Extragalactic Astronomy

System composed by a central BH embedded in Bulge with an Hernquist's density profile - Numerical Study

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1 About the simulations

The simulations are made through the Joshua E. Barnes' *Tree Code* (for more details see `https://www.ifa.hawaii.edu/~barnes/treecode/treeguide.html`).

The system is initialized through the jupyter notebook *BH + Bulge - Initial Conditions Code (Barnerstreecode)*. The execution of that notebook give as output two files,

```
initial_conditions_file.txt
system_properties.txt,
```

where the first one in used by the *Barnes' Tree Code* and contains the number of bodies (number of particles+1, where +1 is for the BH), the number of dimensions, the initial simulation time and the particles' mass, initial positions and velocities; the second one is used by the jupyter notebook made for the analysis.

The execution of the initial condition notebook has to be followed by the run of the *Barnes' Tree Code* compiled from the terminal writing, for example,

```
./treecode in=initial_conditions_file.txt out=output_data.txt dtime=value
 eps=value theta=value options=out-phi tstop=value dtout=value
 > system_description.txt
```

where it is needed to specify the parameters' values. In the *BH + Bulge - Initial Conditions Code (Barnerstreecode)* notebook and in this relation is present a section that gives hints on the choice of the parameters' values.

2 About this relation

The relation presented is very briefly: we report only some plots with a short comment where we verify that the generated system is at equilibrium and we compare qualitatively the results with two plots taken from the article *A family of models for spherical stellar systems* by S. Tremaine, D. Richstone et al.; for the theoretical part see the lectures notes.

3 Numerical parameters' values

3.1 dtime

The *dtime* parameter is the integration time-step and it's convenient to use a value that has an exact representation as a floating-point number; a small value of *dtime* will result into a better precision but a greater computational cost. For the simulations presented in this 'relation', we choose it as a fraction of the system dynamical time t_{dyn}, roughly equal to $\frac{r_{scale}}{v_{circ}(r=r_{scale})} = r_{scale}\left(\frac{GM_{tot}}{r_{scale}}\right)^{-\frac{1}{2}}$ (set, for example, $dtime = t_{dyn}/100$).

3.2 eps

The *eps* parameter is the *Gravitational Softening Parameter*; for more details see the previous relations (ex. *Collapse of an Homogeneous Sphere - Numerical Study*) where we explained in

details what is *eps* and, from the analysis did, found that the better value for the *gravitational softening parameter* has to be chosen in the interval

$$10^{-1}\Delta_{mean} \leq \epsilon \leq \Delta_{mean}, \tag{1}$$

where Δ_{mean} is the mean separation between particles.

In the system under analysis we have also a BH at the center and it's important to resolve its sphere of influence: we decided to choose Δ_{mean} as the minimum value between the BH's influence radius and the mean distance between particles (obtained dividing the sphere into shells, computing the particles' mean distance in each shell and then selecting the smallest one).

Note that a Black Hole of a small mass implies a small softening parameter and so the computational time increases.

3.3 theta

The *theta* parameter is the *opening angle* used to adjust the accuracy of the force calculation; values less than unity produce more accurate forces, albeit at greater computational expense. We usually set $\theta = 0.1$.

3.4 tstop

The *tstop* parameter is the time at which the N-Body integration terminates; we chosen it as a multiple of the dynamical time ($tstop = 2t_{dyn}$).

3.5 dtout

The *dtout* parameter is the time interval between the saving of two complete description of the system.

4 Study of the system's equilibrium

The simulation is made for a Bulge of mass $M = 1$, a BH of mass $M_{BH} = 0.1$, a scale radius $r_{scale} = 10$ and 1000 particles; we runned the Barners' treecode from the terminal with

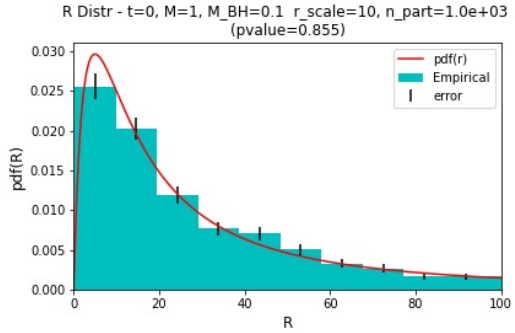
```
./treecode in=initial_conditions_file.txt out=output_data.txt dtime=0.3
 eps=0.25 theta=0.1 options=out-phi tstop=63 dtout=1
 > system_description.txt
```

4.1 Position

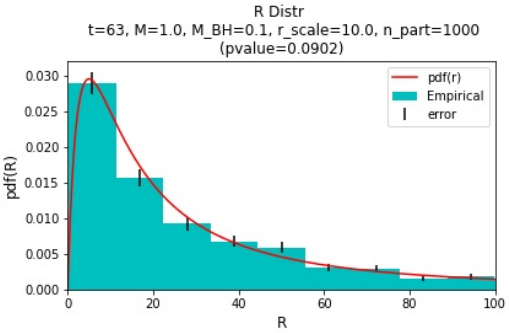
4.1.1 Position Distributions

We know that if the system is at equilibrium already from $t = 0$, the position distributions should be the same at any given time; to check this we compare at $t = 0$ and at another time (in our case, $t = 63$), the empirical distributions with the theorical ones.

We now report the plots obtained in the simulation.

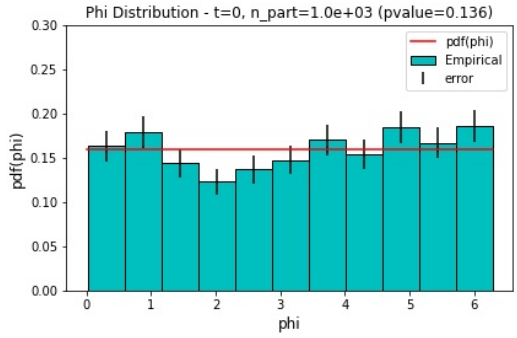


(a) t=0

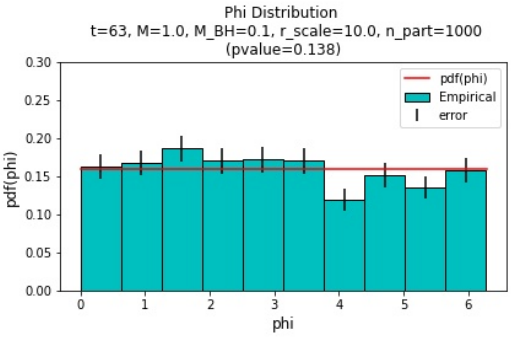


(b) t=63

Figure 1: R distribution

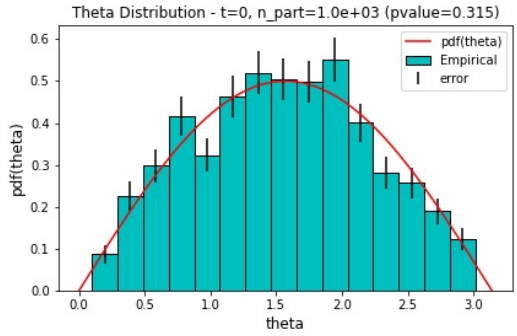


(a) t=0

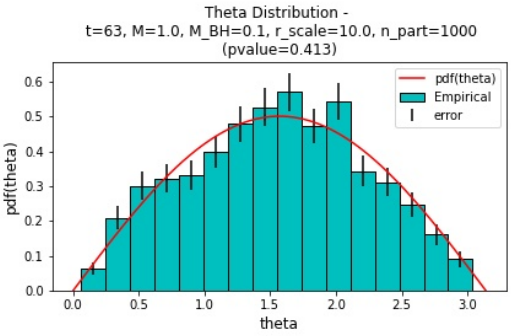


(b) t=63

Figure 2: ϕ distribution



(a) t=0



(b) t=63

Figure 3: θ distribution

We can conclude, analyzing the plots above, that the position distributions do not change over time.

4.1.2 Lagrangian Radius and Sample Distance

We then analyze the evolution over time of the so called *Lagrangian Radius*, which is the radius that contains a certain percentage of the mass (for more details about their computation, see one of the previous relation (ex. *Collapse of an Homogeneous Sphere - Numerical Study*)).

In the following plot are presented the evolution for the Lagrangian radius that embed the 20%, 40%, 60% and 80% of the mass; since the system should be at equilibrium, the Lagrangian radius should be constant at any time and this is, indeed, what we have obtained.

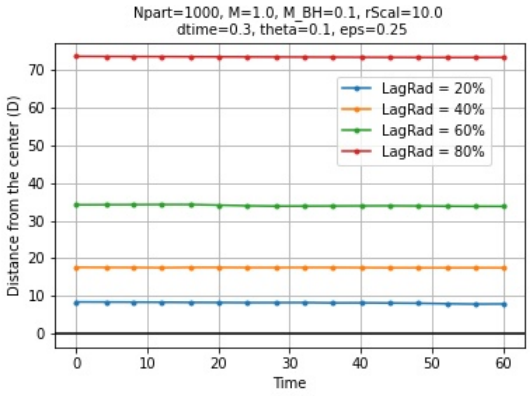


Figure 4: Lagrangian Radius

For a better visualization of the fact that the Lagrangian radius are constant, we focused our attention only on two of them, plotting for each one also a dotted horizontal line that corresponds to the mean value assumed by the Lagrangian radius between $0 < t < 63$.

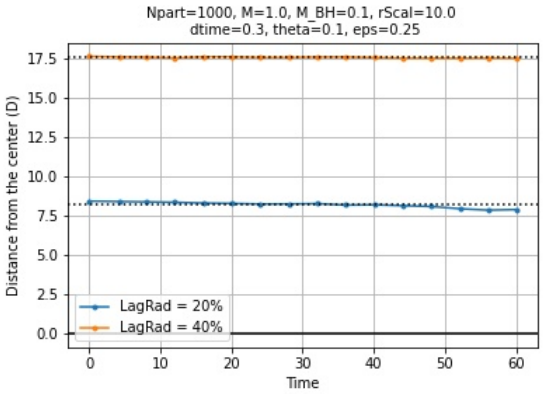


Figure 5: Lagrangian Radius and Mean

We report now the evolution over time of 5 samples' distance from the center of the sphere. You can observe that they made a periodical motion around the center (some of them, at $t = 63$, have not already computed a period).

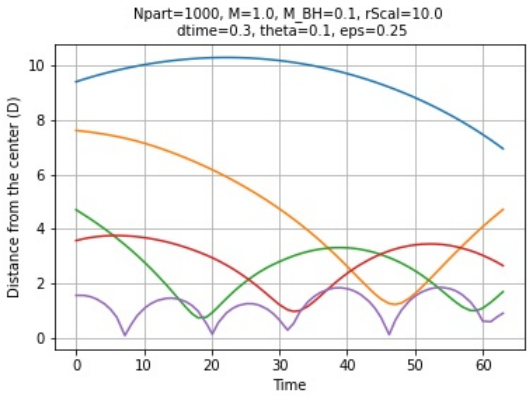


Figure 6: Distance from the center

4.2 Energy

We report here the energy plot for our simulation. If the system is at equilibrium, its position and its velocity distributions do not evolve in time and so do the potential and the kinetic energies. We report the plot obtained.

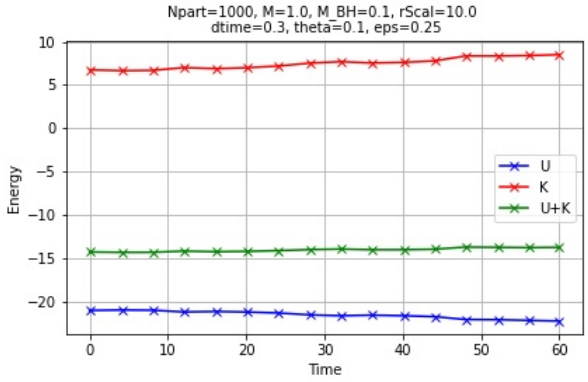
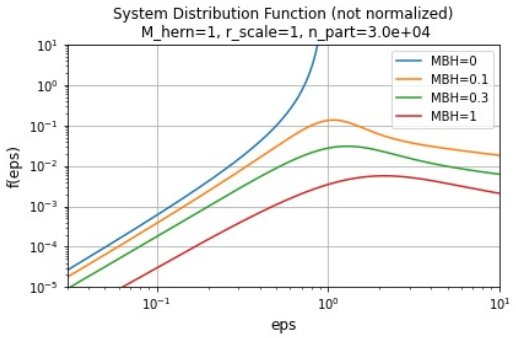


Figure 7: Energies

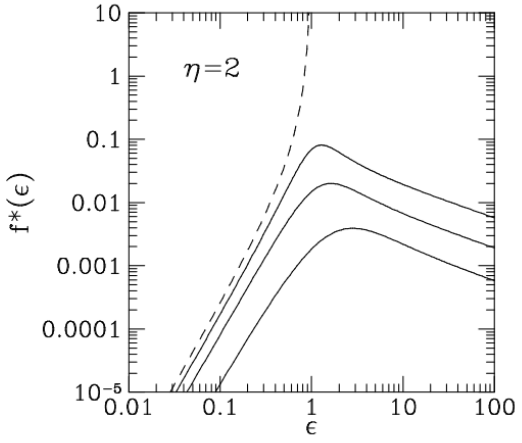
4.3 Comparison with the Tremaine et al. article

We extract two plots from the article *A family of models for spherical stellar systems* by S. Tremaine, D. Richstone et al. and compare them with the ones obtained by us. The plots below are obtained using the Initial Condition code and we made them for 30'000 particles and the system properties as the one chosen in the article: $M_{bulge} = 1$, $r_{scale} = 1$ and a BH of different masses (reported in the legend). Our results are, at least qualitatively, very similar to the ones of the article.

System distribution function:



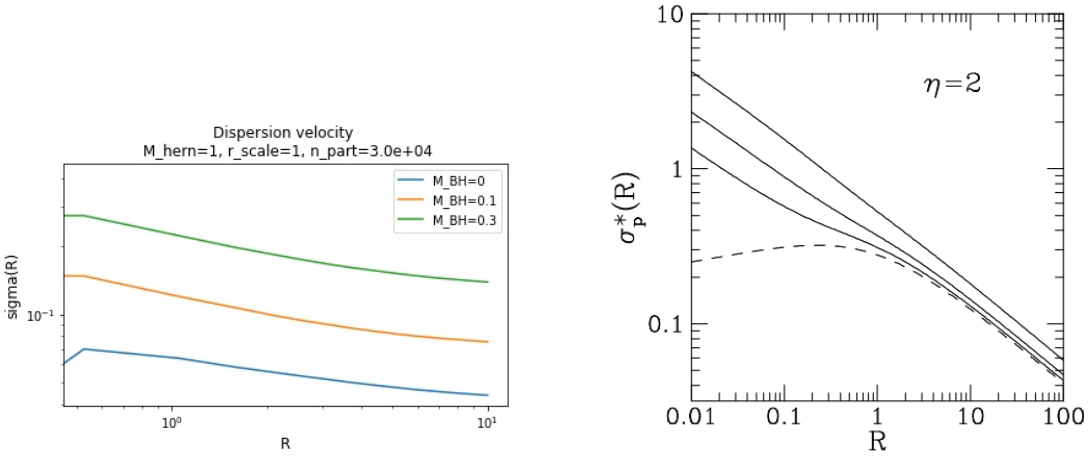
(a) Simulation



(b) Article p.18

Figure 8: Distribution function

Dispersion velocity:



(a) Simulation

(b) Article p.19

Figure 9: Dispersion velocity