
SEM2DPACK

A Spectral Element Method tool for 2D wave propagation
and earthquake source dynamics

User's Guide

Version 2.3.2

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Chapter 1

Introduction

1.1 Overview

The SEM2DPACK package is a set of software tools for the simulation and analysis of the seismic response of sedimentary basins and dynamic earthquake ruptures. Its core is SEM2D, an explicit Spectral Element solver for the 2D elastic wave equation. The general flow of a simulation project is:

1. Mesh generation: a domain decomposition made of quadrilateral elements, linearly (Q4) or quadratically (Q9) deformed.
2. Mesh quality verification, return to previous step if needed.
3. Numerical simulation.
4. Post-processing, analysis and visualization of the output.

SEM2DPACK provides tools for each step. However, no general mesh generation code is included. Instead SEM2D can import unstructured quadrilateral meshes generated externally. As an example we provide an interface to EMC2, one of the few public domain 2D mesh generators including quadrilateral elements and a Graphical User Interface ¹.

Chapter 2 of this User's Guide summarizes the physical assumptions and notations. Chapter 3 explains the usage of the SEM2D solver. For more details, additional results, and for the 3D extension of the algorithm, please refer to Komatitsch (1997), Komatitsch and Vilotte (1998), Komatitsch *et al.* (1999) and also Vai *et al.* (1998). Chapter 4 provides an introduction to mesh generation with EMC2. For more details you must refer to the EMC2 documentation. Chapter 5 gives some tips for advanced users who need to add new features to the code.

This is a research code, constantly under development and provided “as is”, and therefore it should *not* be considered by the user as a 100 % bug-free software package. We welcome comments, suggestions, feature requests, module contributions and bug reports.

¹EMC2 can be downloaded from <http://www-rocq.inria.fr/gamma/cdrom/www/emc2/eng.htm>

1.2 History and credits

The main part of the elastic-isotropic solver was written in the mid 90's by Dimitri Komatitsch while he was preparing his Ph.D. at the *Institut de Physique du Globe de Paris*, under the advise of Prof. Jean-Pierre Vilotte. The elastic-anisotropic solver and several significant improvements to the isotropic code were added by D. Komatitsch later as part of a research contract with DIA Consultants. Further functionalities were added by myself, Jean-Paul Ampuero, while preparing my Ph.D. at IPGP, also under the advise of Prof. Jean-Pierre Vilotte. Most of these additional features were motivated by an ECOS-NORD/FONACYT research project for the study of the seismic response of the valley of Caracas, Venezuela. That became the version 1.0 of the SEM2DPACK, released in April 2002.

For the current version, 2.x, the code was almost completely rewritten in a more object-oriented style in preparation to the implementation of higher level functionalities, such as multigrid, subcycling, adaptivity and multiscale coupling. While the extensive use of object-oriented features of FORTRAN 90 can degrade performance this is not critical in 2D simulation, the emphasis has been rather in facilitating code reuse and expansion.

A simultaneous development for the simulation of earthquake dynamics was undertaken and is the main new feature of the current version. Spontaneous rupture along multiple non-planar faults can be currently modelled. Although there is no intrinsic limitation on applying different friction laws, as of Version 2.2 only linear slip weakening friction is implemented. Dynamic source simulations using methods that discretize the bulk, such as finite difference, finite element and spectral element methods, are more prone to high frequency numerical noise than boundary element methods (e.g. when the size of the process zone is not well resolved). Methods to control this problem were presented in the author's Ph.D. dissertation (Ampuero, 2002)² and in Gaetano Festa's Ph.D. dissertation³, and will be implemented in a forthcoming version of SEM2DPACK.

SEM2DPACK has been used in the following work:

- Madariaga *et al.* (2006): dynamic rupture and seismic wave radiation on faults with geometrical complexities (kinks)
- Haney *et al.* (2007): fault reflections from fluid-infiltrated faults
- De la Puente *et al.* (2007): as benchmark method for anisotropic wave propagation
- Kaneko *et al.* (2008): dynamic earthquake rupture with rate-and-state friction

1.3 Installation

- Uncompress and expand the SEM2DPACK package: `tar xvfz sem2dpack.tgz`
- Go to the source directory: `cd SEM2DPACK/SRC`

²Available in French at <http://www.seg2.ethz.ch/ampueroj/phd.html>

³<http://people.na.infn.it/~festa/>

- Edit the `Makefile` according to your FORTRAN 95 compiler, following the instructions therein.
- Modify the optimization parameters declared and described in `SRC/constant.f90`.
- Compile: `make`
- Move to the `SEM2DPACK/POST` directory, edit the `Makefile` and compile.

On normal termination you should end up with a set of executable files, among which `sem2dsolve`, in `/home/yourhome/bin/`. I have been developping the code with the Lahey/Fujitsu `lf95` compiler and, more recently, with the Intel compiler for Linux ⁴. Other compilers are not being tested on a regular basis, so please report any related problems.

1.4 Help requests, feature requests and bug reports

Since November 2006 (version 2.2.5) current and old versions of SEM2DPACK are hosted by SourceForge at <http://sourceforge.net/projects/sem2d/>. To take advantage of the convenient features offered by this host you must create a SourceForge.net account at <http://sourceforge.net/account/registration/>.

The code repository is at http://sourceforge.net/project/showfiles.php?group_id=182742. To receive notification emails about new releases of SEM2DPACK sign up for the "Package Monitor" at http://sourceforge.net/project/filemodule_monitor.php?filemodule_id=212397.

A "tracking system" is available at http://sourceforge.net/tracker/?group_id=182742, with three separate lists. Requests for implementation of new features must be submitted to the "Feature Requests" tracker. Questions related to the usage of SEM2DPACK must be submitted to the "Support Requests" tracker. Bug reports must be submitted to the "Bugs" tracker. The three tracker lists are browsable and searchable. To browse the complete list of a tracker set "Status" to "Any". Before submitting an issue make sure you are running the most recent version of SEM2DPACK, that you understand the changes listed in SEM2DPACK's `ChangeLog` file and that your problem has not been treated in previous submissions. When relevant, a new submission must include the input files needed to reproduce your problem (`Par.inp`, `*.ftq`, etc). You will receive email notifications of any update of your submitted item, until it is closed. If the item is declared "Pending" you are expected to reply to the last message of the developer within two weeks, otherwise the item will be closed. For more instructions see http://sourceforge.net/support/getsupport.php?group_id=182742.

Contributions to SEM2DPACK by experienced programmers are always welcome and encouraged. A "Developers Forum" is available at http://sourceforge.net/forum/forum.php?forum_id=635737, where the implementation of new features can be discussed. Although the code is stable for my research purposes, there is still a number of missing features. Their

⁴This code works properly with the Intel compiler starting with version 8.0.046_pe047.1, so make sure you have a recent version of `ifort` !

implementation could make SEM2DPACK interesting for a broader audience in mechanical engineering, geotechnical engineering, applied geophysics and beyond. The `ToDo` file included with SEM2DPACK contains a list of missing features that range from basic functionalities to complex code re-engineering. Chapter 5 gives some guidelines for programmers.

1.5 License

This software is freely available for academic research purposes. If you use this software in writing scientific papers include proper attributions to its author, Jean-Paul Ampuero.

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Chapter 2

Physical background

This chapter summarizes the physical assumptions and notations in SEM2DPACK. Footnotes provide reference to the input arguments described in Chapter 3.

2.1 Basic assumptions and conventions

The coordinate system is Cartesian (rectangular). SEM2DPACK works in the two-dimensional (x, z) plane, where x is the horizontal coordinate, with positive direction pointing to the right, and z is the vertical coordinate, with positive direction pointing upwards. The coordinates (x, y, z) will be also denoted as (x_1, x_2, x_3) . This notation carries also for subscripts. For instance, the k -th component of displacement is denoted as u_k , with $k = 1, 2, 3$ or with $k = x, y, z$.

The reference frame is Eulerian. Infinitesimal strain is assumed. The (symmetric) infinitesimal strain tensor ϵ is defined as

$$\epsilon_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \quad (2.1)$$

Material density is denoted $\rho(x, z)$. The displacements and stresses relative to an initial equilibrium configuration are denoted $u_k(x, z, t)$ and $\sigma_{ij}(x, z, t)$, respectively. External forces (sources) are denoted $f_i(x, z, t)$. SEM2DPACK solves the following equations of motion to obtain the relative displacements $u_k(x, z, t)$:

$$\rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial \sigma_{ij}}{\partial x_j} + f_i \quad (2.2)$$

where summation over repeated indices is assumed. The initial conditions are $u_k = 0$ and $\partial u_k / \partial t = 0$. Stresses are related to strain, and possibly to other internal variables, by constitutive equations described in Section 2.2. The governing equations are supplemented by boundary conditions, described in Section 2.3. SEM2DPACK actually solves the governing

equations in variational (weak) form, as described in any textbook on the finite element method.

Two types of 2D problems are solved¹:

- *Plane strain*: Also known in seismology as P-SV, and in fracture mechanics as inplane mode or mode II. It is assumed that $u_3 = 0$ and $\partial/\partial x_3 = 0$. Hence, $\epsilon_{13} = \epsilon_{23} = \epsilon_{33} = 0$ and there are two degrees of freedom per node, u_x and u_z .
- *Antiplane shear*: Also known in seismology as SH, and in fracture mechanics as antiplane mode or mode III. It is assumed that $u_1 = u_2 = 0$ and $\partial/\partial x_3 = 0$. Hence, only ϵ_{13} and ϵ_{23} are non-zero and there is one degree of freedom per node, u_y .

2.2 Material rheologies

We describe here the constitutive equations implemented in SEM2DPACK, relating stress (σ_{ij}), strain (ϵ_{ij}) and internal variables.

2.2.1 Linear elasticity

Linear isotropic elasticity

The stress-strain constitutive relation is Hooke's law:

$$\sigma_{ij} = \lambda \epsilon_{kk} \delta_{ij} + 2\mu \epsilon_{ij} \quad (2.3)$$

where λ and μ are Lamé's first and second parameters, respectively. In 2D plane strain the only relevant stress components are σ_{11} , σ_{22} and σ_{12} . The intermediate stress σ_{33} , although not null, is ignored. The S and P wave speeds are $c_S = \sqrt{\mu/\rho}$ and $c_P = \sqrt{(\lambda + 2\mu)/\rho}$, respectively. In 2D antiplane shear only the stress components σ_{13} and σ_{23} are relevant, and only S waves are generated.

Linear anisotropic elasticity

Transverse anisotropy with vertical symmetry axis is implemented for 2D plane strain (Komatitsch *et al.*, 2000). The stress-strain constitutive relation is:

$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{zz} \\ \sigma_{xz} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{13} & 0 \\ c_{13} & c_{33} & 0 \\ 0 & 0 & c_{55} \end{pmatrix} \begin{pmatrix} \epsilon_{xx} \\ \epsilon_{zz} \\ 2\epsilon_{xz} \end{pmatrix} \quad (2.4)$$

where the c_{ij} are elastic moduli.

¹In the `&GENERAL` input block, plane strain is selected by `ndof=2` and antiplane shear by `ndof=1`.

2.2.2 Linear visco-elasticity

Generalized Maxwell material

Not implemented yet.

Kelvin-Voigt material

Kelvin-Voigt damping can be combined with any of the other constitutive equations by replacing the elastic strain ϵ by $\epsilon^* = \epsilon + \eta \partial \epsilon / \partial t$, where η is a viscosity timescale. The resulting quality factor Q is frequency-dependent, $Q^{-1}(f) = 2\pi\eta f$. This is not convenient to model crustal attenuation with constant Q , unless the source has a narrow frequency band and η is selected to achieve a given Q value at the dominant frequency of the source. Most frequently, a layer of Kelvin-Voigt material is applied as artificial damping close to dynamic faults.

2.2.3 Coulomb plasticity

Perfect plasticity with a Coulomb yield function is implemented for 2D plane strain, as in Andrews (2005).

The total strain is the sum of an elastic and a plastic contribution, $\epsilon = \epsilon^e + \epsilon^p$.

[...]

Plastic yield occurs when

$$\tau = c \cos(\phi) - (\sigma_{xx} + \sigma_{zz})/2 \sin(\phi) \quad (2.5)$$

where c is cohesion, ϕ the internal friction angle and

$$\tau = \sqrt{\sigma_{xz}^2 [(\sigma_{xx} - \sigma_{zz})/2]^2} \quad (2.6)$$

is the maximum shear stress over all planes.

[...]

2.2.4 Continuum damage

The continuum damage formulation by Lyakhovsky *et al.* (1997), including damage-related plasticity as introduced by Hamiel *et al.* (2004), is implemented with modifications for 2D plane strain.

The first and second invariants of the 2D elastic strain tensor are defined as $I_1 = \epsilon_{kk}^e$ and $I_2 = \epsilon_{ij}^e \epsilon_{ij}^e$, respectively. A strain invariant ratio is defined as $\xi = I_1 / \sqrt{I_2}$. The following non-linear stress-strain relation is assumed (Lyakhovsky *et al.*, 1997, eq. 12):

$$\sigma_{ij} = (\lambda - \gamma/\xi) I_1 \delta_{ij} + (2\mu - \gamma\xi) \epsilon_{ij}^e \quad (2.7)$$

where γ is an additional elastic modulus. The elastic moduli depend on a scalar damage variable, $0 \leq \alpha \leq 1$, through (Lyakhovsky *et al.*, 1997, eq. 19):

$$\lambda = \lambda_0 \quad (2.8)$$

$$\mu = \mu_0 + \gamma_r \xi_0 \alpha \quad (2.9)$$

$$\gamma = \gamma_r \alpha \quad (2.10)$$

where λ_0 and μ_0 are Lamé's parameters for the intact material ($\alpha = 0$). The parameter ξ_0 is the threshold value of the strain invariant ratio ξ at the onset of damage. It is related to the internal friction angle ϕ in a cohesionless Mohr-Coulomb yield criterion by the 2D plane strain version of Lyakhovsky *et al.* (1997, eq. 37):

$$\xi_0 = \frac{-\sqrt{2}}{\sqrt{1 + (\lambda_0/\mu_0 + 1)^2 \sin^2 \phi}} \quad (2.11)$$

The scaling factor γ_r is chosen such that convexity is lost at $\alpha = 1$ when $\xi = \xi_0$. It is derived from the 2D plane strain version of Lyakhovsky *et al.* (1997, eq. 15):

$$\gamma_r = p + \sqrt{p^2 + 2\mu_0 q} \quad (2.12)$$

where

$$q = (2\mu_0 + 2\lambda_0)/(2 - \xi_0^2) \quad (2.13)$$

$$p = \xi_0(q + \lambda_0)/2 \quad (2.14)$$

The evolution equation for the damage variable is (Lyakhovsky *et al.*, 1997, eq. 20)

$$\dot{\alpha} = \begin{cases} C_d I_2 (\xi - \xi_0) & \text{if } \xi > \xi_0 \\ 0 & \text{otherwise} \end{cases} \quad (2.15)$$

No healing is assumed below ξ_0 . The evolution of the plastic strain ϵ_{ij}^p is driven by the damage variable α (Hamiel *et al.*, 2004, eq. 9):

$$\dot{\epsilon}_{ij}^p = \begin{cases} \tau_{ij} C_v \dot{\alpha} & \text{if } \dot{\alpha} \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (2.16)$$

where $\tau_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_{kk} \delta_{ij}$ is the deviatoric part of the stress tensor. The parameter C_v is of order $1/\mu_0$ and² is related to the seismic coupling coefficient $0 < \chi < 1$ by (Ben-Zion and Lyakhovsky, 2006)

$$C_v = \frac{1 - \chi}{\chi} \frac{1}{\mu_0} \quad (2.17)$$

2.3 Boundary conditions

2.3.1 Absorbing boundaries

Paraxial

[...]

²In `&MAT_DMG`, the input argument `R` is defined as $R = \mu_0 C_v$.

Clayton-Engquist

[...]

2.4 Fault interface conditions

2.4.1 Linear slip law

See Haney *et al.* (2007) [...]

2.4.2 Normal stress response

Unilateral contact

[...]

Modified Prakash-Clifton regularization

Regularization of the normal stress response, as required for bimaterial rupture problems, is implemented following Rubin and Ampuero (2007). The frictional strength is proportional to a modified normal stress σ^* , related to the real fault normal stress, σ , by either of the following evolution laws:

- Version with a regularization *slip* scale:

$$\dot{\sigma}^* = \frac{|V| + V^*}{L_\sigma} (\sigma - \sigma^*) \quad (2.18)$$

where V is slip rate, and V^* and L_σ are constitutive parameters³.

- Version with a regularization *time* scale:

$$\dot{\sigma}^* = \frac{1}{T_\sigma} (\sigma - \sigma^*) \quad (2.19)$$

where T_σ is a constitutive parameter⁴.

³In `&BC_DYNFLT_NOR`, this law is set by `kind=2`, and the two relevant parameters are `V` and `L`.

⁴In `&BC_DYNFLT_NOR`, this law is set by `kind=3`, and the relevant parameter is `T`.

2.4.3 Friction

Slip-weakening friction

Slip occurs when the fault shear stress reaches the shear strength $\tau = \mu\sigma$ (or $\tau = \mu\sigma^*$ if the Prakash-Clifton law is assumed). [...] The friction coefficient μ is a function of the cumulated slip D , given by one of the following laws:

- Linear slip-weakening law:

$$\mu = \max \left[\mu_d, \mu_s - \frac{\mu_s - \mu_d}{D_c} D \right] \quad (2.20)$$

- Exponential slip-weakening law:

$$\mu = \mu_s - (\mu_s - \mu_d) \exp(-D/D_c) \quad (2.21)$$

“Fast” rate-and-state friction

Friction with fast (power law) velocity weakening at fast slip speed is a first order proxy for physical weakening processes that operate on natural fault zones at seismic slip velocities. A rate-and-state dependent friction law with fast velocity-weakening is implemented in SEM2DPACK, similar to that adopted e.g. by Ampuero and Ben-Zion (2008). The friction coefficient depends on slip velocity (V) and a state variable (θ):

$$\mu_f = \mu_s + a \frac{V}{V + V_c} - b \frac{\theta}{\theta + D_c}. \quad (2.22)$$

The state variable has units of slip and is governed by the evolution equation

$$\dot{\theta} = V - \theta V_c / D_c. \quad (2.23)$$

The friction law is defined by the following constitutive parameters: μ_s is the static friction coefficient, a and b are positive coefficients of a direct effect and an evolution effect, respectively, V_c is a characteristic velocity scale⁵, and D_c is a characteristic slip scale.

The steady-state ($\dot{\theta} = 0$) friction coefficient

$$\mu_f = \mu_s + (a - b) \frac{V}{V + V_c} \quad (2.24)$$

weakens asymptotically as $1/V$ when $V \gg V_c$, if $a < b$, approaching its dynamic value ($\mu_d = \mu_s + a - b$) over a relaxation timescale D_c/V_c . The value of the relaxation time D_c/V_c tunes the weakening mechanism between two limit cases: slip-weakening and velocity-weakening. If D_c/V_c is much longer than the typical time scale of fluctuation of the state variable ($\approx \theta/\dot{\theta}$), Equation 2.23 becomes $\dot{\theta} \approx V$, implying that θ is proportional to slip and that the evolution term of the friction coefficient is effectively slip-weakening, with characteristic slip-weakening distance D_c . Conversely, if D_c/V_c is short the relaxation to steady state is fast, $\theta/D_c \approx V/V_c$ and the friction is effectively velocity-weakening, with characteristic velocity scale V_c .

⁵`vstar` in `&BC_DYNFLT_RSF`

Logarithmic rate-and-state friction

Not implemented yet.

Dieterich and Ruina classical rate-and-state laws, with aging or slip state evolution law.

Chapter 3

The solver SEM2D

3.1 About the method

Given a crustal model meshed with quadrilateral elements and a set of material properties, sources, receivers and boundary conditions, SEM2D solves the elastic wave equation applying the Spectral Element Method (SEM) for the space discretization and a second-order explicit scheme for the time discretization. The range of physical problems solved by SEM2D (material constitutive equations and boundary conditions) is described in more detail in Chapter 2. The SEM, introduced by Patera (1984) in Computational Fluid Dynamics, can be seen as a domain decomposition version of Pseudospectral Methods or as a high order version of the Finite Element Method. It inherits from its parent methods the accuracy (spectral convergence), the geometrical flexibility and the natural implementation of mixed boundary conditions.

Introductory texts to the SEM can be found at www.math.lsa.umich.edu/~karni/m501/boyd.pdf (chapter draft, by J.P. Boyd), at www.mate.tue.nl/people/vosse/docs/vosse96b.pdf (a tutorial exposition of the SEM and its connection to other methods, by F.N. van de Vosse and P.D. Minev) and at www.siam.org/siamnews/01-04/spectral.pdf (a perspective paper). Details about the elastodynamic algorithm and study of some of its properties are presented by Komatitsch (1997), Komatitsch and Vilotte (1998), Komatitsch *et al.* (1999), Komatitsch and Tromp (1999) and Vai *et al.* (1998).

The implementation of fault dynamics is similar to that in FEM with the “traction at split nodes” method explained by Andrews (1999). More details can be found in the author’s Ph.D. dissertation (Ampuero, 2002)¹, in Gaetano Festa’s Ph.D. dissertation² and in Kaneko *et al.* (2008).

A more accessible tutorial code, SBIEMLAB written in Matlab, can be downloaded from the author’s website, at www.seg2.ethz.ch/ampueroj/software.html.

¹www.seg2.ethz.ch/ampueroj/phd.html

²people.na.infn.it/~festa/

3.2 What SEM2D is and what it is not

SEM2D is an explicit spectral element solver for the 2D elastic wave equation and for some problems arising in earthquake source dynamics (see Chapter 2). The code is written in FORTRAN 90, with some (useful but not essential) FORTRAN 95 features.

The SEM requires an initial decomposition of the domain into quadrilateral elements (a quad mesh). Optimal accuracy is obtained when the faces of the elements coincide with the interfaces between different materials. Generating high quality quad meshes for complicated geological models is not yet a fully automated process, and can be very time-consuming. If the user prefers to sacrifice accuracy by generating a simple, logically cartesian mesh where the element faces do not necessarily follow the material interfaces, the basic built-in meshing capabilities of SEM2D are sufficient. However, SEM2D does not include a general mesh generator for unstructured grids. If you want to get the best accuracy out of the spectral element method and your geological model is complicated you need to generate a mesh with some external software. An example using the freely available 2D mesh generation software EMC2 is described in Chapter 4.

A number of post-processing and graphic tools are included, as described in Section 3.7. Most output is in the form of raw binary or ASCII data files. Scripts are provided for graphic display and analysis on Seismic Unix, Gnuplot and Matlab. We recommend the usage of the Matlab functions included, which are well documented (see Section 3.7.6).

3.3 Basic usage flow

In general, a simulation requires the following steps:

1. Prepare the input file `Par.inp` (Section 3.4 and Section 3.5).
2. Run the solver in “check mode” (`iexec=0` in the `GENERAL` input block of `Par.inp`):
`sem2dsolve > info &`.
3. Verify the resolution, stability, estimated CPU cost and memory cost (Section 3.6).
4. If needed go back to step 1 and modify `Par.inp` (Section 3.6), else proceed to next step.
5. Run the solver in “production mode” (`iexec=1`): `sem2dsolve`.
6. Plot and manipulate the solver results (Section 3.7).

Full details are given in the following sections.

3.4 General format of the input file

The input file must be called `Par.inp`. Its typical structure is illustrated by two examples in Figure 3.1 and Figure 3.2. Most of the file is made of standard FORTRAN 90 NAMELIST input blocks. Each block gives input for a specific aspect of the simulation: material properties, sources, receivers, boundary conditions, etc.

The general syntax of a NAMELIST block can be found in any FORTRAN 90 textbook. In summary, a block named STUFF, with possible input arguments *a*, *b* and *c*, must be given as

```
&STUFF a=..., b=..., c=... /
```

where ... are user input values. Line breaks and comments preceded by ! are allowed within an input block.

The complete Reference Guide of the input blocks is presented in Section 3.5. For each block the documentation includes its name, possibly the name of a group of blocks to which it belongs, its purpose, its syntax, the list of its arguments with their description, and some important notes. In the syntax description, a vertical bar | between two arguments means “one or the other”. In the argument list, each item is followed by two informations within brackets []. The first bracketed information is the type of the argument: double precision (*dblr*), integer (*int*), logical (*log*), single character (*char*), fixed length word (e.g. *char*6* is a 6 characters word), arbitrary length word (*name*) or vectors (e.g. *int(2)* is a two element integer vector). The second bracketed information is the default value of the argument. Some arguments are optional, or when absent they are automatically assigned the default values.

Some arguments have a second version with a suffix *H* that allows to set values that are spatially non uniform. The *H*-version of the argument must be set to the name of any of the input blocks of the DISTRIBUTIONS group. The appropriate &DIST_xxxx block must follow immediately. For example, to set the argument *eta* to a Gaussian distribution:

```
&MAT_KV etaH='GAUSSIAN' /
&DIST_GAUSSIAN length=1d6,100d0, ampli=0.1d0 /
```

Arguments that accept an *H*-version are indicated in Section 3.5. When more than one *H*-version argument is present, the &DIST_xxxx blocks must appear in the same order as in the argument list of Section 3.5.

In the next section, Input Block Reference Guide, you should get acquainted with the syntax of the blocks you are most likely to use. The mandatory or more important input blocks are:

- &GENERAL
- &MESH_DEF, followed by a &MESH_Method block
- &MATERIAL, followed by a &MAT_Material block
- &BC_DEF, one for each boundary condition, each followed by a &BC_Kind block
- &TIME
- &SRC_DEF, followed by &STF_SourceTimeFunction and &SRC_Mechanism blocks
- &REC_LINE

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<pre> # Parameter file for SEM2DPACK 2.0 #----- Some general parameters ----- &GENERAL iexec=1, ngll= 6, fmax=1.25d0 , ndof=1 , title = 'Test SH', verbose='1111', ItInfo = 1000 / #----- Build the mesh ----- &MESH_DEF method = 'CARTESIAN' / &MESH_CART xlim=0.d0,30.d0 ,zlim=0.d0,30.d0 , nele=60,60/ #---- Elastic material parameters ----- &MATERIAL tag=1, kind='ELAST' / &MAT_ELASTIC rho=1.d0, cp=1.7321d0, cs=1.d0 / #----- Boundary conditions ----- &BC_DEF tag = 2 , kind = 'ABSORB' / &BC_ABSORB stacey=F/ &BC_DEF tag = 3 , kind = 'ABSORB' / &BC_ABSORB stacey=F/ #---- Time scheme settings ----- &TIME TotalTime=35.d0, courant = 0.3d0 / #---- Sources ----- &SRC_DEF stf= 'RICKER' ,coord= 0.d0,0.d0 , mechanism= 'FORCE' / &STF_RICKER f0= 0.5d0, onset = 3.d0, ampli = 0.25d0 / &SRC_FORCE angle = 0d0/ #----- Receivers ----- &REC_LINE number = 7 , field='D', first = 0.d0,0.d0, last = 30d0,0.d0, isamp=1 / #----- Plots settings ----- &SNAP_DEF itd=100000, ps=F , bin=F / </pre>		
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Figure 3.1: Input file `Par.inp` for an elementary example in `EXAMPLES/TestSH/` : a boxed region with a structured mesh.

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<pre> #----- Some general parameters ----- &GENERAL Iexec =0 , Ngll = 6 , fmax = 1.5 , ndof=1, Title = 'Palos Grandes NS meshed with EMC2' , Verbose='1111', ItInfo = 1000/ #----- Build the mesh ----- &MESH_DEF Method = 'EMC2' / &MESH_EMC2 File= 'NS03qb.ftq' / #----- Elastic material parameters ----- &MATERIAL tag=1, kind='ELAST' / &MAT_ELASTIC rho=1800.d0, cp=850.d0, cs=450.d0/ &MATERIAL tag=2, kind='ELAST' / &MAT_ELASTIC rho=2100.d0, cp=1800.d0, cs=650.d0/ &MATERIAL tag=3, kind='ELAST' / &MAT_ELASTIC rho=2400.d0, cp=2300.d0, cs=850.d0/ &MATERIAL tag=4, kind='ELAST' / &MAT_ELASTIC rho=2600.d0, cp=3800.d0, cs=2200.d0/ #&MAT_ELASTIC rho=2500.d0, cp=5000.d0, cs=2900.d0/ #----- Boundary conditions ----- &BC_DEF Tag = 2, Kind = 'ABSORB' / &BC_ABSORB Stacey=F / &BC_DEF Tag = 3, Kind = 'ABSORB' / &BC_ABSORB Stacey=F, let_wave=T / &BC_DEF Tag = 4, Kind = 'ABSORB' / &BC_ABSORB Stacey=F / #----- Time scheme settings ----- &TIME TotalTime=25.d0, Courant = 0.55d0 / &TIME_NEWMARK alpha=1.d0, beta=0.d0, gamma=0.5d0 / #----- Sources ----- &SRC_DEF stf='RICKER', Mechanism='WAVE' Coord= -1160000.d0,-2000.d0 / &STF_RICKER f0 = 1.d0 , Onset = 1.5d0 ,Ampli = 1.d0 / &SRC_FORCE Angle = 90. / &SRC_WAVE Angle = 30. , phase='S' / #----- Receivers ----- # receivers located at the surface by giving a very large vertical position # locating them at the nearest computational node (AtNode=.true. is the default) &REC_LINE Number = 31 , First = -1163068.0d0,1.d3, Last = -1159697.36d0,1.d3, Isamp=10 / #----- Plots settings ----- &SNAP_DEF itd=100000, fields='V', components='x' / # itd = 3500 &SNAP_PS Mesh=T, Vectors=F, Color=T, Interpol = T, DisplayPts=7, ScaleField=0.2d0 / </pre>		
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Figure 3.2: Input file `Par.inp` for a more realistic example: a sedimentary basin with an unstructured mesh generated by EMC2. Available in `EXAMPLES/UsingEMC2/`.

3.5 Input Blocks Reference Guide

```
=====
= Self-documentation for the INPUT BLOCKS of the SEM2D code =
=====
```

```
-----

NAME   : BC_ABSORB
GROUP  : BOUNDARY_CONDITION
PURPOSE: Absorbing boundary
SYNTAX : &BC_ABSORB stacey, let_wave /
```

```

stacey  [log] [F] Apply Stacey absorbing conditions for P-SV.
           Higher order than Clayton-Engquist (the default).
let_wave [log] [T] Allow incident waves across this boundary
```

```
NOTE   : Only implemented for vertical and horizontal boundaries.
```

```
-----

NAME   : BC_DIRNEU
GROUP  : BOUNDARY_CONDITION
PURPOSE: Dirichlet (null displacement)
           and/or Neumann (null or time-dependent traction)
           boundary conditions on vertical or horizontal boundaries
SYNTAX : &BC_DIRNEU h, v, hsrc, vsrc /
           possibly followed by one or two STF_XXXX blocks

h       [char] ['N'] Boundary condition on the horizontal component
v       [char] ['N'] Boundary condition on the vertical component :
           'N' : Neumann
           'D' : Dirichlet
hsrc    [name] ['null'] Name of the source time function for a
           time-dependent horizontal traction:
           'RICKER', 'TAB', 'USER', etc (see STF_XXXX input blocks)
vsrc    [name] ['null'] Same for the vertical component
```

```
-----

NAME   : BC_DYNFLT
GROUP  : BOUNDARY_CONDITION, DYNAMIC_FAULT
PURPOSE: Dynamic fault with friction
SYNTAX : &BC_DYNFLT friction, Tn|TnH, Tt|TtH,
           Sxx|SxxH, Sxy|SxyH, Sxz|SxzH, Syz|SyzH, Szz|SzzH
```

```

        ot1, otd, oxi /
followed, in order, by:
1. &DIST_XXX blocks (from the DISTRIBUTIONS group) for arguments
   with suffix H, if present, in the order listed above.
2. &BC_DYNFLT_SWF, &BC_DYNFLT_TWF or &BC_DYNFLT_RSF block(s)
   (if absent, default values are used)
3. &BC_DYNFLT_NOR block (if absent, default values are used)

friction [name(2)] ['SWF',''] Friction law type:
        SWF = slip weakening friction
        TWF = time weakening friction
        RSF = rate and state dependent friction
Some friction types can be combined. E.g. to set the
friction coefficient to the minimum of SWF and TWF, set
friction='SWF','TWF'
Tn      [dble] [0d0] Initial normal traction (positive = tensile)
Tt      [dble] [0d0] Initial tangent traction (positive antiplane: y>0)
Sxx     [dble] [0d0] Initial stress sigma_xx
Sxy     [dble] [0d0] Initial stress sigma_xy
Sxz     [dble] [0d0] Initial stress sigma_xz
Syz     [dble] [0d0] Initial stress sigma_yz
Szz     [dble] [0d0] Initial stress sigma_zz
otd     [dble] [0.d0] Time lag between outputs (in seconds)
        Internally adjusted to the nearest multiple of the timestep
        Its value can be found in the output file FltXX_sem2d.hdr
        The default internally resets otd = timestep
ot1     [dble] [0.d0] Time of first output (in seconds)
        Internally adjusted to the nearest multiple of the timestep
        Its value can be found in the output file FltXX_sem2d.hdr
oxi     [int(3)] [(1,huge,1)] First, last node and stride for output
        The default resets oxi(2) = last fault node

```

NOTE: the initial stress can be set as a stress tensor (Sxx,etc), as
initial tractions on the fault plane (Tn and Tt) or as the sum of both.

NOTE: we recommend to use dynamic faults with the leapfrog time scheme
and a layer of Kelvin-Voigt damping material near the fault.

```

-----
NAME    : BC_DYNFLT_NOR
GROUP   : DYNAMIC_FAULT
PURPOSE: Normal stress response for dynamic faults.
SYNTAX  : &BC_DYNFLT_NOR kind, V, L, T /

```

```

kind    [int] [1] Type of normal stress response:
          1 = Coulomb

```

2 = Prakash-Clifton with regularizing time scale
 3 = Prakash-Clifton with regularizing length scale
 T [dble] [1d0] Regularization time scale if kind=2
 V [dble] [1d0] Characteristic velocity if kind=3
 L [dble] [1d0] Regularization length scale if kind=3

NAME : BC_DYNFLT_RSF
 GROUP : DYNAMIC_FAULT
 PURPOSE: Velocity and state dependent friction
 SYNTAX : &BC_DYNFLT_RSF kind, Dc | DcH, Mus | MusH ,
 a | aH, b | bH, Vstar | VstarH /
 followed by &DIST_XXX blocks (from the DISTRIBUTIONS group) for
 arguments with suffix H, if present, in the order listed above.

kind [int] [1] Type of rate-and-state friction law:
 1 = strong velocity-weakening at high speed
 as in Ampuero and Ben-Zion (2008)
 Dc [dble] [0.5d0] Critical slip
 MuS [dble] [0.6d0] Static friction coefficient
 a [dble] [0.01d0] Direct effect coefficient
 b [dble] [0.02d0] Evolution effect coefficient
 Vstar [dble] [1d0] Characteristic or reference slip velocity

NAME : BC_DYNFLT_SWF
 GROUP : DYNAMIC_FAULT
 PURPOSE: Slip-weakening friction
 SYNTAX : &BC_DYNFLT_SWF Dc | DcH, MuS | MuSH , MuD | MuDH /
 followed by &DIST_XXX blocks (from the DISTRIBUTIONS group) for
 arguments with suffix H, if present, in the order listed above.

kind [int] [1] Type of slip weakening function:
 1 = linear
 2 = exponential
 Dc [dble] [0.5d0] Critical slip
 MuS [dble] [0.6d0] Static friction coefficient
 MuD [dble] [0.5d0] Dynamic friction coefficient

NAME : BC_DYNFLT_TWF
 GROUP : DYNAMIC_FAULT
 PURPOSE: Time weakening friction for dynamic faults
 with prescribed rupture speed.

SYNTAX : &BC_DYNFLT_TWF MuS, MuD, X, Z, V, L, T /

MuS	[dble]	[0.6d0]	Static friction coefficient
MuD	[dble]	[0.5d0]	Dynamic friction coefficient
Mu0	[dble]	[0.6d0]	Friction coefficient at the hypocenter at time=0
X,Z	[dble]	[0d0]	Position of hypocenter
V	[dble]	[1d3]	Rupture propagation speed
L	[dble]	[1d0]	Size of weakening zone
T	[dble]	[huge]	Total duration

NAME : BC_DEF

PURPOSE: Define a boundary condition

SYNTAX : &BC_DEF tag, tags, kind /
possibly followed by &BC_kind blocks

tag	[int]	[none]	A number assigned to the boundary. If you are using SEM2D built-in structured mesher the conventions are:
	1		bottom
	2		right
	3		up
	4		left

If you are importing a mesh, you must use the tags assigned to the boundaries during the mesh construction.

tags	[int(2)]	[none]	Two tags are needed for interfaces (split-node) and for periodic boundaries.
------	----------	--------	--

kind	[char*6]	[none]	Type of boundary condition. The following are implemented: 'DIRNEU', 'ABSORB', 'PERIOD', 'LISFLT', 'DYNFLT'
------	----------	--------	--

NOTE : Most of the boundary conditions need additional data, given in a BC_kind input block of the BOUNDARY_CONDITIONS group immediately following the BC_DEF block.

NAME : BC_LSF

GROUP : BOUNDARY_CONDITION

PURPOSE: Linear slip fault, a displacement discontinuity interface where stress and slip are linearly related

SYNTAX : &BC_LSF Ktang | Ctang, Knorm | Cnorm /

Ktang	[dble]	[Inf]	Tangential stiffness
Ctang	[dble]	[0d0]	Tangential compliance
Knorm	[dble]	[Inf]	Normal stiffness
Cnorm	[dble]	[0d0]	Normal compliance

NOTE: For each component:

You can set K_or_C, but _not_both_

If C=0d0 or K=Inf then no discontinuity is allowed (transparent)

If K=0d0 the fault is free stress boundary

NAME : DIST_GAUSSIAN

GROUP : DISTRIBUTIONS_2D

PURPOSE: Bell shaped Gaussian 2D distribution

SYNTAX : &DIST_GAUSSIAN centered_at, length, offset, ampli, order /

centered_at	[dble(2)] [0,0]	Coordinates of the center point.
length	[dble(2)] [1]	Characteristic lengths on each axis.
offset	[dble] [0]	Background level.
ampli	[dble] [1]	Amplitude from background.
order	[dble] [1]	Exponent

NAME : DIST_GRADIENT

GROUP : DISTRIBUTIONS_2D

PURPOSE: Constant gradient 2D distribution.

SYNTAX : &DIST_GRADIENT file,valref ,grad,angle/

file	[name] [none]	Name of the file containing the coordinates of the points defining the reference line. It is an ASCII file with 2 columns per line: (1) X position (in m) and (2) Z position (in m)
valref	[dble] [none]	Value along the reference line
grad	[dble >0] [none]	Positive gradient (valref_units/meter)
angle	[dble] [none]	Angle (degrees) between the vertical down and the grad+ direction. Anticlockwise convention (grad+ points down if 0, right if 90)

NOTE : Be sure that your angle and ref-line are compatible. The code will abort if the ref-line is too short: some points of the domain cannot be projected to ref-line in the angle direction.

NAME : DIST_HETE1

GROUP : DISTRIBUTIONS_2D

PURPOSE: Linear interpolation of values from a regular 2D grid.

SYNTAX : &DIST_HETE1 file, col /

file [name] [none] Name of the file containing the definition
 of the regular grid and values at grid points.
 The format of this ASCII file is:
 Line 1 : ncol nx nz x0 z0 dx dz
 ncol = [int] number of data columns
 nx,nz = [2*int] number of nodes along x and z
 x0,z0 = [2*double] bottom-left corner
 dx,dz = [2*double] spacing along x and z
 Line 2 to nx*nz+1 : [ncol*double] values at grid points
 listed from left to right (x0 to x0+nx*dx),
 then from bottom to top (z0 to z0+nz*dx)

col [int] [1] Column of the file to be read

NOTE : The same file can contain values for (ncol) different properties,
 (e.g. rho, vp, vs) but each DIST_HETE1 block will read only one.

NOTE : Even if the original model domain has an irregular shape,
 the regular grid where input values are defined must be rectangular
 and large enough to contain the whole model domain.
 The regular grid possibly contains buffer areas with dummy values.
 These dummy values should be assigned carefully (not random nor zero)
 because SEM2D might use them during nearest-neighbor interpolation.

NAME : DIST_LINEAR
 GROUP : DISTRIBUTIONS_1D
 PURPOSE: Piecewise linear 1D distribution along X.
 SYNTAX : &DIST_LINEAR file,length /

file [name] [none] Name of the ASCII file containing
 the data to be interpolated, two columns per line:
 (1) X position, sorted in increasing order, and
 (2) data value at X

length [double] [0] Smoothing length for sliding average window
 No smoothing if length=0

NAME : DIST_ORDERO
 GROUP : DISTRIBUTIONS_2D
 PURPOSE: Blockwise constant 2D distribution.
 SYNTAX : &DIST_ORDERO xn, zn /
 x(1) ... x(xn-1)
 z(1) ... z(zn-1)
 v(1,1) ... v(xn,1)

```

      ...      ...      ...
v(1,zn) ... v(xn,zn)

```

```

xn      [int] [none] Number of zones along X
zn      [int] [none] Number of zones along Z
x       [dble(xn-1)] [none] Boundaries of X-zones: first zone  $X < x(1)$ ,
                        second zone  $x(1) < X < x(2)$ , ... , last zone  $x(xn-1) < X$ 
z       [dble(zn-1)] [none] Boundaries of Z-zones
v       [dble(xn,zn)] [none] Values inside each zone

```

```

NAME    : DIST_PWCONR
GROUP   : DISTRIBUTIONS_2D
PURPOSE: Piecewise constant radial (2D) distribution.
SYNTAX  : &DIST_PWCONR num, ref /
          r(1) ... .. r(num-1)
          v(1) v(2) ... v(num-1) v(num)

```

```

num      [int] [none] Number of radial zones (including outermost)
ref      [dble(2)] [(0d0,0d0)] Reference point: center of radial zones
r        [dble(num-1)] [none] External radius of zones:
          first zone  $R \leq r(1)$ ,
          second  $r(1) < R \leq r(2)$ , ...
          last  $r(num-1) < R$ 
v        [dble(num)] [none] Values inside each zone

```

```

NAME    : DIST_SPLINE
GROUP   : DISTRIBUTIONS_1D
PURPOSE: Spline interpolated 1D distribution along X.
SYNTAX  : &DIST_SPLINE file /

```

```

file     [name] [none] Name of the ASCII file containing
          the data to be interpolated, two columns per line:
          (1) X position, sorted in increasing order, and
          (2) data value at X

```

```

NAME    : GENERAL
PURPOSE: General parameters
SYNTAX  : &GENERAL iexec, ngll, fmax, title, verbose, itInfo /

```

```

iexec    [int] [0] Run level:
          0 = just check

```

```

1 = solve
ngll    [int] [9] Number of GLL nodes per edge on each spectral element
        ( polynomial order +1 ). Usually 5 to 9.
fmax    [dble] [0.d0] Maximum frequency to be well resolved. Mandatory.
        This is a target frequency, the code will check if it is
        compatible with the mesh and issue a warning if not. To
        improve the resolution for a given fmax you must increase ngll
        (but you will have to use shorter timesteps) or refine/redesign
        the mesh.
ndof    [int] [2] Number of degrees of freedom per node
        1 = SH waves, anti-plane
        2 = P-SV waves, in-plane
title   [word] [none] Title of the simulation
verbose [char(4)] ['1101'] Print progress information during each phase:
        verbose(1) = input phase
        verbose(2) = initialization phase
        verbose(3) = check phase
        verbose(4) = solver phase
        Example: '0001' is verbose only during solver.
itInfo  [int] [100] Frequency (in number of timesteps) for printing
        progress information during the solver phase.

```

```

NAME    : MAT_DAMAGE
GROUP   : MATERIALS
PURPOSE: Set material properties for the damage rheology of
        Lyakhovsky, Ben-Zion and Agnon (J. Geophys. Res. 1997)
        and Hamiel et al (Geophys. J. Int. 2004)
SYNTAX  : &MAT_DAMAGE cp,cs,rho,phi,alpha,Cd,R,e0,ep /

cp      [dble][0d0] P wave velocity
cs      [dble][0d0] S wave velocity
rho     [dble][0d0] density
phi     [dble][0d0] internal friction angle
alpha   [dble][0d0] initial value of damage variable
Cd      [dble][0d0] damage evolution coefficient
R       [dble][0d0] damage-related plasticity coefficient Cv
        normalized by the inverse of the intact shear modulus
e0      [dble(3)][0d0] initial total strain (11, 22 and 12)
ep      [dble(3)][0d0] initial plastic strain (11, 22 and 12)

```

```

NAME    : MAT_ELASTIC
GROUP   : MATERIALS
PURPOSE: Set material properties for a linear elastic medium

```

SYNTAX : For isotropic material:

&MAT_ELASTIC rho|rhoH, cp|cpH, cs|csH /
 followed by DIST_XXXX blocks, for arguments with suffix H,
 if present, in the same order as listed above.

For transverse anisotropy with vertical symmetry axis:

&MAT_ELASTIC rho, c11,c13,c33,c55 /

cp [dble][0d0] P wave velocity
 cs [dble][0d0] S wave velocity
 rho [dble][0d0] density
 c11,c13,c33,c55 [dble][0d0] anisotropic elastic moduli

NAME : MATERIAL

PURPOSE: Define the material type of a tagged domain

SYNTAX : &MATERIAL tag, kind /
 followed by one or two MAT_XXXX input blocks.

tag [int] [none] Number identifying a mesh domain
 kind [name(2)] ['ELAST',''] Material types:
 'ELAST', 'DMG', 'PLAST', 'KV'

NOTE : Some combinations of material kinds can be assigned to the same domain.
 Any material type can be combined with 'KV', for instance:

&MATERIAL tag=1, kind='ELAST','KV' /
 followed by a &MAT_ELAST block and a &MAT_KV block
 sets an elastic material with Kelvin-Voigt damping.

NAME : MAT_KV

GROUP : MATERIALS

PURPOSE: Set material properties for Kelvin-Voigt viscosity

Adds a damping term $C*v = K*eta*v$, where eta is a viscous time
 This produces attenuation with frequency-dependent quality factor
 $Q(f) = 1/(eta*2*pi*f)$

This is often useful as an artificial damping layer in fault zones
 to control high-frequency numerical artifacts, setting $eta=0.1*dt$
 and a layer thickness of 4 to 5 GLL nodes.

SYNTAX : &MAT_KV eta|etaH, ETAXDT /
 followed by a DIST_XXX input block if etaH is present.

eta [dble][0d0] Viscosity coefficient
 ETAXDT [log][T] If eta is given in units of dt (timestep)

NAME : MAT_PLASTIC
 GROUP : MATERIALS
 PURPOSE: Set material properties for elasto-plastic material
 with Mohr-Coulomb yield criterion
 and non-dilatant (null volumetric plastic strain)
 SYNTAX : &MAT_PLASTIC cp,cs,rho,phi,coh,e0,ep /

cp [dble][0d0] P wave velocity
 cs [dble][0d0] S wave velocity
 rho [dble][0d0] density
 phi [dble][0d0] internal friction angle
 coh [dble][0d0] cohesion
 e0 [dble(4)][0d0] initial total strain (11, 22 and 12)
 ep [dble(4)][0d0] initial plastic strain (11, 22 and 12)

NAME : MESH_CART
 GROUP : MESH_DEF
 PURPOSE: Rectangular box with structured mesh.
 SYNTAX : &MESH_CART xlim, zlim, nelem, FaultX /

xlim [dble(2)] [none] X limits of the box (min and max)
 zlim [dble(2)] [none] Z limits of the box (min and max)
 nelem [int(2)] [none] Number of elements along each direction
 FaultX [log] [F] Cut the box in the middle by a horizontal fault
 If FaultX=T, nelem(2) must be even

NOTE: the following tags are automatically assigned to the boundaries:

1	Bottom
2	Right
3	Top
4	Left
5	Fault, bottom side
6	Fault, top side

NAME : MESH_CART_DOMAIN
 PURPOSE: Define a subdomain within a structured meshed box.
 SYNTAX : &MESH_CART_DOMAIN tag,ex,eZ /

tag [int] [none] Tag number assigned to this domain.
 ex [int(2)] [none] First and last element along the X direction.
 eZ [int(2)] [none] First and last element along the Z direction.

NOTE : If you ignore this input block a single domain (tag=1) will span the whole box

NAME : MESH_EMC2

GROUP : MESH_DEF

PURPOSE: Imports a mesh from INRIA's EMC2 mesh generator in FTQ format

SYNTAX : &MESH_EMC2 file /

file [name] [none] Name of the FTQ file, including suffix

NAME : MESH_DEF

PURPOSE: Selects a method to import/generate a mesh.

SYNTAX : &MESH_DEF method /

method [name] [none] 'CARTESIAN', 'LAYERED' or 'EMC2'

The &MESH_DEF input block must be followed by a
&MESH_method input block

NAME : MESH_LAYERED

GROUP : MESH_DEF

PURPOSE: Structured mesh for layered medium
with surface and interface topography.

SYNTAX : &MESH_LAYERED xlim,zmin,nx,file,nlayer /

xlim [dble(2)] [none] X limits of the box (min and max)

zmin [dble] [none] bottom Z limit of the box

nx [int] [none] Number of elements along X direction

file [string] [''] Only for flat layers,
name of ASCII file containing layer parameters,
one line per layer, listed from top to bottom,
3 columns per line:
(1) vertical position of top boundary,
(2) number of elements along Z direction
(3) material tag

nlayer [int] [none] Number of layers

If a file name is not given the layer parameters
must be given immediately after the &MESH_LAYERED block
by nlayer &MESH_LAYER input blocks,
one for each layer, listed from top to bottom.

NOTE: the following tags are automatically assigned to the boundaries:

1	Bottom
2	Right
3	Top
4	Left
5	Fault, bottom side
6	Fault, top side

NAME : MESH_LAYER
GROUP : MESH_DEF
PURPOSE: Define mesh parameters for one layer
SYNTAX : &MESH_LAYER nz, ztop|ztopH, tag /
followed by a DIST_XXXX block if ztopH is set

nz [int] [none] Number of elements in layer along Z direction
ztop [dble] [none] Only for layers with flat top surface:
vertical position of top boundary
ztopH [string] ['none'] Only for layers with irregular top boundary:
name of distribution, 'LINEAR', 'SPLINE' or any other
1D distribution available through a DIST_XXXX block.
tag [int] [none] Material tag
If not given, a tag is automatically assigned to the layer,
sequentially numbered from top to bottom (top layer tag =1)

NAME : SNAP_DEF
GROUP : SNAPSHOT_OUTPUTS
PURPOSE: Set preferences for exporting snapshots
SYNTAX : &SNAP_DEF it1, itd, fields, components, bin, visual3, avs, ps, gmt /
Followed by a &POSTSCRIPT block if ps=T.

it1 [int] [0] Time step of first snapshot output
itd [int] [100] Number of timesteps between snapshots
fields [char*] ['V'] fields to export in snapshots (the prefix of the
output file names is given in parenthesis):
'D' displacements (dx,dy,dz,da)
'V' velocity (vx,vy,vz,va)
'A' acceleration (ax,ay,az,aa)
'E' strain (e11,e22,e12,e23,e13)
'S' stress (s11,s22,s12,s33,e13,e23)
components [char*] ['ya'] components for PostScript outputs:
in P-SV: 'x','z' and/or 'a' (amplitude). 'y' is ignored
in SH: 'y' only. Other values are ignored.
ps [log] [T] PostScript (see &SNAP_PS input block)
gmt [log] [F] output triangulation file grid_sem2d.gmt

to be used in "pscontour -T" of the General Mapping Tool (GMT)

avs	[log] [F]	AVS (only for D,V and A fields)
visual3	[log] [F]	Visual3 (only for D,V and A fields)
bin	[log] [T]	binary

NOTE : E and S fields are exported only as binary.

NAME : SNAP_PS
GROUP : SNAPSHOT_OUTPUTS
PURPOSE: Preferences for PostScript snapshots
SYNTAX : &SNAP_PS vectors, mesh, background, color,
isubsamp, boundaries, symbols, numbers, legend,
ScaleField, Interpol, DisplayPts /

vectors	[log] [F]	Plots a vectorial field with arrows
mesh	[log] [F]	Plots the mesh on background
background	[char] ['']	Filled background, only for vector plots: '' none 'P' P-velocity model 'S' S-velocity model 'T' domains
isubsamp	[int] [3]	Subsampling of the GLL nodes for the output of velocity model. The default samples every 3 GLL points.
boundaries	[log] [T]	Colors every tagged boundary
symbols	[log] [T]	Plots symbols for sources and receivers
numbers	[log] [F]	Plots the element numbers
legend	[log] [T]	Writes legends
color	[log] [T]	Color output
ScaleField	[dble] [0d0]	Fixed amplitude scale (saturation), convenient for comparing snapshots and making movies. The default scales each snapshot by its maximum amplitude
Interpol	[log] [F]	Interpolate field on a regular subgrid inside each element
DisplayPts	[log] [3]	Size of interpolation subgrid inside each element is DisplayPts*DisplayPts. The default plots at vertices, mid-edges and element center.

NAME : REC_LINE
PURPOSE: Defines a line of receivers
SYNTAX : &REC_LINE number,isamp,field,first,last,file,AtNode,irepr /

number	[int] [0]	Number of stations in the line
--------	-----------	--------------------------------

isamp [int] [1] Sampling stride (in number of timesteps). Note that
 for stability reasons the timestep can be very small.
 field [char] ['V'] The field in the seismogram:
 'D' displacement
 'V' velocity
 'A' acceleration
 first [dble(2)] Receivers can be located along a line,
 this is the position (x,z) of the first receiver
 last [dble(2)] Position (x,z) of the last receiver,
 other receivers will be located with regular spacing
 between First and Last.
 file [name] ['none'] Station positions can instead be read
 from an ASCII file, with 2 columns per line:
 (1) X position (in m) and
 (2) Z position (in m)
 AtNode [log] [T] Relocate the stations at the nearest GLL node
 irepr [char] ['D'] Abscissa for the seismic multitrace plot:
 'X' Horizontal position
 'Z' Depth
 'D' Distance to the first station

NOTE : to locate receivers at the free surface set their vertical position
 above the free surface and AtNode=T

NAME : SRC_FORCE
 GROUP : SOURCE MECHANISM
 PURPOSE: Point force source
 SYNTAX : &SRC_FORCE angle /

angle [dble] [0d0] For P-SV, the angle of the applied force,
 in degrees, counterclockwise from Z-UP, e.g.:
 (90 points left, 180 points down)
 For SH, angle is ignored.

NAME : SRC_DEF
 PURPOSE: Define the sources.
 SYNTAX : &SRC_DEF stf, mechanism, coord, file /
 followed by one SOURCE MECHANISM block (SRC_XXXX)
 and one SOURCE TIME FUNCTION block (STF_XXXX)

stf [name] [none] Name of the source time function:
 'RICKER', 'TAB', 'HARMONIC' or 'USER'
 mechanism [name] [none] Name of the source mechanism:

'FORCE', 'EXPLOSION', 'DOUBLE_COUPLE', 'MOMENT' or 'WAVE'
 coord [dble(2)] [huge] Location (x,z) of the source (m).
 file [string] ['none'] Station coordinates and delay times can
 be read from an ASCII file, with 3 columns per line:
 (1) X position (in m),
 (2) Z position (in m) and
 (3) time delay (in seconds)

NAME : SRC_DOUBLE_COUPLE
 GROUP : SOURCE MECHANISM
 PURPOSE: Define a double-couple source
 SYNTAX : &SRC_DOUBLE_COUPLE dip /

dip [dble] [90] Dip angle, in degrees, clockwise
 from the positive X direction

NOTE : Sign convention: if the source amplitude is positive the right block
 moves up (positive Z direction) in PSV and forward (positive Y
 direction) in SH.

NAME : SRC_MOMENT
 GROUP : SOURCE
 PURPOSE: Define a moment tensor source
 SYNTAX : &SRC_MOMENT Mxx,Mxz,Mzx,Mzz , Myx,Myz /

Mxx,Mxz,Mzx,Mzz [dble] [0] Tensor components for PSV
 Myx,Myz [dble] [0] Tensor components for SH

NAME : SRC_WAVE
 GROUP : SOURCE MECHANISM
 PURPOSE: Incident plane wave through the absorbing boundaries
 SYNTAX : &SRC_WAVE angle, phase /

angle [dble] [0d0] Incidence angle in degrees within [-180,180]
 counterclockwise from the positive Z (up) direction
 to the wave vector direction:
 Exs: incidence from below if angle in]-90,90[
 normal incidence from below if angle=0
 from bottom right if angle=+45
 from bottom left if angle=-45
 phase [char] ['S'] 'S' or 'P' (only needed in PSV, ignored in SH)

NOTE : Incident waves enter through the absorbing boundaries.
 An incident wave is applied on every absorbing boundary
 unless "let_wave = F" in the respective BC_ABSO block.
 Incident waves are not implemented for "Stacey" absorbing boundaries.

NAME : STF_HARMONIC
 GROUP : SOURCE TIME FUNCTIONS
 PURPOSE: Harmonic source time function $f(t) = \text{ampli} \cdot \sin(2\pi \cdot t \cdot f0)$
 SYNTAX : &STF_HARMONIC ampli, f0 /

ampli [dble] [0d0] Amplitude
 f0 [dble] [0d0] Frequency

NAME : STF_RICKER
 GROUP : SOURCE TIME FUNCTIONS
 PURPOSE: The Ricker wavelet is the second derivative of a gaussian.
 SYNTAX : &STF_RICKER ampli, f0, onset /

ampli [real] [1.] Signed amplitude of the central peak
 f0 [real >0] [0] Fundamental frequency (Hz).
 distribution: it has a peak at f0 and an exponential
 decay at high frequency. The cut-off high frequency is usually
 taken as $f_{\text{max}} = 2.5 \times f0$.
 onset [real >1/f0] [0] Delay time (secs) with respect to the peak value.

NOTE : The spectrum has a peak at f0 and decays exponentially at high
 frequencies. Beyond $2.5 \cdot f0$ there is little energy, this is a
 recommended value for f_{max} .
 NOTE : onset > 1/f0 is needed to avoid a strong jump at $t=0$, which can cause
 numerical oscillations. Ignore if using incident waves.

NAME : STF_TAB
 GROUP : SOURCE TIME FUNCTIONS
 PURPOSE: Source time function spline-interpolated from values in a file
 SYNTAX : &STF_TAB file /

file [string] ['stf.tab'] ASCII file containing the source time function,
 two columns per line:
 (1) time
 (2) value

NOTE : time can be irregularly sampled
 NOTE : assumes value=0 before min(time) and after max(time)

NAME : STF_USER
 GROUP : SOURCE TIME FUNCTIONS
 PURPOSE: A template for user-supplied source time function.
 File stf_user.f90 must be modified by the user to fit
 special needs.
 SYNTAX : &STF_USER ampli, onset, par1, par2, ipar1, ipar2 /

ampli [db] [1.] Amplitude
 onset [db] [0] Delay time (secs)
 par1 [db] [0] Example parameter
 par1 [db] [0] Example parameter
 par1 [db] [0] Example parameter
 par1 [db] [0] Example parameter
 par1 [db] [0] Example parameter

NAME : TIME
 PURPOSE: Defines time integration scheme
 SYNTAX : &TIME kind, NbSteps, Dt, Courant, TotalTime /

kind [db] ['leapfrog'] Type of scheme:
 'newmark' Newmark-alpha
 'leapfrog' Central difference
 'symp_PV' Position Verlet
 'symp_PFR' Position Forest-Ruth (4th order)
 'symp_PEFRL' Extended PFR (4th order)
 NbSteps [db] [none] Number of timesteps to be performed
 Dt [db] [none] Amplitude of the timestep
 Courant [db] [0.5d0] Courant stability number: the maximum ratio
 Dt*wave_velocity/dx where dx is the inter-GLL node distance
 Typically <= 0.5
 TotalTime[db] [none] Total duration (in seconds) of simulation

NOTE : Not all combinations of parameters need to be set at once.
 You can set the total duration (secs) or the number of steps.
 You can set the timestep or the Courant number (or use default).

NOTE : The leap-frog scheme is equivalent to the Newmark scheme
 with alpha=1, beta=0, gamma=1/2. However it is faster and requires
 less memory. Dynamic faults require this scheme.

NAME : TIME_NEWMARK

PURPOSE: Parameters of the explicit Newmark or HHT-alpha time scheme

SYNTAX : &TIME_NEWMARK alpha|gamma, beta|rho /

beta [dble] [0.5d0] The algorithm is fully explicit if beta=0
otherwise it is a single-predictor-corrector scheme
gamma [dble] [1.d0]
alpha [dble] [0.5d0] parameter in the Hilber-Hughes-Taylor method
Actually, here $\alpha = 1 + \text{their original definition of } \alpha$
rho [dble] [1.d0] high frequencies are damped by a factor $\geq \rho$.
The default is non-dissipative. Dissipation is limited however
to $\rho \geq 0.5$. For max dissipation you should work close to
the stability limit (Courant around 0.56 for $\rho=0.5$).

NOTE: For second order schemes only two parameters need to be set:
(alpha OR gamma) AND (beta OR rho)

NOTE: Dissipative schemes ($0.5 \leq \rho < 1$) are slightly more unstable,
i.e. they require slightly smaller Courant number
(0.56 for $\rho=0.5$, compared to 0.6 for $\rho=1$)

3.6 Verifying the settings and running a simulation

Once the code has been successfully compiled, the simulation can be started by typing `sem2dsolve` from your working directory, which contains the file `Par.inp`. The computations can be run in background and the screen output saved in a file (e.g. `info`) by typing `sem2dsolve > info &`.

A typical screen output of SEM2D, corresponding to the first example, is shown on the following pages. The parameters of the simulation and some verification information are reported there in a self-explanatory form. You are advised to do a first run with `iexec=0` in the **GENERAL** input block and check all these informations prior to the real simulation. You should always verify the following:

- **Stability:** the CFL stability number should be smaller than $0.55 \sim 0.60$ for second order time schemes. This number is defined at each computational node as

$$\text{CFL} = c_P \Delta t / \Delta x$$

where Δt is the timestep, c_P the P-wave velocity and Δx the local grid spacing. Note that Δx is usually much smaller than the element size h ($\approx \text{Ng11}^2$ times smaller) because SEM internally subdivides each element onto a non-regular grid of $\text{Ng11} \times \text{Ng11}$ nodes clustered near the element edges (Gauss-Lobatto-Legendre nodes). If the computation is unstable, the maximum displacement, printed every `ItInfo` time steps, increases exponentially with time. Stability can be controlled by decreasing `Dt` or `Courant` in `Par.inp`.

- **Resolution:** the number of nodes per shortest wavelength λ_{min} should be larger than $4.5 \sim 5$. The minimum wavelength is defined as

$$\lambda_{min} = \min(c_S) / f_{max}$$

where c_S is the S-wave velocity and f_{max} the highest frequency you would like to resolve, e.g. the maximum frequency at which the source spectrum has significant power (for a Ricker wavelet $f_{max} = 2.5 \times f_0$). For an element of size h and polynomial order $p = \text{Ng11} - 1$, the number of nodes per wavelength G is

$$G = \frac{p \lambda_{min}}{h}.$$

Typical symptoms of poor resolution are ringing and dispersion of the higher frequencies. However, in heterogeneous media these spurious effects might be hard to distinguish from a physically complex wavefield, so mesh resolution must be checked beforehand. If resolution is too low the mesh might be refined by increasing `Ng11` in `Par.inp` (p -refinement) or by generating a denser mesh (h -refinement). If you were using EMC2 as a mesh generator, the script `PRE/href.csh` can be useful for h -refinement.

- **Cost:** the total CPU time and memory required for the simulation are as much as you can afford. Estimates of total CPU time are printed at the end of check mode. Details about memory usage can be found in `MemoryInfo_sem2d.txt`.

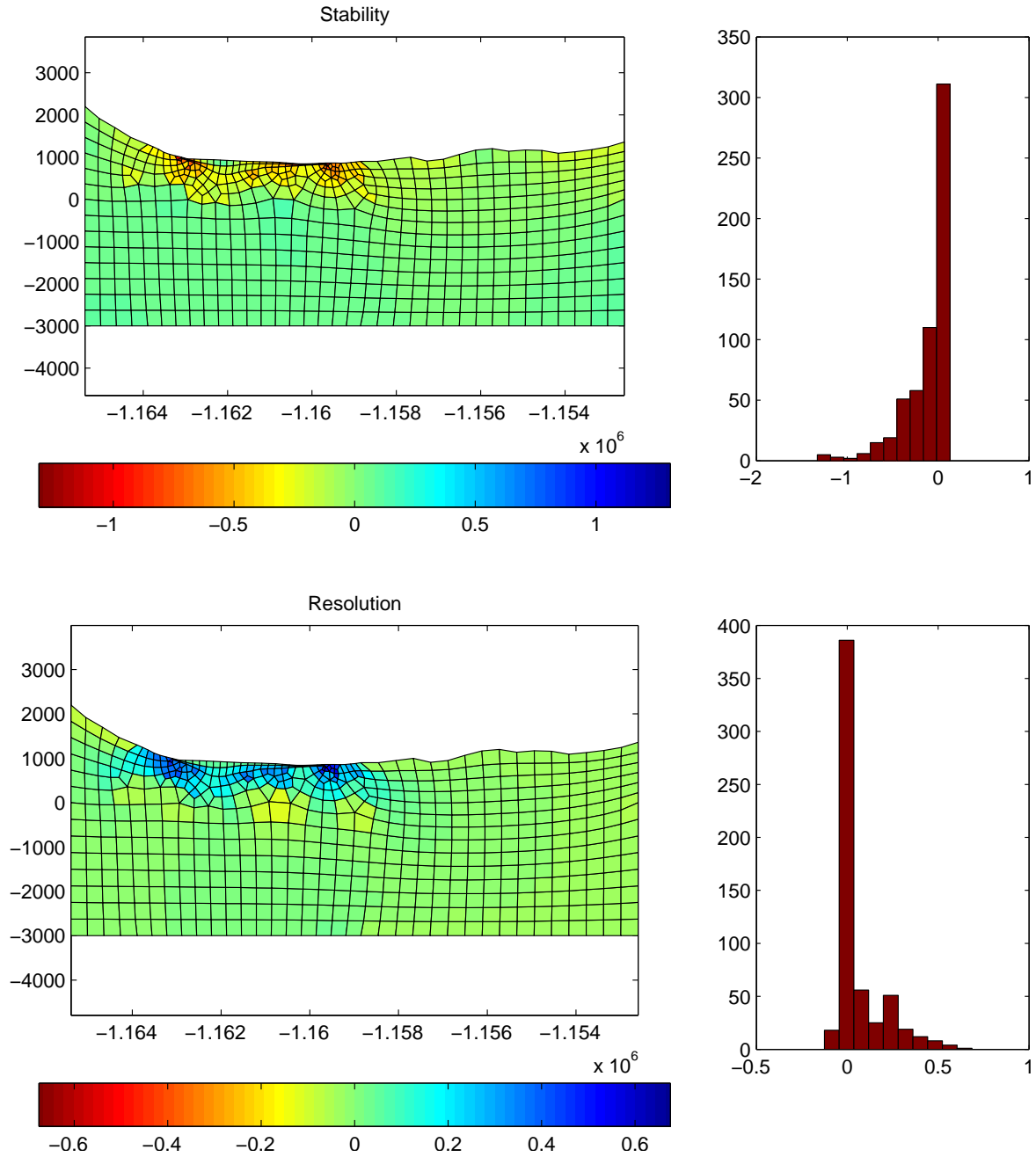


Figure 3.3: Checking the quality of a mesh with `PRE/ViewMeshQuality.m` for the example in `EXAMPLES/UsingEMC2/`. The balance of the stability and resolution properties of the mesh can be analyzed: logarithmic stability index (top) and logarithmic resolution index (bottom). Histograms of these indices (in number of elements) are shown on the right.

The quality of the mesh can be inspected with the Matlab script `PRE/ViewMeshQuality.m` which produces plots like in Figure 3.3. The proper balance of the mesh with respect to the following two criteria can be analyzed:

- **Stability criterion**, related to the largest stable timestep. On each element we define a stability index as the logarithm of $\min(\Delta x/c_P)$ normalized by its median value over the whole mesh. Red elements (small stability index) are relatively unstable and require small timesteps Δt . Because Δt is constant over the whole mesh and the computational cost is inversely proportional to Δt these red elements penalize the computational efficiency. The mesh should be redesigned to increase their size, as much as possible, while keeping them small enough to resolve the shortest wavelength (see next).
- **Resolution criterion**, related to the number of nodes per shortest wavelength. On each element we define a resolution index as the logarithm of $\min(c_S/h)$ normalized by its median value over the whole mesh. Red elements (small resolution index) have relatively poor resolution, in their vicinity the maximum frequency resolvable by the mesh is limited. The mesh should be redesigned to decrease their size, as much as possible. Conversely, elements with very high resolution index (blue) are smaller than required and might increase the computational cost.

To minimize the CPU and memory cost of a simulation an ideal mesh design should minimize the spread of the two indices above, by aiming at a ratio of element size to wave velocity, h/c , as uniform as possible across the whole mesh. However, in some cases a poorly balanced mesh is inevitable: in the example of Figure 3.3 the worst elements are near the edges of the sedimentary basin, at a sharp velocity contrast. Small element sizes on the rock side are inherited from the sediment mesh.³

Similar information is plotted by `gv Stability_sem2d.ps` and `gv Resolution_sem2d.ps`. The indices in these files are however not logarithmic and are not normalized by the median.

³In future releases of SEM2DPACK this penalty on computational efficiency will be reduced by non-conformal meshing with mortar elements, by timestep subcycling or by implicit/explicit timestep partitioning.

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```

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-----
Program S P E C F E M : start
-----

Date : 06 - 03 - 2008                           Time : 18:03:23
-----

*****
*           Input phase                         *
*****

General Parameters
=====
Execution mode . . . . . (iexec) = solve
Number of nodes per edge . . . . . (ngll) = 6
Number of d.o.f per node . . . . . (ndof) = 1
Highest frequency to be resolved . . . . . (fmax) = 1.250E+00
Print progress information during
    input phase . . . . . (verbose(1)) = T
    initialization phase . . . . . (verbose(2)) = T
    checking phase . . . . . (verbose(3)) = T
    solver phase . . . . . (verbose(4)) = T
Frequency for solver progress information . (itInfo) = 1000

Mesh Generation
=====
Method . . . . . (method) = CARTESIAN
Minimum X . . . . . (xlim(1)) = 0.000E+00
Maximum X . . . . . (xlim(2)) = 3.000E+01
Minimum Z . . . . . (zlim(1)) = 0.000E+00
Maximum Z . . . . . (zlim(2)) = 3.000E+01
Number of elements along X . . . . . (nelem(1)) = 60
Number of elements along Z . . . . . (nelem(2)) = 60
Cut by horizontal fault . . . . . (faultx) = F

Time integration
=====
Scheme . . . . . (kind) = leapfrog
Number of time steps . . . . . (NbSteps) = will be set later
Time step increment . . . . . (Dt) = will be set later
Courant number . . . . . (Courant) = 0.30
Total simulation duration . (TotalTime) = 35.000E+00

Material Properties
=====
Number of materials . . . . . = 1

Material index . . . . . (tag) = 1
Material type . . . . . (kind) = Elastic
P-wave velocity . . . . . (cp) = 1.732E+00
S-wave velocity . . . . . (cs) = 1.000E+00
Mass density . . . . . (rho) = 1.000E+00

```

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```

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Poisson's ratio . . . . . = 250.021E-03
First Lamé parameter Lambda . . . . . = 1.000E+00
Second Lamé parameter Mu . . . . . = 1.000E+00
Bulk modulus K . . . . . = 1.667E+00
Young's modulus E . . . . . = 2.500E+00

Boundary Conditions
=====

Boundary tag . . . . . (tag) = 2
Boundary condition . . . . . (kind) = ABSORB
Type of absorbing boundary . . . (stacey) = Clayton-Engquist
Allow incident wave . . . . . (let_wave) = T

Boundary tag . . . . . (tag) = 3
Boundary condition . . . . . (kind) = ABSORB
Type of absorbing boundary . . . (stacey) = Clayton-Engquist
Allow incident wave . . . . . (let_wave) = T

Sources
=====
X-position (meters) . . . . (coord(1)) = 0.000E+00
Y-position (meters) . . . . (coord(2)) = 0.000E+00
Source time function . . . . . = Ricker
Fundamental frequency (Hz) . . . . (f0) = 500.000E-03
Time delay (s) . . . . . (onset) = 3.000E+00
Multiplying factor . . . . . (ampli) = 250.000E-03
Source Type . . . . . = Collocated Force
If P-SV: counterclockwise angle / up . = 0.00

Receivers
=====
Number of receivers . . . . . (number) = 7
Subsampling for seismograms recording . . . (isamp) = 1
Field recorded . . . . . (field) = D
Axis of the seismogram plot . . . . . (irepr) = D

Snapshot Outputs
=====
Timestep of first snapshot output . . . . . (it1) = 0
Number of timesteps between snapshots . . . . (itd) = 100000
Save results in PS file or not . . . . . (ps) = F
Save grid triangulation for GMT . . . . . (gmt) = F
Save results in AVS file or not . . . . . (avs) = F
Save results in Visual3 file or not . . . . . (visual3) = F
Save results in binary file or not . . . . . (bin) = F
Selected fields :
    Displacement . . . . . = F
    Velocity . . . . . = T
    Acceleration . . . . . = F
    Strain . . . . . = F
    Stress . . . . . = F
Selected components for PostScript snapshots :
    X . . . . . = F
    Y . . . . . = T
    Z . . . . . = F
    Amplitude . . . . . = F

*****
* Initialization phase *
*****

```

info

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```

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Defining the FEM mesh ..... [OK]
Saving node coordinates in file MeshNodesCoord_sem2d.tab ..... [OK]
Saving element connectivity in file ElmtNodes_sem2d.tab ..... [OK]

S p e c t r a l   e l e m e n t s   g r i d
=====

Numbering GLL points ..... [OK]
Total number of GLL points. . . . . = 90601

Saving element/node table in binary file ibool_sem2d.dat ..... [OK]
Defining nodes coordinates ..... [OK]

Saving the grid coordinates (coord) in a text file ..... [OK]
Saving the grid coordinates (coord) in a binary file ..... [OK]

M a t e r i a l   p r o p e r t i e s
=====

Translating input model ..... [OK]
Exporting model ..... [OK]

M e s h   p r o p e r t i e s
=====

Checking mesh ..... [OK]
Max mesh size = 142.616E-03
Min mesh size = 58.736E-03
Ratio max/min = 2.428E+00

RESOLUTION: nodes per min wavelength = 8.000E+00
              for maximum frequency   = 1.250E+00 Hz
              minimum wavelength      = 1.600E+00 m

Dump PostScript Resolution_sem2d.ps ..... [OK]
Dump PostScript Stability_sem2d.ps ..... [OK]

T i m e   s o l v e r
=====

Time step (secs)      = 17.621E-03
Number of time steps  = 1987
Total duration (secs) = 35.013E+00
Courant number        = 300.000E-03

STABILITY: CFL number = 300.000E-03

Defining work arrays for elasticity ..... [OK]
Initializing kinematic fields ..... [OK]
  Max displ = 0.000E+00
  Max veloc = 0.000E+00

Building the mass matrix ..... [OK]
Defining boundary conditions ..... [OK]
Initializing receivers ...

R e c e i v e r s
=====

Receivers have been relocated to the nearest GLL node

Receiver  x-requested  z-requested  x-obtained  z-obtained  distance
-----
1         0.000E+00    0.000E+00    0.000E+00  0.000E+00    0.000E+00
2         5.000E+00    0.000E+00    5.000E+00  0.000E+00    0.000E+00
3        10.000E+00    0.000E+00   10.000E+00  0.000E+00    0.000E+00
4        15.000E+00    0.000E+00   15.000E+00  0.000E+00    0.000E+00

```

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```

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5         20.000E+00    0.000E+00    20.000E+00  0.000E+00    0.000E+00
6         25.000E+00    0.000E+00    25.000E+00  0.000E+00    0.000E+00
7         30.000E+00    0.000E+00    30.000E+00  0.000E+00    0.000E+00

Maximum distance between asked and real = 0.000E+00

Sampling rate (Hz)      = 56.751E+00
Sampling timestep (secs) = 17.621E-03
Total number of samples = 1988
Number of receivers     = 7

... [OK]
  Initializing sources ...

S o u r c e s
=====

Sources have been relocated to the nearest GLL node

Source  x-requested  z-requested  x-obtained  z-obtained  distance
-----
1         0.000E+00    0.000E+00    0.000E+00  0.000E+00    0.000E+00

Maximum distance between requested and real = 0.000E+00
... [OK]
Timestep #          0  t = 0.000E+00  vmax = 0.000E+00  dmax = 0.000E+00

*****
*           S o l v e r   p h a s e           *
*****

--- CPU TIME ESTIMATES (in seconds) :
CPU time for initialization . . 806.877E-03
CPU time per timestep . . . . 18.997E-03
Total solver CPU time . . . . 37.747E+00
(mins) . . . . 629.118E-03
(hours) . . . . 10.485E-03

Timestep #        1000  t = 17.621E+00  vmax = 91.661E-03  dmax = 28.653E-03

--- CPU TIME INFORMATION (in seconds) :
CPU time for initialization . . 806.877E-03
CPU time per timestep . . . . 20.395E-03
Total solver CPU time . . . . 40.524E+00
(mins) . . . . 675.397E-03
(hours) . . . . 11.257E-03

Storing sismos data (SEP format) ...

-----
Program  S P E C F E M :  end
-----

Test SH

-----
Date : 06 - 03 - 2008                               Time : 18:04:11
-----

```

info

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3.7 Outputs, their visualization and manipulation

In addition to the screen output described above, `sem2dsolve` generates different files and scripts that allow the user to control the parameters of the simulation and to display the results. All the outputs files follow the naming convention `SomeName_sem2d.xxx`, where `xxx` is one of the following extensions: `tab` for ASCII data files, `txt` for other text files, `dat` for binary data files, etc. This makes it easy to clean a working directory with a single command like `rm -f *_sem2d*`.

3.7.1 Spectral element grid

As explained in the previous section, `sem2dsolve` generates two PostScript files for mesh quality checking purposes: `Stability_sem2d.ps` and `Resolution_sem2d.ps`. The relevant information is contained in the files `Stability_sem2d.tab` and `Resolution_sem2d.tab` and can also be inspected with the Matlab script `PRE/ViewMeshQuality.m`.

3.7.2 Source time function

`sem2dsolve` generates a file called `SourcesTime_sem2d.tab` containing the source time function sampled at the same rate as the receivers. It is important to verify that the spectrum of the source has little power at those high frequencies that are not well resolved by the mesh (those that correspond to less than 5 nodes per wavelength). If this is not the case you must be very cautious in the interpretation of the seismograms in the high frequency range, or low-pass filter the results.

3.7.3 Snapshots

`sem2dsolve` generates snapshots at a constant interval defined, in number of solver timesteps, by the input parameter `ItSnapshots`. An example is shown in Figure 3.4. Requested fields are exported in binary data files called `xx_XXX_sem2d.dat`, where `xx` is the field code defined in the documentation of the `PLOTS` input block and `XXX` is the 3-digit snapshot number. The user is encouraged to inspect the Matlab script `POST/sample_snapshots.m` and function `POST/sem2d_snapshot_read.m` to find more about the data formats and their manipulation.

If requested PostScript files `xx_XXX_sem2d.ps` are also dumped. A movie file `movie.gif` can be generated by the script `POST/movie.csh` and displayed by `xanim movie.gif`.

3.7.4 Seismograms

The seismograms are stored using the SEP format, a simple binary block of single precision floats. The components of the vector field (velocity by default) are stored in separate files `U*_sem2d.dat`, where `*` is `x` or `z` in P-SV and `y` in SH. The seismograms header is in the file

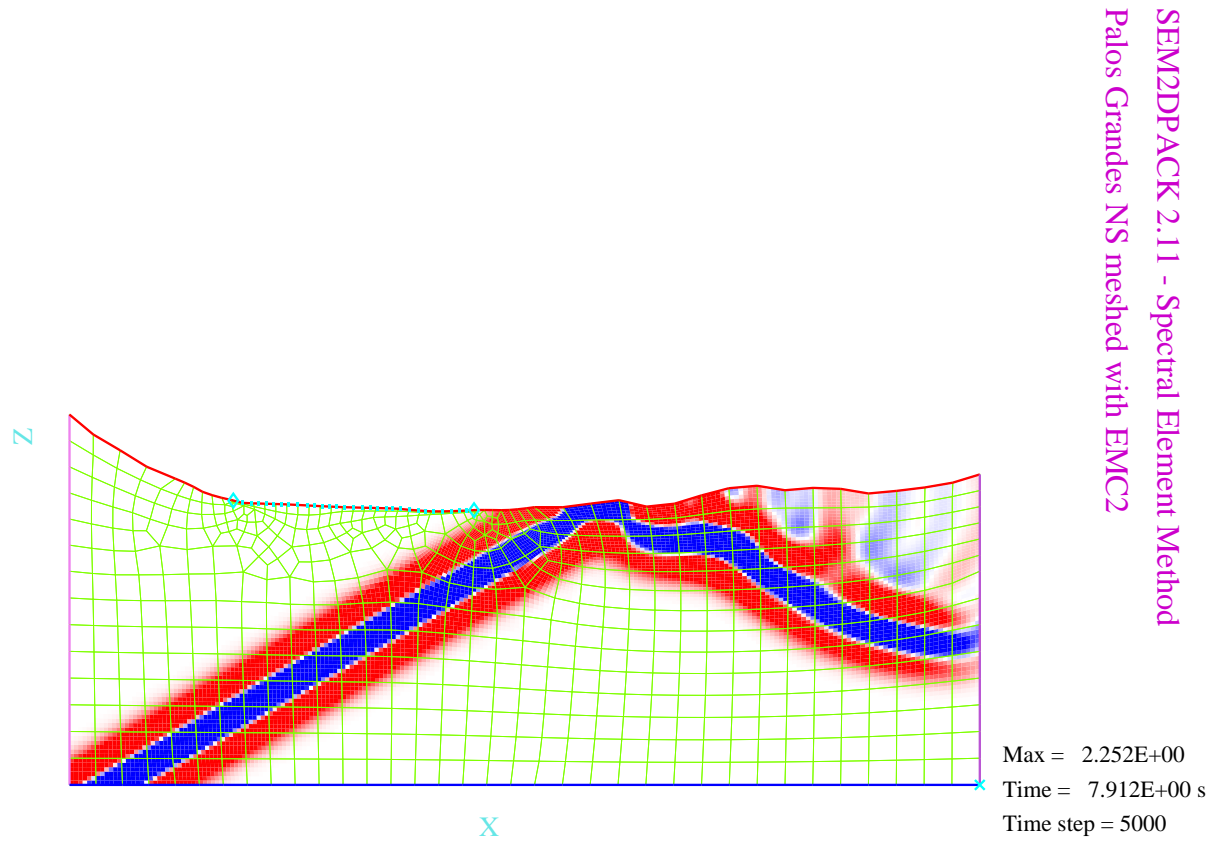


Figure 3.4: Sample snapshot from `EXAMPLES/UsingEMC2/` : an obliquely incident SH plane wave impinging on a sedimentary basin. The unstructured mesh of spectral elements is plotted on background.

`SeisHeader_sem2d.hdr`. Its second line contains the sampling timestep `DT`, the number of samples `NSAMP` and the number of stations `NSTA`. The stations coordinates, `XSTA` and `ZSTA`, are listed from the third line to the end of file. With this notations, `U*_sem2d.dat` contains a $\text{NSAMP} \times \text{NSTA}$ single precision matrix.

You can view the seismograms using any tool that is able to read the SEP format, which is the case of almost all the softwares able to deal with seismic data. `sem2dsolve` generates scripts for the XSU-Seismic Unix visualization tool⁴:

- `Xline_sem2d.csh` displays all seismograms together on screen
- `PSline_sem2d.csh` plots all seismograms on PostScript files `U*Poly_sem2d.ps`
- `Xtrace_sem2d.csh` prompts the user for a trace number (between 1 and `NSTA`) and then displays this particular trace on screen
- `PStrace_sem2d.csh` does the same as `Xtrace`, but exports the traces as PostScript files `U*TraceXXX_sem2d.ps` where `XXX` is the number of that particular trace

The program `post_seis.exe` performs similar basic manipulation and plotting (through `gnuplot`) of the seismograms. Its interactive menu is self-explanatory. It is usually called inside a script, as in `POST/seis_b2a.csh` (converts all seismograms to ASCII) or `POST/seis_plot.csh` (plots all seismograms together, an example is shown in Figure 3.5).

The script `POST/sample_seis.m` shows how to manipulate and plot seismogram data in Matlab. It uses the functions `POST/sem2d_read_seis.m` and `POST/plot_seis.m`.

3.7.5 Fault outputs

Fault data from dynamic rupture simulations is stored in `F1tXX_sem2d.dat`, where `XX` is the boundary tag of the first side of the fault, `tags(1)` of the `BC_SWFFLT` input block. Every `DELT` seconds (`NSAMP` total output times) `NDAT` lines with `NPTS` columns, one per fault node, are written⁵. Stress fields are relative to their initial values, which are contained in the first `NDAT` lines. The header file `F1tXX_sem2d.hdr` contains the information needed to read the data file. Its format, line by line, is:

1. `NPTS NDAT NSAMP DELT` (name of parameters)
2. Value of parameters above
3. Name of data fields, separated by ":"
4. `XPTS ZPTS` (name of coordinate axis)

⁴Seismic Unix is freely available from the Colorado School of Mines at <http://timna.mines.edu/cwpcodes>

⁵The actual number of columns is `NPTS + 2`: Fortran adds a one-word tag at the front and end of each record.

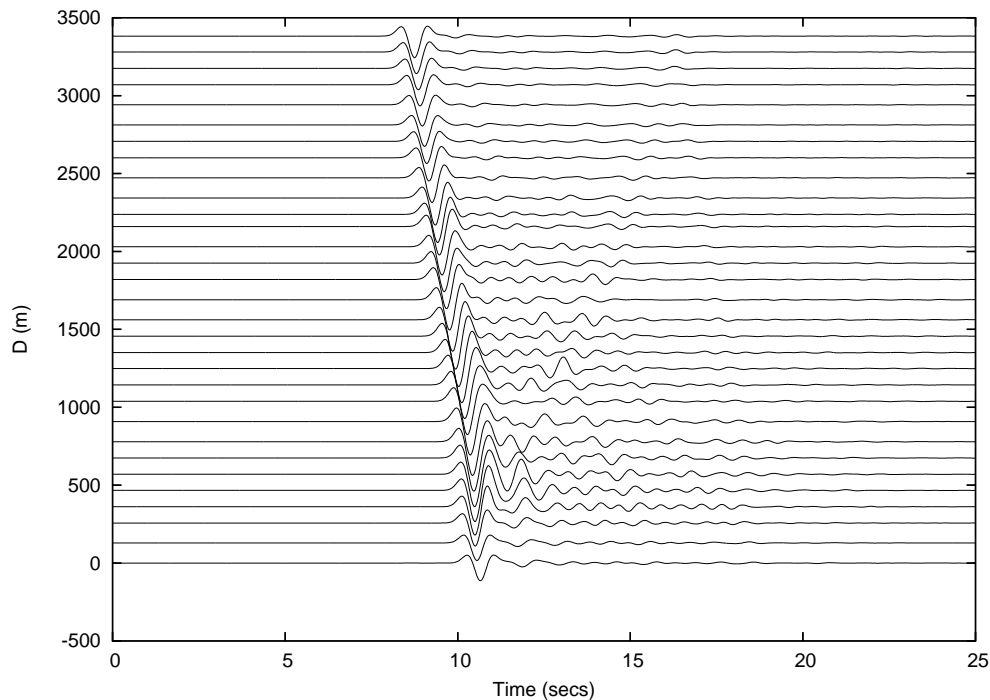


Figure 3.5: Sample seismograms from `EXAMPLES/UsingEMC2/` generated with `POST/seis_plot.csh`.

5. from here to the end of file, a two-column table of coordinates of the output fault nodes

The script `FltXX_sem2d.csh` shows how to extract ASCII time series of different fields at given locations on the fault, using Seismic Unix tools.

The program `post_fault.exe` performs basic manipulations of the fault data, including conversion to an ASCII file readable by `gnuplot`. Its interactive menu is self-explanatory.

The script `POST/sample_fault.m` and function `POST/sem2d_read_fault.m` show how to manipulate and plot fault data in Matlab.

3.7.6 Matlab utilities

A range of functions and sample scripts for Matlab are available to read, manipulate and plot output data. Add the directory `POST/` to your Matlab path (`addpath`). For an overview of existing utilities, type `help POST`:

```
XCORRSHIFT Cross-correlation time-delay measurement
SPECSHIFT signal time shift by non-integer lag via spectral domain
SPECFILTER zero-phase Butterworth filter via spectral domain
PLOT_SEIS plots multiple seismogram traces, with a vertical offset
SAMPLE_SNAPSHOTS example of visualization of snapshot outputs from SEM2DPACK
```

SAMPLE_FAULT example of visualization of fault output from SEM2DPACK
SAMPLE_SEIS example of visualization of seismogram output from SEM2DPACK
SEM2D_SNAPSHOT_GUI interactive plot of snapshot outputs from SEM2DPACK
PLOT_FRONTS space-time plot of rupture front and process zone tail
SEM2D_SNAPSHOT_READ reads a snapshot output from SEM2DPACK
SEM2D_READ_SPECGRID reads a spectral element grid
SEM2D_SNAPSHOT_PLOT color-plots a snapshot output from SEM2DPACK
SEM2D_READ_SEIS reads seismogram outputs from SEM2DPACK
SEM2D_READ_FAULT reads fault outputs from SEM2DPACK

Chapter 4

Generating a mesh with EMC2

To obtain optimal accuracy the spectral element method requires a mesh in which the faces of the quadrilateral elements coincide with the interfaces between different materials. For dynamic faulting problems the element faces must necessarily coincide with the fault planes. SEM2DPACK provides only basic meshing capabilities and does not include an unstructured mesh generator for complicated, realistic geological models. This chapter describes how to achieve that task with an external software, EMC2.

Generating a high quality unstructured quad mesh can be a time-consuming task. Let's note that, for wave propagation problems without dynamic faults, if the acceptable accuracy is low (or large computational resources are available to work with a very fine mesh) a structured mesh in which the element faces do not necessarily follow the material interfaces can be generated with the basic built-in meshing capabilities of SEM2D.

4.1 The mesh generator EMC2

EMC2 is one of the few public domain 2D mesh generators including quadrilateral elements and a Graphical User Interface. It is a C code which sources and executables can be freely downloaded from <http://www-rocq.inria.fr/gamma/cdrom/www/emc2/eng.htm> . Although a complete documentation can be found in that package, we show here an example featuring the most useful functionalities.

Before starting you must provide files containing 2-column data (X,Z), without headers, of all the points needed to define the geometry of the model (topography, sediment bottom).

Once installed, you can run EMC2 by typing `emc2`.

4.2 Notations

The following notations are assumed in this chapter:

- (XXX) = click XXX on top menu bar
- (xxx) = click xxx on bottom menu bar
- <XXX< = click XXX on left menu bar
- >XXX> = click XXX on right menu bar
- \$xxx\$ = enter xxx from keyboard or from the calculator in the right panel
- "xxx" = type xxx in bottom prompt
- {xxx} = perform action xxx
- *xxx = do xxx as many times as needed
- n*xxx = do xxx n times

4.3 Basic step-by-step

A typical EMC2 session has three steps:

STEP I: CONSTRUCT, defines the geometry of the model

1. Switch to the construction tool:
`<CONSTRUCTION<`
2. Load the points:
`(POINT) (xy file) "palosgrandes.dat"`
 You must give the full path to your points-file, the root directory being the one where you launched `emc2`.
3. Reset the figure window to fit all points:
`>SHOW ALL>`

The original data has some geometrical features that are too complex to be meshed by quadrilaterals, for instance the corners at the N and S ends of the basin, you may want to smooth out these features. You also need to define the extreme boundaries of the region to be modelled (N,S and bottom absorbing boundaries) and some additional points on the free surface outside the basin. You must modify the data set (add and delete points):

4. Add new points:
 - a. with the mouse:
`(POINT) (mouse) *{click in figure window}`

- b. by coordinates:
 (POINT) (xy pt) *{ \$x=y=\$ }
 This is the safest way to get really vertical and horizontal boundaries needed for the absorbing conditions in SPECSEM90. You probably need to get the coordinates of an existent reference point:
 (POINT) <QUERY< (point) *{click on point}
- c. you can also reload another point-file (I2)
- 5. Delete points,
 (POINT) <DESTRUCT< (point) *{click on point}

Now you must define the geometry of the domains. These macro-blocks are intended to be internally meshed by deformed quadrilaterals. Their geometry follows the geometry of the geological model (one domain per material). Each domain must be bounded by segments or splines:

- 6. Segments:
 (SEGMENT) (point) 2*{click extreme point}
- 7. Splines:
 (SPLINE) (point) *{click point}
 You will see the spline evolve as you click points.

STEP II: PREPARE, defines the properties of the discrete spectral element mesh

- 1. Switch to the preparing mesh tool:
 <PREP MESH<
- 2. Define domains with rock n:
 (DOMAIN REF) \$n=\$ (any) *{click inside domain}
 You will see the domains edges get colored and the domains get numbered with n.
- 3. At any moment you can decide to show or not the domain decomposition:
 To hide the domain decomposition:
 >REFRESH>
 Show the domain decomposition:
 (SHOW) (ALL)
- 4. Remove a domain definition:
 (REMOVE) (DOMAINE) (any) {click inside domain}
 WARNING: corrections to the domain decomposition are sometimes displayed only after refreshing the figure window.
- 5. Now you must define the subdivision of each domain in quadrilateral finite elements. Define the number n of elements on each edge:
 (NB INTERVAL) \$n=\$ (any) {click edge}
 You will see the intermediate points appear. The number of intervals n is mainly dictated by the resolution criterion: elements should be smaller than the smallest wavelength you want to propagate. Moreover, **a domain can be quadrangulated only if the total number of intervals along its perimeter is even** (the sum of all n along its boundaries). However, a

quality mesh is not always guaranteed and you need to proceed by trial and error (emc2 allows you to jump back and forth between the different steps of the meshing procedure).

6. Finally you must define the external boundaries of the modelled region which will have a special treatment. You must associate a tag (a number) to each absorbing boundary. No convention is assumed but you should remember those tags later when setting the boundary conditions in SEM2D. It is also useful to assign a tag to the free surface boundary, that will be eventually used by SEM2D to locate the receivers or sources.

Define a boundary with index n:

(LINE REF) \$n=\$ (any) *{click edge}

Of course each boundary can be composed of many domain edges. Refresh the display to better see the boundaries. *The same procedure applies to define split-node interfaces such as faults and cracks: you must assign a different tag to each side of the fault.*

7. Save your work in EMC2 format:
 <SAVE< "name"
 The resulting file is **name.emc2.bd**

STEP III: EDIT, generates the mesh

1. Switch to the edit mesh tool:
 <EDIT_MESH<
 Press ENTER 4 times.
 A triangles mesh appears. You must convert it to a quad mesh:
2. Convert the triangle mesh to a quad mesh:
 <QUADRANGULATE> <ALL>
 You can smooth the mesh with: <REGULARIZE> *<ALL>

The final mesh is displayed. If there remain some triangles come back to the previous step and figure out how to modify the points per edge to help the mesher. Some experience is needed here.

3. Renumber the mesh, in order to optimize computations:
 *<RENUMBER>
4. Define the boundary condition for the 4 corner nodes of the model: (these nodes belong to 2 external boundaries so they were given a reference number =0)
 (MODIF_REF) \$n=\$ (corner) {click close to corner, inside element}
 Where n is the reference number of one of the 2 boundaries containing the corner node. Zooming can be useful. The same operation must be performed for the corner nodes of the subdomains belonging to an external boundary, and for the crack tip nodes. *However, as a special case, crack tip nodes must be assigned the -1 tag.*
5. Export the mesh:
 <SAVE<
 Two questions are asked in the bottom prompt:

- Format of the file, you must select:
"ftq"
- Prefix name for the file
"name"
The resulting file name will be `name.ftq`

4.4 Further tips

- Whenever possible it is better to mesh a domain with a *structured* mesh (a deformed cartesian grid). This can be done with (QUADRANGULATE), during the PREPARE step. See our FAQ for further details.
- To load an existent project, in the construction tool or in the preparation mesh tool:
<RESTORE< "name"
EMC2 will look for the file `name.emc2.bd`. Beware: the project loaded will replace the actual project if any, there is no superposition.
- BUG WARNING (13/07/01): the Sun release of EMC2 has a bug with the reference indices in the ftq format This bug is fixed in the 2.12c version. If you work on a Sun station, download the most recent version of the sources, rather than the executable, and compile it yourself.
- To densify (h-refinement) an existent mesh use the script `SEM2DPACK/POST/href.csh`. It edits the `*.emc2.bd` file, then you can restore it in EMC2 and save it in `*.ftq` format.
- To create a fault, in EDIT_MESH mode:
 - a. Crack an existent edge:
(CRACK) (segment)
 - b. Give a reference number to each side of the fault :
(MODIF_REF) \$n=\$ (segment)
 - c. Give the tag "-1" to crack tip nodes:
(MODIF_REF) \$-1=\$ (corner) *{click close to crack tip node, inside element}
- Note that only Q4 elements (4 control nodes) are supported. For a smoother description of boundaries Q9 would be desirable.

Chapter 5

Adding features to SEM2D (notes for advanced users)

Sometimes you will need to add new capabilities to the SEM2DPACK solver, by modifying the program. The following notes are intended to guide you through this process. We will not give here a comprehensive description of the code architecture, only enough details to get you started in performing safely the most usual and evident modifications.

5.1 Overview of the code architecture

[... in progresss ...]

This code uses a mixture of procedural (imperative) and object-oriented paradigms. Historically, it evolved from a purely procedural code.

Extensive use of modularity.

Object Oriented Programming (OOP) features (principles) applied in this code: encapsulation, classes, static polymorphism. These are not applied everywhere in the code, for different reasons: reusage of legacy code, performance, difficulty related to the limits of Fortra 90, or sections of code yet to be updated.

Added cost of structures containing pointer components: the possibility of pointer aliasing prevents more aggressive compiler optimizations and adds overhead for safety checks.

5.2 Accessible areas of the code

Some areas of the code have been written in such a way that a moderately experienced Fortran 95 programmer, with a limited understanding of the code architecture, can introduce new fea-

tures without breaking the whole system. This is achieved through modularity, encapsulation and templates. The modifications that are currently accessible are:

- boundary conditions, see `bc_gen.f90`
- material rheology, see `mat_gen.f90`
- source time functions, see `stf_gen.f90`
- spatial distributions, see `distribution_general.f90`

The source files listed above contain step-by-step instructions, just follow the comments starting by `!!`.

Chapter 6

Frequently Asked Questions

6.1 SEM2D

Segmentation fault

This problem is often related to a small stack size in your computer settings. In your Linux shell do: `ulimit -s unlimited`. Place this command in your `.bashrc`.

6.2 EMC2

I can't get rid of a few triangles

Obtaining a quality quad mesh is not always a trivial task. Trial and error and experience is needed. This can be by far the most time consuming stage of modeling.

First make sure that the total number of element edges along the perimeter of each mesh domain is even. This is a necessary topological condition to generate a quad-only mesh.

When the geometry seems too complicated for quad meshing you should consider simplifying the geometry, especially those details that are much smaller than the dominant wavelength.

If the above fails or does not apply, you have to help the mesher. The recommended procedure in EMC2 is:

1. Divide your original mesh into simple domains, in such a way that *most* domains have exactly four sides (possibly curved) and the remaining non-four-sided domains are as small as possible.
2. Generate a structured quad-mesh (a regular grid) inside each four-sided domain with the (QUADRANGULATE) tool of the PREP_MESH mode, as described in section 5.2.13

of EMC2's manual (note that this is *not* the same as the <QUADRANGULATE> button in the EDIT_MESH mode).

3. Proceed as usual (triangulation followed by quadrangulation) inside the remaining non-four-sided domains. If these are small enough EMC2 should not have problems doing a correct tri-to-quad meshing.

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