Homework 5

1. Consider an M/M/1/2 system with input rate λ. When a customer arrives to an empty system, the server proceeds at rate μ. However, if a second customer arrives while the first is still in service, the server speeds up to a rate 2μ, and continues at rate 2μ until the system empties again. Only one customer will be in service at a time. Note that the system has limited storage and can hold at most one in service and one in queue. Find $P_0 = P[systemisempty]$. Be sure to express your answer explicitly in terms of λ, and μ only.

2. In a TA-office there are half as many workstations as there are TA's. Assume that the amount of time each TA spends away from the office is an exponentially distributed random variable with mean $1/\lambda$. Also assume that each time the TA returns to the office s/he needs to use a workstation for an exponentially distributed amount of time with mean $1/\mu$. (Also assume the TA will wait for and use the workstation and then leave the office on each visit.) As a TA, you would prefer the office in which the probability of finding an idle workstation in the office is highest. Based on these assumptions:

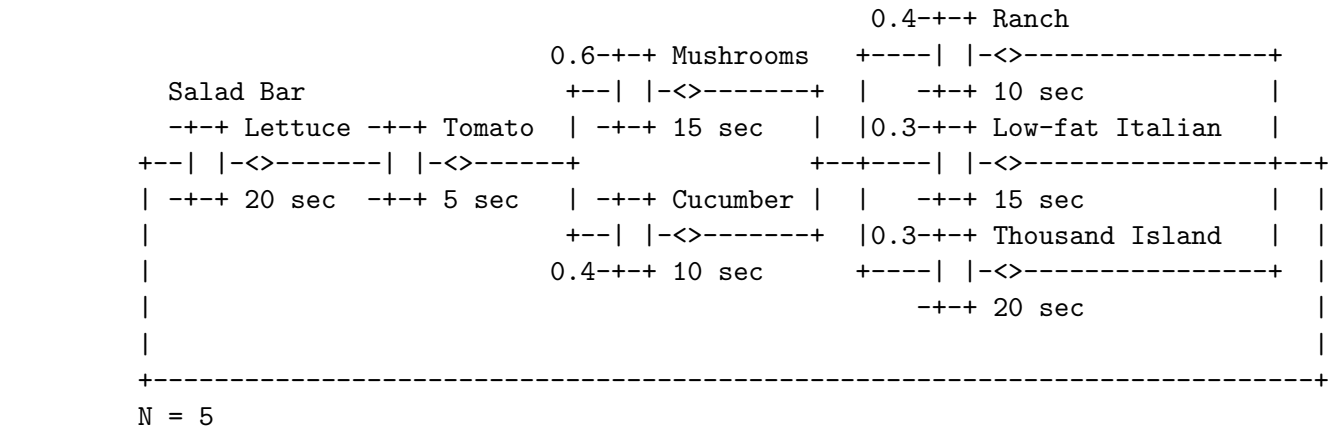
   (a) Find the probability that a workstation is idle in a two-TA office, in terms of λ and μ.

   (b) Find the probability that a workstation is idle in a four-TA office in terms of λ and μ.

   (c) If the two workstations in the four-TA office are replaced by a workstation that is twice as fast (so that a TA only needs it for $1/(2\mu)$ on average), what is the probability of finding it idle?

3. Write a mean value analysis program. The program should take as input the number of devices K, the mean service time S_i at each device i, the visit ratio V_i at each device i, and the customer population N. The program should output the system throughput and the average queue length at each device.

   Use your program to solve the following system:



   You should submit your source code and your program output from this problem.