Yirong Wang

Homework 2 Source code: https://github.com/Effygal/EECE5643/tree/main/hw2

Problem 1 Ex1.2.2

1.1 (a) for 1000 jobs:
average interarrival time = 9.87
average service time = 7.12
average delay = 18.59
average wait = 25.72
maximum delay = 118.76
num of jobs in the service node at (t=400) = 7
proportion of jobs delayed = 0.72

1.2 (b) maximum delay = 118.76

1.3 (c) num of jobs in the service node at (t=400) = 7.
To intergrate the indicator function $\varphi_i(t)$ over the job domain is the process that counts the number of jobs that arrives before t and leaves after t for $\forall i \in \{1...n\}$.

$$l(t) = \sum_{i=1}^{n} \varphi_i(t)$$

Let $t = 400$, the simulation results indicates that

$$l(t = 400) = 7$$

1.4 (d) The simulation results shows that the proportion of jobs delayed = 0.72. Given that

$$\overline{x} = \frac{1}{c_n} \int_0^{c_n} x(t)dt$$

and,

$$\int_0^{c_n} x(t)dt = \sum_{i=1}^{n} s_i$$

therefore,

$$\overline{x} = \frac{n\overline{s}}{c_n} = \frac{1000 \times 7.12}{9897.22} \approx 0.72$$

Observe that this number is equal to the proportion of job delayed. An intuiative explanation is that whenever the server is busy, there are jobs get delayed in the queue; when the server is idle, there is no job in the queue. The two ratios converge with each other.

Problem 2 Ex.1.2.6

2.1 (a) for 500 jobs
average service time = 3.03
server's utilization = 0.74
traffic intensity = 0.74

2.2 (b)

$$s_i = c_i - a_i - d_i$$

where

$$d_i = c_{i-1} - a_i$$

therefore,

$$s_i = c_i - a_i - c_{i-1} + a_i = c_i - c_{i-1}$$

Problem 3 Ex.2.3.4

3.1 (a) Let $X \in \mathbb{Z}$ denote the r.v. of the sum of the two up-faces, based on 1000000 replications the estimated probabilities are:

$$\begin{aligned}
P[X=2] &= 0.006 \\
P[X=3] &= 0.024 \\
P[X=4] &= 0.048 \\
P[X=5] &= 0.071 \\
P[X=6] &= 0.095 \\
P[X=7] &= 0.142 \\
P[X=8] &= 0.165 \\
P[X=9] &= 0.142 \\
P[X=10] &= 0.119 \\
P[X=11] &= 0.095 \\
P[X=12] &= 0.094
\end{aligned}$$

3.2 (b) Axiomatically,

$$\begin{aligned}
P[X=7] &= P[1 \wedge 6] + P[2 \wedge 5] + P[3 \wedge 4] \\
&= \frac{1}{13} \times \frac{4}{13} \times 2 + \frac{2}{13} \times \frac{2}{13} \times 2 + \frac{2}{13} \times \frac{2}{13} \times 2 \\
&= \frac{8}{169} + \frac{8}{169} + \frac{8}{169} \\
&= \frac{24}{169} \\
&\approx 0.142
\end{aligned}$$

Problem 4 Ex.2.3.5

4.1 (a) Let $\overrightarrow{v}$ and $\overrightarrow{w}$ denote two randomly selected 2D-vectors land on the circumference of the circle centering at the origin of a two-dimensional coordinate plane, of radius $\rho = 1$. Let $d(\overrightarrow{v}, \overrightarrow{w})$ denote the distance between $\overrightarrow{v}$ and $\overrightarrow{w}$. Based on 100000 replications of Monte Carlo simulation, the estimated probabilities are:

$$P[d(\overrightarrow{v}, \overrightarrow{w}) \geqslant \rho] = 0.667$$

4.2 (b) The event that $d(\overrightarrow{v}, \overrightarrow{w}) \geqslant \rho$ is independent of the value of ρ. The reason is that as we do Monte Carlo simulation, we draw sample of $\theta \sim Uniform(-\pi, \pi)$, and $d(\overrightarrow{v}, \overrightarrow{w})$ scales at the same rate with ρ as ρ changes.