

Yirong Wang

Homework 2 Source code: <https://github.khoury.northeastern.edu/yirongwn/EECE5643/tree/master/hw2>

Problem 1 Ex1.2.2

1.1 (a) for 1000 jobs:

average interarrival time = 9.87

average service time = 7.12

average delay = 18.59

average wait = 25.72

maximum delay = 118.76

num of jobs in the service node at (t=400) = 7

proportion of jobs delayed = 0.72

1.2 (b) maximum delay = 118.76

1.3 (c) num of jobs in the service node at (t=400) = 7.

To intergrate the indicator function $\varphi_i(t)$ over the job domain is the process that counts the number of jobs that arrives before t and leaves after t for $\forall i \in \{1 \dots n\}$.

$$l(t) = \sum_{i=1}^n \varphi_i(t)$$

Let $t = 400$, the simulation results indicates that

$$l(t = 400) = 7$$

1.4 (d) The simulation results shows that the proportion of jobs delayed = 0.72. Given that

$$\bar{x} = \frac{1}{c_n} \int_0^{c_n} x(t) dt$$

and,

$$\int_0^{c_n} x(t) dt = \sum_{i=1}^n s_i$$

therefore,

$$\bar{x} = \frac{n\bar{s}}{c_n} = \frac{1000 \times 7.12}{9897.22} \approx 0.72$$

Observe that this number is equal to the proportion of job delayed. The explanation is that ...

Problem 2 Ex.1.2.6

2.1 (a) for 500 jobs

average service time = 3.03

server's utilization = 0.74

traffic intensity = 0.74

2.2 (b)

$$s_i = c_i - a_i - d_i$$

where

$$d_i = c_{i-1} - a_i$$

therefore,

$$s_i = c_i - a_i - c_{i-1} + a_i = c_i - c_{i-1}$$

Problem 3 Ex.2.3.4

3.1 (a) Let $X \in \mathbb{Z}$ denote the r.v. of the sum of the two up-faces, based on 1000000 replications the estimated probabilities are:

$$P[X = 2] = 0.006$$

$$P[X = 3] = 0.024$$

$$P[X = 4] = 0.048$$

$$P[X = 5] = 0.071$$

$$P[X = 6] = 0.095$$

$$P[X = 7] = 0.142$$

$$P[X = 8] = 0.165$$

$$P[X = 9] = 0.142$$

$$P[X = 10] = 0.119$$

$$P[X = 11] = 0.095$$

$$P[X = 12] = 0.094$$

3.2 (b)

$$\begin{aligned} P[X = 7] &= P[1 \wedge 6] + P[2 \wedge 5] + P[3 \wedge 4] \\ &= \frac{1}{13} \times \frac{4}{13} \times 2 + \frac{2}{13} \times \frac{2}{13} \times 2 + \frac{2}{13} \times \frac{2}{13} \times 2 \\ &= \frac{8}{169} + \frac{8}{169} + \frac{8}{169} \\ &= \frac{24}{169} \\ &\approx 0.142 \end{aligned}$$

Problem 4 Ex.2.3.5

4.1 (a) Let \vec{v} and \vec{w} denote two randomly selected 2D-vectors land on the circumference of the circle centering at the origin of a two-dimensional coordinate plane, of radius $\rho = 1$. Let $d(\vec{v}, \vec{w})$ denote the distance between \vec{v} and \vec{w} . Based on 100000 replications of Monte Carlo simulation, the estimated probabilities are:

$$P[d(\vec{v}, \vec{w}) \geq \rho] = 0.667$$

4.2 (b) The event that $d(\overrightarrow{v}, \overrightarrow{w}) \geq \rho$ is independent of the value of ρ .