Yirong Wang

Homework 2

Problem 1 Oscar #1

1.1 If Oscar searchs for A in the first day:

$$\begin{aligned} P[F_1|SA_1] &= P[A] \times P[F_1|SA_1 \wedge A] \\ &= 0.25 + \times 0.4 \\ &= 0.1 \end{aligned}$$

If Oscar searchs for B in the first day:

$$\begin{aligned} P[F_1|SB_1] &= P[B] \times P[F_1|SB_1 \wedge B] \\ &= 0.15 \times 0.6 \\ &= 0.09 \end{aligned}$$

Therefore, Oscar should search for A in the first day to maximize the probability he finds his dog on the first day.

1.2 According to Bayes'rule:

$$\begin{aligned} &P[A|SA_1 \wedge \neg F_1] \\ &= \frac{P[SA_1 \wedge \neg F_1|A] \times P[A]}{P[SA_1 \wedge \neg F_1]} \\ &= 0.75 \times 0.4 \div 0.9 \\ &= 0.33 \end{aligned}$$

1.3 According to Bayes'rule:

$$\begin{aligned} P[SA_1|F_1] &= \frac{P[F_1|SA_1] \times P[SA_1]}{P[F_1]} \\ &= 0.1 \times \frac{1}{2} \div (0.1 \times \frac{1}{2} + 0.09 \times \frac{1}{2}) \\ &= 0.526 \end{aligned}$$

Problem 2 Oscar #2

2.1 Let X denote the value that Oscar earned up to the end of day 2. Let $\mathbb{S}$ denote the state space of X.

images/image1.jpg

Figure 1: Decision Tree

$\mathbb{S}$ has only 6 states i.e. $\mathbb{S} = \{s_1, s_2, s_3, s_4, s_5, s_6\}$, where

$$\begin{aligned}
s_1 &:= Val(A \wedge \neg F_1 \wedge \neg F_2) = -19 \\
s_2 &:= Val(A \wedge \neg F_1 \wedge F_2 \wedge D_1) = -9 \\
s_3 &:= Val(A \wedge \neg F_1 \wedge F_2 \wedge \neg D_2) = 51 \\
s_4 &:= Val(B \wedge \neg F_1 \wedge \neg F_2) = -19 \\
s_5 &:= Val(B \wedge F_1 \wedge D_1) = -3 \\
s_6 &:= Val(B \wedge F_1 \wedge \neg D_1) = 57
\end{aligned}$$

Given that:

$$\begin{aligned}
&P[A \wedge \neg F_1 \wedge \neg F_2] = P[X = s_1] = \frac{1}{3} \times 1 \times \frac{1}{2} = \frac{1}{6} \\
&P[A \wedge \neg F_1 \wedge F_2 \wedge D_2] = P[X = s_2] = \frac{1}{3} \times \frac{1}{2} \times \frac{1}{5} = \frac{1}{30} \\
&P[A \wedge \neg F_1 \wedge F_2 \wedge \neg D_2] = P[X = s_3] = \frac{1}{3} \times 1 \times \frac{1}{2} \times \frac{4}{5} = \frac{4}{30} \\
&P[B \wedge \neg F_1 \wedge \neg F_2] = P[X = s_4] = \frac{2}{3} \times \frac{3}{5} \times 1 = \frac{6}{15} \\
&P[B \wedge F_1 \wedge D_1] = P[X = s_5] = \frac{2}{3} \times \frac{2}{5} \times \frac{2}{5} = \frac{8}{75} \\
&P[B \wedge F_1 \wedge \neg D_1] = P[X = s_6] = \frac{2}{3} \times \frac{2}{5} \times \frac{3}{5} = \frac{12}{75}
\end{aligned}$$

$$
\begin{aligned}
\mathbb{E}(X) &= P[X = s_1] \times (-19) + P[X = s_2] \times (-9) + P[X = s_3] \times 51 \\
&+ P[X = s_4] \times (-19) + P[X = s_5] \times (-3) + P[X = s_6] \times 57 \\
&= \frac{1}{6} \times (-19) + \frac{1}{30} \times (-9) + \frac{4}{30} \times 51 + \frac{6}{15} \times (-19) + \frac{8}{75} \times (-3) + \frac{12}{75} \times (57) \\
&= \frac{68}{15}
\end{aligned}
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