UNIVERSITY OF GHANA

Estimation of the Lending Phase and Interest Rate in Microfinance Instituition

BY

Efua Ainooson Noonoo (10676881)

Rachel Makafui Kotoku (10683471)

David Clark Anani (10661650)

September, 2021

THE PROJECT IS SUBMITTED TO THE UNIVERSITY OF GHANA, DEPARTMENT OF STATISTICS AND ACTUARIAL SCIENCE, IN PARTIAL REQUIREMENT FOR THE AWARD OF DEGREE IN BSc. ACTUARIAL SCIENCE

DECLARATION

We hereby declare that this dissertation is a presentation of our original research work, in partial fulfilment for the requirement for Bachelor of Science degree. Wherever contributions of others are involved, every effort is made to indicate this clearly, with due reference to the literature and acknowledgement of research work and articles of relevant authors. This dissertation was done at the University of Ghana, Legon under the supervision of Dr. Perpetual Andam Boiquaye.

Student: Efua Ainooson Noonoo (10676881) 06/10/2021

Student: Rachel Makafui Kotoku (10683471) 06/10/2021

Student:David Clark Anani (10661650) 06/10/2021

Supervisor: Dr. Perpetual Andam Boiquaye 06/10/2021

ACKNOWLEDGEMENT

We would first of all like to thank our Almighty God for making this project a success. Secondly, we would like to thank our families for their immense prayers and support towards this project. A very big thank you goes to our supervisor, Dr. Perpetual Andam Boiquaye for her enormous support and counsel. We also like to thank all lectures and Teaching Assistants for their help in making this project a success.

DEDICATION

We dedicate this project to God, families and friends.

Abstract

Microfinance institutions also known as microcredit are institutions or a banking services which give loans to the less privileged in societies. They give out loans without collateral. One of the techniques used in the microfinancing is banning individuals from taking loans once they default. The microfinances give the borrowers ample time to pay back their loans. This project seeks to calculate the suitable length of time of punishment in the lending phase and the optimal interest rate by maximizing both borrower and lender's payoff. Upon maximizing the borrowers' payoff, we realized that the interest rate does not only affect the borrowers and lenders payoff but also the length off punishment phase. It also shows that the aim of the lender is to make profit after repayment of loans. The optimal interest rate of a loan helps to prevent strategic defaulting of borrowers.

Contents

1 Introduction

This research work presents findings from a quantitative research study. It highlights the theoretical background of the study and problem statement.The objectives and significance of the work are also explained and finally, an overview of the way in which the study was conducted is provided.

1.1 Background of Study

The formal and informal financial institutions established to provide services to the public have grown significantly over time. The majority of structured financial products, on the other hand, are profit-driven. As a result, they tend to overlook the weakest members of society who are unable to meet their collateral criteria. The majority of people in the informal sector borrow money from their friends, family, and neighbors (McGuire et al., 1998). Although legal and informal income loans are available, the poor are frequently discouraged from taking advantage of them due to exorbitant interest rates. In the financial sector, inflation and temporal value theories of money are becoming increasingly important considerations when dealing with money and related operations. The same is true of loans. Today's money as a modest principal is not the same as tomorrow's. As a result, as the customer repays the debt, a creditor wants to ensure that he does not lose money on a loan he makes to an individual. He will then charge an interest rate that will make the loan worth as much as, or more than, the current loan value in the future. The optimal loan rate is the interest rate that makes the loan's value equal to tomorrow's today (Bennett et al.,1996)

1.2 Problem Statement

Microfinance institutions usually support poor microentrepreneurs by offering them loans to support their ventures. It is only right that borrowers pay back the loans taken when payment is due. Unfortunately, some borrowers default as a result of negative economic shock because microfinance institutions lack sound legal systems, advanced credit scoring mechanisms and collateral requirements in ascertaining the credit worthiness of a borrower to ensure payment is done. There is therefore the need for microfinance institutions to obtain an optimal interest rate and as well as derive the length of time a borrower would spend in the punishment phase so borrowers will desist from defaulting strategically.

1.3 Objectives of Study

The objective of the study is to calculate the duration of the lending phase and the optimal interest rate when taking a loan

1.4 Significance of Study

Microfinance institutions provide microcredit to micro-entrepreneurs and low-income individuals to support their ventures and also enhance their living conditions. In as much as microfinance institutions would want to meet their social mission obligations, they are not to give out loans at the risk of the sustainability of the institution. A low interest rate would result in microfinance institutions running at a loss and a high interest rate will cause borrowers to default strategically. There is therefore the need to estimate the optimal interest rate and suitable length of time in the punishment phase to ensure the sustainability and liquidity of the institution and the repayment of incentives respectively.

1.5 Organisation of Work

The final research study is organized into five chapters. Chapter one deals with the introduction, chapter two consists mainly of the literature review, chapter three explains the research methodology and the final chapter presents the summary of the study and conclusions.

2 Literature Review

Developing countries deal with a number of social cankers and a major one faced is the inaccessibility of bank loans. This is only due to the fact that most individuals cannot provide collateral to secure a loan. Due to this, most poor households are usually excluded from the formal banking system as a result of the inability of formal credit institutions to monitor and enforce loan repayments. These people are left with no option than to borrow from the informal sector and lenders at usurious interest rates or just be denied access to credit and therefore investment. (Morduch,2000) opines that it is at this point that the microfinance movement step in with new contractual structures and organizational forms that reduce costs of making small and uncollateralized loans and the risk involved in providing such loans. Microfinance does not only promise to combat poverty but to also develop the institutional capacity of micro financial systems to cost-effectively lend money to poor households. From the discussions so far, one would agree with (Dierner & Mauk, 2013) who define microcredits as very affordable loans given by microfinance institutions (MFIs) to poor people to help them establish micro enterprises. Again, (Macchiavello, 2012) indicates that microcredit goes beyond offering affordable loans by providing services such as coaching which encompasses business training and financial education with women being the main focus. He also argues that microcredit advocates against high interest rate charges on loans offered. Its principal objective is to raise incomes and broaden financial markets by providing financial services to small-scale entrepreneurs who most often than not lack access to capital markets. (Aghion & Morduch 2000) explicitly state the two main responsibilities of microfinance institutions which are; poverty alleviation by reaching out to the means-tested and to promote private sector activity of curbing unemployment and underemployment.

Microfinance institutions were set up to help the poor by providing or offering very small loans which most traditional banks avoid because it is a huge risk. These institutions also empower women who are being treated as outcastes in their societies which are normally ruled by men and also fund minor investments which end up prospering. Over the years, due to the success stories of microfinance institutions, it has led to the proliferation of lending programs across the globe. According to information from Distribution, Growth and Performance of Microfinance, there are 1500 institutions in 85 developing countries with 688 situated in Indonesia and are mostly supported by international organizations. They have about 54 million members, 44 million savers and 23 million borrowers. The main reason of the success of microfinance institution is difficult to pinpoint because of the variety of contracts. The idea of microfinance institutions was first introduced into the world by an economist named Muhammad Yunus. Yunus and some of his students came up with this idea when a famine hit the town which their university was located in 1974. They made a list of items the people need as it was a town of craftsmen and also decided to give out small loans to the people starting with 42 women. He then went further and pitched his ideas to local banks so they make policies that would consider the vulnerable ones. Sadly, his plans were not considered so he established his own bank in 1983 known as the Grameen Bank which catered for such people. The model of Grameen Bank had some important elements and they are the repayment of loan begins once the loan has been given out and it continues over the period of time agreed upon in the contract but with the maximum of one year. The bank also brought into play the group lending method and then reduced operating expenses which is indirect expenses of running a business by having regular meetings held by loan officers in the towns and rural areas to receive

and distribute payment and hence banned the need for the creation of physical branches. It is from this bank that other microfinance institutions started to develop. Early microfinance concentrated on group lending methods which helped with the issue of moral hazard and adverse selection. To reduce adverse selection, members are jointly liable for each other's loans, compelling group members, who have better information than the lender, to choose individuals they believe most likely to repay. When the groups are formed, each member has the motive to monitor the others' behavior which reduces both moral hazard and the lender's monitoring costs which forms part of the transaction costs. In addition to these mechanisms always present in the group lending programs, newer structures of microfinance, such as individual lending and traditional banks, cannot rely on these incentives, as the groups do not exist or members are too many to have the necessary information to ensure repayment. Upon further research on incentives present under both group and individual lending schemes, a good amount of this research concentrated on the repeated interaction between the borrowers and the lenders also known as "dynamic incentives". Although first attributed to (Besley, 1995) only, (Morduch, 1999) has also actively been instrumental in recording the dynamic incentives at play in microfinance. When a borrower has a continuous need to be credited, access to future loans provide a strong reason to avoid default on a current loan. Moreover, continual increases in loan size, or "progressive lending", improve a borrower's incentives to repay over time. Ensuring repayment incentives through refinancing is designed in the context of microfinance by (Hulme & Mosley 1996) and (Armenda´riz de Aghion & Morduch 2000). In a two-period model, repayment of the first loan is induced with the promise of a second loan. However, in these two period models, the borrower or micro entrepreneur always defaults on the second loan. In practice, microfinance institutions state that once a borrower defaults on a

loan, he becomes forever ineligible for future loans. This procedure is almost globally employed by microfinance institutions and is quite a harsh means to ensure repayment. (Madajewicz, 1997) notes that "these incentives are often quite extreme", maybe too extreme, considering the lack of other sources of credit for the poor in developing countries. Mosely, in his study of the Bolivian microfinance sector, cites an example of one micro entrepreneur who lost his investment in a burglary. As a result of one member being burglarized, the entire group fell into default, and remains so, banned from borrowing any more from BancoSolQ (Mosley, 1999). A micro entrepreneur with repayment issues faces a difficult choice: do I sell productive assets or withdraw my children from school in order to pay my loan, or do I surrender all access to future loans, are some tough decisions one has to make. Unfortunately, some successful households may decide to slide back into poverty in order to retain the possibility of loans in the future. Although often used and considered successful, the non-refinancing threats may unnecessarily reduce borrower welfare. This paper will question the need for severe non-refinancing threats in microfinance, especially in countries with well-developed financial sectors, where default today does not mean a lifetime without credit or loans. Endogenizing the default penalty allows micro entrepreneurs who fall into misfortunes obtain loans in the future, clearly improving outcomes for the poor. Finally, academic models of microfinance base moral hazard on the choice of project to undertake but assume that a borrower will repay when he or she is capable. Two deviations from this model are (Besley & Coate, 1995) and (Armenda´riz de Aghion, 1999), where both address the strategic default in group lending contracts. In a case where successful group members are capable of repaying the portion of the loan for a defaulting group member, but refuse to do so, can be construed as strategic default. Besley and Coate show how social collateral can reduce strategic default

in group lending situations. Furthermore, Armenda´riz de Aghion finds that peer monitoring reduces strategic default when groups are externally formed. This model will show how features such as additional loans among others which are known as "dynamic incentives", can reduce strategic default without relying on the group incentives currently used in the literature. The decision to repay the loan can then be modeled as a choice of whether or not to divert funds meant for loan repayments to other urgent wants or needs. There are two types of default, strategic default and default due to a negative economic shock. One can easily construct either case: a family may choose to buy household luxury items instead of repaying a loan, or they may need to pay the hospital bills of a family member. Due to the risks that poor entrepreneurs in developing countries face and the asymmetric information between borrower and lender regarding such risks, a model in which the lender provides motives to discourage strategic default may be most appropriate. There are the well-known, systemic shocks, such as the flooding in Bangladesh, Hurricane Mitch, which struck micro entrepreneurs in Honduras, or armed conflict, which can upset an entire economy. Nevertheless, there is an increasing amount of evidence that large systemic shocks are not the only things that put micro entrepreneurs at risk. A study in Peru found that half of the sample experienced one or more shocks in the 2 years prior to the survey, the largest of which was burglary (Dunn, 1999). Without functioning insurance markets, theft, illness of a family member, or loss of another source of household income can quickly destroy the ability to repay a loan on time and in full. As a result of these risks, they become ineligible for future loans, and, as the system currently functions, the dent remains on their credit record indefinitely. We suggest an amendment over lending contracts currently used in practice, by including several features of microfinance not yet brought forth in the literature in addition to the moral hazard and ad-

verse selection which are always present in microfinance models. First, we can develop a lending contract that continues for a very long time and provides repayment incentives at all points in the time period. Secondly, we can model the fundamental riskiness of the income streams of the poor by including a negative income shock. Lastly, the model will allow us to endogenize the amount of time that a micro entrepreneur who defaults must stay without a loan. It will be shown that the necessary punishment phase can be less than infinity, especially when a borrower has much to gain from the lending relationship. Microfinance institutions like any other type of business venture have had their own share of challenges over the years. The cost of lending to the poor is one of the major challenges faced by these institutions. This is because, gaining profits from giving out loans seems impossible as the loans are of very little amount which makes the operating expenses higher. Another issue they face is the non-repayment of loans given out as most of the borrowers do not have a steady flow of income and some are hit by an unexpected negative shock. Credit markets for the poor suffer from the same informational asymmetries which occur in formal credit markets-lenders must ascertain a borrower's creditworthiness as well as ensure repayment once a loan is disbursed. However, most microfinance institutions do not have the advanced credit scoring mechanisms, collateral requirements, and sound legal systems that allow banks to overcome these difficulties in developed markets. Innovative measures seen in microfinance help subdue these problems.

Microcredit, according to (Dierner, 2013), is a series of contracts designed to provide very small loans to very poor individuals in society in order to help them grow small companies or income-generating activities. This concept arose from research that revealed that people in rural areas, in particular, had little chance of obtaining traditional credit, and that banks imposed strict requirements such

as being literate (able to read and write), possessing certain identification documents, and, most importantly, having a minimum deposit in an active account. Yunus founded the organization, a world-renowned economist who witnessed a famine in the village where his institution was located with some of his students in 1974. Yunus and his colleagues decided to list the demands of the people and chose to give out modest loans to them, with his sample being 42 women, in a town populated by peasants and craftspeople. Yunus then decided to take things a step further by attempting to persuade banks to create policies that would assist the most disadvantaged people. When he realized that his ideas were being ignored, he decided to start his own bank, the Grameen Bank, in 1983. Currently, microcredit activity is widespread over the world, with 10,000 organizations providing over fifty million euros in loans to address the needs of over 500 million people. Microfinance encompasses a broader range than microcredit. Microcredit services, microinsurance, deposits, settlements, and other money items are all confined to microcredit. In a nutshell, (Ledgerwood, 1998) discusses the two most important aspects of microfinance: financial and social intermediation. Social intermediation, according to (Qudrat-I Elahi & Rahman, 2006), is about sorting out and raising the voice of the poor to address whatever challenges they face. Microcredit and microfinance should be employed in accordance with their respective ranges. Because most Microfinance Institutions (MFIs) have not yet created a significant deposit activation structure, (Jain & Moore 2003) recommend microcredit rather than microfinance. In either case, their reasoning can be used in the existing setting, as most MFIs already provide banking administrations comparable to those found in the corporate banking system. All MFIs in Bangladesh lend, accept deposits, and provide other monetary services to their consumers at the same time. Commercial banks, non-bank monetary agencies, and credit agencies, all of which

provide a wide range of services, have entered the microfinance industry, according to (Mia, 2016). In either case, we wish to use the term'microfinance' instead of'microcredit.' Though MFIs have expanded to include a wide range of goods and services, their original goal was to help the most vulnerable and underprivileged members of society by allowing them to participate in certain industries. Ventures and this goal are still very much a component of their business. According to (Dunford, 2012), transformation is linked to the example idea motivating microfinance, a financial sector movement centered on disadvantaged people that most traditional financial institutions ignore. Microfinance schemes, on the other hand, do not view the poor to be victims who require charitable assistance (Nasrin et al. 2017). Microfinance programs, on the other hand, consider the vulnerable as ordinary people who have a unique way of accessing the world's resources (Hickel, 2015). Microfinance assumes a relationship between the poor and the financial sector because the poor cannot access loans from banks without collateral. The Muhammad Yunus microfinance approach allows loans to be obtained through a 'peer monitoring system' that acts as a form of'money-related guarantee,' something that the traditional banking system lacks. Microfinance has made issuing loans to the needy more flexible as a result. As a result, microfinance has grown in popularity. Microfinance enables the poor to generate money, create resources, and minimize their helplessness in monetary situations, among other things. Furthermore, one of microfinance's most significant contributions is its commitment to involving women, a fundamental social reform in male-dominated social structures that aims to increase well-being and accomplish the Sustainable Development Goals. Sustainable Development Goals (SDGs) (Loewe and Rippin, 2015; Mull, 2016; Pitt, Khandker and Cartwright, 2006; Pronyk, Hargreaves and Morduch, 2007; Weber and Ahmad, 2014). In addition, microfinance developed widespread pop-

ularity towards the end of the twentieth century, particularly after early research by economists commended it as not merely a way of aiding the destitute, but also a strategy of expanding economic progress. (Hashemi, Schuler, and Riley, 1996; Morduch, 1998, 1999a, 1999b, 2000; Pitt and Khandker, 1998; Pitt, Khandker, and Mundial, 1996; Schuler and Hashemi, 1994; Pitt and Khandker, 1998; Pitt, Khandker, and Mundial, 1996; Schuler and Hashemi, 1994). (Duvendack et al. 2011) focused on the lack of techniques in microfinance evaluations. However, one of the most modern, advanced, and precise ways to measure the impact of microfinance is to employ Randomized Control and Trial (RCT). (Karlan & Zinman 2009) discovered that when the impoverished have access to flexible credit facilities, their well-being is maximized. They used data and the RCT approach to arrive at this conclusion. In addition, another study they conducted in the Philippine microfinance sector (Karlan & Zinman 2011) indicated that microcredit led to enhanced society interactions, effective risk hedging, and more flexible loan access. After evaluating the after-effects of six distinct RCT microfinance tests in India, Banerjee, Dufflo, Glennerster, and (Kinnan 2015) discovered that the microfinance influence was unassumingly hopeful but not revolutionary. Despite these RCT evaluation reports, (Khandker & Samad (2014) conducted the most thorough study to date in Bangladesh, which included over 3,000 family units in 87 towns over a 20-year period. They emphasized that microfinance has continued to aid the poor by increasing family government assistance in a variety of ways. These factors include increased individual spending, family resource gathering, female empowerment, and increased work accessibility, among others. In male-dominated social orders, adequate proof-based work supports the claim that microfinance loans are primarily intended to empower women (Laha and Kuri, 2014; Nilakantan, Datta, Sinha, and Datta, 2013; Rehman, Moazzam, and Ansari, 2015; Weber and Ahmad, 2014), which represents a sig-

nificant social change. Furthermore, Weber and (Ahmad 2014) discovered that in Pakistan, women in higher credit cycles felt more powerful than those who are new to the loaning cycle. Grameen-based microfinance programs have focused on eliminating poverty and enabling individuals to become self-sufficient as a result of these notable benefits. Developed countries, such as the United States, Canada, Germany, and a few other European countries, have reflected this. Microfinance, on the other hand, is argued by (Bateman & Chang, 2012) to be a barrier to achieving self-sustaining economic goals. Microfinance Institutions' high interest rates on loans tend to work against the core goals of microfinance, rather than alleviating poverty. As a result, some people default on their credit cards because they can't keep up with the high cost of borrowing. This has an impact on the other members of the group since they will have to shoulder the additional cost of this defaulting individual. In the worst-case situation, they will no longer be able to use the credit facility. Microfinance concerns grew in 2007 when Compartamos, a Mexican microfinance institution, went public with interest rates as high as 195 percent (Bateman & Chang, 2012). Professor Muhammad Yunus called these despicable advantage boosting MFIs 'new credit sharks' taking advantage of needy individuals (Mitra, 2009). Somewhere in the range of 2005 and 2014, there was a reliable loan cost in the microfinance area in Bangladesh (22% to 26%). The exorbitant loan fees charged by MFIs are because of significant expense of activities like endorsing cost. Theoretically, a MFI that tries to advance the underlying thought of microfinance could give credit offices to the poor at sensible rates and thus put in measures to boost the utilization of their assets to decrease working expenses (Mia & Soltane, 2016). For specific countries, where customers are immensely over-obliged and default rates have expanded, the microfinance area in like manner faces monetary challenges. In any case, (Ashta et al, 2015) did an exploration that showed that there was

a feeble relationship among's microfinance and self destruction occurrences yet rather showed high connection between's self destruction episodes and banking area. (Moh'd Al-Azzam, 2016) and (Sinclair, 2012) have focused on debasement in microfinance. Microfinance was introduced in Bangladesh during the mid-1970s, in the midst of financial hardship rising up out of the united difficulties of being brought into the world as another country. Bangladesh had a financial circumstance where banking activities were restricted and dejected people were kept away from such offices. After Bangladesh got free in 1971, the country ascended out of the conflict of opportunity with Pakistan, which effects affected the economy which made the degree of destitution rise particularly in rustic regions. The issue was moreover bothered by the amazing starvation of 1974, when a considerable number of Bangladeshis kicked the pail in 1973–1974 due to the feebleness of the as of late settled government to give sufficient assistance to more than 80% of the general population living underneath the destitution line (Hossain, 2014). Overthrows in 1975 further weakened the monetary area. In the period 1971–1975, money related advancement of about 2% consistently denied policymakers the ability to help poor people. To meet its headway costs, Bangladesh expected to depend upon remote assistance from the USA, Japan, the Soviet Union and India (Racioppi, 1994). Grameen Bank (GB) was officially settled in 1983 with the execution of the Grameen Bank Ordinance as an independent bank administered with the council following a lot of courses of action with attentive financial backers and managing reluctant government legislators and executives. From the outset, the plan of Yunus' Grameen Bank used a gathering loaning model in which five individuals obstinately make a gathering, and a standard branch contains seven or eight gatherings (Morduch, 1999). The underlying two individuals get advances for any arbitrary gathering, followed by the accompanying two and over the long haul the last individual,

when in doubt for a one year or 50week time period. Should any gathering part default, each and every other gathering part will be turned down regarding credit. The branch set up in the normal MFI contains a field administrator and different field authorities who spread the zone from 15 to 22 towns by making primer visits to the towns to become familiar with the lifestyle and necessities of anticipated customers (Kabir Hasan, 2002; Morduch, 1999a). Microfinance advancement got balance during the 1980s as tantamount sorts of MFIs were set up from one side of the country to the other (Ahmed, 2009), which turned out to be emphatically through 'enhancing' as new branches reiterated the frameworks and rules of various pieces of their parent affiliation (Zaman, 2004). Regardless of the way that the Grameen Bank model is Bangladesh 's driving microfinance structure, the methodology hasn't proceeded as in the past. Reformist overhauls have been noted consistently, as analysis from field agents, land region and social guidelines have remarkably influenced the model. Some MFI's have adjusted the principal importance of microfinance to follow their functional thinking (Khan & Ashta, 2013) and have better options for the close by network, prompting a wide scope of enhancements in credit things and how monetary services are delivered. Laborers, for example, got analysis that different helpless ladies are untalented and need ambitious abilities; thus MFIs presented preparing and coordinated monetary education programs for them. In addition to providing entrepreneurial loans, MFIs recognized that residents in disaster-prone areas would require catastrophe loans if any unanticipated calamities occurred (Matin & Taher 2001). MFIs have begun to provide disaster-stricken people loans with flexible repayment periods to help them deal with the aftermath of a natural disaster. The Grameen Bank's working model has also been revised from a fundamental standpoint in order to respond to diverse types of risks (Khan Ashta, 2013). Overall, due to community members'

rising concerns about freeriders, the group-based lending method has given way to individual loans. Weak social relationships have also resulted in group lending's equivalent inability (Lehnar, 2009). Group loans include high operational costs and a relatively high loan installment pace, fines that deter high credit risk bearers (Gine & Karlan, 2014), and key defaults and lower reimbursement rates. After 2002, GB rejected shared responsibility arrangements that other financial sector MFIs had emulated (Kono, 2006). Individual loans quickly acquired popularity in Bangladesh, contributing to the microfinance sector's rapid expansion. Because there were so many requests, (Lehnar, 2009) predicted that the sector's demand for individual loans would continue to grow. Although women remain the primary focus of the microfinance system, loan services have been expanded to include males as well. Women continue to outnumber males when it comes to microfinance. During the development stage, finance sources also played a vital role in the expansion of MFI's loan activities. The sector exploded after the Palli Karma-Sahayak Foundation (PKSF) was established. The foundation, which was founded in 1990 to assist MFI financing efforts, is a major institution. The World Bank, the United Nations, the Ford Foundation, Oxfam, the Aga Khan Foundation, and other national and international private donors have all contributed to the financing of microfinance initiatives. However, the profitable monetarily stable MFIs, notably large and driving MFIs like as BRAC, ASA, and Grameen Bank, would not accept additional gifts (Zaman, 2004). As a result, the majority of international donor assistance went to new MFIs. Furthermore, the sector's development during this period was aided by the poor people's well-documented and imaginative actions of overcoming adversity. (Ledgerwood, 1998). Microfinance has emerged as an important financial tool for policymakers seeking to alleviate poverty. More importantly, from 1986 to 1995, the lack of effective organizational management and govern-

mental monitoring fostered the growth of microfinance in Bangladesh (Conroy MacGuire, 2000).

3 Estimating The Lending Phase and the Interest Rate

3.1 The Borrower's Payoff From Accepting A Loan

In the business world the aim of an entrepreneur is to increase his or her wealth whiles providing people with the services or products needed. To be able to keep the business running and make money, investments must be made into the business. One way of making an investment is by taking a loan from an individual or an institution for example banks, microfinance among others. In taking a loan two parties are involved the lender (microfinance institution) and the borrower (micro entrepreneur). Borrowers are assumed to have undertaken identical projects or assumed to be in the same business and the only difference is the risk each of them face. The lenders aim is to maximize each borrower's payoff at the end of the period. For this to happen, the borrower must be willing to accept the loan, have the right incentives to repay the loan and the lending rate over the entire loan portfolio must be reasonable (Green and Porter's, 1984). A financial relationship is maintained between the two parties if they both benefit from the first transaction.

Now lets suppose at the beginning of a lending phase, a borrower takes K unit of credit. It is expected that after the end of the stipulated period T, the borrower makes a payoff of K(1 + r) with r being the interest rate on the credit taken. It would be prudent for the borrower who is assumed to the K unit credit to obtain proceeds of ωK from investment, where $\omega = 1 + j$ and j is the interest rate on investment and j >r.

Realistically, the borrower may default in payment during the lending phase. This default may be as a result of the borrower channeling ωK into a pressing

need and foregoing repayment or the borrower gets affected by a negative economic shock such as fire outbreak causing the borrower to run into losses and consequently missing out on the repayment of the loan. Upon defaulting, the borrower is said to enter into a new phase known as the punishment phase in the following period Q where Q > T. If at time T, ωK > K(1 + r) then the borrower is successful in repaying the loan thus can receive a new loan. Otherwise, the borrower enters the punishment phase of at length Q which is determined by the lender.

During this period, he or she is not given any loan until the punishment phase is served. We shall denote the probability of the borrower being successful in repaying her loan thus being lent another loan as $(1 - \alpha_i)$ and α_i as the probability of the borrower not being able to pay the loan hence entering the punishment phase. Also, let L and B denote the lending state and borrowing state respectively. It is also worth noting that moving from the borrowing state to the lending state is not guaranteed. During the punishment period, there is a set of defined punishment states B^i, i> 1 hence restricting the defaulter from obtaining another loan. It is also assumed that during the punishment phase transitioning from one punishment state to another is certain. γ is modeled as the probability that the borrower returns to the lending phase in any period after Q and $\gamma(1 - \gamma)$ probability of getting a loan in Q+2.

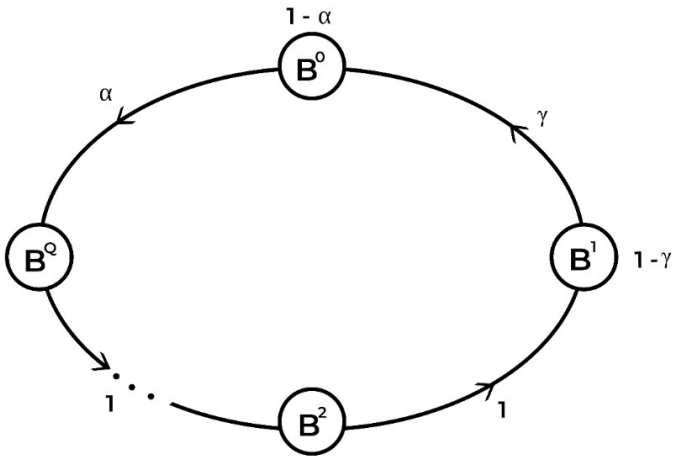
The information above is represented in the Markov chain X_t

$$(X_t)_{t\epsilon N}, (X_t)\epsilon S = \{L, B^1, B^2, ..., B^Q\} \equiv \{B^0, B^1, B^2, ..., B^Q\}$$

The transition matrix of the Markov chain X_t :

$$P= \begin{pmatrix} (1-\alpha) & 0 & 0 & . & . & . & 0 & \alpha \\ \gamma & (1-\gamma) & 0 & . & . & . & 0 & 0 \\ 0 & 1 & 0 & . & . & . & 0 & 0 \\ 0 & 0 & 1 & . & . & . & 0 & 0 \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & . & . & . & 1 & 0 \end{pmatrix}$$

The multigraph is also presented below:



The possible transitions by a borrower within a definite period is represented by the arrows in the multigraph.

From the transition matrix we deduce that;

The probability the borrower pays his or her loan is

$P(X_{t+1} = B^0 | X_t = B^0) = (1 - \alpha)$

The probability of the borrower not being able to pay the loan is

$P(X_{t+1} = B^T | X_t = B^0) = \alpha$

The probability of transitions of a borrower in punishment period of at least Q is $P(X_{t+1} = B^{i-1} | X_t = B^i) = 1$ for i = 2,..., T.

The probability that the borrower gets a loan is

$P(X_{t+1} = B^0 | X_t = B^1) = \gamma$

The probability that the borrower fails to acquire a loan after applying is

$P(X_{t+1} = B^1 | X_t = B^1) = (1 - \gamma)$

We define two function for a borrower of type i;

V_i^+ = payoff at the beginning of a period in the lending phase

V_i^- = payoff at the beginning of the punishment phase

δ = discounted factor

But $V_i^+ = (1-\alpha_i)\left[(w-(1+r))K + \delta V_i^+\right] + \alpha_i \delta V_i^-$ (1)

Also,

$$V_i^- = \gamma\delta^Q V_i^+ \sum_{t=0}^{\infty} \delta^Q (1-\gamma)^Q = \frac{\gamma\delta^Q}{1-\delta(1-\gamma)} V_i^+ \quad (2)$$

According to (Tedeschi, 2006) V_i^+ and V_i^- is given by the equations above because for a borrower, there is a $(1-\alpha)$ chance that the borrower earns ωB, repays the loan, receives another loan in the following period and α chance that borrower will experience a negative shock and default and enter into the punishment phase in the following period. In either case, the next period's expected payoff is discounted by the borrower by δ.

Making V_i^+ the subject will yield

$$V_i^+ = \left(\frac{\gamma\delta^Q}{1-\delta(1-\gamma)}\right)^{-1} V_i^-$$

Substituting V_i^- into equation (1)

$$V_i^+ = (1-\alpha_i)\left[(w-(1+r))K + \delta V_i^+\right] + \alpha_i\delta\left[\frac{\gamma\delta^Q}{1-\delta(1-\gamma)}\right] V_i^+$$

$$V_i^+ = (1-\alpha_i)\left(w-(1+r)\right)K + (1-\alpha_i)(\delta V_i^+) + \left[\frac{V_i^+ \alpha_i \gamma \delta^Q}{1-\delta(1-\gamma)}\right]$$

$$V_i^+ = (1-\alpha_i)\left(w-(1+r)\right)K + \delta(1-\alpha_i)(V_i^+) + \left[\frac{V_i^+ \alpha_i \gamma \delta^Q}{1-\delta(1-\gamma)}\right]$$

$$V_i^+ - \delta(1-\alpha_i)V_i^+ - \frac{V_i^+ \alpha_i \gamma \delta^{Q+1}}{1-\delta(1-\gamma)} = (1-\alpha_i)\left(w-(1+r)\right)K$$

$$V_i^+ \left[1-\delta(1-\alpha_i) - \frac{\alpha_i \gamma \delta^{Q+1}}{1-\delta(1-\gamma)}\right] = (1-\alpha_i)\left(w-(1+r)\right)K$$

$$V_i^+ = \frac{(1-\alpha_i)(w-(1+r))K}{1-\delta(1-\alpha_i)-\frac{\alpha_i \gamma \delta^{Q+1}}{1-\delta(1-\gamma)}} \tag{3}$$

Also,

$$V_i^- = \frac{\gamma \delta^Q}{1-\delta(1-\gamma)} V_i^+ \tag{4}$$

Substituting (1) into (4);

$$V_i^- = \frac{\gamma \delta^Q}{1-\delta(1-\gamma)}\left[(1-\alpha_i)[(w-(1+r))K + \delta V_i^+] + \alpha_i \delta V_i^-\right]$$

$$V_i^- = \frac{\gamma\delta^Q}{1-\delta(1-\gamma)}\left[(1-\alpha_i)\left(w-(1+r)\right)K + (1-\alpha_i)\delta V_i^+ + \alpha_i\delta V_i^-\right]$$

$$V_i^- = \frac{\gamma\delta^Q(1-\alpha_i)\left(w-(1+r)\right)K}{1-\delta(1-\gamma)} + \frac{\gamma(1-\alpha_i)\delta^{Q+1}V_i^+}{1-\delta(1-\gamma)} + \frac{\gamma\alpha_i\delta^{Q+1}V_i^-}{1-\delta(1-\gamma)}$$

But,

$$\frac{\gamma(1-\alpha_i)\delta^{Q+1}V_i^+}{1-\delta(1-\gamma)} = \frac{\gamma(1-\alpha_i)\delta^{Q+1}}{1-\delta(1-\gamma)}\left[\frac{1-\delta(1-\gamma)}{\gamma\delta^Q}V_i^-\right] = (1-\alpha_i)\delta V_i^-$$

$$\implies V_i^- = \frac{\gamma\delta^Q(1-\alpha_i)\left(w-(1+r)\right)K}{1-\delta(1-\gamma)} + (1-\alpha_i)\delta V_i^- + \frac{\gamma\alpha_i\delta^{Q+1}V_i^-}{1-\delta(1-\gamma)}$$

$$V_i^- - (1-\alpha_i)\delta V_i^- - \frac{\gamma\alpha_i\delta^{Q+1}V_i^-}{1-\delta(1-\gamma)} = \frac{\gamma\delta^Q(1-\alpha_i)\left(w-(1+r)\right)K}{1-\delta(1-\gamma)}$$

$$V_i^-\left[1-(1-\alpha_i)\delta - \frac{\gamma\alpha_i\delta^{Q+1}}{1-\delta(1-\gamma)}\right] = \frac{\gamma\delta^Q(1-\alpha_i)\left(w-(1+r)\right)K}{1-\delta(1-\gamma)}$$

$$V_i^-\left[\frac{1-\delta(1-\gamma)-(1-\alpha_i)\delta\left(1-\delta(1-\gamma)\right)-\gamma\delta^{Q+1}\alpha_i}{1-\delta(1-\gamma)}\right] = \frac{\gamma\delta^Q(1-\alpha_i)\left(w-(1+r)\right)K}{1-\delta(1-\gamma)}$$

$$V_i^- \left[\frac{[1-\delta(1-\gamma)][1-\delta(1-\alpha_i)]-\gamma\alpha_i\delta^{Q+1}}{1-\delta(1-\gamma)}\right] = \frac{\gamma\delta^Q(1-\alpha_i)\left(w-(1+r)\right)K}{1-\delta(1-\gamma)}$$

$$V_i^- = \frac{1-\delta(1-\gamma)}{[1-\delta(1-\gamma)][1-\delta(1-\alpha_i)]-\gamma\alpha_i\delta^{Q+1}} \times \frac{\gamma\delta^Q(1-\alpha_i)\left(w-(1+r)\right)K}{1-\delta(1-\gamma)}$$

$$\implies V_i^- = \frac{\gamma\delta^Q(1-\alpha_i)(w-(1+r))K}{[1-\delta(1-\gamma)][1-\delta(1-\alpha_i)]-\gamma\alpha_i\delta^{Q+1}} \tag{5}$$

3.2 The Borrower's Constraint

Certain factors may spur borrowers to obtain loans or prevent them from refinancing lenders. These factors are categorized in two: Participation constraints and incentive constraints. A borrower would only be in a position to take a loan if they are certain on making profit from projects they will undertake and this situation is referred to as the participation constraint.This is the case for all types of borrowers because repayment is only possible in the no-shock state and this is mathematically represented as $\omega \geq$ (1+r) (Tedeschi, 2006).

The best lenders can do when borrowers want to obtain loans is to introduce incentives that that would induce them to pay, but usually borrowers fail to refinance lenders thus imposing much threat to most microfinance institutions.Failure to repay loans is usually as a result of no collaterals being demanded hence nothing to hold borrowers accountable for. To ensure borrowers act in the interest of the lenders at periods when they are able to, lenders should see to it that the punishment phase must be made unattractive to the borrower. This is to say that, payoff from investment, repayment of loan and entering the lending phase in the next period should be more beneficial as compared to the

borrower opting to go into the punishment phase in the next period. This is also mathematically represented as

$$[\omega \text{ - } (1+\text{r})]\text{K} + \delta \text{V}_i^+ \geq \omega \text{K} + \delta \text{V}_i^-] \tag{6}$$

In most cases, a potential defaulter though has earned returns from investing money without experiencing shocks would still not refinance the microfinance institution. This would alter the equation (6) so we obtain

$$\delta[\text{V}_i^+ - V_i^-] \geq (1+\text{r})\text{K} \tag{7}$$

To avoid this mishappening, the microfinance is to ensure that the borrowers are not in a position to benefit more from future earnings as compared to being in a defaulting period.

Substituting equation (3) and (6) into equation (7)

$$\frac{\delta(1-\alpha_i)[1-\delta(1-\gamma)-\gamma\delta^Q}{1-\delta(1-\gamma)-\gamma\delta^Q+1} \geq \frac{(1+r)}{\omega} \tag{8}$$

It is observed from the equation that a relationship exist between the length of punishment phase and the interest rate. This suggests that the higher the frequency a borrower spends serving the punishment phase would necessitate an increase in interest rate when he or she demands a loan since the likelihood of the borrower defaulting keeps increasing.

3.3 The Microfinance Institution's Problem

To obtain the lender's problem in any period, the portfolio risk and distribution of borrowers must be stable. In the process of achieving stability in the distribution, (Tedeschi 2006) assumes that borrowers have the same profitability, loan needs and discount factor, but can either be safe borrowers or risky borrowers in terms of risk. Safe borrowers, β_0 is a portion of borrowers who face lower risk and a portion of borrowers who face higher risk are risky borrowers and it is denoted by $(1 - \beta_0)$. It is also assumed that general population consists of safe borrowers and risky borrowers. Most individual borrowers are fully aware of the kind of borrowers they are but unfortunately the lender knows only the distribution of types of borrowers in the population and not their kind.

So, during the lending or punishment phase the microfinance should then be able to anticipate from the general population its safe borrowers to avoid running into losses thus stabilising the distribution. To predict the proportion of the safe borrowers belonging to the portfolio, the lender must determine the proportion of safe borrowers within the much risky population of the entire population. This is mathematically represented as

$$\beta = \frac{\alpha_R \beta_0}{\alpha_S(1-\beta_0)+\alpha_R \beta_0}$$

β_0 denotes safe types in the entire population and $(1 - \beta_0)$ denotes risky types in the entire population.

It is also worth noting that, lenders are more interested in the safe borrowers because safe types default less often than risky types.

As explained previously, some borrowers may default and will be sent to the punishment phase while the others will remain in the lending phase. A fraction

â of the borrowers that will be sent to punishment phase is estimated. This is mathematically represented as $\hat{a} = \beta\alpha_S + (1-\beta)\alpha_R$. The reason for the estimation is also to average the risk of the portfolio.

Now that portfolio risk and distribution are stable, we can continue with lender's problem in any period. Now if K unit of credit is lent to the borrower, the lender expects (1+r)K from the borrower as the future value of the money lent or nothing in the case of defaulting. Also, the lender incurs (1+z)K as cost of lending to the borrower. z is the lending costs per cedi lent and $(1-\hat{a})$ the fraction of borrowers in the lending phase.

Putting all these together, the lender's net gain in any period is $[(1+r)(1-\hat{a})-(1+z)]K$ and whatever returns the lender expects to get in the subsequent periods will further be added. Since returns from the subsequent periods is yet to be obtained, the value is discounted by δ. This is then mathematically represented as

$$V_i^+ = [(1+r)(1-\hat{a})-(1+z)]K+\delta[(1-\hat{a})V_i++\hat{a}V_i^-] \tag{9}$$

with V_i+ and V_i- denoting payoff in the lending phase, and in the punishment phase respectively.

We also need to note that during the punishment phase, the lender gives no loans to the defaulting borrower for some Q periods, thus nothing is earned from the borrower. We also assume that unlike the lending phase there are no lending costs involved in the punishment phase since no loans are given out. With a probability γ a newly eligible borrower is also then ushered in the Q + 1st period, so we can now define the payoff for the lender at the beginning of the punishment as

$$V_i^- = \frac{\gamma\delta^Q}{1-\delta(1-\gamma)}V_i + \tag{10}$$

Combining equations (9) and (10) the lender's value functions which is also the payoff is defined in the lending phase, V_i^+ and punishment phase V_i^- in terms of interest rate, length of the punishment phase, constants and mathematically as

$$V_i^+ = \frac{(1+r)(1-\hat{a})-(1+z)K}{1-\delta(1-\hat{a})-\frac{\hat{a}\gamma\delta^{Q+1}}{1-\delta(1-\gamma)}}$$

Sustainable operations are very important to lenders in order to maximize borrower profit hence the lender's sustainability constraints must be averaged so that the MFI does not lose out on profits if not the MFI faces a huge challenge.

3.4 Solving The Model

To find the optimal length of the punishment phase, we would maximize the borrower's payoff (3) choosing r and Q, subject to the incentive constraint (8) and the lender's sustainability constraint. The borrower's participation constraint must be checked to ensure a sustainable borrowing equilibrium exists. The answer can most easily be found by noting the properties of the borrower's value function and incentive constraint and the lender's sustainability constraint.

Modelling Payoff For Punishment Phase

As explained, borrowers may default in their lending phase thus their payment period may be extended and this leads to the punishment phase. Though it is required that borrowers make payments in the early stages of their punishment phase, they may delay in paying as required in accordance to regular settlement dates, Q_w. Q_w are weekly settlement dates that are variably dependent on the borrowers ability to pay her loan when it is due. Considering the fact that a borrower may make payments right after entering the punishment phase at any given settlement date Q_w; w = 1,2,. . . ,N, two events are likely to occur during each settlements dates and these events are detailed below:

1. The borrower is likely to pay the installments. This event is assigned a probability p.

2. The borrower is not likely to pay the installments. This event is assigned a probability (1- p).

This random process follows a Bernoulli process $\beta = (\beta_n)_{n\geq 0}$.

n denotes any weekly settlement beginning from the first payment week of the punishment phase. Hence β_n is a sequence of Bernoulli random variables with distribution $\beta(0, p)$, that is,

$\beta_n \sim$ B (0,p).

The notations are therefore defined to make a model:
If $\beta_n = 1$: The borrower is capable of paying at week n.
If $\beta_n = 0$: The borrower is not capable of paying at week n.

It is expected that the loan is repaid in w finite settlements. These settlement will usually span from 1 to some large number N, hence w can be expressed as w = 1,2,3,4,...,N. These are the fixed number of weekly settlement dates if the borrower does not delay but realistically, some borrowers if not all borrowers may delay payment when settlement dates are due. Considering these, it can be concluded that the total number of payments Q_w will either be w payments in n weeks given that borrowers delay payments or a borrower would not delay hence w payments in Z weeks. This is mathematically expressed as

$$Q_w = min\{n|\beta_1 + ... + \beta_n\}, w = 1, 2, ..., N$$

We can express the X_w as the sequence of the wth settlement made by the borrower and considering that settlements would be made weekly then X_w becomes dependent on Q_w.

$X_w = Q_w$ - Q_{w-1}

Lets also denote the sequence of random settlements with X and note R_x, where R_x = 0,1 and p ∈ (0,1). If x also denotes the total settlements made, then given that installments payments are made on time, then x - 1 settlements would have to be made before the x – th settlement. Each x – 1 with probability (1 - p) denotes failure to make settlements until the x – th settlement which is the success is made. x – th settlement also has a probability of p. This suggests that:

P{X=x} = $(1 - p)^{x-1}p$

⇒ $X\sim$ GEO(p).

Theorem: The Strong Markov Property

Let X_0, X_1, X_2, X_3, ... be a Markov chain with a transition matrix G. As defined Q_w is the stopping of X and X as early on described is a sequence of random settlements each being assigned value in the set R_x = {0,1}. Then

P{X_w = j/ $X_0 = x_0$,..., X_{k-1} = i} ∀x_0,...,x_{k-1}, i, j ∈R_x and w = 1,2, ... N

X_w is a Markov chain with state space R_x.

X has a transition probability GEO(p) so by the Strong Markov Property,

$X_w \sim$ GEO(p)

Solving For The Optimal Interest Rate

Firstly, it is easy to see how the interest rate should be as low as possible so as to maximize the borrower's payoff, but an interest rate of zero and below will not satisfy the sustainability constraint of the lender. Therefore, the interest rate must be positive, but only so large so as to allow the lender to break even, or lender's value function = 0. This applies when $(1+r)(1-\hat{a})-(1-z)=0$, which implies an interest rate of

$$r^* = \frac{\alpha+z}{1-\alpha} \tag{11}$$

Derivation of Equation II

The optimal interest rate is an interest rate that maximises the borrower's welfare and makes lending sustainable as well.

The borrower's payoff is given as

$$V_i^+ = \frac{(1-\alpha_i)(w-(1+r))K}{1-\delta(1-\alpha_i)-\frac{\alpha_i\gamma\delta^{Q+1}}{1-\delta(1-\gamma)}}$$

Differentiating V_i^+ with respect to r

$$\frac{\partial}{\partial r}\left[\frac{K(1-\alpha)(r-\omega-1)}{-\frac{\alpha\delta^{Q+1}\gamma}{1-\delta(1-\gamma)}-(1-\alpha)\delta+1}\right]$$

$$\frac{\partial V_i^+}{\partial r} = \frac{K(1-\alpha)}{-\frac{\alpha\delta^{Q+1}\gamma}{1-\delta(1-\gamma)}-(1-\alpha)\delta+1}\left(\frac{\partial}{\partial r}[-r]+\frac{\partial}{\partial r}[w]+\frac{\partial}{\partial r}[-1]\right)$$

$$\frac{\partial V_i^+}{\partial r} = \frac{((-\frac{\partial}{\partial r}[r]+0+0)K(1-\alpha)}{-\frac{\alpha\delta^{Q+1}\gamma}{1-\delta(1-\gamma)}-(1-\alpha)\delta+1}$$

$$\frac{\partial V_i^+}{\partial r} = -\frac{1K(1-\alpha)}{-\frac{\alpha\delta Q+1_\gamma}{1-\delta(1-\gamma)}-(1-\alpha)\delta+1}$$

$$\frac{\partial V_i^+}{\partial r} = -\frac{K(1-\alpha)}{-\frac{\alpha\delta Q+1_\gamma}{1-\delta(1-\gamma)}-(1-\alpha)\delta+1}$$

$$\therefore \frac{\partial V_i^+}{\partial r} = (-\frac{K(1-\alpha)}{-\frac{\alpha\delta Q+1_\gamma}{1-\delta(1-\gamma)}-(1-\alpha)\delta+1}) < 0$$

Thus V_i^+ is a decreasing function of r implying that an increase in the interest rate would mean a decrease in the borrower's payoff. This further means that a high interest rate will cause the borrower to default strategically.

Considering the lender's sustainability constraint

The lender's sustainability constraint is a positive lender's value function. That is

$$V_i^+ = \frac{(1+r)(1-\alpha)-(1+z)K}{1-\delta(1-\alpha)-\frac{\alpha\gamma\delta Q+1}{1-\delta(1-\gamma)}} > 0$$

It is also an increasing function of r.

Proof

Differentiating V_i^+ with respect to r.

$$\frac{\partial}{\partial r}[\frac{(1-\alpha)(r+1)-K(z+1)}{-\frac{\alpha\delta Q+1_\gamma}{1-\delta(1-\gamma)}-(1-\alpha)\delta+1}]$$

$$\frac{\partial V_i^+}{\partial r} = \frac{1}{-\frac{\alpha\delta Q+1_\gamma}{1-\delta(1-\gamma)}-(1-\alpha)\delta+1}(\frac{\partial}{\partial r}[(1-\alpha)(r+1)] + \frac{\partial}{\partial r}[-K(z+1)])$$

$$\frac{\partial V_i^+}{\partial r} = \frac{(1-\alpha)(\frac{\partial}{\partial r}[r]+\frac{\partial}{\partial r}[1])+0}{-\frac{\alpha\delta Q+1_\gamma}{1-\delta(1-\gamma)}-(1-\alpha)\delta+1}$$

$$\frac{\partial V_i^+}{\partial r} = \frac{(1+0)(1-\alpha)}{-\frac{\alpha\delta^{Q+1}\gamma}{1-\delta(1-\gamma)}-(1-\alpha)\delta+1}$$

$$\frac{\partial V_i^+}{\partial r} = \frac{(1-\alpha)}{-\frac{\alpha\delta^{Q+1}\gamma}{1-\delta(1-\gamma)}-(1-\alpha)\delta+1}$$

$\therefore \frac{(1-\alpha)}{-\frac{\alpha\delta^{Q+1}\gamma}{1-\delta(1-\gamma)}-(1-\alpha)\delta+1} > 0$ for $\alpha \epsilon (0,1)$, $\delta \epsilon (0,1)$ and $\gamma \epsilon (0,1)$ since α_i, δ and γ are probabilities.

This implies that a positive interest rate will yield a positive lender's payoff.

Hence the interest rate, r, that will solve the maximisation problem will be the r^* for which the lender's sustainability binds.

$$\frac{(1+r^*)(1-\alpha)-(1+z)K}{1-\delta(1-\alpha)-\frac{\alpha\gamma\delta^{Q+1}}{1-\delta(1-\gamma)}} = 0$$

$$(1+r^*)(1-\alpha)-(1+z)K = 0$$

$$(1+r^*) = \frac{K(1+z)}{(1-\alpha}$$

$$r^* = \frac{K(1+z)-(1-\alpha)}{(1-\alpha)}$$

And in the case where the borrower is lent a cedi

$$r^* = \frac{(\alpha+z)}{(1-\alpha)}$$

The interest rate that maximizes borrower welfare and makes lending sustainable, will exactly cover the total cost of lending, adjusted for the portion of the portfolio that is non-performing.

3.5 Solving For The Length Of The Punishment Phase

It is easy to show that a borrower would to continue to borrow even when he or she defaults. However, for most borrowers, an important punishment phase will be crucial to preserve repayment incentives. In equilibrium, we would like Q to be small to maximize the payoff of the borrower but must also be large enough to meet the incentive constraints of both the safe and risky borrower types. The binding incentive constraint for the risky borrower or the borrower with the weaker repayment incentives, Q* which is enough length of time in the punishment phase to maintain repayment incentive, is given below;

$$\text{Given } \mathrm{V}_i^+ = \frac{(1-\alpha_i)(\omega-(1+r))K}{1-\delta(1-\alpha_i)-\frac{\alpha_i\gamma\delta^{Q+1}}{1-\delta(1-\gamma)}} \tag{12}$$

we have to differentiate V_i^+ and Q.

$$\frac{\partial V_i^+}{\partial Q} = \frac{\partial}{\partial Q}\left[\frac{(1-\alpha_i)(\omega-(1+r))K}{1-\delta(1-\alpha_i)-\frac{\alpha_i\gamma\delta^{Q+1}}{1-\delta(1-\gamma)}}\right] \tag{13}$$

$$\text{Apply quotient rule.}\left(\frac{f}{g}\right)' = \frac{f'g-g'f}{(g)^2} \tag{14}$$

here $f = (1-\alpha_i)(w-(1+r))K$

$$\text{Now } f' = \frac{\partial f}{\partial Q} = 0 \tag{15}$$

and $g = 1-\delta(1-\alpha_i)-\frac{\alpha_i\gamma\delta^{Q+1}}{1-\delta(1-\gamma)}$

$g' = \frac{\partial g}{\partial Q} =$ Apply sum differential rule $(f \pm g)' = \mathrm{f}' \pm \mathrm{g}'$

then $g' = \frac{\partial}{\partial Q}(1-\delta(1-\alpha_i)) - \frac{\partial}{\partial Q}\left[\frac{\alpha_i\gamma\delta^{Q+1}}{1-\delta(1-\gamma)}\right]$

we know $\frac{d}{dx}(c) = 0$

$\therefore$c = constant

also $\frac{d}{dx}(ax) = a\frac{d}{dx}x$

$$g^{'} = 0 - \frac{\alpha_i \gamma}{1-\delta(1-\gamma)} \frac{\partial}{\partial Q} \delta^{Q+1} \tag{16}$$

from basic differentiation $[\frac{d}{dx}a^{x+1} - a^{x+1}ln(a)]$

then equation (16) becomes

$$g^{'} = \frac{-\alpha_i \gamma}{1-\delta(1-\gamma)} \delta^{Q+1} \ln(\delta) \tag{17}$$

$$\frac{\partial V_i^+}{\partial Q} = \frac{0(g) - g^{'} f}{g^2}$$

$$\frac{\partial V_i^+}{\partial Q} = \frac{-(\frac{-\alpha_i \gamma}{1-\delta(1-\gamma)} \delta^{Q+1} ln(\delta))(1-\alpha_i)(\omega-(1-r))K}{[1-\delta(1-\gamma) - \frac{\alpha_i \gamma \delta^{Q+1}}{1-\delta(1-\gamma}]^2}$$

It should be known that over several loan cycles, safe borrowers spend less time in the punishment phase as compared to the risky borrowers who spend quite a longer time. This is similar to a finding by (Ghatak, 1999) that the effective cost of borrowing may be lower for safe types, even with a single group lending contract. The time spent in the punishment phase is not optimal but sufficient for this model. Both types of borrowers could be better off by using borrower histories. For Q* to be positive and exist and therefore a true punishment, few conditions must be followed. For this to exist, we expect that the lowest expected benefit of taking a loan should be greater than the cost of the

loan. Mathematically, $\omega(1-\alpha_R) > 1 + r^*$ and the borrowers must sufficiently value the future (large enough). For $Q^* > 0$, there is a final restriction that , the probability of returning to the lending phase is large enough. As γ decreases, the probability that borrower returns to the lending phase from the punishment phase reduces, which in effect increases the punishment even after the borrower is done serving their time in the punishment phase. In this situation, there is a gap in the required Q^* against the actual time a person is without a loan after default, and $Q^*= 0$ may still satisfy the incentive constraint. Note that the length of the punishment phase intrinsically depends on the interest rate, as it was a prominent factor in the borrower's incentive constraint (8). If the interest rate is to rise, the benefits of strategic default also rise. This will entail a longer length of time without loans. On the other hand, if the interest rate falls, benefits of strategic default are lower and the length of the punishment phase will decrease.

4 Conclusion

The borrower's payoff was analysed. From the analysis we realized that the lender's aim is to maximise the borrower's payoff so return made would exceed loan received. We also observed that the random settlement made by the various borrowers in loan repayment takes the form of a Markov chain. Also, the punishment phase must be unattractive to the borrower so the borrower's constraint was analysed from which we realized that the interest rate on loan demanded by a borrower who frequently serves the punishment phase must increase. Furthermore, the lender's payoff and payoff for the punishment phase was obtained. The punishment phase was observed to have a transition probability which follows a geometric distribution. Though the lender must make profit and the borrower's payoff must be maximized, an optimal interest rate must be obtained so borrowers do not default strategically and lenders would not incur losses as well so this project work calculated for the optimal loan rate of a microfinance institution. From our findings, we also realized the interest rate does not only affect the borrower's and lender's payoff but the length of the punishment phase as well, that is an increase in interest rate would cause strategic default to rise hence a longer length of time without loans. So an optimal punishment phase was obtained as well.

References

Armendáriz de Aghion, B., Morduch, J. (2000). Microfinance beyond group lending. Economics of transition, 8(2), 401-420.

Ashta, A., Khan, S., Otto, P. (2015). Does Microfinance Cause or Reduce Suicides? Policy Recommendations for Reducing Borrower Stress. Strategic Change, 24(2), 165–190.

Bateman, M. and H. J. C. (2012). Microfinance and the Illusion of Development: From Hubris to Nemesis in Thirty Years. World Social and Economic Review, 2012(1), 13–36.

Conroy, J. D., MacGuire, P. B. (2000). The role of Central Banks in microfinance in Asia and the pacific. Asian Development Bank.

Diener, F., Diener, M., Khodr, O., Protter, P. (2009). Mathematical Models for Microlending. Proceedings of the 16th Mathematical Conference of Bangladesh Mathematical Society. http://math1.unice.fr/ diener/MiFi/Dhaka09.pdf.

Dunford, C. (2012). First step in the microfinance theory of change. Take a Loan or Save–Do They.

Duvendack, M., Palmer-Jones, R., Copestake, J. G., Hooper, L., Loke, Y., Rao, N. (2011).

Gin´e, X., Karlan, D. S. (2014). Group versus individual liability: Short and long term evidence from Philippine microcredit lending groups. Journal of De-

velopment Economics, 107, 65–83. https://doi.org/10.1080/00076791.2017.1413096

Jain, P., Moore, M. (2003). What makes microcredit programmes effective? Fashionable fallacies and workable realities. IDS Working Paper 177, 1–41. https://doi.org/1858644798

Kabir, M. (2002). The Microfinance Revolution and the Grameen Bank Experience in Bangladesh. Financial Markets, Institutions and Instruments, 11(3), 205–265.

Khan, S., Ashta, A. (2013). Managing multi-faceted risks in microfinance operations. Strategic Change, 22(1–2), 1–16.

Kono, H. (2006). Is group lending a good enforcement scheme for achieving high repayment rates?: Evidence from field experiments in Vietnam. Discussion Paper No. 061.

Ledgerwood, J. (1998). Microfinance handbook. An Institutional and Financial Perspective.

Lehner, M. (2009). Group lending versus individual lending in microfinance. SFB/TR 15 Discussion Paper, 299.

Loewe, M., Rippin, N. (2015). The Sustainable Development Goals of the Post-2015 Agenda: Comments on the OWG and SDSN Proposals. SSRN Electronic Journal.

Macchiavello, E.(2012). Microfinance regulation and supervision: A multi-faced prism of structures, levels and issues.

Matin, N., Taher, M. (2001). The changing emphasis of disasters in Bangladesh NGOs. Disasters, 25(3), 227–239. https://doi.org/10.1111/1467-7717.00174

Mia, M. A. (2016). Microfinance Institutions and Legal Status: An Overview of the Microfinance Sector in Bangladesh. The Journal of Asian Finance, Economics and Business, 3(2), 21–31. https://doi.org/10.13106/jafeb.2016.vol3.no2.21.

Mia, M. A., Ben Soltane, B. I. (2016). Productivity and its determinants in microfinance institutions (MFIs): Evidence from South Asian countries. Economic Analysis and Policy, 51, 32–45. https://doi.org/10.1016/j.eap.2016.05.003

Mia, M. A., Lee, H. A., Chandran, V. G. R., Rasiah, R., & Rahman, M. (2019). History of microfinance in Bangladesh: A life cycle theory approach. Business History, 61(4), 703–733.

Morduch, Jonathan, 2000. The microfinance schism. World Development 28 (4), 617 – 629

Nasrin, S., Baskaran, A., Rasiah, R. (2017). Microfinance and savings among the poor: Evidence from Bangladesh microfinance sector. Quality Quantity, 51(4), 1435–1448.

Pitt, M. M., Khandker, S. R., Cartwright, J. (2006). Empowering women with micro finance: Evidence from Bangladesh. Economic Development and

Cultural Change, 54(4), 791–831. https://doi.org/10.1086/503580

Qudrat-I Elahi, K., Rahman, M. L. (2006). Micro-credit and micro-finance: Functional and conceptual differences. Development in Practice, 16(5), 476–483.

Racioppi, L., Linda, R. (1994). (1994). Soviet policy towards South Asia since 1970. 91.

Rehman, H., Moazzam, A., Ansari, N. (2015). Role of Microfinance Institutions in Women Empowerment: A Case Study of Akhuwat, Pakistan. South Asian Studies, 30(1), 107.

What is the evidence of the impact of microfinance on the well-being of poor people? Systematic review (Issue August).

Zaman, H. (2004). Microfinance in Bangladesh: growth, achievements, and lessons. Scaling up poverty and reduction, 47.