('(variance)', 'fontweight', 'bold');
ylabel('(expected return)', 'fontweight', 'bold');
title('Efficient Frontier', 'fontweight', 'bold');
legend('short selling', 'no short selling', 'Location', 'best');
grid on;
hold off;
saveas(gcf, 'efficientPairsPortfolio.png');

fprintf('%s\n', date);
diary off
```

B Results

The attached results of Matlab running sessions contain only short messages concerning the feasibility or infeasibility of the optimization problems. The long log messages have been removed for the needs of this report, but they are provided as a link at the end.

```
03-Dec-2014
---- Portfolio Optimization ----

 running...
---- Question 2, Results ----


w_lambda_sol =

    0.3198
    0.7213
    0.0350
   -0.0708
   -0.0052
    0.0039


variance_question2 =

    0.0020

The linear constraint is satisfied.

Minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the default value of the function tolerance,
and constraints are satisfied to within the selected value of the constraint tolerance.


w_quadprog =

    0.3198
    0.7213
    0.0350
   -0.0708
   -0.0052
```

```
variance_quadprog =

    0.0020



---- Question 3, Results ----

--------- p = 0.10 ---------
---short selling---
weights, variance, expected rate

Minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the default value of the function tolerance,
and constraints are satisfied to within the selected value of the constraint tolerance.


w =

    0.3738
    0.4444
    0.1266
   -0.1000
    0.1552


variance =

    0.0031


exitflag =

     1


expectedReturn =

    0.1000

---no short selling---
weights, variance, expected rate

Minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the default value of the function tolerance,
and constraints are satisfied to within the selected value of the constraint tolerance.


w =

    0.3362
    0.3403
    0.1192
    0.0000
```

```
    0.2043


variance =

    0.0032


exitflag =

     1


expectedReturn =

    0.1000

--------- p = 0.20 ---------
---short selling---
weights, variance, expected rate

Minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the default value of the function tolerance,
and constraints are satisfied to within the selected value of the constraint tolerance.


w =

    0.6346
   -0.8917
    0.5688
   -0.2410
    0.9293


variance =

    0.0398


exitflag =

     1


expectedReturn =

    0.2000

---no short selling---
weights, variance, expected rate

No feasible solution found.

quadprog stopped because it was unable to find a point that satisfies
the constraints within the selected value of the constraint tolerance.
```

```
w =

    2.0937
    0.0000
    0.0000
    0.0000
    0.0000


variance =

    0.0879


exitflag =

    -2


expectedReturn =

    0.2722



---- Question 4, Results ----

Short selling allowed.

Minimum found that satisfies the constraints.

Minimum found that satisfies the constraints.

Minimum found that satisfies the constraints.

Minimum found that satisfies the constraints.

Minimum found that satisfies the constraints.

Minimum found that satisfies the constraints.

Minimum found that satisfies the constraints.

Minimum found that satisfies the constraints.

Minimum found that satisfies the constraints.

Minimum found that satisfies the constraints.

Minimum found that satisfies the constraints.

Minimum found that satisfies the constraints.

Minimum found that satisfies the constraints.

Minimum found that satisfies the constraints.

Minimum found that satisfies the constraints.

Minimum found that satisfies the constraints.
```

```
Minimum found that satisfies the constraints.

Minimum found that satisfies the constraints.

Minimum found that satisfies the constraints.

Minimum found that satisfies the constraints.


Short selling not allowed.

Minimum found that satisfies the constraints.

Minimum found that satisfies the constraints.

Minimum found that satisfies the constraints.

Minimum found that satisfies the constraints.

Minimum found that satisfies the constraints.

Minimum found that satisfies the constraints.

Minimum found that satisfies the constraints.

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Minimum found that satisfies the constraints.

Minimum found that satisfies the constraints.

Minimum found that satisfies the constraints.

Minimum found that satisfies the constraints.

Minimum found that satisfies the constraints.

03-Dec-2014
```

The following link contains the folder of Matlab implementation for this assignment. I have attached my source code and detailed output results for further testing and validation, if required.

https://www.dropbox.com/sh/s90hc1rc8d90xys/AAD6Lg5HFvuJRXLCfHzhA6z8a?dl=0