Homework 1

Purpose: Probability Review Exercises.

Oscar # 1

Oscar has lost his dog in either forest A (with a priori probability 0.4) or in forest B (with a priori probability 0.6). If the dog is alive and not found by the Nth day of the search, it will die that evening with probability $N/(N+2)$.

If the dog is in A (either dead or alive) and Oscar spends a day searching for it in A, the conditional probability that he will find the dog that day is 0.25. Similarly, if the dog is in B and Oscar spends a day looking for it there, he will find the dog that day with probability 0.15.

The dog cannot go from one forest to the other. Oscar can search only in the daytime, and he can travel from one forest to the other only at night.

All parts of this problem are to be worked separately. Please give the answers and explain why.

Let the events be denoted as follows:

A - the dog is in forest A
B - the dog is in forest B
D_n - the dog dies on the nth evening
SA_n - Oscar searches in forest A on the nth day
SB_n - Oscar searches in forest B on the nth day
F_n - Oscar finds the dog on the nth day

In terms of these events,

$P(A) = 0.4$
$P(B) = 0.6$
$P(D_n|\text{not}F_1\&\text{not}F_2\&...\&\text{not}F_n) = n/(n+2)$
$P(F_n|SA_n\&A) = 0.25$, for all n
$P(F_n|SB_n\&B) = 0.15$, for all n

1. In which forest should Oscar look to maximize the probability he finds his dog on the first day of the search?

2. Given that Oscar looked in A on the first day but didn't find his dog, what is the probability that the dog is in A?

3. If Oscar flips a fair coin to determine where to look on the first day and finds the dog on the first day, what is the probability that he looked in A?

Oscar #2

Oscar has lost his dog either in forest A (with a priori probability 1/3) or forest B (with a priori probability 2/3). The probability that the dog will survive any particular night in forest A is 4/5 and in forest B is 3/5.

If the dog is in A (either dead or alive) and Oscar spends a day searching for him in A, the probability that he will find the dog that day is 1/2. The similar detection probability for a day of search in forest B is 2/5.

The dog cannot go from one forest to the other. Oscar can search only in the daytime and can travel from one forest to the other only at night.

Let the events be denoted as follows:

A - the dog is in forest A
B - the dog is in forest B
D_n - the dog dies on the nth evening
SA_n - Oscar searches in forest A on the nth day
SB_n - Oscar searches in forest B on the nth day
F_n - Oscar finds the dog on the nth day

The probabilities are given as follows:

$P(A) = 1/3$
$P(B) = 2/3$
$P(notD_n|A) = 4/5$
$P(notD_n|B) = 3/5$
$P(F_n|A\&SA_n) = 1/2$
$P(F_n|B\&SB_n) = 2/5$

Coolheaded Oscar has established the following values (in dollars):

Finding dog alive +60
Each day (or part thereof) of search -3
Finding dog dead 0
Not finding dog -10
Additional cost if Oscar must actually search in both forests -3

Oscar is incapable of figuring it all out; so he decides that he will search for just two days – looking in B on the first day and, if necessary, looking in A on the second day.

1. Determine the expected value of this policy.