SEM2DPACK

A Spectral Element Method tool for 2D wave propagation and earthquake source dynamics

User's Guide

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Chapter 1

Introduction

1.1 Overview

The SEM2DPACK package is a set of software tools for the simulation and analysis of the seismic response of sedimentary basins and dynamic earthquake ruptures. Its core is SEM2D, an explicit Spectral Element solver for the 2D elastic wave equation. The general flow of a simulation project is:

- 1. Mesh generation: a domain decomposition made of quadrilateral elements, linearly (Q4) or quadratically (Q9) deformed.
- 2. Mesh quality verification, return to previous step if needed.
- 3. Numerical simulation.
- 4. Post-processing, analysis and visualization of the output.

SEM2DPACK provides tools for each step. However, no general mesh generation code is included. Instead SEM2D can import unstructured quadrilateral meshes generated externally. As an example we provide an interface to EMC2, one of the few public domain 2D mesh generators including quadrilateral elements and a Graphical User Interface ¹.

Chapter 2 of this User's Guide summarizes the physical assumptions and notations. Chapter 3 explains the usage of the SEM2D solver. For more details, additional results, and for the 3D extension of the algorithm, please refer to Komatitsch (1997), Komatitsch and Vilotte (1998), Komatitsch et al. (1999) and also Vai et al. (1998). Chapter 4 provides an introduction to mesh generation with EMC2. For more details you must refer to the EMC2 documentation. Chapter 5 gives some tips for advanced users who need to add new features to the code.

This is a research code, constantly under development and provided "as is", and therefore it should not be considered by the user as a 100 % bug-free software package. We welcome comments, suggestions, feature requests, module contributions and bug reports.

¹EMC2 can be downloaded from http://www.ann.jussieu.fr/~hecht/ftp/emc2/

1.2 History and credits

The main part of the elastic-isotropic solver was written in the mid 90's by Dimitri Komatitsch while he was preparing his Ph.D. at the *Institut de Physique du Globe de Paris*, under the advise of Prof. Jean-Pierre Vilotte. The elastic-anisotropic solver and several significant improvements to the isotropic code were added by D. Komatitsch later as part of a research contract with DIA Consultants. Further functionalities were added by myself, Jean-Paul Ampuero, while preparing my Ph.D. at IPGP, also under the advise of Prof. Jean-Pierre Vilotte. Most of these additional features were motivated by an ECOS-NORD/FONACYT research project for the study of the seismic response of the valley of Caracas, Venezuela. That became the version 1.0 of the SEM2DPACK, released in April 2002.

For the current version, 2.x, the code was almost completely rewritten in a more object-oriented style in preparation to the implementation of higher level functionalities, such as multigrid, subcycling, adaptivity and multiscale coupling. While the extensive use of object-oriented features of FORTRAN 90 can degrade performance this is not critical in 2D simulation, the emphasis has been rather in facilitating code reuse and expansion.

A simultaneous development for the simulation of earthquake dynamics was undertaken and is the main new feature of the current version. Spontaneous rupture along multiple non-planar faults can be currently modelled. Although there is no intrinsic limitation on applying different friction laws, as of Version 2.2 only linear slip weakening friction is implemented. Dynamic source simulations using methods that discretize the bulk, such as finite difference, finite element and spectral element methods, are more prone to high frequency numerical noise than boundary element methods (e.g. when the size of the process zone is not well resolved). Methods to control this problem were presented in the author's Ph.D. dissertation (Ampuero, 2002)² and in Gaetano Festa's Ph.D. dissertation³, and will be implemented in a forthcoming version of SEM2DPACK.

SEM2DPACK has been used in the following work:

- Madariaga et al. (2006): dynamic rupture and seismic wave radiation on faults with geometrical complexities (kinks)
- Haney et al. (2007): fault reflections from fluid-infiltrated faults
- De la Puente et al. (2007): as benchmark method for anisotropic wave propagation
- Kaneko et al. (2008): dynamic earthquake rupture with rate-and-state friction

1.3 Installation

- Uncompress and expand the SEM2DPACK package: tar xvfz sem2dpack.tgz
- Go to the source directory: cd SEM2DPACK/SRC

²Available in French at http://www.seg2.ethz.ch/ampueroj/phd.html

 $^{^3}$ http://people.na.infn.it/ $^{\sim}$ festa/

- Edit the Makefile according to your FORTRAN 95 compiler, following the instructions therein.
- Modify the optimization parameters declared and described in SRC/constant.f90.
- Compile: make
- Move to the SEM2DPACK/POST directory, edit the Makefile and compile.

On normal termination you should end up with a set of executable files, among which sem2dsolve, in /home/yourhome/bin/. I have been developping the code with the Lahey/Fujitsu 1f95 compiler and, more recently, with the Intel compiler for Linux ⁴. Other compilers are not being tested on a regular basis, so please report any related problems.

1.4 Help requests, feature requests and bug reports

Since November 2006 (version 2.2.5) current and old versions of SEM2DPACK are hosted by SourceForge at http://sourceforge.net/projects/sem2d/. To take advantage of the convenient features offered by this host you must create a SourceForge.net account at http://sourceforge.net/account/registration/.

The code repository is at http://sourceforge.net/project/showfiles.php?group_id=182742. To receive notification emails about new releases of SEM2DPACK sign up for the "Package Monitor" at http://sourceforge.net/project/filemodule_monitor.php?filemodule_id=212397.

A "tracking system" is available at http://sourceforge.net/tracker/?group_id=182742, with three separate lists. Requests for implementation of new features must be submitted to the "Feature Requests" tracker. Questions related to the usage of SEM2DPACK must be submitted to the "Support Requests" tracker. Bug reports must be submitted to the "Bugs" tracker. The three tracker lists are browsable and searchable. To browse the complete list of a tracker set "Status" to "Any". Before submitting an issue make sure you are running the most recent version of SEM2DPACK, that you understand the changes listed in SEM2DPACK's ChangeLog file and that your problem has not been treated in previous submissions. When relevant, a new submission must include the input files needed to reproduce your problem (Par.inp, *.ftq, etc). You will receive email notifications of any update of your submitted item, until it is closed. If the item is declared "Pending" you are expected to reply to the last message of the developer within two weeks, otherwise the item will be closed. For more instructions see http://sourceforge.net/support/getsupport.php?group_id=182742.

Contributions to SEM2DPACK by experienced programmers are always welcome and encouraged. A "Developers Forum" is available at http://sourceforge.net/forum/forum.php? forum_id=635737, where the implementation of new features can be discussed. Although the code is stable for my research purposes, there is still a number of missing features. Their

 $^{^4}$ This code works properly with the Intel compiler starting with version 8.0.046-pe047.1, so make sure you have a recent version of ifort!

1.5 License 8

implementation could make SEM2DPACK interesting for a broader audience in mechanical engineering, geotechnical engineering, applied geophysics and beyond. The ToDo file included with SEM2DPACK contains a list of missing features that range from basic functionalities to complex code re-engineering. Chapter 5 gives some guidelines for programmers.

1.5 License

This software is freely available for academic research purposes. If you use this software in writing scientific papers include proper attributions to its author, Jean-Paul Ampuero.

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Chapter 2

Physical background

This chapter summarizes the physical assumptions and notations in SEM2DPACK. Footnotes provide reference to the input arguments described in Chapter 3.

2.1 Basic assumptions and conventions

The coordinate system is Cartesian (rectangular). SEM2DPACK works in the two-dimensional (x, z) plane, where x is the horizontal coordinate, with positive direction pointing to the right, and z is the vertical coordinate, with positive direction pointing upwards. The coordinates (x, y, z) will be also denoted as (x_1, x_2, x_3) . This notations carry also for subscripts. For instance, the k-th component of displacement is denoted as u_k , with k = 1, 2, 3 or with k = x, y, z.

The reference frame is Eulerian. Infinitesimal strain is assumed. The (symmetric) infinitesimal strain tensor ϵ is defined as

$$\epsilon_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \tag{2.1}$$

Material density is deonted $\rho(x,z)$. The displacements and stresses relative to an initial equilibrium configuration are denoted $u_k(x,z,t)$ and $\sigma_{ij}(x,z,t)$, respectively. External forces (sources) are denoted $f_i(x,z,t)$. SEM2DPACK solves the following equations of motion to obtain the relative displacements $u_k(x,z,t)$:

$$\rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial \sigma_{ij}}{\partial x_j} + f_i \tag{2.2}$$

where summation over repeated indices is assumed. The initial conditions are $u_k = 0$ and $\partial u_k/\partial t = 0$. Stresses are related to strain, and possibly to other internal variables, by constitutive equations described in Section 2.2. The governing equations are supplemented by boundary conditions, described in Section 2.3. SEM2DPACK actually solves the governing

equations in variational (weak) form, as described in any textbook on the finite element method.

Two types of 2D problems are solved¹:

- Plane strain: Also known in seismology as P-SV, and in fracture mechanics as inplane mode or mode II. It is assumed that $u_3 = 0$ and $\partial/\partial x_3 = 0$. Hence, $\epsilon_{13} = \epsilon_{23} = \epsilon_{33} = 0$ and there are two degrees of freedom per node, u_x and u_z .
- Antiplane shear: Also known in seismology as SH, and in fracture mechanics as antiplane mode or mode III. It is assumed that $u_1 = u_2 = 0$ and $\partial/\partial x_3 = 0$. Hence, only ϵ_{13} and ϵ_{23} are non-zero and there is one degree of freedom per node, u_y .

2.2 Material rheologies

We describe here the constitutive equations implemented in SEM2DPACK, relating stress (σ_{ij}) , strain (ϵ_{ij}) and internal variables.

2.2.1 Linear elasticity

Linear isotropic elasticity

The stress-strain constitutive relation is Hooke's law:

$$\sigma_{ij} = \lambda \epsilon_{kk} \delta_{ij} + 2\mu \epsilon_{ij} \tag{2.3}$$

where λ and μ are Lamé's first and second parameters, respectively. In 2D plane strain the only relevant stress components are σ_{11} , σ_{22} and σ_{12} . The intermediate stress σ_{33} , although not null, is ignored. The S and P wave speeds are $c_S = \sqrt{\mu/\rho}$ and $c_P = \sqrt{(\lambda + 2\mu)/\rho}$, respectively. In 2D antiplane shear only the stress components σ_{13} and σ_{23} are relevant, and only S waves are generated.

Linear anisotropic elasticity

Transverse anisotropy with vertical symmetry axis is implemented for 2D plane strain (Komatitsch et al., 2000). The stress-strain constitutive relation is:

$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{zz} \\ \sigma_{xz} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{13} & 0 \\ c_{13} & c_{33} & 0 \\ 0 & 0 & c_{55} \end{pmatrix} \begin{pmatrix} \epsilon_{xx} \\ \epsilon_{zz} \\ 2\epsilon_{xz} \end{pmatrix}$$
(2.4)

where the c_{ij} are elastic moduli.

¹In the &GENERAL input block, plane strain is selected by ndof=2 and antiplane shear by ndof=1.

2.2.2 Linear visco-elasticity

Generalized Maxwell material

Not implemented yet.

Kelvin-Voigt material

Kelvin-Voigt damping can be combined with any of the other constitutive equations by replacing the elastic strain ϵ by $\epsilon^* = \epsilon + \eta \partial \epsilon / \partial t$, where η is a viscosity timescale. The resulting quality factor Q is frequency-dependent, $Q^{-1}(f) = 2\pi \eta f$. This is not convenient to model crustal attenuation with constant Q, unless the source has a narrow frequency band and η is selected to achieve a given Q value at the dominant frequency of the source. Most frequently, a layer of Kelvin-Voigt material is applied as artificial damping close to dynamic faults.

2.2.3 Coulomb plasticity

Perfect plasticity with a Coulomb yield function is implemented for 2D plane strain, as in Andrews (2005).

The total strain is the sum of an elastic and a plastic contribution, $\epsilon = \epsilon^e + \epsilon^p$.

[...]

Plastic yield occurs when

$$\tau = c\cos(\phi) - (\sigma_{xx} + \sigma_{zz})/2 \sin(\phi) \tag{2.5}$$

where c is cohesion, ϕ the internal friction angle and

$$\tau = \sqrt{\sigma_{xz}^2 [(\sigma_{xx} - \sigma_{zz})/2]^2} \tag{2.6}$$

is the maximum shear stress over all planes.

[...]

2.2.4 Continuum damage

The continuum damage formulation by Lyakhovsky et al. (1997), including damage-related plasticity as introduced by Hamiel et al. (2004), is implemented with modifications for 2D plane strain.

The first and second invariants of the 2D elastic strain tensor are defined as $I_1 = \epsilon_{kk}^e$ and $I_2 = \epsilon_{ij}^e \epsilon_{ij}^e$, respectively. A strain invariant ratio is defined as $\xi = I_1/\sqrt{I_2}$. The following non-linear stress-strain relation is assumed (Lyakhovsky *et al.*, 1997, eq. 12):

$$\sigma_{ij} = (\lambda - \gamma/\xi) I_1 \delta_{ij} + (2\mu - \gamma\xi) \epsilon_{ij}^e$$
(2.7)

where γ is an additional elastic modulus. The elastic moduli depend on a scalar damage variable, $0 \le \alpha \le 1$, through (Lyakhovsky *et al.*, 1997, eq. 19):

$$\lambda = \lambda_0 \tag{2.8}$$

$$\mu = \mu_0 + \gamma_r \xi_0 \ \alpha \tag{2.9}$$

$$\gamma = \gamma_r \alpha \tag{2.10}$$

where λ_0 and μ_0 are Lamé's parameters for the intact material ($\alpha = 0$). The parameter ξ_0 is the threshold value of the strain invariant ratio ξ at the onset of damage. It is related to the internal friction angle ϕ in a cohensionless Mohr-Coulomb yield criterion by the 2D plane strain version of Lyakhovsky *et al.* (1997, eq. 37):

$$\xi_0 = \frac{-\sqrt{2}}{\sqrt{1 + (\lambda_0/\mu_0 + 1)^2 \sin^2 \phi}}$$
 (2.11)

The scaling factor γ_r is chosen such that convexity is lost at $\alpha = 1$ when $\xi = \xi_0$. It is derived from the 2D plane strain version of Lyakhovsky *et al.* (1997, eq. 15):

$$\gamma_r = p + \sqrt{p^2 + 2\mu_0 q} \tag{2.12}$$

where

$$q = (2\mu_0 + 2\lambda_0)/(2 - \xi_0^2) \tag{2.13}$$

$$p = \xi_0(q + \lambda_0)/2 \tag{2.14}$$

The evolution equation for the damage variable is (Lyakhovsky et al., 1997, eq. 20)

$$\dot{\alpha} = \begin{cases} C_d I_2(\xi - \xi_0) & \text{if } \xi > \xi_0 \\ 0 & \text{otherwise} \end{cases}$$
 (2.15)

No healing is assumed below ξ_0 . The evolution of the plastic strain ϵ_{ij}^p is driven by the damage variable α (Hamiel *et al.*, 2004, eq. 9):

$$\dot{\epsilon}_{ij}^p = \begin{cases} \tau_{ij} C_v \dot{\alpha} & \text{if } \dot{\alpha} \ge 0\\ 0 & \text{otherwise} \end{cases}$$
 (2.16)

where $\tau_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_{kk} \, \delta_{ij}$ is the deviatoric part of the stress tensor. The parameter C_v is of order $1/\mu_0$ and 2 is related to the seismic coupling coefficient $0 < \chi < 1$ by (Ben-Zion and Lyakhovsky, 2006)

$$C_v = \frac{1-\chi}{\chi} \frac{1}{\mu_0} \tag{2.17}$$

2.3 Boundary conditions

2.3.1 Absorbing boundaries

Paraxial

[...]

²In &MAT_DMG, the input argument R is defined as $R = \mu_0 C_v$.

Clayton-Engquist

[...]

2.4 Fault interface conditions

2.4.1 Linear slip law

See Haney et al. (2007) [...]

2.4.2 Normal stress response

Unilateral contact

[...]

Modified Prakash-Clifton regularization

Regularization of the normal stress response, as required for bimaterial rupture problems, is implemented following Rubin and Ampuero (2007). The frictional strength is proportional to a modified normal stress σ^* , related to the real fault normal stress, σ , by either of the following evolution laws:

- Version with a regularization *slip* scale:

$$\dot{\sigma^*} = \frac{|V| + V^*}{L_{\sigma}} \left(\sigma - \sigma^*\right) \tag{2.18}$$

where V is slip rate, and V^* and L_{σ} are constitutive parameters³.

– Version with a regularization *time* scale:

$$\dot{\sigma^*} = \frac{1}{T_\sigma} \left(\sigma - \sigma^* \right) \tag{2.19}$$

where T_{σ} is a constitutive parameter⁴.

³In &BC_DYNFLT_NOR, this law is set by kind=2, and the two relevant parameters are V and L.

 $^{^4}$ In &BC_DYNFLT_NOR, this law is set by kind=3, and the relevant parameter is T.

2.4.3 Friction

Slip-weakening friction

Slip occurs when the fault shear stress reaches the shear strength $\tau = \mu \sigma$ (or $\tau = \mu \sigma^*$ if the Prakash-Clifton law is assumed). [...] The friction coefficient μ is a function of the cumulated slip D, given by one of the following laws:

- Linear slip-weakening law:

$$\mu = \max \left[\mu_d, \mu_s - \frac{\mu_s - \mu_d}{D_c} D \right] \tag{2.20}$$

Exponential slip-weakening law:

$$\mu = \mu_s - (\mu_s - \mu_d) \exp(-D/D_c)$$
 (2.21)

"Fast" rate-and-state friction

Friction with fast (power law) velocity weakening at fast slip speed is a first order proxy for physical weakening processes that operate on natural fault zones at seismic slip velocities. A rate-and-state dependent friction law with fast velocity-weakening is implemented in SEM2DPACK, similar to that adopted e.g. by Ampuero and Ben-Zion (2008). The friction coefficient depends on slip velocity (V) and a state variable (θ) :

$$\mu_f = \mu_s + a \frac{V}{V + V_c} - b \frac{\theta}{\theta + D_c}.$$
 (2.22)

The state variable has units of slip and is governed by the evolution equation

$$\dot{\theta} = V - \theta V_c / D_c. \tag{2.23}$$

The friction law is defined by the following constitutive parameters: μ_s is the static friction coefficient, a and b are positive coefficients of a direct effect and an evolution effect, respectively, V_c is a characteristic velocity scale⁵, and D_c is a characteristic slip scale.

The steady-state $(\dot{\theta} = 0)$ friction coefficient

$$\mu_f = \mu_s + (a - b) \frac{V}{V + V_c} \tag{2.24}$$

weakens asymptotically as 1/V when $V \gg V_c$, if a < b, approaching its dynamic value ($\mu_d = \mu_s + a - b$) over a relaxation timescale D_c/V_c . The value of the relaxation time D_c/V_c tunes the weakening mechanism between two limit cases: slip-weakening and velocity-weakening. If D_c/V_c is much longer than the typical time scale of fluctuation of the state variable ($\approx \theta/\dot{\theta}$), Equation 2.23 becomes $\dot{\theta} \approx V$, implying that θ is proportional to slip and that the evolution term of the friction coefficient is effectively slip-weakening, with characteristic slip-weakening distance D_c . Conversely, if D_c/V_c is short the relaxation to steady state is fast, $\theta/D_c \approx V/V_c$ and the friction is effectively velocity-weakening, with characteristic velocity scale V_c .

 $^{^{5}}$ vstar in &BC_DYNFLT_RSF

Logarithmic rate-and-state friction

 $Not\ implemented\ yet.$

Dieterich and Ruina classical rate-and-state laws, with aging or slip state evolution law.

Chapter 3

The solver SEM2D

3.1 About the method

Given a crustal model meshed with quadrilateral elements and a set of material properties, sources, receivers and boundary conditions, SEM2D solves the elastic wave equation applying the Spectral Element Method (SEM) for the space discretization and a second-order explicit scheme for the time discretization. The range of physical problems solved by SEM2D (material constitutive equations and boundary conditions) is described in more detail in Chapter 2. The SEM, introduced by Patera (1984) in Computational Fluid Dynamics, can be seen as a domain decomposition version of Pseudospectral Methods or as a high order version of the Finite Element Method. It inherits from its parent methods the accuracy (spectral convergence), the geometrical flexibility and the natural implementation of mixed boundary conditions.

Introductory texts to the SEM can be found at www.math.lsa.umich.edu/~karni/m501/boyd.pdf (chapter draft, by J.P. Boyd), at www.mate.tue.nl/people/vosse/docs/vosse96b.pdf (a tutorial exposition of the SEM and its connection to other methods, by F.N. van de Vosse and P.D. Minev) and at www.siam.org/siamnews/01-04/spectral.pdf (a perspective paper). Details about the elastodynamic algorithm and study of some of its properties are presented by Komatitsch (1997), Komatitsch and Vilotte (1998), Komatitsch et al. (1999), Komatitsch and Tromp (1999) and Vai et al. (1998).

The implementation of fault dynamics is similar to that in FEM with the "traction at split nodes" method explained by Andrews (1999). More details can be found in the author's Ph.D. dissertation (Ampuero, 2002)¹, in Gaetano Festa's Ph.D. dissertation² and in Kaneko et al. (2008).

A more accesible tutorial code, SBIEMLAB written in Matlab, can be downloaded from the author's website, at www.seg2.ethz.ch/ampueroj/software.html.

¹ www.seg2.ethz.ch/ampueroj/phd.html

²people.na.infn.it/~festa/

3.2 What SEM2D is and what it is not

SEM2D is an explicit spectral element solver for the 2D elastic wave equation and for some problems arising in earthquake source dynamics (see Chapter 2). The code is written in FORTRAN 90, with some (useful but not essential) FORTRAN 95 features.

The SEM requires an initial decomposition of the domain into quadrilateral elements (a quad mesh). Optimal accuracy is obtained when the faces of the elements coincide with the interfaces between different materials. Generating high quality quad meshes for complicated geological models is not yet a fully automated process, and can be very time-consuming. If the user prefers to sacrifice accuracy by generating a simple, logically cartesian mesh where the element faces do not necessarily follow the material interfaces, the basic built-in meshing capabilities of SEM2D are sufficient. However, SEM2D does not include a general mesh generator for unstructured grids. If you want to get the best accuracy out of the spectral element method and your geological model is complicated you need to generate a mesh with some external software. An example using the freely available 2D mesh generation software EMC2 is described in Chapter 4.

A number of post-processing and graphic tools are included, as decribed in Section 3.7. Most output is in the form of raw binary or ASCII data files. Scripts are provided for graphic display and analysis on Seismic Unix, Gnuplot and Matlab. We recommend the usage of the Matlab functions included, which are well documented (see Section 3.7.6).

3.3 Basic usage flow

In general, a simulation requires the following steps:

- 1. Prepare the input file Par.inp (Section 3.4 and Section 3.5).
- 2. Run the solver in "check mode" (iexec=0 in the GENERAL input block of Par.inp): sem2dsolve > info &.
- 3. Verify the resolution, stability, estimated CPU cost and memory cost (Section 3.6).
- 4. If needed go back to step 1 and modify Par.inp (Section 3.6), else proceed to next step.
- 5. Run the solver in "production mode" (iexec=1): sem2dsolve.
- 6. Plot and manipulate the solver results (Section 3.7).

Full details are given in the following sections.

3.4 General format of the input file

The input file must be called Par.inp. Its typical structure is illustrated by two examples in Figure 3.1 and Figure 3.2. Most of the file is made of standard FORTRAN 90 NAMELIST input blocks. Each block gives input for a specific aspect of the simulation: material properties, sources, receivers, boundary conditions, etc.

The general syntax of a NAMELIST block can be found in any FORTRAN 90 textbook. In summary, a block named STUFF, with possible input arguments a, b and c, must be given as

```
&STUFF a=..., b=..., c=... /
```

where ... are user input values. Line breaks and comments preceded by ! are allowed within an input block.

The complete Reference Guide of the input blocks is presented in Section 3.5. For each block the documentation includes its name, possibly the name of a group of blocks to which it belongs, its purpose, its syntax, the list of its arguments with their description, and some important notes. In the syntax description, a vertical bar | between two arguments means "one or the other". In the argument list, each item is followed by two informations within brackets []. The first bracketed information is the type of the argument: double precision (dble), integer (int), logical (log), single character (char), fixed length word (e.g. char*6 is a 6 characters word), arbitrary length word (name) or vectors (e.g. int(2) is a two element integer vector). The second bracketed information is the default value of the argument. Some arguments are optional, or when absent they are automatically assigned the default values.

Some arguments have a second version with a suffix H that allows to set values that are spatially non uniform. The H-version of the argument must be set to the name of any of the input blocks of the DISTRIBUTIONS group. The appropriate &DIST_xxxx block must follow immediately. For example, to set the argument eta to a Gaussian distribution:

```
&MAT_KV etaH='GAUSSIAN' /
&DIST_GAUSSIAN length=1d6,100d0, ampli=0.1d0 /
```

Arguments that accept an H-version are indicated in Section 3.5. When more than one H-version argument is present, the &DIST_xxxx blocks must appear in the same order as in the argument list of Section 3.5.

In the next section, Input Block Reference Guide, you should get acquainted with the syntax of the blocks you are most likely to use. The mandatory or more important input blocks are:

- &GENERAL
- &MESH_DEF, followed by a &MESH_Method block
- &MATERIAL, followed by a &MAT_Material block
- &BC_DEF, one for each boundary condition, each followed by a &BC_Kind block
- &TIME
- &SRC_DEF, followed by &STF_SourceTimeFunction and &SRC_Mechanism blocks
- &REC_LINE

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```
Par.inp
 Mar 06, 08 10:48
                                                                Page 1/1
# Parameter file for SEM2DPACK 2.0
#---- Some general parameters -----
&GENERAL iexec=1, ng\overline{l}l= 6, fmax=1.25d0 , ndof=1
  title = 'Test SH', verbose='1111', ItInfo = 1000 /
#---- Build the mesh -----
&MESH_DEF method = 'CARTESIAN' /
&MESH_CART xlim=0.d0,30.d0 ,zlim=0.d0,30.d0 , nelem=60,60/
#--- Elastic material parameters -----&MATERIAL tag=1, kind='ELAST' /
&MAT_ELASTIC rho=1.d0, cp=1.7321d0, cs=1.d0 /
#---- Boundary conditions -----
&BC_DEF tag = 2 , kind = 'ABSORB' /
&BC_ABSORB stacey=F/
&BC_DEF tag = 3 , kind = 'ABSORB' /
&BC_ABSORB stacey=F/
#---- Time scheme settings ------
&TIME TotalTime=35.d0, courant = 0.3d0 /
\&SRC_FORCE angle = 0d0/
#---- Receivers ------
&REC_LINE number = 7 , field='D', first = 0.d0,0.d0, last = 30d0,0.d0, isamp=1 /
#----- Plots settings -----
&SNAP_DEF itd=100000, ps=F , bin=F /
Thursday March 06, 2008
```

Figure 3.1: Input file Par.inp for an elementary example in EXAMPLES/TestSH/: a boxed region with a structured mesh.

Printed by Jean Paul Ampuero

```
Mar 06, 08 10:49
                                      Par.inp
                                                                      Page 1/1
#---- Some general parameters -----
&GENERAL lexec =0 , Ngll = 6 , fmax = 1.5 , ndof=1,
      Title = 'Palos Grandes NS meshed with EMC2',
      {\tt Verbose='1111',\ ItInfo=1000/}
#---- Build the mesh ----
&MESH_DEF Method = 'EMC2' /
&MESH_EMC2 File= 'NS03qb.ftq'
#---- Elastic material parameters ------
&MATERIAL tag=1, kind='ELAST'
&MAT_ELASTIC rho=1800.d0, cp=850.d0, cs=450.d0/
&MATERIAL tag=2, kind='ELAST' /
&MAT_ELASTIC rho=2100.d0, cp=1800.d0, cs=650.d0/
&MATERIAL tag=3, kind='ELAST' /
&MAT_ELASTIC rho=2400.d0, cp=2300.d0, cs=850.d0/
&MATERIAL tag=4, kind='ELAST'
&MAT_ELASTIC rho=2600.d0, cp=3800.d0, cs=2200.d0/
#&MAT_ELASTIC rho=2500.d0, cp=5000.d0, cs=2900.d0/
#---- Boundary conditions -----
&BC_DEF Tag = 2, Kind = 'ABSORB' /
&BC_ABSORB Stacey=F /
&BC_DEF Tag = 3, Kind = 'ABSORB' /
&BC_ABSORB Stacey=F, let_wave=T /
&BC_DEF Tag = 4, Kind = 'ABSORB' /
&BC_ABSORB Stacey=F /
#---- Time scheme settings ------
&TIME TotalTime=25.d0, Courant = 0.55d0 /
&TIME_NEWMARK alpha=1.d0, beta=0.d0, gamma=0.5d0 /
#---- Sources ------
&SRC_DEF stf='RICKER', Mechanism='WAVE' Coord= -1160000.d0,-2000.d0 /
&STF_RICKER f0 = 1.d0 , Onset = 1.5d0 , Ampli = 1.d0 / &SRC_FORCE Angle = 90. /
&SRC_WAVE Angle = 30. , phase='S' /
#---- Receivers ------
# receivers located at the surface by giving a very large vertical position
# locating them at the nearest computational node (AtNode=.true. is the default)
&REC_LINE Number = 31 , First = -1163068.0d0,1.d3, Last = -1159697.36d0,1.d3,
         Isamp=10 /
#----- Plots settings -----
&SNAP_DEF itd=100000, fields='V', components='x' / \# itd = 3500
&SNAP_PS Mesh=T, Vectors=F, Color=T, Interpol = T, DisplayPts=7,
          ScaleField=0.2d0 /
Thursday March 06, 2008
```

Figure 3.2: Input file Par.inp for a more realistic example: a sedimentary basin with an unstructured mesh generated by EMC2. Available in EXAMPLES/UsingEMC2/.

3.5 Input Blocks Reference Guide

= Self-documentation for the INPUT BLOCKS of the SEM2D code =

NAME : BC_ABSORB

GROUP : BOUNDARY_CONDITION PURPOSE: Absorbing boundary

SYNTAX : &BC_ABSORB stacey, let_wave /

stacey [log] [F] Apply Stacey absorbing conditions for P-SV.

Higher order than Clayton-Engquist (the default).

let_wave [log] [T] Allow incident waves across this boundary

NOTE : Only implemented for vertical and horizontal boundaries.

NAME : BC_DIRNEU

GROUP : BOUNDARY_CONDITION

PURPOSE: Dirichlet (null displacement)

and/or Neumann (null or time-dependent traction)

boundary conditions on vertical or horizontal boundaries

SYNTAX : &BC_DIRNEU h, v, hsrc, vsrc /

possibly followed by one or two STF_XXXX blocks

h [char]['N'] Boundary condition on the horizontal component v [char]['N'] Boundary condition on the vertical component :

'N' : Neumann
'D' : Dirichlet

hsrc [name]['null'] Name of the source time function for a

time-dependent horizontal traction:

'RICKER', 'TAB', 'USER', etc (see STF_XXXX input blocks)

vsrc [name]['null'] Same for the vertical component

NAME : BC_DYNFLT

GROUP : BOUNDARY_CONDITION, DYNAMIC_FAULT

PURPOSE: Dynamic fault with friction

SYNTAX : &BC_DYNFLT friction, Tn|TnH, Tt|TtH,

Sxx|SxxH, Sxy|SxyH, Sxz|SxzH, Syz|SyzH, Szz|SzzH

```
ot1, otd, oxi, osides /
```

followed, in order, by:

- 1. &DIST_XXX blocks (from the DISTRIBUTIONS group) for arguments with suffix H, if present, in the order listed above.
- 2. &BC_DYNFLT_SWF, &BC_DYNFLT_TWF or &BC_DYNFLT_RSF block(s) (if absent, default values are used)
- 3. &BC_DYNFLT_NOR block (if absent, default values are used)

friction [name(2)] ['SWF',''] Friction law type:

SWF = slip weakening friction

TWF = time weakening friction

RSF = rate and state dependent friction

Some friction types can be combined. E.g. to set the friction coefficient to the minimum of SWF and TWF, set friction='SWF', 'TWF'

Tn [dble] [0d0] Initial normal traction (positive = tensile)

Tt [dble] [0d0] Initial tangent traction (positive antiplane: y>0)

Sxx [dble] [0d0] Initial stress sigma_xx

Sxy [dble] [0d0] Initial stress sigma_xy

Sxz [dble] [0d0] Initial stress sigma_xz

Syz [dble] [0d0] Initial stress sigma_yz Szz [dble] [0d0] Initial stress sigma_zz

otd [dble] [0.d0] Time lag between outputs (in seconds)

Internally adjusted to the nearest multiple of the timestep Its value can be found in the output file FltXX_sem2d.hdr The default internally resets otd = timestep

ot1 [dble] [0.d0] Time of first output (in seconds)

Internally adjusted to the nearest multiple of the timestep Its value can be found in the output file FltXX_sem2d.hdr

oxi [int(3)] [(1,huge,1)] First, last node and stride for output

The default resets oxi(2) = last fault node [log] [F] Export displacement and velocities on each side

osides [log] [F] Export displacement and velocities on each side of the fault

NOTE: the initial stress can be set as a stress tensor (Sxx,etc), as initial tractions on the fault plane (Tn and Tt) or as the sum of both.

NOTE: we recommend to use dynamic faults with the leapfrog time scheme and a layer of Kelvin-Voigt damping material near the fault.

NAME : BC_DYNFLT_NOR GROUP : DYNAMIC_FAULT

PURPOSE: Normal stress response for dynamic faults.

SYNTAX : &BC_DYNFLT_NOR kind, V, L, T /

```
[int] [1] Type of normal stress response:
 kind
                      1 = Coulomb
                      2 = Prakash-Clifton with regularizing time scale
                      3 = Prakash-Clifton with regularizing length scale
 Τ
          [dble] [1d0] Regularization time scale if kind=2
 V
          [dble] [1d0] Characteristic velocity if kind=3
          [dble] [1d0] Regularization length scale if kind=3
NAME
       : BC_DYNFLT_RSF
GROUP : DYNAMIC_FAULT
PURPOSE: Velocity and state dependent friction
SYNTAX: &BC_DYNFLT_RSF kind, Dc | DcH, Mus | MusH ,
                        a | aH, b | bH, Vstar | VstarH /
         followed by &DIST_XXX blocks (from the DISTRIBUTIONS group) for
         arguments with suffix H, if present, in the order listed above.
 kind
          [int] [1] Type of rate-and-state friction law:
                      1 = strong velocity-weakening at high speed
                          as in Ampuero and Ben-Zion (2008)
 Dc
          [dble] [0.5d0] Critical slip
 MuS
          [dble] [0.6d0] Static friction coefficient
 a
          [dble] [0.01d0] Direct effect coefficient
          [dble] [0.02d0] Evolution effect coefficient
 h
 Vstar
          [dble] [1d0] Characteristic or reference slip velocity
       : BC_DYNFLT_SWF
NAME
GROUP : DYNAMIC_FAULT
PURPOSE: Slip-weakening friction
SYNTAX : &BC_DYNFLT_SWF Dc | DcH, MuS | MuSH , MuD | MuDH /
         followed by &DIST_XXX blocks (from the DISTRIBUTIONS group) for
         arguments with suffix H, if present, in the order listed above.
 kind
          [int] [1] Type of slip weakening function:
                      1 = linear
                      2 = exponential
 Dc
          [dble] [0.5d0] Critical slip
          [dble] [0.6d0] Static friction coefficient
 MuS
 MuD
          [dble] [0.5d0] Dynamic friction coefficient
```

NAME : BC_DYNFLT_TWF GROUP : DYNAMIC_FAULT PURPOSE: Time weakening friction for dynamic faults

with prescribed rupture speed.

SYNTAX: &BC_DYNFLT_TWF MuS, MuD, X, Z, V, L, T /

MuS [dble] [0.6d0] Static friction coefficient
MuD [dble] [0.5d0] Dynamic friction coefficient

Mu0 [dble] [0.6d0] Friction coefficient at the hypocenter at time=0

X,Z [dble] [0d0] Position of hypocenter
V [dble] [1d3] Rupture propagation speed
L [dble] [1d0] Size of weakening zone

T [dble] [huge] Total duration

NAME : BC_DEF

PURPOSE: Define a boundary condition
SYNTAX : &BC_DEF tag, tags, kind /

possibly followed by &BC_kind blocks

tag [int] [none] A number assigned to the boundary. If you are using SEM2D built-in structured mesher the conventions are:

1 bottom
2 right
3 up
4 left

If you are importing a mesh, you must use the tags assigned to the boundaries during the mesh construction.

tags [int(2)] [none] Two tags are needed for interfaces (split-node) and for periodic boundaries.

kind [char*6] [none] Type of boundary condition. The following are implemented:

'DIRNEU', 'ABSORB', 'PERIOD', 'LISFLT', 'DYNFLT'

NOTE : Most of the boundary conditions need additional data, given in a BC_kind input block of the BOUNDARY_CONDITIONS group immediately following the BC_DEF block.

NAME : BC_LSF

GROUP : BOUNDARY_CONDITION

PURPOSE: Linear slip fault, a displacement discontinuity interface

where stress and slip are linearly related

SYNTAX : &BC_LSF Ktang | Ctang, Knorm | Cnorm /

Ktang [dble] [Inf] Tangential stiffness Ctang [dble] [0d0] Tangential compliance

Knorm [dble] [Inf] Normal stiffness
Cnorm [dble] [Od0] Normal compliance

NOTE: For each component:

You can set K _or_ C, but _not_both_

If C=0d0 or K=Inf then no discontinuity is allowed (transparent)

If K=OdO the fault is free stress boundary

NAME : DIST_GAUSSIAN
GROUP : DISTRIBUTIONS_2D

PURPOSE: Bell shaped Gaussian 2D distribution

SYNTAX: &DIST_GAUSSIAN centered_at, length, offset, ampli, order /

centered_at [dble(2)] [0,0] Coordinates of the center point. length [dble(2)] [1] Characteristic lengths on each axis.

offset [dble] [0] Background level.

ampli [dble] [1] Amplitude from background.

order [dble] [1] Exponent

NAME : DIST_GRADIENT
GROUP : DISTRIBUTIONS_2D

PURPOSE: Constant gradient 2D distribution.

SYNTAX : &DIST_GRADIENT file, valref ,grad, angle/

file [name] [none] Name of the file containing the coordinates

of the points defining the reference line. It is an ASCII file with 2 columns per line:

(1) X position (in m) and(2) Z position (in m)

valref [dble] [none] Value along the reference line

grad [dble >0] [none] Positive gradient (valref_units/meter) angle [dble] [none] Angle (degrees) between the vertical down

and the grad+ direction. Anticlockwise convention (grad+

points down if 0, right if 90)

 ${\tt NOTE}$: Be sure that your angle and ref-line are compatible. The code will

abort if the ref-line is too short: some points of the domain

cannot be projected to ref-line in the angle direction.

NAME : DIST_HETE1

GROUP : DISTRIBUTIONS_2D

```
PURPOSE: Linear interpolation of values from a regular 2D grid.
SYNTAX : &DIST_HETE1 file, col /
 file
                  [name] [none] Name of the file containing the definition
                      of the regular grid and values at grid points.
                      The format of this ASCII file is:
                         Line 1: ncol nx nz x0 z0 dx dz
                           ncol = [int] number of data columns
                           nx,nz = [2*int] number of nodes along x and z
                           x0,z0 = [2*dble] bottom-left corner
                           dx,dz = [2*dble] spacing along x and z
                         Line 2 to nx*nz+1 : [ncol*dble] values at grid points
                           listed from left to right (x0 to x0+nx*dx),
                           then from bottom to top (z0 to z0+nz*dx)
 col
                  [int] [1] Column of the file to be read
NOTE.
       : The same file can contain values for (ncol) different properties,
         (e.g. rho, vp, vs) but each DIST_HETE1 block will read only one.
NOTE
       : Even if the original model domain has an irregular shape,
         the regular grid where input values are defined must be rectangular
         and large enough to contain the whole model domain.
         The regular grid possibly contains buffer areas with dummy values.
         These dummy values should be assigned carefully (not random nor zero)
         because SEM2D might use them during nearest-neighbor interpolation.
NAME
       : DIST_LINEAR
GROUP : DISTRIBUTIONS_1D
PURPOSE: Piecewise linear 1D distribution along X.
SYNTAX : If the data is given in Par.inp:
           &DIST_LINEAR n,length /
           followed immediately by the data points, two columns per line:
              (1) X position, sorted in increasing order, and
              (2) data value at X
         If the data is given in a separate file, same format as above:
           &DIST_LINEAR file, length /
          [int] [0] Number of points to be interpolated
 n
          [name] [none] Name of the ASCII file containing the data
 file
          [dble] [0] Smoothing length for sliding average window
 length
              No smoothing if length=0
```

NAME : DIST_ORDERO

```
GROUP : DISTRIBUTIONS_2D
PURPOSE: Blockwise constant 2D distribution.
SYNTAX : &DIST_ORDERO xn, zn /
         x(1) ... x(xn-1)
         z(1) ... z(zn-1)
         v(1,1) \dots v(xn,1)
               . . .
         v(1,zn) \dots v(xn,zn)
          [int] [none] Number of zones along X
 xn
          [int] [none] Number of zones along Z
 zn
          [dble(xn-1)] [none] Boundaries of X-zones: first zone X < x(1),
               second zone x(1) < X < x(2), \ldots, last zone x(xn-1) < X
          [dble(zn-1)] [none] Boundaries of Z-zones
          [dble(xn,zn)] [none] Values inside each zone
NAME
       : DIST_PWCONR
GROUP : DISTRIBUTIONS_2D
PURPOSE: Piecewise constant radial (2D) distribution.
SYNTAX : &DIST_PWCONR num, ref /
            r(1) ... r(num-1)
        v(1) v(2) ... v(num-1) v(num)
          [int] [none] Number of radial zones (including outermost)
 num
          [dble(2)] [(0d0,0d0)] Reference point: center of radial zones
 ref
          [dble(num-1)] [none] External radius of zones:
               first zone R \leq r(1),
               second r(1) < R <= r(2), ...
               last r(num-1) < R
          [dble(num)] [none] Values inside each zone
NAME
       : DIST_SPLINE
GROUP : DISTRIBUTIONS_1D
PURPOSE: Spline interpolated 1D distribution along X.
SYNTAX : &DIST_SPLINE file /
 file
          [name] [none] Name of the ASCII file containing
              the data to be interpolated, two columns per line:
              (1) X position, sorted in increasing order, and
              (2) data value at X
```

```
: GENERAL
NAME
PURPOSE: General parameters
SYNTAX : &GENERAL iexec, ngll, fmax, title, verbose, itInfo /
          [int] [0] Run level:
 iexec
                      0 = just check
                      1 = solve
          [int] [9] Number of GLL nodes per edge on each spectral element
 ngll
               (polynomial order +1). Usually 5 to 9.
 fmax
          [dble] [0.d0] Maximum frequency to be well resolved. Mandatory.
               This is a target frequency, the code will check if it is
               compatible with the mesh and issue a warning if not. To
               improve the resolution for a given fmax you must increase ngll
               (but you will have to use shorter timesteps) or refine/redesign
               the mesh.
 ndof
          [int] [2] Number of degrees of freedom per node
                      1 = SH waves, anti-plane
                      2 = P-SV waves, in-plane
 title
          [word] [none] Title of the simulation
          [char(4)] ['1101'] Print progress information during each phase:
 verbose
                      verbose(1) = input phase
                      verbose(2) = initialization phase
                      verbose(3) = check phase
                      verbose(4) = solver phase
               Example: '0001' is verbose only during solver.
 itInfo
          [int] [100] Frequency (in number of timesteps) for printing
               progress information during the solver phase.
       : MAT_DAMAGE
NAME
GROUP : MATERIALS
PURPOSE: Set material properties for the damage rheology of
         Lyakhovsky, Ben-Zion and Agnon (J. Geophys. Res. 1997)
         and Hamiel et al (Geophys. J. Int. 2004)
SYNTAX: &MAT_DAMAGE cp,cs,rho,phi,alpha,Cd,R,e0,ep /
          [dble][0d0] P wave velocity
 ср
 cs
          [dble] [0d0] S wave velocity
          [dble][0d0] density
 rho
          [dble][0d0] internal friction angle
 phi
          [dble][0d0] initial value of damage variable
 alpha
 Cd
          [dble] [0d0] damage evolution coefficient
 R
          [dble][0d0] damage-related plasticity coefficient Cv
                normalized by the inverse of the intact shear modulus
          [dble(3)][0d0] initial total strain (11, 22 and 12)
 e0
          [dble(3)][0d0] initial plastic strain (11, 22 and 12)
 ер
```

NAME : MAT_ELASTIC GROUP : MATERIALS

PURPOSE: Set material properties for a linear elastic medium

SYNTAX : For isotropic material:

&MAT_ELASTIC rho|rhoH, cp|cpH, cs|csH /

followed by DIST_XXXX blocks, for arguments with suffix H,

if present, in the same order as listed above.

For transverse anisotropy with vertical symmetry axis:

&MAT_ELASTIC rho, c11,c13,c33,c55 /

cp [dble][0d0] P wave velocity cs [dble][0d0] S wave velocity

rho [dble][0d0] density

c11,c13,c33,c55 [dble][0d0] anisotropic elastic moduli

NAME : MATERIAL

PURPOSE: Define the material type of a tagged domain

SYNTAX: &MATERIAL tag, kind /

followed by one or two MAT_XXXX input blocks.

tag [int] [none] Number identifying a mesh domain

kind [name(2)] ['ELAST',''] Material types:

'ELAST', 'DMG', 'PLAST', 'KV'

NOTE : Some combinations of material kinds can be assigned to the same domain.

Any material type can be combined with 'KV', for instance:

&MATERIAL tag=1, kind='ELAST', 'KV' /

followed by a &MAT_ELAST block and a &MAT_KV block sets an elastic material with Kelvin-Voigt damping.

NAME : MAT_KV GROUP : MATERIALS

PURPOSE: Set material properties for Kelvin-Voigt viscosity

Adds a damping term C*v = K*eta*v, where eta is a viscous time This produces attenuation with frequency-dependent quality factor

Q(f) = 1/(eta*2*pi*f)

This is often useful as an artificial damping layer in fault zones to control high-frequency numerical artifacts, setting eta=0.1*dt

and a layer thickness of 4 to 5 GLL nodes.

SYNTAX : &MAT_KV eta|etaH, ETAxDT /

```
followed by a DIST_XXX input block if etaH is present.
  eta [dble][0d0] Viscosity coefficient
  ETAxDT [log][T] If eta is given in units of dt (timestep)
NAME
        : MAT_PLASTIC
GROUP : MATERIALS
PURPOSE: Set material properties for elasto-plastic material
          with Mohr-Coulomb yield criterion
          and non-dilatant (null volumetric plastic strain)
SYNTAX : &MAT_PLASTIC cp,cs,rho,phi,coh,Tv,e0,ep /
           [dble] [0d0] P wave velocity
  ср
  cs
           [dble] [0d0] S wave velocity
           [dble][0d0] density
  rho
           [dble][0d0] internal friction angle
  phi
  coh
           [dble][0d0] cohesion
           [dble][0d0] Maxwellian visco-plastic timescale
  Tv
           [dble(3)][0d0] initial total strain (11, 22 and 12)
  e0
           [dble(3)][0d0] initial plastic strain (11, 22 and 12)
  ер
NAME
        : MESH_CART
GROUP : MESH_DEF
PURPOSE: Rectangular box with structured mesh.
SYNTAX : &MESH_CART xlim, zlim, nelem, ezflt, FaultX /
  xlim
           [dble(2)] [none] X limits of the box (min and max)
           [dble(2)] [none] Z limits of the box (min and max)
  7.1 i m
           [int(2)] [none] Number of elements along each direction
  nelem
  ezflt
           [int][0] introduce a horizontal fault between the ezflt-th
               and the (ezflt+1)-th element rows.
If ezflt=0, no fault is introduced (default).
If ezflt=-1, a fault is introduced at (or near) the middle of the box
           [log] [F] (obsolete, will be deprecated, same as ezflt=-1)
  FaultX
NOTE: the following tags are automatically assigned to the boundaries:
               1
                       Bottom
               2
                       Right
               3
                       Top
                       Left
                       Fault, bottom side
                       Fault, top side
```

NAME : MESH_CART_DOMAIN

PURPOSE: Define a subdomain within a structured meshed box.

SYNTAX : &MESH_CART_DOMAIN tag,ex,ez /

tag [int] [none] Tag number assigned to this domain.

ex [int(2)] [none] First and last element along the X direction. ez [int(2)] [none] First and last element along the Z direction.

NOTE : If you ignore this input block a single domain (tag=1) will span

the whole box

NAME : MESH_EMC2 GROUP : MESH_DEF

PURPOSE: Imports a mesh from INRIA's EMC2 mesh generator in FTQ format

SYNTAX : &MESH_EMC2 file /

file [name] [none] Name of the FTQ file, including suffix

NAME : MESH_DEF

PURPOSE: Selects a method to import/generate a mesh.

SYNTAX : &MESH_DEF method /

method [name] [none] 'CARTESIAN', 'LAYERED' or 'EMC2'

The &MESH_DEF input block must be followed by a

&MESH_method input block

NAME : MESH_LAYERED GROUP : MESH_DEF

PURPOSE: Structured mesh for layered medium

with surface and interface topography.

SYNTAX : &MESH_LAYERED xlim,zmin,nx,file,nlayer,ezflt /

xlim [dble(2)] [none] X limits of the box (min and max)

zmin [dble] [none] bottom Z limit of the box

nx [int] [none] Number of elements along X direction

file [string] [''] Only for flat layers,

name of ASCII file containing layer parameters, one line per layer, listed from top to bottom, $\,$

3 columns per line:

```
(1) vertical position of top boundary,
```

(2) number of elements along Z direction

(3) material tag

nlayer [int] [none] Number of layers

If a file name is not given the layer parameters must be given immediately after the &MESH_LAYERED block by nlayer &MESH_LAYER input blocks,

one for each layer, listed from top to bottom.

ezflt [int][0] introduce a fault between the ezflt-th and the (ezflt+1)-th element rows. If ezflt=0 (default), no fault is introduced.

NOTE: the following tags are automatically assigned to the boundaries:

1 Bottom

2 Right

3 Top

4 Left

5 Fault, bottom side

6 Fault, top side

NAME : MESH_LAYER GROUP : MESH_DEF

PURPOSE: Define mesh parameters for one layer SYNTAX : &MESH_LAYER nz, ztop|ztopH, tag /

followed by a DIST_XXXX block if ztopH is set

nz [int] [none] Number of elements in layer along Z direction

ztop [dble] [none] Only for layers with flat top surface:

vertical position of top boundary

ztopH [string] ['none'] Only for layers with irregular top boundary:
name of distribution, 'LINEAR', 'SPLINE' or any other
1D distribution available through a DIST_XXXX block.

[int] [none] Material tag

If not given, a tag is automatically assigned to the layer, sequentially numbered from top to bottom (top layer tag =1)

NAME : SNAP_DEF

tag

GROUP : SNAPSHOT_OUTPUTS

PURPOSE: Set preferences for exporting snapshots

SYNTAX: &SNAP_DEF it1, itd, fields, components, bin, visual3, avs, ps, gmt /

Followed by a &SNAP_PS block if ps=T.

it1 [int] [0] Time step of first snapshot output

```
[int] [100] Number of timesteps between snapshots
 itd
 fields
          [char*] ['V'] fields to export in snapshots (the prefix of the
               output file names is given in parenthesis):
                'D'
                        displacements (dx,dy,dz,da)
                ٠V,
                        velocity (vx,vy,vz,va)
                , Α,
                        acceleration (ax,ay,az,aa)
                'E'
                        strain (e11,e22,e12,e23,e13)
                s,
                        stress (s11,s22,s12,s33,e13,e23)
                'd'
                        divergence rate (dvx/dx + dvz/dz)
                , c,
                        curl rate (dvx/dz - dvz/dx)
 components [char*] ['ya'] components for PostScript outputs:
                in P-SV: 'x', 'z' and/or 'a' (amplitude). 'y' is ignored
                         'y' only. Other values are ignored.
 ps
          [log] [T] PostScript (see &SNAP_PS input block)
          [log] [F] output triangulation file grid_sem2d.gmt
 gmt
                to be used in "pscontour -T" of the General Mapping Tool (GMT)
          [log] [F] AVS (only for D,V and A fields)
 avs
          [log] [F] Visual3 (only for D,V and A fields)
 visual3
 bin
          [log] [T] binary
NOTE
       : E and S fields are exported only as binary.
NAME
       : SNAP_PS
GROUP : SNAPSHOT_OUTPUTS
PURPOSE: Preferences for PostScript snapshots
SYNTAX: &SNAP_PS vectors, mesh, background, color,
              isubsamp, boundaries, symbols, numbers, legend,
              ScaleField, Interpol, DisplayPts /
 vectors
                  [log] [F] Plots a vectorial field with arrows
                  [log] [F] Plots the mesh on background
 mesh
                  [char] [''] Filled background, only for vector plots:
 background
                                   , ,
                                        none
                                   P'
                                       P-velocity model
                                       S-velocity model
                                   'Τ'
                                       domains
 isubsamp
                  [int] [3] Subsampling of the GLL nodes for the
                                output of velocity model.
                                The default samples every 3 GLL points.
 boundaries
                  [log] [T] Colors every tagged boundary
                  [log] [T] Plots symbols for sources and receivers
 symbols
 numbers
                  [log] [F] Plots the element numbers
 legend
                  [log] [T] Writes legends
 color
                  [log] [T] Color output
 ScaleField
                  [dble] [0d0] Fixed amplitude scale (saturation),
```

convenient for comparing snapshots and making movies. The default scales each snapshot by its maximum amplitude Interpol [log] [F] Interpolate field on a regular subgrid inside each element [log] [3] Size of interpolation subgrid inside each DisplayPts element is DisplayPts*DisplayPts. The default plots at vertices, mid-edges and element center. NAME : REC_LINE PURPOSE: Defines a line of receivers SYNTAX : If single receiver line: &REC_LINE number, first, last, AtNode, isamp, field, irepr / If receiver locations from file: &REC_LINE file, AtNode, isamp, field, irepr / [int] [0] Number of stations in the line number first [dble(2)] Receivers can be located along a line, this is the position (x,z) of the first receiver last [dble(2)] Position (x,z) of the last receiver, other receivers will be located with regular spacing between First and Last. file [name] ['none'] Station positions can instead be read from an ASCII file, with 2 columns per line: (1) X position (in m) and (2) Z position (in m) AtNode [log] [T] Relocate the stations at the nearest GLL node [int] [1] Sampling stride (in number of timesteps). Note that isamp for stability reasons the timestep can be very small. field [char] ['V'] The field in the seismogram: 'D' displacement ,۸, velocity 'A' acceleration [char] ['D'] Abscissa for the seismic multitrace plot: irepr 'X' Horizontal position 'Z' Depth 'D' Distance to the first station NOTE : to locate receivers at the free surface set their vertical position above the free surface and AtNode=T

NAME : SRC_FORCE

GROUP : SOURCE MECHANISM PURPOSE: Point force source

SYNTAX : &SRC_FORCE angle /

angle [dble] [0d0] For P-SV, the angle of the applied force,

in degrees, counterclockwise from Z-UP, e.g.:

(90 points left, 180 points down)

For SH, angle is ignored.

NAME : SRC_DEF

PURPOSE: Define the sources.

SYNTAX : &SRC_DEF stf, mechanism, coord, file /

followed by one SOURCE MECHANISM block (SRC_XXXX) and one SOURCE TIME FUNCTION block (STF_XXXX)

stf [name] [none] Name of the source time function:

'RICKER', 'TAB', 'HARMONIC' or 'USER'

mechanism [name] [none] Name of the source mechanism:

'FORCE', 'EXPLOSION', 'DOUBLE_COUPLE', 'MOMENT' or 'WAVE'

coord [dble(2)] [huge] Location (x,z) of the source (m).

file [string] ['none'] Station coordinates and delay times can

be read from an ASCII file, with 3 columns per line:

(1) X position (in m),

(2) Z position (in m) and

(3) time delay (in seconds)

NAME : SRC_DOUBLE_COUPLE
GROUP : SOURCE MECHANISM

PURPOSE: Define a double-couple source
SYNTAX : &SRC_DOUBLE_COUPLE dip /

dip [dble] [90] Dip angle, in degrees, clockwise

from the positive X direction

NOTE : Sign convention: if the source amplitude is positive the right block

moves up (positive Z direction) in PSV and forward (positive Y

direction) in SH.

NOTE : The source time function gives the seismic moment, NOT the moment rate

NAME : SRC_MOMENT

GROUP : SOURCE MECHANISM

PURPOSE: Define a moment tensor source

```
SYNTAX : &SRC_MOMENT Mxx, Mxz, Mzx, Mzz , Myx, Myz /
Mxx,Mxz,Mzx,Mzz [dble] [0] Tensor components for PSV
               [dble] [0] Tensor components for SH
NAME
       : SRC_WAVE
GROUP : SOURCE MECHANISM
PURPOSE: Incident plane wave through the absorbing boundaries
SYNTAX : &SRC_WAVE angle, phase /
 angle
          [dble] [0d0]
                          Incidence angle in degrees within [-180,180]
                counterclockwise from the positive Z (up) direction
                to the wave vector direction:
                Exs: incidence from below if angle in ]-90,90[
                     normal incidence from below if angle=0
                     from bottom right if angle=+45
                     from bottom left if angle=-45
                          'S' or 'P' (only needed in PSV, ignored in SH)
          [char] ['S']
 phase
NOTE
       : Incident waves enter through the absorbing boundaries.
         An incident wave is applied on every absorbing boundary
         unless "let_wave = F" in the respective BC_ABSO block.
         Incident waves are not implemented for "Stacey" absorbing boundaries.
NAME : STF_BRUNE
GROUP : SOURCE TIME FUNCTIONS
PURPOSE: Brune (1970)'s model with omega-squared spectral fall-off:
           stf(t) = ampli*(1 + (1+fc*t)*exp(-fc*t))
SYNTAX : &STF_BRUNE ampli, fc /
          [dble] [1d0] Amplitude (usually the seismic moment)
 ampli
          [dble] [1d0] Corner frequency (Hz)
NAME
       : STF_HARMONIC
GROUP : SOURCE TIME FUNCTIONS
PURPOSE: Harmonic source time function f(t) = ampli*sin(2*pi*t*f0)
SYNTAX : &STF_HARMONIC ampli, f0 /
 ampli
          [dble] [0d0] Amplitude
 f0
          [dble] [0d0] Frequency
```

NAME : STF_RICKER

GROUP : SOURCE TIME FUNCTIONS

PURPOSE: The Ricker wavelet is the second derivative of a gaussian.

SYNTAX : &STF_RICKER ampli, f0, onset /

ampli [real] [1.] Signed amplitude of the central peak

f0 [real >0] [0] Fundamental frequency (Hz).

distribution: it has a peak at f0 and an exponential

decay at high frequency. The cut-off high frequency is usually

taken as $fmax = 2.5 \times f0$.

onset [real >1/f0] [0] Delay time (secs) with respect to the peak value.

NOTE : The spectrum has a peak at fO and decays exponentially at high

frequencies. Beyond 2.5*f0 there is little energy, this is a

recommended value for fmax.

NOTE : onset>1/f0 is needed to avoid a strong jump at t=0, which can cause

numerical oscillations. Ignore if using incident waves.

NAME : STF_TAB

GROUP : SOURCE TIME FUNCTIONS

PURPOSE: Source time function spline-interpolated from values in a file

SYNTAX : &STF_TAB file /

file [string] ['stf.tab'] ASCII file containing the source time function, two columns per line:

(1) time

(2) value

NOTE : assumes value(t<min(time))=value(min(time))</pre>

and value(t>max(time))=value(max(time))

NOTE : time can be irregularly sampled

NAME : STF_USER

GROUP : SOURCE TIME FUNCTIONS

PURPOSE: A template for user-supplied source time function.

File stf_user.f90 must be modified by the user to fit

special needs.

SYNTAX : &STF_USER ampli, onset, par1, par2, ipar1, ipar2 /

ampli [dble] [1.] Amplitude

onset [dble] [0] Delay time (secs)

```
par1
           [dble] [0] Example parameter
           [dble] [0] Example parameter
 par1
 par1
           [int] [0] Example parameter
           [int] [0] Example parameter
 par1
NAME
        : TIME
PURPOSE: Defines time integration scheme
SYNTAX: &TIME kind, {Dt or Courant}, {NbSteps or TotalTime} /
          Possibly followed by a TIME_XXXX block.
 kind
           [char*10] ['leapfrog'] Type of scheme:
                                       Explicit Newmark
                       'newmark'
                                       Explicit HHT-alpha
                       'HHT-alpha'
                       'leapfrog'
                                       Central difference
                       'symp_PV'
                                       Position Verlet
                       'symp_PFR'
                                       Position Forest-Ruth (4th order)
                       'symp_PEFRL'
                                       Extended PFR (4th order)
 Dt
           [dble] [none] Timestep (in seconds)
           [dble] [0.5d0] the maximum value of the Courant-Friedrichs-Lewy
stability number (CFL), defined as
                 CFL = Dt*wave_velocity/dx
where dx is the distance between GLL nodes. Tipically CFL<= 0.5
 NbSteps [int] [none] Total number of timesteps
 TotalTime[int] [none] Total duration (in seconds)
NOTE
        : The leap-frog scheme is recommended for dynamic faults. It is equivalent
          to the default Newmark scheme (beta=0, gamma=1/2). However it is
          faster and requires less memory.
NAME
        : TIME_NEWMARK
GROUP : TIME SCHEMES
PURPOSE: Explicit Newmark time integration scheme
SYNTAX : &TIME_NEWMARK gamma, beta /
 beta
           [dble] [0d0] First Newmark parameter.
If beta=0 the scheme is fully explicit (the update of
               displacement depends only on the last value of acceleration),
               otherwise it is a single-predictor-corrector scheme
           [dble] [0.5d0] Second Newmark parameter.
Second order requires gamma=1/2.
```

NAME : TIME_HHTA
GROUP : TIME SCHEMES

PURPOSE: Explicit HHT-alpha time integration scheme, second order

SYNTAX : &TIME_HHTA alpha, rho /

alpha [dble] [0.5d0] Parameter in the HHT-alpha method. Values in [0,1].

Defined here as 1 + HHT's original definition of alpha. When alpha=1 it reduces to second order explicit Newmark

(beta=0, gamma=0.5).

rho [dble] [0.5d0] Minimum damping factor for high frequencies.

Values in [0.5,1]. Rho=1 is non-dissipative.

NOTE: We consider only second order schemes, for which alpha+gamma=3/2

If alpha<1, the Newmark parameter beta is related to the HHT parameters by

beta = 1 -alpha -rho^2*(rho-1)/[(1-alpha)*(1+rho)^3]

If alpha=1, we set rho=1 (beta=0, gamma=0.5)

NOTE: Dissipative schemes (rho<1) require slightly smaller Courant number (0.56 for rho=0.5, compared to 0.6 for rho=1)

NOTE: This is an explicit version of the HHT-alpha scheme of
H.M. Hilber, T.J.R. Hughes and R.L. Taylor (1977) "Improved numerical
dissipation for time integration algorithms in structural dynamics"
Earthquake Engineering and Structural Dynamics, 5, 283-292
implemented with a slightly different definition of alpha (1+original).

Its properties can be derived from the EG-alpha scheme of G.M. Hulbert and J. Chung (1996) "Explicit time integration algorithms for structural dynamics with optimal numerical dissipation" Comp. Methods Appl. Mech. Engrg. 137, 175-188 by setting alpha_m=0 and alpha=1-alpha_f.

3.6 Verifying the settings and running a simulation

Once the code has been successfully compiled, the simulation can be started by typing sem2dsolve from your working directory, which contains the file Par.inp. The computations can be run in background and the screen output saved in a file (e.g. info) by typing sem2dsolve > info &.

A typical screen output of SEM2D, corresponding to the first example, is shown on the following pages. The parameters of the simulation and some verification information are reported there in a self-explanatory form. You are advised to do a first run with <code>iexec=0</code> in the <code>GENERAL</code> input block and check all these informations prior to the real simulation. You should always verify the following:

• Stability: the CFL stability number should be smaller than $0.55 \sim 0.60$ for second order time schemes. This number is defined at each computational node as

$$CFL = c_P \Delta t / \Delta x$$

where Δt is the timestep, c_P the P-wave velocity and Δx the local grid spacing. Note that Δx is usually much smaller than the element size h ($\approx \text{Ng11}^2$ times smaller) because SEM internally subdivides each element onto a non-regular grid of Ng11×Ng11 nodes clustered near the element edges (Gauss-Lobatto-Legendre nodes). If the computation is unstable, the maximum displacement, printed every ItInfo time steps, increases exponentially with time. Stability can be controlled by decreasing Dt or Courant in Par.inp.

• Resolution: the number of nodes per shortest wavelength λ_{min} should be larger than $4.5 \sim 5$. The minimum wavelength is defined as

$$\lambda_{min} = \min(c_S)/f_{max}$$

where c_S is the S-wave velocity and f_{max} the highest frequency you would like to resolve, e.g. the maximum frequency at which the source spectrum has significant power (for a Ricker wavelet $f_{max} = 2.5 \times f_0$). For an element of size h and polynomial order p = Ngll - 1, the number of nodes per wavelength G is

$$G = \frac{p \lambda_{min}}{h}$$
.

Typical symptoms of poor resolution are ringing and dispersion of the higher frequencies. However, in heterogeneous media these spurious effects might be hard to distinguish from a physically complex wavefield, so mesh resolution must be checked beforehand. If resolution is too low the mesh might be refined by increasing Ngll in Par.inp (p-refinement) or by generating a denser mesh (h-refinement). If you were using EMC2 as a mesh generator, the script PRE/href.csh can be useful for h-refinement.

• Cost: the total CPU time an memory required for the simulation are as much as you can afford. Estimates of total CPU time are printed at the end of check mode. Details about memory usage can be found in MemoryInfo_sem2d.txt.

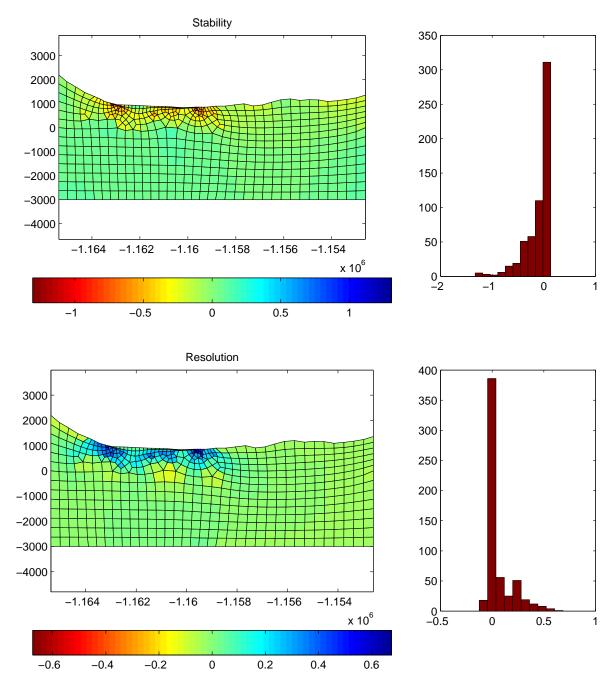


Figure 3.3: Checking the quality of a mesh with PRE/ViewMeshQuality.m for the example in EXAMPLES/UsingEMC2/. The balance of the stability and resolution properties of the mesh can be analyzed: logarithmic stability index (top) and logarithmic resolution index (bottom). Histograms of these indices (in number of elements) are shown on the right.

The quality of the mesh can be inspected with the Matlab script PRE/ViewMeshQuality.m which produces plots like in Figure 3.3. The proper balance of the mesh with respect to the following two criteria can be analyzed:

- Stability criterion, related to the largest stable timestep. On each element we define a stability index as the logarithm of $\min(\Delta x/c_P)$ normalized by its median value over the whole mesh. Red elements (small stability index) are relatively unstable and require small timesteps Δt . Because Δt is constant over the whole mesh and the computational cost is inversely proportional to Δt these red elements penalize the computational efficiency. The mesh should be redesigned to increase their size, as much as possible, while keeping them small enough to resolve the shortest wavelength (see next).
- Resolution criterion, related to the number of nodes per shortest wavelength. On each element we define a resolution index as the logarithm of $\min(c_S/h)$ normalized by its median value over the whole mesh. Red elements (small resolution index) have relatively poor resolution, in their vicinity the maximum frequency resolvable by the mesh is limited. The mesh should be redesigned to decrease their size, as much as possible. Conversely, elements with very high resolution index (blue) are smaller than required and might increase the computational cost.

To minimize the CPU and memory cost of a simulation an ideal mesh design should minimize the spread of the two indices above, by aiming at a ratio of element size to wave velocity, h/c, as uniform as possible across the whole mesh. However, in some cases a poorly balanced mesh is inevitable: in the example of Figure 3.3 the worst elements are near the edges of the sedimentary basin, at a sharp velocity contrast. Small element sizes on the rock side are inherited from the sediment mesh.³

Similar information is plotted by gv Stability_sem2d.ps and gv Resolution_sem2d.ps. The indices in these files are however not logarithmic and are not normalized by the median.

³In future releases of SEM2DPACK this penalty on computational efficiency will be reduced by non-conformal meshing with mortar elements, by timestep subcycling or by implicit/explicit timestep partitioning.

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```
Mar 06. 08 18:04
                                info
                                                        Page 1/4
Program SPECFEM: start
____
Date: 06 - 03 - 2008
                                             Time: 18:03:23
* Input phase *
General Parameters
______
   Execution mode . . . . . . . . . . . . . (iexec) = solve
   Highest frequency to be resolved . . . . . . (fmax) = 1.250E+00
   Print progress information during
            input phase . . . . (verbose(1)) = T
initialization phase . . (verbose(2)) = T
checking phase . . . (verbose(3)) = T
solver phase . . . (verbose(4)) = T
   Frequency for solver progress information .(itInfo) = 1000
Mesh Generation
-----
   Method. . . . . . . . . . . . . . . . . (method) = CARTESIAN
   Number of elements along X.....(nelem(1)) = 60
   Number of elements along Z. . . . . (nelem(2)) = 60
Cut by horizontal fault . . . . . . . (faultx) = F
Time integration
-----
   Time step increment . . . . . . (Dt) = will be set later

Courant number . . . . . . . . (Courant) = 0.30

Total simulation duration . (TotalTime) = 35.000E+00
Material Properties
-----
   Number of materials . . . . . . . . . . = 1
   Material index. . . . . . . . . (tag) = 1
   Material type (kind) = Elastic
P-wave velocity (cp) = 1.732E+00
S-wave velocity (cs) = 1.000E+00
   Mass density. . . . . . . . . . (rho) = 1.000E+00
```

```
info
 Mar 06. 08 18:04
                                                                  Page 2/4
    Poisson's ratio . . . . . . . . = 250.021E-03
First Lame parameter Lambda . . . . . = 1.000E+00
     Second Lame parameter Mu. . . . . . . . =
    Young's modulus E . . . . . . . . . . = 2.500E+00
Boundary Conditions
 . . . . . . . . . (tag) = 2
    Boundary tag. .
    Boundary condition. . . . (kind) = ABSORB
Type of absorbing boundary. . .(stacey) = Clayton-Engquist
Allow incident wave . . . .(let_wave) = T
    Boundary condition. . . . . . . (kind) = ABSORB
     Type of absorbing boundary. . . . (stacey) = Clayton-Engquist
    Allow incident wave . . . . . (let_wave) = T
Sources
    Source time function . . . . . . . . . . . . Ricker Fundamental frequency (Hz) . . . (f0) = 500.000E-03
    Time delay (s) . . . . . . . . . (onset) = 3.000E+00 Multiplying factor . . . . . (ampli) = 250.000E-03
    Source Type. . . . . . . . . . . = Collocated Force
    If P-SV: counterclockwise angle / up . = 0.00
Receivers
    Number of receivers . . . . . . . . . . . (number) = 7
    Subsampling for seismograms recording . . . . (isamp) = 1
    Field recorded. . . . . . . . . . . . . (field) = D
Axis of the seismogram plot . . . . . . (irepr) = D
Snapshot Outputs
_____
     Timestep of first snapshot output . . . . . (it1) = 0
    Number of timesteps between snapshots. . . . (itd) = 100000 Save results in PS file or not . . . . . . . . . . . . . . (ps) = F
    Save grid triangulation for GMT. . . . . . (gmt) = F
    Save results in AVS file or not. . . . . . (avs) = F Save results in Visual3 file or not. . . . (visual3) = F
    Save results in binary file or not . . . . . (bin) = F
     Selected fields :
      Displacement . . . . . . . . . . . . . . . . . . = F
      Velocity . . . . . = T
Acceleration . . . . = F
      Strain . . . . . . . . . . . . . . . . . . = F
    Amplitude . . . . . . . . . . . . . . . . . . = F
* Initialization phase *
```

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Mar 06, 08 18:04 info	Page 3/4
Defining the FEM mesh [OK] Saving node coordinates in file MeshNodesCoord_sem2d.tab Saving element connectivity in file ElmtNodes_sem2d.tab [[OK]
Spectral elements grid	
Numbering GLL points [OK] Total number of GLL points = 90601	
Saving element/node table in binary file ibool_sem2d.dat Defining nodes coordinates [OK]	[OK]
Saving the grid coordinates (coord) in a text file [OK] Saving the grid coordinates (coord) in a binary file [OK]	
Material properties	
Translating input model [OK] Exporting model [OK]	
Mesh properties	
Checking mesh [OK] Max mesh size = 142.616E-03 Min mesh size = 58.736E-03 Ratio max/min = 2.428E+00	
RESOLUTION: nodes per min wavelength = 8.000E+00 for maximum frequency = 1.250E+00 Hz minimum wavelength = 1.600E+00 m	
<pre>Dump PostScript Resolution_sem2d.ps [OK] Dump PostScript Stability_sem2d.ps [OK]</pre>	
Time solver	
Time step (secs) = 17.621E-03 Number of time steps = 1987 Total duration (secs) = 35.013E+00 Courant number = 300.000E-03	
STABILITY: CFL number = 300.000E-03	
Defining work arrays for elasticity [OK] Initializing kinematic fields [OK] Max displ = 0.000E+00 Max veloc = 0.000E+00	
Building the mass matrix [OK] Defining boundary conditions [OK] Initializing receivers	
Receivers	
Receivers have been relocated to the nearest GLL node	
Receiver x-requested z-requested x-obtained z-obtained dista	ance
2 5.000E+00 0.000E+00 5.000E+00 0.000E+00 0.00 3 10.000E+00 0.000E+00 10.000E+00 0.000E+00 0.00	00E+00 00E+00 00E+00 00E+00

Mar 06, 08 18:04		info		Page 4/4
5 20.000E+00 6 25.000E+00 7 30.000E+00	0.000E+00 0.000E+00 0.000E+00	20.000E+00 25.000E+00 30.000E+00	0.000E+00 0.000E+00 0.000E+00	0.000E+00 0.000E+00 0.000E+00
Maximum distance betwe	en asked and	real = 0.0	00E+00	
Sampling rate (Hz) Sampling timestep (sec Total number of sample Number of receivers	= 56.79 s) = 17.63 s = 1988 = 7	51E+00 21E-03		
[OK] Initializing source	s			
Sources				
Sources have been reloc	ated to the 1	nearest GLL n	iode	
Source x-requested	z-requested	x-obtained	z-obtained	distance
1 0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
Maximum distance betwe	en requested	and real =	0.000E+00	
[OK] Timestep # 0 t =	0.000E+00	vmax = 0.0	00E+00 dmax =	0.000E+00
**************************************	phase	*		
CPU TIME ESTIMATES CPU time for initializ CPU time per timestep Total solver CPU time (mins (hour	ation 806	5.877E-03 3.997E-03 7.747E+00 9.118E-03		
Timestep # 1000 t =	17.621E+00	vmax = 91.6	61E-03 dmax =	28.653E-03
CPU TIME INFORMATIO CPU time for initializ CPU time per timestep Total solver CPU time (mins (hour	ation 800	5.877E-03 0.395E-03 0.524E+00 5.397E-03		
Storing sismos data (SE	P format)			
Program SPECFEM	: end			
Test SH				
Date: 06 - 03 - 200	8		Tim	e : 18:04:11

3.7 Outputs, their visualization and manipulation

In addition to the screen output described above, sem2dsolve generates different files and scripts that allow the user to control the parameters of the simulation and to display the results. All the outputs files follow the naming convention SomeName_sem2d.xxx, where xxx is one of the following extensions: tab for ASCII data files, txt for other text files, dat for binary data files, etc. This makes it easy to clean a working directory with a single command like rm -f *_sem2d*.

3.7.1 Spectral element grid

As explained in the previous section, sem2dsolve generates two PostScript files for mesh quality checking purposes: Stability_sem2d.ps and Resolution_sem2d.ps. The relevant information is contained in the files Stability_sem2d.tab and Resolution_sem2d.tab and can also be inspected with the Matlab script PRE/ViewMeshQuality.m.

3.7.2 Source time function

sem2dsolve generates a file called SourcesTime_sem2d.tab containing the source time function sampled at the same rate as the receivers. It is important to verify that the spectrum of the source has little power at those high frequencies that are not well resolved by the mesh (those that correspond to less than 5 nodes per wavelength). If this is not the case you must be very cautious in the interpretation of the seismograms in the high frequency range, or low-pass filter the results.

3.7.3 Snapshots

sem2dsolve generates snapshots at a constant interval defined, in number of solver timesteps, by the input parameter ItSnapshots. An example is shown in Figure 3.4. Requested fields are exported in binary data files called xx_XXX_sem2d.dat, where xx is the field code defined in the documentation of the PLOTS input block and XXX is the 3-digit snapshot number. The user is encouraged to inspect the Matlab script POST/sample_snapshots.m and function POST/sem2d_snapshot_read.m to find more about the data formats and their manipulation.

If requested PostScript files xx_XXX_sem2d.ps are also dumped. A movie file movie.gif can be generated by the script POST/movie.csh and displayed by xanim movie.gif.

3.7.4 Seismograms

The seismograms are stored using the SEP format, a simple binary block of single precision floats. The components of the vector field (velocity by default) are stored in separate files U*_sem2d.dat, where * is x or z in P-SV and y in SH. The seismograms header is in the file

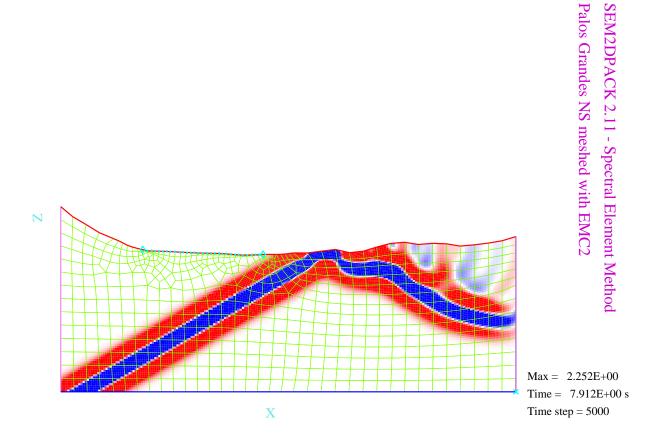


Figure 3.4: Sample snapshot from EXAMPLES/UsingEMC2/: an obliquely incident SH plane wave impinging on a sedimentary basin. The unstructured mesh of spectral elements is plotted on background.

SeisHeader_sem2d.hdr. Its second line contains the sampling timestep DT, the number of samples NSAMP and the number of stations NSTA. The stations coordinates, XSTA and ZSTA, are listed from the third line to the end of file. With this notations, U*_sem2d.dat contains a NSAMP×NSTA single precision matrix.

You can view the seismograms using any tool that is able to read the SEP format, which is the case of almost all the softwares able to deal with seismic data. sem2dsolve generates scripts for the XSU-Seismic Unix visualization tool⁴:

- Xline_sem2d.csh displays all seismograms together on screen
- PSline_sem2d.csh plots all seismograms on PostScript files U*Poly_sem2d.ps
- Xtrace_sem2d.csh prompts the user for a trace number (between 1 and NSTA) and then displays this particular trace on screen
- PStrace_sem2d.csh does the same as Xtrace, but exports the traces as PostScript files U*TraceXXX_sem2d.ps where XXX is the number of that particular trace

The program post_seis.exe performs similar basic manipulation and plotting (through gnuplot) of the seismograms. Its interactive menu is self-explanatory. It is usually called inside a script, as in POST/seis_b2a.csh (converts all seismograms to ASCII) or POST/seis_plot.csh (plots all seismograms together, an example is shown in Figure 3.5).

The script POST/sample_seis.m shows how to manipulate and plot seismogram data in Matlab. It uses the functions POST/sem2d_read_seis.m and POST/plot_seis.m.

3.7.5 Fault outputs

Fault data from dynamic rupture simulations is stored in FltXX_sem2d.dat, where XX is the boundary tag of the first side of the fault, tags(1) of the BC_SWFFLT input block. Every DELT seconds (NSAMP total output times) NDAT lines with NPTS columns, one per fault node, are written ⁵. Stress fields are relative to their initial values, which are contained in the first NDAT lines. The header file FltXX_sem2d.hdr contains the information needed to read the data file. Its format, line by line, is:

- 1. NPTS NDAT NSAMP DELT (name of parameters)
- 2. Value of parameters above
- 3. Name of data fields, separated by ":"
- 4. XPTS ZPTS (name of coordinate axis)

⁴Seismic Unix is freely available from the Colorado School of Mines at http://timna.mines.edu/cwpcodes ⁵The actual number of columns is NPTS +2: Fortran adds a one-word tag at the front and end of each record.

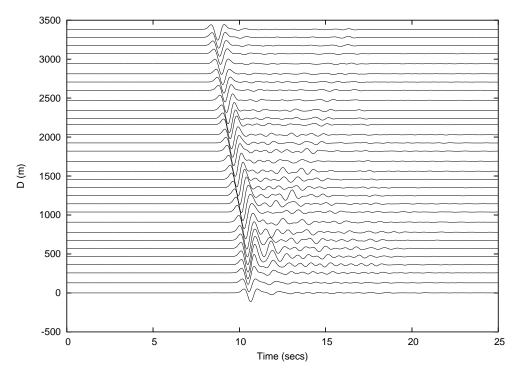


Figure 3.5: Sample seismograms from EXAMPLES/UsingEMC2/ generated with POST/seis_plot.csh.

5. from here to the end of file, a two-column table of coordinates of the output fault nodes

The script FltXX_sem2d.csh shows how to extract ASCII time series of different fields at given locations on the fault, using Seismic Unix tools.

The program post_fault.exe performs basic manipulations of the fault data, including conversion to an ASCII file readable by gnuplot. Its interactive menu is self-explanatory.

The script POST/sample_fault.m and function POST/sem2d_read_fault.m show how to manipulate and plot fault data in Matlab.

3.7.6 Matlab utilities

A range of functions and sample scripts for Matlab are available to read, manipulate and plot output data. Add the directory POST/ to your Matlab path (addpath). For an overview of existing utilities, type help POST:

XCORRSHIFT Cross-correlation time-delay measurement
SPECSHIFT signal time shift by non-integer lag via spectral domain
SPECFILTER zero-phase Butterworth filter via spectral domain
PLOT_SEIS plots multiple seismogram traces, with a vertical offset
SAMPLE_SNAPSHOTS example of visualization of snapshot outputs from SEM2DPACK

SAMPLE_FAULT example of visualization of fault output from SEM2DPACK SAMPLE_SEIS example of visualization of seismogram output from SEM2DPACK SEM2D_SNAPSHOT_GUI interactive plot of snapshot outputs from SEM2DPACK PLOT_FRONTS space-time plot of rupture front and process zone tail SEM2D_SNAPSHOT_READ reads a snapshot output from SEM2DPACK SEM2D_READ_SPECGRID reads a spectral element grid SEM2D_SNAPSHOT_PLOT color-plots a snapshot output from SEM2DPACK SEM2D_READ_SEIS reads seismogram outputs from SEM2DPACK SEM2D_READ_FAULT reads fault outputs from SEM2DPACK

Chapter 4

Generating a mesh with EMC2

To obtain optimal accuracy the spectral element method requires a mesh in which the faces of the quadrilateral elements coincide with the interfaces between different materials. For dynamic faulting problems the element faces must necessarily coincide with the fault planes. SEM2DPACK provides only basic meshing capabilities and does not include an unstructured mesh generator for complicated, realistic geological models. This chapter describes how to achieve that task with an external software, EMC2.

Generating a high quality unstructured quad mesh can be a time-consuming task. Let's note that, for wave propagation problems without dynamic faults, if the acceptable accuracy is low (or large computational resources are available to work with a very fine mesh) a structured mesh in which the element faces do not necessarily follow the material interfaces can be generated with the basic built-in meshing capabilities of SEM2D.

4.1 The mesh generator EMC2

EMC2 is one of the few public domain 2D mesh generators including quadrilateral elements and a Graphical User Interface. It is a C code which sources and executables can be freely downloaded from http://www.ann.jussieu.fr/~hecht/ftp/emc2/. Although a complete documentation can be found in that package, we show here an example featuring the most useful functionalities.

Before starting you must provide files containing 2-column data (X,Z), without headers, of all the points needed to define the geometry of the model (topography, sediment bottom).

Once installed, you can run EMC2 by typing emc2.

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4.2 Notations

The following notations are assumed in this chapter:

- (XXX) = click XXX on top menu bar
- (xxx) = click xxx on bottom menu bar
- \bullet <XXX< = click XXX on left menu bar
- \bullet >XXX> = click XXX on right menu bar
- \$xxx\$ = enter xxx from keyboard or from the calculator in the right panel
- "xxx" = type xxx in bottom prompt
- $\{xxx\}$ = perform action xxx
- *xxx = do xxx as many times as needed
- n*xxx = do xxx n times

4.3 Basic step-by-step

A typical EMC2 session has three steps:

STEP I: CONSTRUCT, defines the geometry of the model

1. Switch to the construction tool:

<CONSTRUCTION<

2. Load the points:

(POINT) (xy file) "palosgrandes.dat"

You must give the full path to your points-file, the root directory being the one where you launched emc2.

3. Reset the figure window to fit all points:

>SHOW ALL>

The original data has some geometrical features that are too complex to be meshed by quadrilaterals, for instance the corners at the N and S ends of the basin, you may want to smooth out these features. You also need to define the extreme boundaries of the region to be modelled (N,S and bottom absorbing boundaries) and some additional points on the free surface outside the basin. You must modify the data set (add and delete points):

- 4. Add new points:
 - a. with the mouse:

```
(POINT) (mouse) *{click in figure window}
```

b. by coordinates:

(POINT) (xy pt)
$$*{ x=y=}$$

This is the safest way to get really vertical and horizontal boundaries needed for the absorbing conditions in SPECFEM90. You probably need to get the coordinates of an existent reference point:

- c. you can also reload another point-file (I2)
- 5. Delete points,

```
(POINT) < DESTRUCT < (point) *{click on point}
```

Now you must define the geometry of the domains. These macro-blocks are intended to be internally meshed by deformed quadrilaterals. Their geometry follows the geometry of the geological model (one domain per material). Each domain must be bounded by segments or splines:

6. Segments:

```
(SEGMENT) (point) 2*{click extreme point}
```

7. Splines:

You will see the spline evolve as you click points.

STEP II: PREPARE, defines the properties of the discrete spectral element mesh

1. Switch to the preparing mesh tool:

```
<PREP MESH<
```

2. Define domains with rock n:

```
(DOMAIN REF) $n=$ (any) *{click inside domain}
```

You will see the domains edges get colored and the domains get numbered with n.

3. At any moment you can decide to show or not the domain decomposition:

To hide the domain decomposition:

```
>REFRESH>
```

Show the domain decomposition:

(SHOW) (ALL)

4. Remove a domain definition:

```
(REMOVE) (DOMAINE) (any) {click inside domain}
```

WARNING: corrections to the domain decomposition are sometimes displayed only after refreshing the figure window.

5. Now you must define the subdivision of each domain in quadrilateral finite elements. Define the number n of elements on each edge:

You will see the intermediate points appear. The number of intervals n is mainly dictated by the resolution criterion: elements should be smaller than the smallest wavelength you want to propagate. Moreover, a domain can be quadrangulated only if the total number of intervals along its perimeter is even (the sum of all n along its boundaries). However, a

quality mesh is not always guaranteed and you need to proceed by trial and error (emc2 allows you to jump back and forth between the different steps of the meshing procedure).

6. Finally you must define the external boundaries of the modelled region which will have a special treatment. You must associate a tag (a number) to each absorbing boundary. No convention is assumed but you should remember those tags later when setting the boundary conditions in SEM2D. It is also useful to assign a tag to the free surface boundary, that will be eventually used by SEM2D to locate the receivers or sources.

Define a boundary with index n:

(LINE REF) \$n=\$ (any) *{click edge}

Of course each boundary can be composed of many domain edges. Refresh the display to better see the boundaries. The same procedure applies to define split-node interfaces such as faults and cracks: you must assign a different tag to each side of the fault.

7. Save your work in EMC2 format:

<SAVE< "name"

The resulting file is name.emc2_bd

STEP III: EDIT, generates the mesh

1. Switch to the edit mesh tool:

<EDIT_MESH<

Press ENTER 4 times.

A triangles mesh appears. You must convert it to a quad mesh:

2. Convert the triangle mesh to a quad mesh:

<QUADRANGULATE> <ALL>

You can smooth the mesh with: <REGULARIZE> *<ALL>

The final mesh is displayed. If there remain some triangles come back to the previous step and figure out how to modify the points per edge to help the mesher. Some experience is needed here.

3. Renumber the mesh, in order to optimize computations:

*<RENUMBER>

4. Define the boundary condition for the 4 corner nodes of the model: (these nodes belong to 2 external boundaries so they were given a reference number =0)

(MODIF_REF) \$n=\$ (corner) {click close to corner, inside element}

Where n is the reference number of one of the 2 boundaries containing the corner node. Zooming can be useful. The same operation must be performed for the corner nodes of the subdomains belonging to an external boundary, and for the the crack tip nodes. However, as a special case, crack tip nodes must be assigned the -1 taq.

5. Export the mesh:

<SAVE<

Two questions are asked in the bottom prompt:

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- Format of the file, you must select: "fta"
- Prefix name for the file "name" The resulting file name will be name.ftq

4.4 Further tips

- Whenever possible it is better to mesh a domain with a *structured* mesh (a deformed cartesian grid). This can be done with (QUADRANGULATE), during the PREPARE step. See our FAQ for further details.
- To load an existent project, in the construction tool or in the preparation mesh tool: <RESTORE< "name"</pre>
 - EMC2 will look for the file name.emc2_bd. Beware: the project loaded will replace the actual project if any, there is no superposition.
- BUG WARNING (13/07/01): the Sun release of EMC2 has a bug with the reference indices in the ftq format This bug is fixed in the 2.12c version. If you work on a Sun station, download the most recent version of the sources, rather than the executable, and compile it yourself.
- To densify (h-refinement) an existent mesh use the script SEM2DPACK/POST/href.csh. It edits the *.emc2_bd file, then you can restore it in EMC2 and save it in *.ftq format.
- To create a fault, in EDIT_MESH mode:
 - a. Crack an existent edge: (CRACK) (segment)
 - b. Give a reference number to each side of the fault : $(MODIF_REF) n=$ (segment)$
 - c. Give the tag "-1" to crack tip nodes:
 (MODIF_REF) \$-1=\$ (corner) *{click close to crack tip node, inside element}
- Note that only Q4 elements (4 control nodes) are supported. For a smoother description of boundaries Q9 would be desirable.

Chapter 5

Adding features to SEM2D (notes for advanced users)

Sometimes you will need to add new capabilities to the SEM2DPACK solver, by modifying the program. The following notes are intended to guide you through this process. We will not give here a comprehensive description of the code architecture, only enough details to get you started in performing safely the most usual and evident modifications.

5.1 Overview of the code architecture

```
[ ... in progresss ...]
```

This code uses a mixture of procedural (imperative) and object-oriented paradigms. Historically, it evolved from a purely procedural code.

Extensive use of modularity.

Object Oriented Programming (OOP) features (principles) applied in this code: encapsulation, classes, static polymorphism. These are not applied everywhere in the code, for different reasons: reusage of legacy code, performance, difficulty related to the limits of Fortra 90, or sections of code yet to be updated.

Added cost of structures containing pointer components: the possibility of pointer aliasing prevents more agressive compiler optimizations and adds overhead for safety checks.

5.2 Accessible areas of the code

Some areas of the code have been written in such a way that a moderately experienced Fortran 95 programmer, with a limited understanding of the code architecture, can introduce new fea-

tures without breaking the whole system. This is achieved through modularity, encapsulation and templates. The modifications that are currently accessible are:

- boundary conditions, see bc_gen.f90
- material rheology, see mat_gen.f90
- source time functions, see stf_gen.f90
- spatial distributions, see distribution_general.f90

The source files listed above contain step-by-step instructions, just follow the comments starting by !!.

Chapter 6

Frequently Asked Questions

6.1 SEM2D

Segmentation fault

This problem is often related to a small stack size in your computer settings. In your Linux shell do: ulimit -s unlimited under bash or limit stacksize unlimited under csh. Place this command in your startup files (.login, .bashrc or .cshrc).

Instabilities on very distorted elements

Very distorted elements (with very small or very large angles) are usual close to wedges of sedimentary basins, fault branching points, etc. In general, distorted elements are less stable than square elements: spurious motions with exponentially increasing amplitude might appear in their vicinity. In most cases these instabilities can be suppressed by reducing the Courant input parameter. There is currently no simple recipe to determine the maximum value of this parameter, so trial and error is required.

6.2 EMC2

I can't get rid of a few triangles

Obtaining a quality quad mesh is not always a trivial task. Trial and error and experience is needed. This can be by far the most time consuming stage of modeling.

First make sure that the total number of element edges along the perimeter of each mesh domain is even. This is a necessary topological condition to generate a quad-only mesh.

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When the geometry seems too complicated for quad meshing you should consider simplifying the geometry, especially those details that are much smaller than the dominant wavelength.

If the above fails or does not apply, you have to help the mesher. The recommended procedure in EMC2 is:

- 1. Divide your original mesh into simple domains, in such a way that *most* domains have exactly four sides (possibly curved) and the remaining non-four-sided domains are as small as possible.
- 2. Generate a structured quad-mesh (a regular grid) inside each four-sided domain with the (QUADRANGULATE) tool of the PREP_MESH mode, as described in section 5.2.13 of EMC2's manual (note that this is *not* the same as the <QUADRANGULATE> button in the EDIT_MESH mode).
- 3. Proceed as usual (triangulation followed by quadrangulation) inside the remaining non-four-sided domains. If these are small enough EMC2 should not have problems doing a correct tri-to-quad meshing.

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