1. Introduction

This project follows the procedure of establishing an investment portfolio: selecting assets, exploratory analysis, assigning weights and risk management.

We chose 10 stocks from Yahoo Finance to analyze their monthly performances over 10 years from November 1st 2005 to October 31st 2015. These 10 stocks have less risk among stocks we were interested in. Specifically, they are Royal Caribbean Cruises Ltd. (RCL), Nasdaq, Inc. (NDAQ), Vertex Pharmaceuticals Incorporated (VRTX), WEC Energy Group, Inc. (WEC), Wells Fargo & Company (WFC), The Walt Disney Company (DIS), Kohl's Corp. (KSS), Level 3 Communications, Inc. (LVLT), Stericycle, Inc. (SRCL), Walgreens Boots Alliance, Inc. (WBA). A risk free asset from 4-week Treasury bill of secondary market are used to build efficient portfolio. These stocks cover diverse industries, including consumer discretionary, finance, utilities, telecommunication, industrials and health care, to reduce correlation of returns.

2. Summary

Firstly, minimum variance portfolio and tangency portfolio with and without short sell are built. Sharpe ratio of these portfolios are calculated to give a better comparison. Tangency portfolio with short sell has highest sharpe ratio, 0.286. We also construct the portfolio under a given expect return.

3. Descriptive Statistics

	RCL	NDAQ	VRTX	WEC	WFC	DIS	KSS	LVLT	SRCL	WBA
Mean	1.83	0.76	2.17	0.51	0.5	1.49	0.31	15.4	0.87	0.84
Std	15.76	9.09	13.3	6.34	10.48	6.47	7.89	155.8	6.8	8.04
Skewness	1.01	-0.49	0.83	-3.86	-0.96	-0.34	0.07	10.6	-3.68	0.33
Kurtosis	6.5	1.38	1.76	28.31	8.04	0.93	0.38	111.5	27.42	0.42

TABLE 3.1 MEAN, STANDARD, DEVIATION, SKEWNESS AND KURTOSIS OF 10 ASSETS

From table above, LVLT has the highest mean return, along with the highest risk. KSS has the lowest mean return and relatively lower variance. Overall, the 10 assets go with "the higher return, the higher risk".

As for the Skewness Coefficients, NDAQ, VRTX, WFC, DIS, KSS and WBA are around 0, which means these assets have a symmetrical distribution. RCL and LVLT have Skewness Coefficients are larger than 1, indicating an asymmetrical distribution with a long tail to the right. WEC and SRCL have Skewness Coefficients are less than -1, which means these two assets have asymmetrical distribution with a long tail to the left.

At last, for Kurtosis Coefficients, DIS, KSS and WBA are around 0, which indicates their distributions are close to Gaussian distribution. The rest have positive Kurtosis Coefficients, showing their distributions are more peak than a Gaussian distribution.

From the all of the analysis, LVLT is most noticeable, with significant higher statistics. The cause is there is an astonishing high return on October 2011, which is 16.91. From news, on October 20, 2011, LVLT has a 1 for 15 reverse split, which means for each 15 shares of LVLT owned pre-split, the shareholder now owned 1 share (Source: https://www.splithistory.com/lvlt/). This explains the reason.

From equity curve shown in appendix, although there are several significant drops, LVLT grows constantly in general. Growing speed and variation tendency of VRTX and DIS are quite similar. And their prices also tend to be similar to each other over these 10 years. KSS remain stable over 10 years. But equity of WFC drops a bit. As for S&P500, although it remains stable, there is a slight trend of growth over time.

From histograms, except LVLT, most assets have bell shapes. The reason of LVLT does not have a "bell shape" is that it contains a quite obvious outlier. The cause has been explained earlier.

From boxplots, all of the assets have outliers. The outliers mostly caused by stock split. LVLT has the most significant outlier. The reason is a reverse split on October 2011. WEC also has an obvious outlier which happens on March 2011. The reason is that 2 for 1 split happens (Source: https://www.stocksplithistory.com/wec-energy-group/). And On August 2006, WFC had 2 for 1 split, causing rate dropped significantly.

And for qq-plots, except RCL, WFC, SRCL, other plots have nearly straight lines. For RCL, WFC and SRCL, the lines almost like "S" shape.

Stationary Test

	RCL	NDAQ	VRTX	WEC	WFC	DIS	KSS	LVLT	SRCL	WBA
DF	-3.89	-5.25	-6.23	-5.36	-6.46	-4.57	-4.12	-4.63	-5.68	-4.61
P-Value	0.017	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01

TABEL 3.2 AUGMENTED DICKEY-FULLER TEST FOR 10 ASSETS' RETURNS

From table above, under 0.05 significant level, the p-values are all small enough to accept they are stationary.

Normality Test

	RCL	NDAQ	VRTX	WEC	WFC	DIS	KSS	LVLT	SRCL	WBA
W	0.9	0.98	0.96	0.73	0.82	0.98	0.99	0.13	0.73	0.98
P-Value	2.19E-07	0.05	0.0025	1.76E-13	1.12E-10	0.12	0.52	2.2E-16	1.9E-13	0.31

TABEL 3.3 SHARIRO-WILK NORMALITY TEST FOR 10 ASSETS' RETURNS

Under 0.05 significant level, RCL, NDAQ, VRTX, WEC, WFC, LVLT and SRCL's p values are less than 0.05, then their null hypothesis is rejected. So these assets are not normally distributed. As for DIS, KSS and WBA, they have p values larger than 0.05. So DIS, KSS and WBA are normally distributed. Comparing with other distributions, the normal distribution fits those best.

From pairwise scatter plot, S&P500 has linear relationships with all of assets, which could be explained by S&P500 could reflect overall stock fluctuate. And all of stocks have positive correlation between each other. This could also be proved by the correlation matrix.

4. Portfolio Theory

In this part, we find the efficient frontier, minimum variance portfolio and tangency portfolio with short-selling and without short-selling. Sharpe ratios of these portfolios and assets are also calculated to give better comparison among portfolios.

4.1. Minimum Variance Portfolio

We develop the minimum variance portfolio both with and without short sell. The weight of each asset is shown as below:

	RCL	NDAQ	VRTX	WEC	WFC	DIS	KSS	LVLT	SRCL	WBA	Std	Expected Return
Short	0.085	0.063	0.047	0.268	0.074	0.166	0.182	-0.002	0.224	0.062	3.873	0.693
No Short	0.000	0.030	0.062	0.295	0.038	0.108	0.169	0.000	0.258	0.040	4.012	0.796

TABEL 4.1 THE MVP PORTFOLIO WITH AND WITHOUT SHORT SELL

With short sell, annualized mean return is 8.316 with standard deviation equals 46.476, while without short sell, annualized mean return is 9.552 and standard deviation is 48.150.

Compared with the variance of 10 original assets, variances of two MVP portfolios with and without short sell are much lower. This shows that by diversifying the assets, investors can reduce their risk.

Comparing two MVP portfolios, the one without short sell has higher variance. Short sell allows a wider weight range from -1 to 1. This means less diversity, so the variance becomes slightly higher. In another perspective, we can hedge against risk with short sell, and thus has a lower variance.

4.2. Tangency Portfolio

Now we use R to find the tangency portfolio with and without short sell. The result is as follows.

	RCL	NDAQ	VRTX	WEC	WFC	DIS	KSS	LVLT	SRCL	WBA	Variance	Expected Return
Short	0.083	-0.087	0.210	0.028	-0.26	0.865	-0.252	0.009	0.386	0.021	51.000	2.130
No Short	0.000	0.000	0.181	0.044	0.000	0.494	0.000	0.009	0.272	0.000	29.300	1.520

TABLE 4.2 TANGENCY PORTFOLIOS WITH AND WITHOUT SHORT SELL

From Table 4.2, when there is short sell the tangency portfolio has higher expected return and variance. As for the efficient frontiers of each portfolio, they are listed in appendix.

4.3. Portfolio Comparison

From previous parts, four different portfolios have been built. In this part, sharpe ratio and value-at-risk are calculated to get deeper insight into these four portfolios.

	RCL	NDAQ	VRTX	WEC	WFC	DIS	KSS	LVLT	SRCL	WBA
Sharpe Ratio	0.1106	0.0730	0.1559	0.0656	0.0385	0.2164	0.0278	0.0986	0.1139	0.0924

TABLE 4.3 SHARPE RATIOS OF 10 ASSETS

	MVP with short	MVP without short	Tangency with short	Tangency without short
Sharpe Ratio	0.155	0.176	0.286	0.264
Expected Return(%)	0.693	0.796	2.130	1.520
Variance (0.0001)	15.000	16.100	51.000	29.300

TABLE 4.4 SHARPE RATIOS OF FOUR PORTFOLIOS

From Table 4.3 and Table 4.4, tangency portfolios have higher sharpe ratios overall. Also, tangency portfolio with short has higher sharpe ratio than the one without short. Comparing sharpe ratios of 10 assets and their weights on these two portfolios, it is quite clear that portfolio with short sell has higher sharpe ratio in general.

	MVP with short	MVP without short	Tangency with short	Tangency without short
VaR	5678.714	5796.222	7386.98	9609.528

TABLE 4.5 VAR OF FOUR PORTFOLIOS

Compared with 10 individual assets, four portfolios' VaR becomes higher. And comparing four assets, tangency portfolio without short sell has highest VaR. This tell us two things. First, combination of assets could reduce risk. Secondly, when taking risk management in to consideration, portfolios without short sell perform better.

Appendix

1. Descriptive Statistics

10 Stocks' Monthly Closing Prices

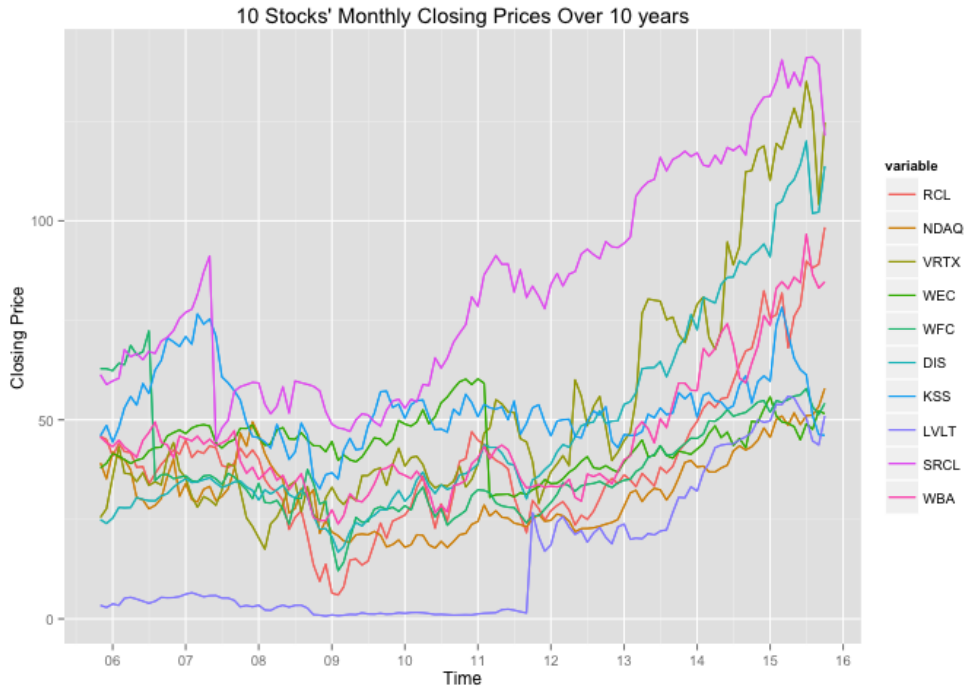

FIGURE 3.1 10 STOCKS' MONTHLY CLOSING PRICES OVER 10 YEARS

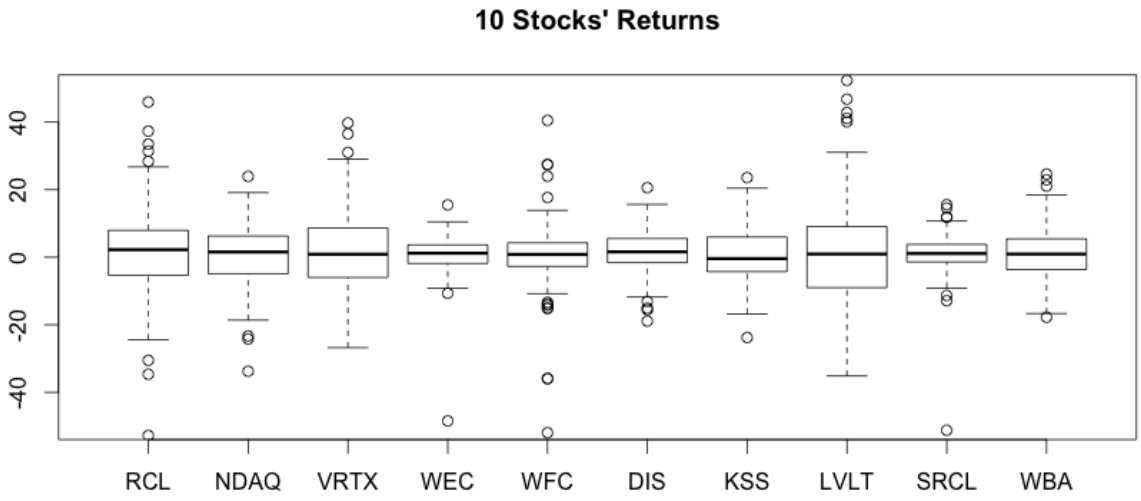

FIGURE 3.2 BOXPLOT OF RETURNS OVER 10 YEARS

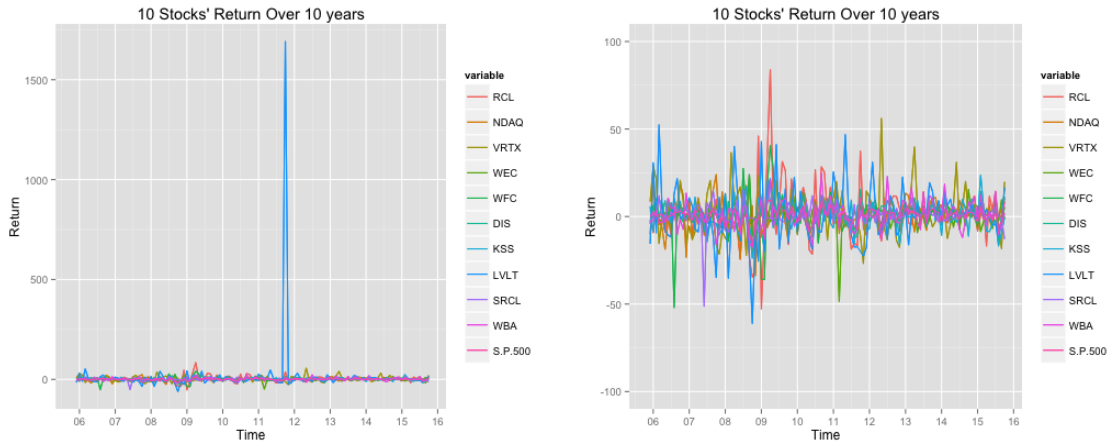

FIGURE 3.3 RETURNS OVER 10 YEARS AND RETURNS LEFT SIGNIFICANT OUTLIER

Equity Curve

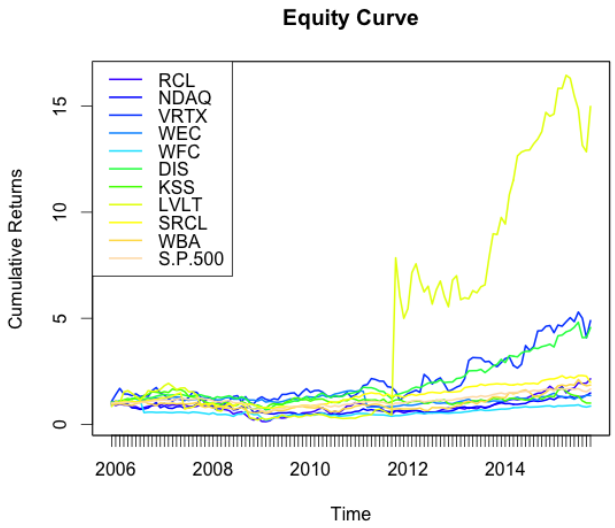

FIGURE 3.4 EQUITY CURVE OF 10 STOCKS OVER 10 YEARS

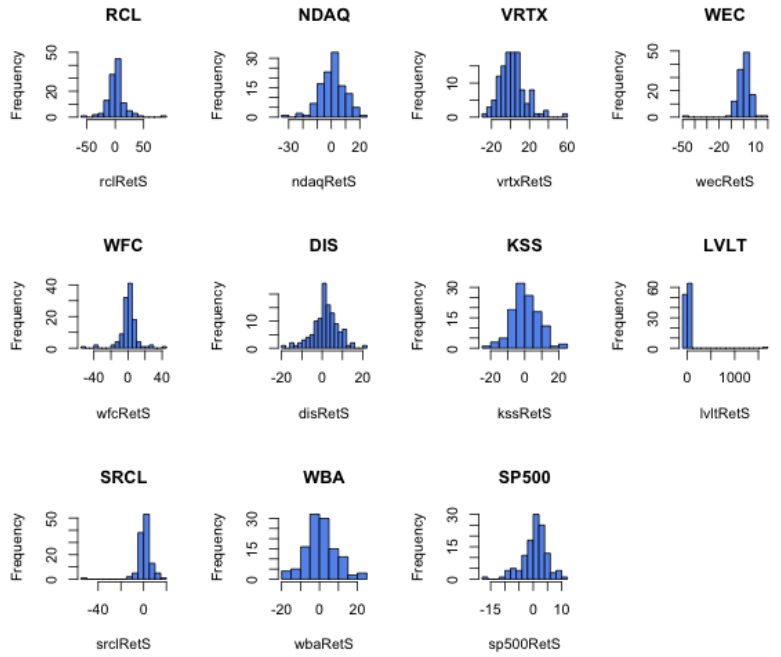

FIGURE 3.5 HISTOGRAMS FOR EACH RETURN SERIES

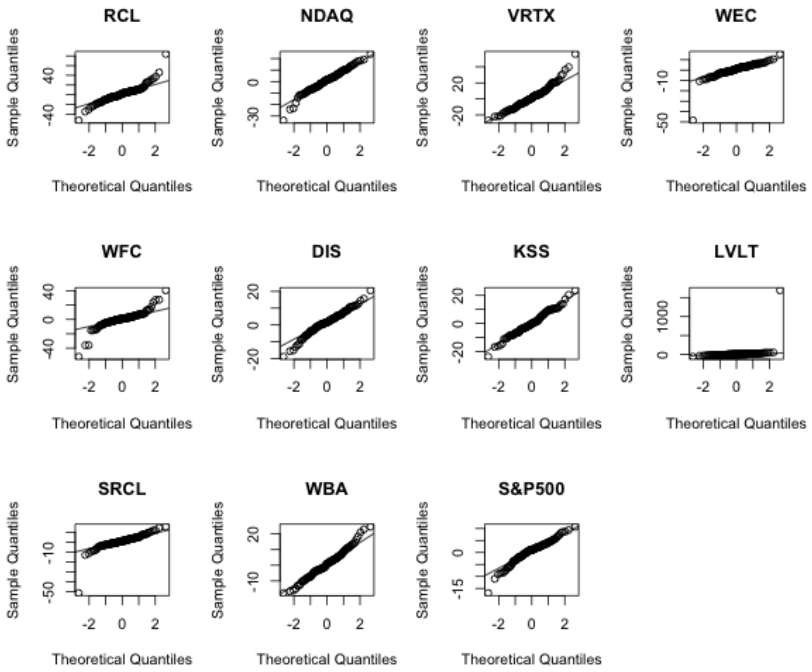

FIGURE 3.6 QQPLOTS FOR EACH RETURN SERIES

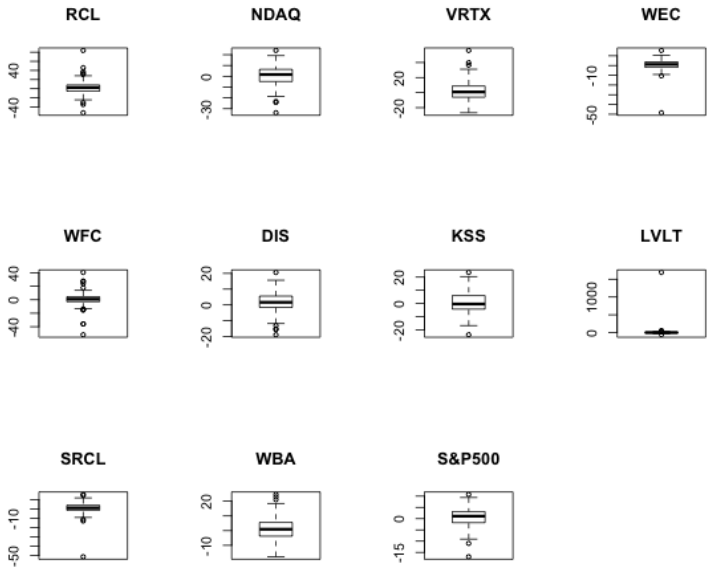

FIGURE 3.7 BOXPLOTS FOR EACH RETURN SERIES

Pairwise scatter plot

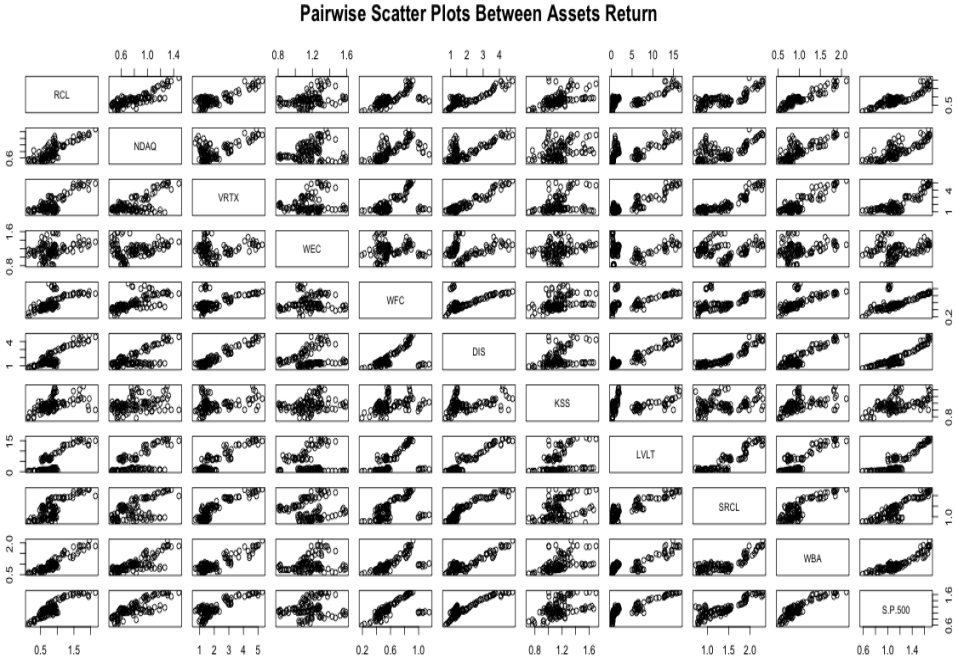

FIGURE 3.8 PAIRWISE SCATTER PLOTS

Sample covariance matrix

	RCL	NDAQ	VRTX	WEC	WFC	DIS	KSS	LVLT	SRCL	WBA	S.P.500
RCL	1.0000000	0.8343732	0.8214040	0.35570197	0.70478946	0.8589596	0.5157692	0.77348992	0.77283837	0.9266863	0.8913585
NDAQ	0.8343732	1.0000000	0.6495274	0.21418080	0.68894050	0.6997101	0.3371982	0.65733903	0.59345069	0.7984963	0.8128865
VRTX	0.8214040	0.6495274	1.0000000	0.16508284	0.58427551	0.9603069	0.3156486	0.90658879	0.89724254	0.8923763	0.8535131
WEC	0.3557020	0.2141808	0.1650828	1.00000000	0.08750108	0.2023086	0.2479535	0.07806391	0.05854805	0.2602766	0.1774511
WFC	0.7047895	0.6889405	0.5842755	0.08750108	1.00000000	0.5853363	0.3617842	0.58188597	0.54899006	0.7098185	0.6685365
DIS	0.8589596	0.6997101	0.9603069	0.20230857	0.58533630	1.0000000	0.3305786	0.95393885	0.93203252	0.9179423	0.9235966
KSS	0.5157692	0.3371982	0.3156486	0.24795350	0.36178418	0.3305786	1.0000000	0.29675793	0.33811553	0.4974434	0.4819252
LVLT	0.7734899	0.6573390	0.9065888	0.07806391	0.58188597	0.9539388	0.2967579	1.00000000	0.90632319	0.8499600	0.8908024
SRCL	0.7728384	0.5934507	0.8972425	0.05854805	0.54899006	0.9320325	0.3381155	0.90632319	1.00000000	0.8266680	0.8803334
WBA	0.9266863	0.7984963	0.8923763	0.26027659	0.70981848	0.9179423	0.4974434	0.84996003	0.82666797	1.0000000	0.9086065
S.P.500	0.8913585	0.8128865	0.8535131	0.17745112	0.66853648	0.9235966	0.4819252	0.89080237	0.88033344	0.9086065	1.0000000

2. Portfolio Theory

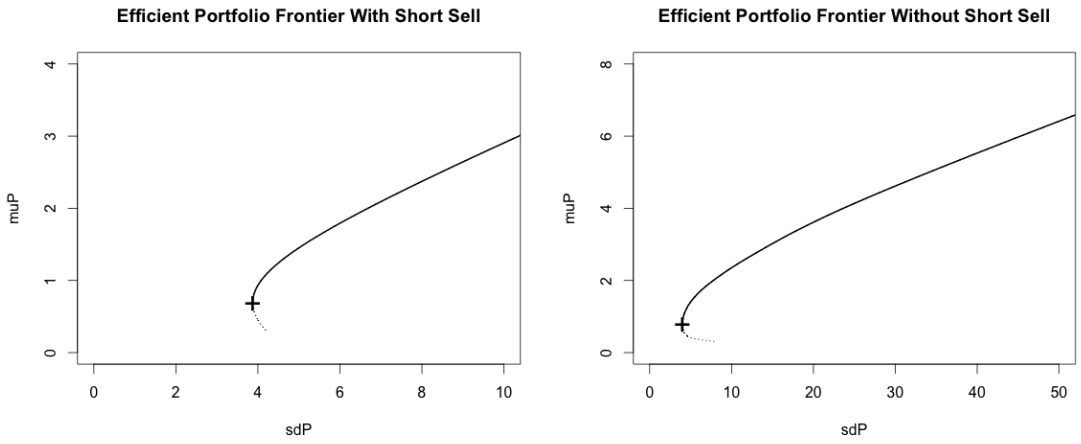

FIGURE 4.1 EFFICIENT PORTFOLIO FRONTIERS WITH AND WITHOUT SHORT SELL

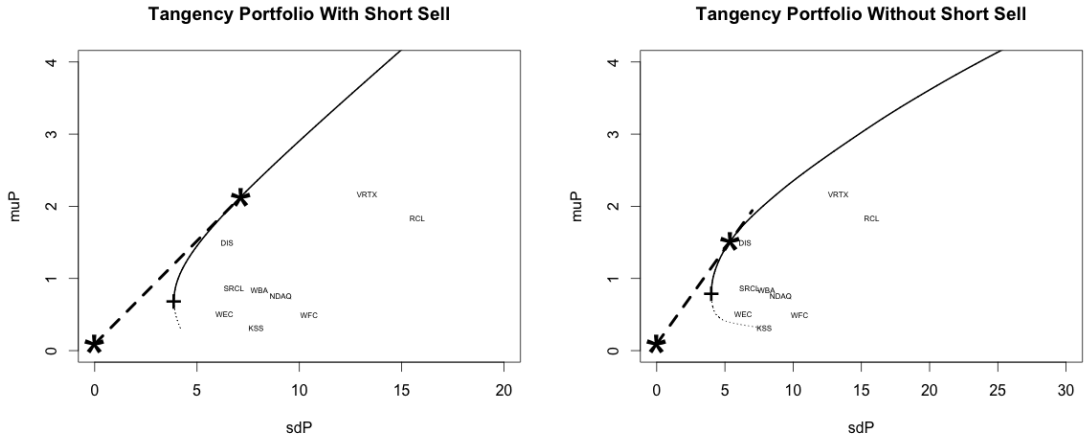

FIGURE 4.2 TANGENCY PORTFOLIO WITH AND WITHOUT SHORT SELL