ANALOG TECHNOLOGY

1973 – MODELING OF A GYRATOR CIRCUIT

EGE SARICA - 244573

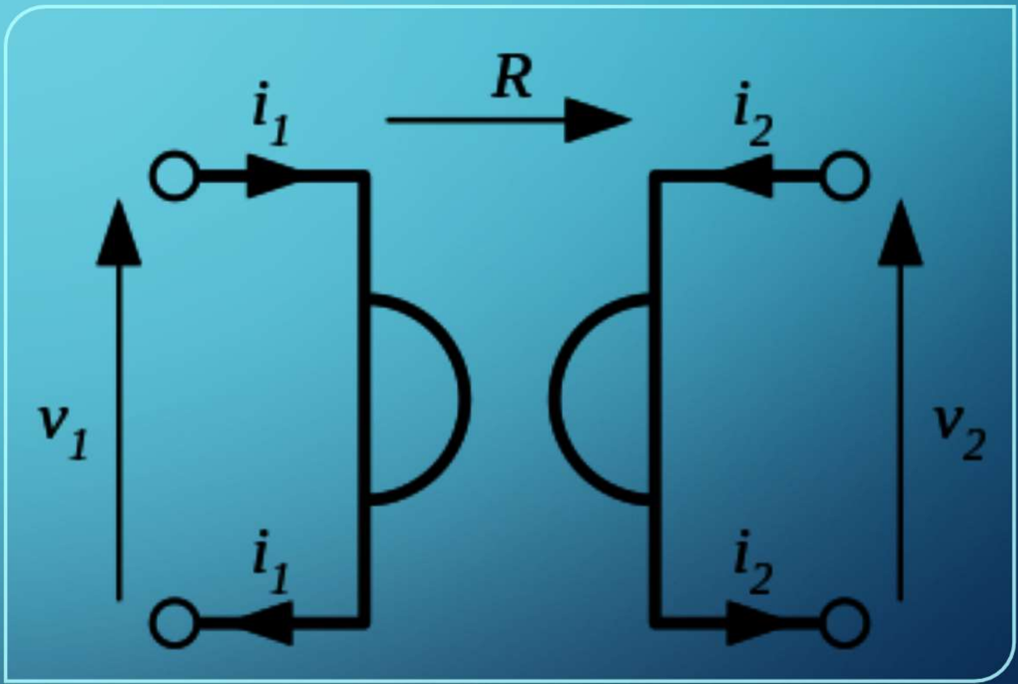
OUTLINE

- Gyrator Circuit – Introduction
- Gyrator Circuit Types
- Operational Amplifiers
- Gyrator Circuit Built with OpAmp
- Mathematical Analysis of the Circuit
- Model for the Gyrator Circuit
- Graphical Representations
- Conclusion
- References

GYRATOR CIRCUIT - INTRODUCTION

A **gyrator** is a passive, linear, lossless, two-port electrical network element proposed in 1948 by Bernard D. H. Tellegen as a hypothetical fifth linear element after the resistor, capacitor, inductor and ideal transformer.

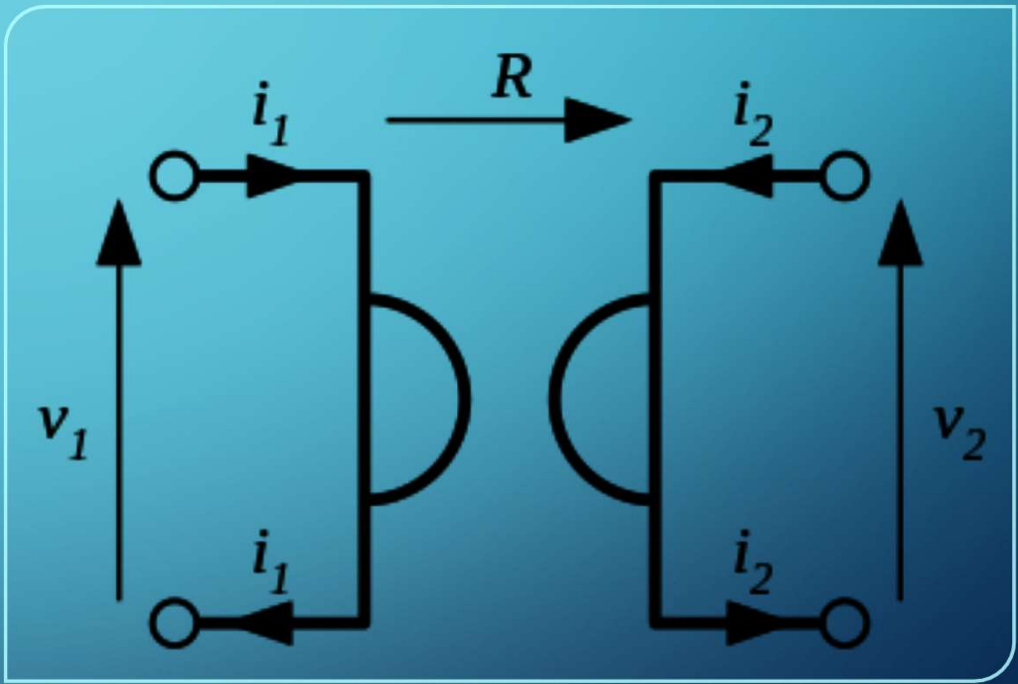
Gyrator is a device that reverses the polarity of the signal traversing or travelling from one side to the other side but not the other way.



$$V_2 = R * i_1$$
$$V_1 = -R * i_2$$

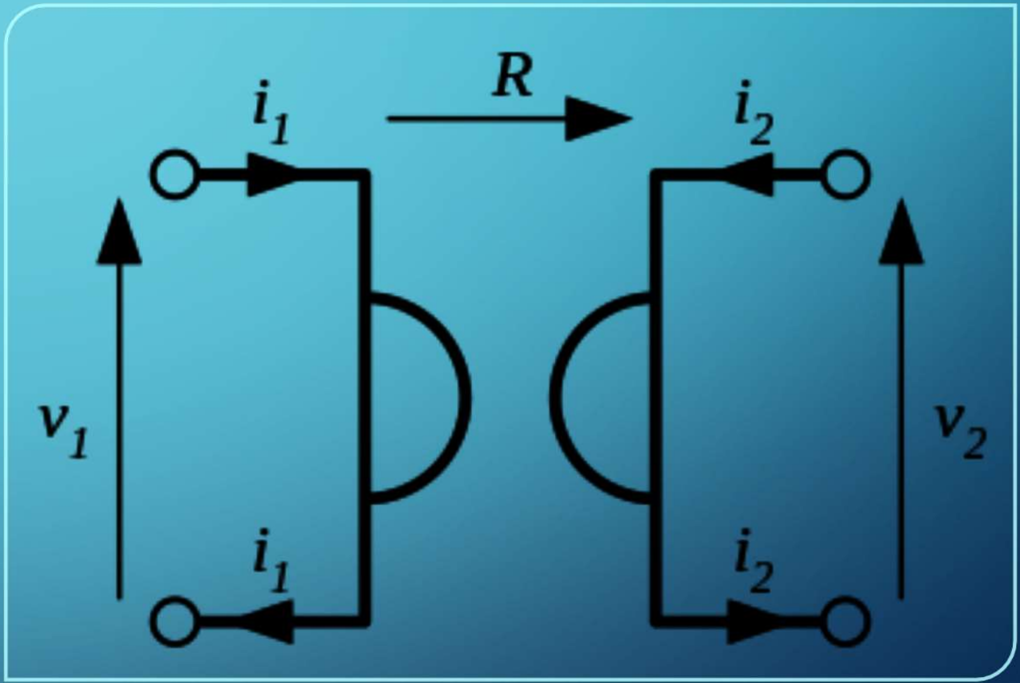
Impedence matrix, Z is anti-symetric, so the gyrator circuit is non-reciprocal circuit unlike the four conventional elements.

- $V_2 = R * i_1$
- $V_1 = -R \, * \, i_2$
- $\begin{matrix} V_1 \\ V_2 \end{matrix} = \begin{matrix} 0 & -R \\ R & 0 \end{matrix} = \begin{matrix} i_1 \\ i_2 \end{matrix}$



INVERTING CHARACTERICTIC

An important property of a gyrator is that it inverts the current–voltage characteristic of an electrical component or network. In the case of linear elements, the impedance is also inverted. Gyrator can make a capacitive circuit behave inductively, a series LC circuit behave like a parallel LC circuit, and so on. It is primarily used in active filter design and miniaturization.



GYRATOR TYPES

Gyrators can be built by using different circuit elements. The three gyrator types are:

- Vacuum-tube gyrators
- Transistor gyrators
- Operational-amplifier gyrators

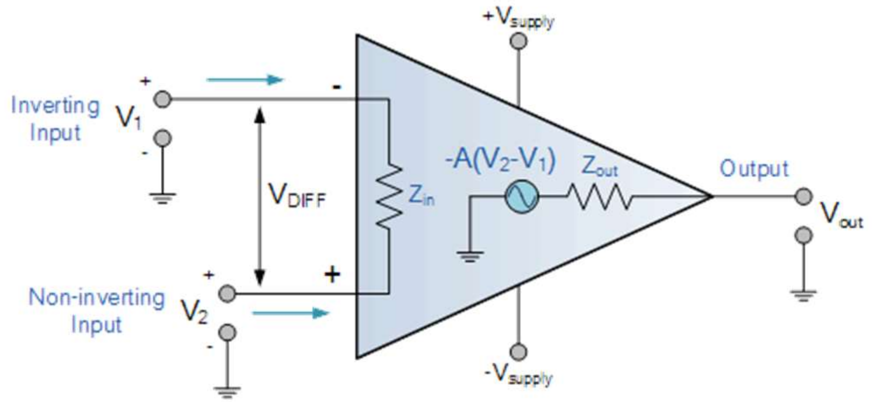
The model we examine is operational-amplifier gyrator.

OPERATIONAL-AMPLIFIER GYRATORS

The operational amplifier has been used for network realization of gyrator circuits since 1964. The operational amplifier is commercially available and economic. It is usually fabricated as an integrated circuit and has:

- High open- loop gain
- Low dc drift
- High input impedance
- Low output impedance
- Excellent bandwidth
- Low cost

OPERATIONAL AMPLIFIERS



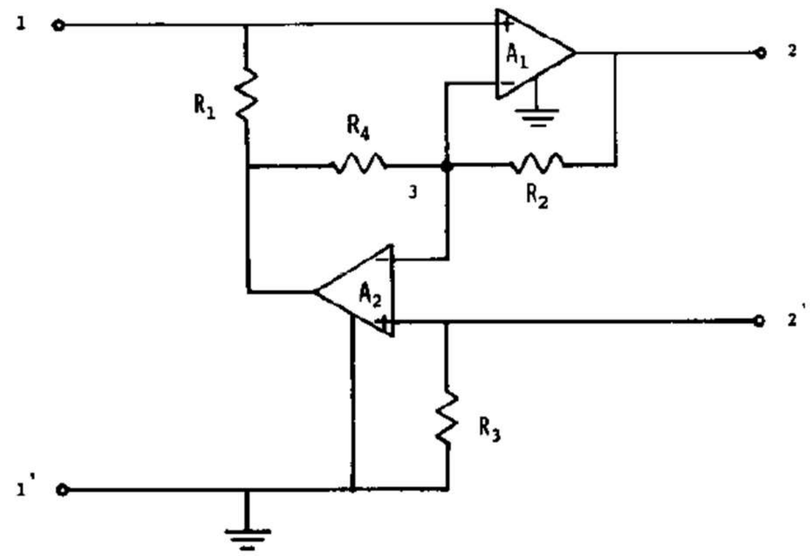
- *Operational amplifiers* are linear devices that have all the properties required for nearly ideal DC amplification and are therefore used extensively in signal conditioning, filtering or to perform mathematical operations such as add, subtract, integration and differentiation.

OP-AMP PARAMETERS AND IDEALISED CHARACTERISTIC

- Open Loop Gain (A_{V0}): **Infinite** – The main function of an operational amplifier is to amplify the input signal and the more open loop gain it has the better. Typical real values range from about 20,000 to 200,000.
- Input Impedance (Z_{in}): **Infinite** – Input impedance is the ratio of input voltage to input current and is assumed to be infinite to prevent any current flowing from the source supply into the amplifiers input circuitry (IIN = 0). Real op-amps have input leakage currents from a few pico-amps to a few milli-amps.

OP-AMP PARAMETERS AND IDEALISED CHARACTERISTIC

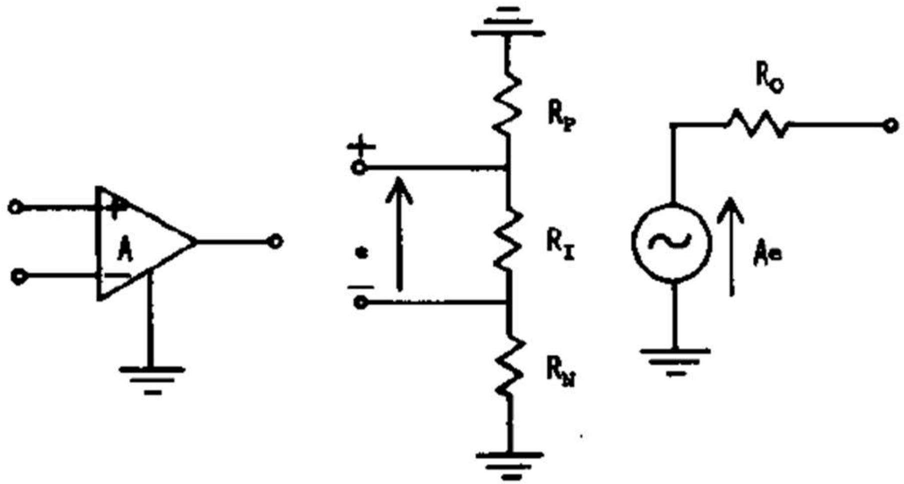
- Output Impedance (Z_{out}): **Zero –** The output impedance of the ideal operational amplifier is assumed to be zero acting as a perfect internal voltage source with no internal resistance so that it can supply as much current as necessary to the load. This internal resistance is effectively in series with the load thereby reducing the output voltage available to the load. Real op−amps have output impedances in the 100−20kΩ range.
- Bandwidth: **Infinite –** An ideal operational amplifier has an infinite frequency response and can amplify any frequency signal from DC to the highest AC frequencies so it is therefore assumed to have an infinite bandwidth. With real op-amps, the bandwidth is limited by the Gain-Bandwidth product (GB), which is equal to the frequency where the amplifiers gain becomes unity.
- Offset Voltage: **Zero –** The amplifiers output will be zero when the voltage difference between the inverting and the non-inverting inputs is zero, the same or when both inputs are grounded. Real op-amps have some amount of output offset voltage.



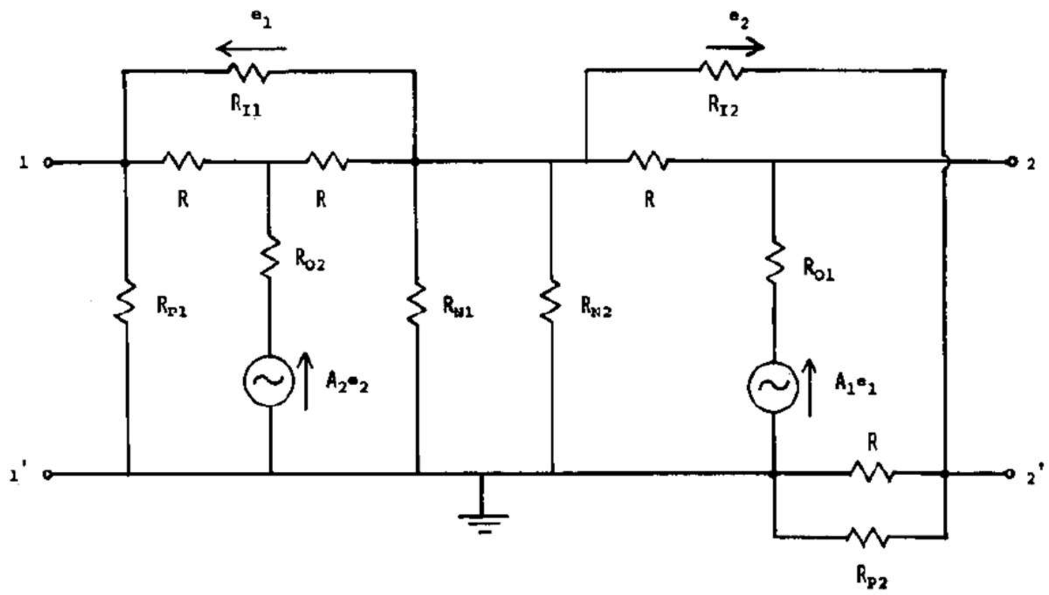
GYRATOR CIRCUIT BUILT WITH OPAMP

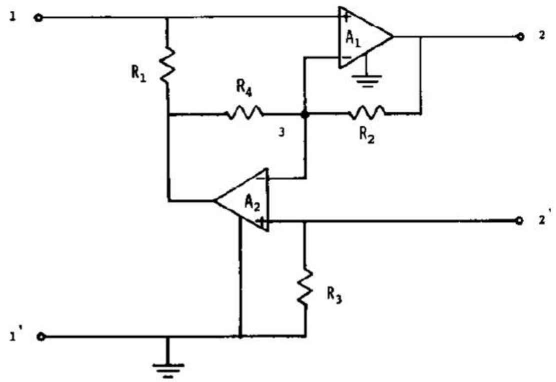
- The gyrator circuit we focus explains a model for the gyrator circuit of which additionally calculates the effect of amplifier imperfections on the performance of the gyrator circuit.

NON-IDEAL OP-AMP



EQUIVALENT CIRCUIT OF GYRATOR

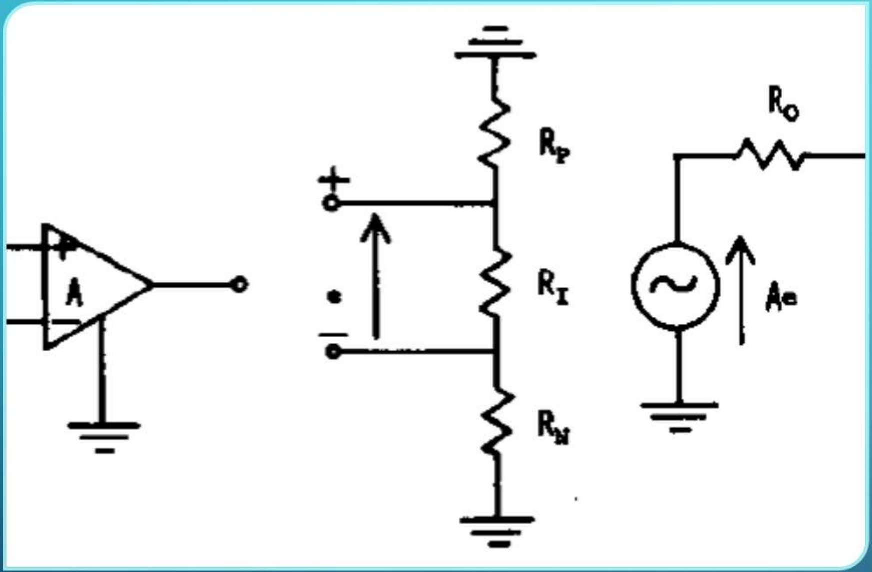




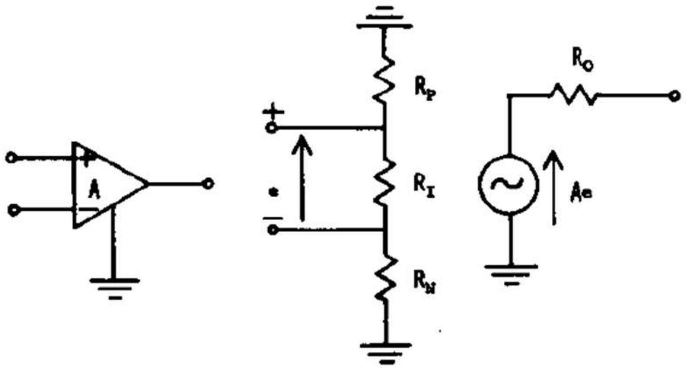
- $[y] = \begin{bmatrix} 0 & \frac{R_4}{R_1 * R_2} \\ -\frac{1}{R_3} & 0 \end{bmatrix}$
- Input impedence is:
- $z_i = \frac{k}{Z_L} = \frac{R_1 * R_2 * R_3}{R_4 * Z_L}$, Z_L *load impedance*
- $k = \frac{R_1 * R_2 * R_3}{R_4}$, Gyration constant
- *Capacitance at the output port will be*
- *converted into an input inductance*
- $L_i = k * C_L$

- $k = \frac{R_1 * R_2 * R_3}{R_4}$
- According to gyration constant, simulated inductance can be easily adjusted by changing only one of the four resistors. This gyrator circuit gives stable and high Q-factor inductances.

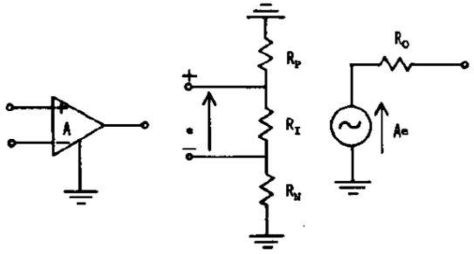
MATHEMATICAL ANALYSIS OF THE CIRCUIT



- It is of significant practical interest to examine the influence of amplifier imperfections on the performance of gyrator circuits. The amplifiers are assumed to have finite input and output resistances and also frequency-dependent finite gains. The analysis is concerned with the derivation of a gyrator model comprising resistances, capacitances, and inductances.
- $A = \frac{A_0 * \omega_0}{s + \omega_0}$, A_0 is dc gain, ω_0 is cutoff frequency



- It is assumed that the resistances are all equal and two opamps are identical. The usual parameters are:
- A_0 = 3000-200000,
- ω_0 = 30 – 15*10^3 rad/s
- $R_P, R_N = 50 - 500 MOhm,$
- $R_I = 50k\ - 2M\ Ohm,\ \ R_O = 50 - 200\ Ohm$



- It's assumed that frequency range is much smaller than the gain-bandwidth product:
- $\omega \ll A_0\omega_0$
- k has range 10^6 – 25*10^6, R has range 1-5k Ohm, so:
- $|A| \gg 1;\ R_I \gg R;\ R_P, R_N \gg R_I; R \gg R_0$

- The y-parameters of the equivalent gyrator circuit is obtained by using an exact signal flow-graph analysis. And using the equations from previous slides:

$$y_{11} \simeq \frac{1}{R_{P1}} + \frac{1}{DR}\left(\frac{2A}{R} + \frac{AR_0}{R^2} - \frac{A^2}{R_5} + \frac{2}{R}\right) \tag{8}$$

$$y_{12} \simeq \frac{A}{DR}\left(\frac{2+A}{R} + \frac{2R_0}{R^2}\right) \tag{9}$$

$$y_{21} \simeq -\frac{A}{DR^2}\left(A + 2 + \frac{2R_0}{R} + \frac{R}{R_I}\right) \tag{10}$$

$$y_{22} \simeq \frac{1}{DR}\left(\frac{2+2A}{R} + \frac{3AR_0}{R^2} + \frac{A}{R_I} + \frac{2AR_0}{RR_I}\right) \tag{11}$$

where

$$D \simeq \frac{1}{R}\left(2 + 2A + A^2 + \frac{3AR_0}{R}\right) \tag{12}$$

$$R_5 = \frac{R_{N1}R_{N2}}{R_{N1} + R_{N2}} \tag{13}$$

- By letting $s = j\omega$ and using the first assumption equations:

$$y_{11} \simeq G_a + G_b + G_c + \frac{s}{A_0\omega_0 R}\left(2 + \frac{R_0}{R}\right) \quad (14)$$

$$y_{12} \simeq \frac{1}{R} + \frac{2\omega^2}{A_0^2\omega_0^2 R}\left(1 - \frac{R_0}{R}\right) - \frac{sR_0}{A_0\omega_0 R^2} \quad (15)$$

$$y_{21} \simeq -\frac{1}{R} - \frac{2\omega^2}{A_0^2\omega_0^2 R^2}(R - R_0) + \frac{s}{A_0\omega_0 R^2 R_I}(R_0 R_I - R^2) \quad (16)$$

$$y_{22} \simeq \frac{2}{A_0 R} + \frac{3R_0}{A_0 R^2} + \frac{2\omega^2}{A_0^2\omega_0^2 R^2}(R + 6R_0) + \frac{s}{A_0\omega_0 R^2} \cdot (2R + 3R_0) \quad (17)$$

where

$$G_a = \frac{1}{R_{P_1}} + \frac{1}{A_0 R}\left(2 + \frac{R_0}{R}\right) \quad (18)$$

$$G_b = -\left(\frac{1}{R_{N_1}} + \frac{1}{R_{N_2}}\right) \quad (19)$$

$$G_c = \frac{2\omega^2}{A_0^2\omega_0^2 R}\left(1 + \frac{4R_0}{R}\right). \quad (20)$$

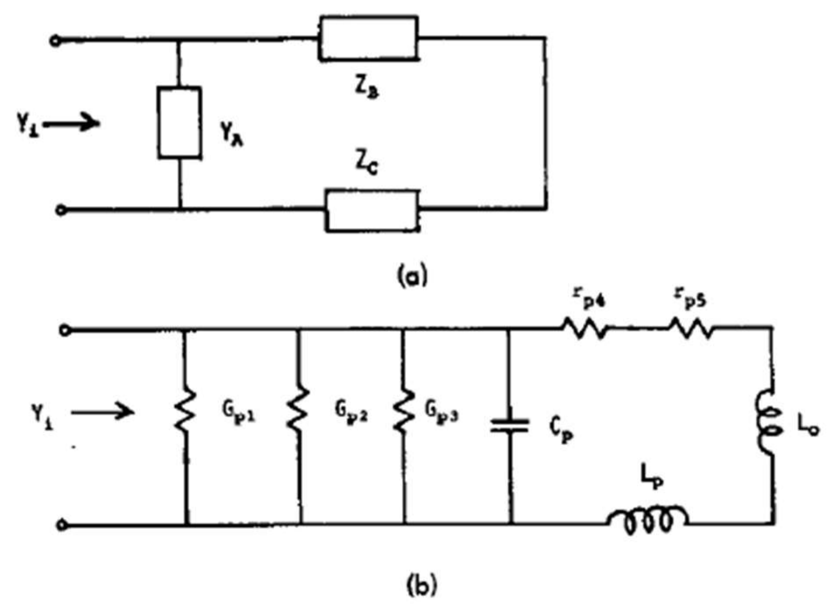


Fig. 2. Model for the capacitively terminated gyrator.

MODEL FOR THE GYRATOR CIRCUIT

- With the help of previous equations, we can derive a gyrator model comprising resistances, capacitances and inductances.

- The advantages of this model are:

1) The influence of amplifier imperfections on the performance of the gyrator circuit can be clearly understood.

2) It can be used for the optimization of the gyrator circuit.

3) It can be used for computer-aided analysis of gyrator-C filters.

- Consider a gyrator circuit terminated by a lossy capacitor such that
- $Y_L = G_L + sC$
- Input admittance is given by
- $Y_i = y_{11} - \dfrac{y_{12}y_{21}}{y_{22} + G_L + sC}$
- $Y_i = Y_A + \dfrac{1}{Z_B + Z_C}$, which gives us Fig. 2(a)

$$Y_A = y_{11} + Hs + HK_1 \tag{24}$$

$$Z_B = \frac{s}{HK_2} \tag{25}$$

$$Z_C = \frac{K_3}{HK_2} \tag{26}$$

$$H = \frac{R_0(R_0R_I - R^2)}{A_0\omega_0R^2R_I(2R + 3R_0 + A_0\omega_0R^2C)} \tag{27}$$

$$K_1 \simeq \frac{1}{A_0\omega_0R_0(R_0R_I - R^2)}\{2A_0^2\omega_0^2RR_0R_I - A_0^2\omega_0^2R^3 + \omega^2(4RR_0R_I - 4R_0^2R_I - 2R^3 + 2R^2R_0)\} \tag{28}$$

$$K_2 \simeq \frac{A_0\omega_0^2R^2R_I}{R_0(R_0R_I - R^2)} \tag{29}$$

$$K_3 \simeq \frac{2A_0\omega_0^2 + 2\omega^2 + G_LA_0^2\omega_0^2R}{A_0\omega_0(2 + A_0\omega_0RC)}. \tag{30}$$

- From (24), (14), (18)-(20) (27) and (28)

  We can write:

- $Y_A = G_{p1} + G_{p2} + G_{p3} + sC_p$
- The admittance Y_A, can thus be split into three parallel conductances G_{p1}, G_{p2}, and G_{p3} and a parallel capacitance C_p .

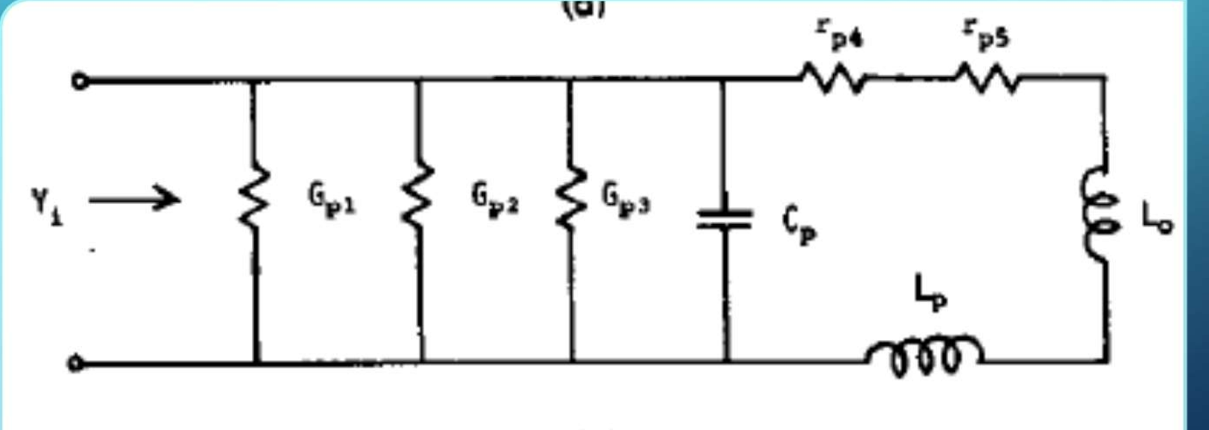
$$G_{p1} = \frac{1}{R_{P1}} + \frac{1}{A_0 R}\left(2 + \frac{R_0}{R}\right) + \frac{R}{R_I(2R + 3R_0 + A_0\omega_0 R^2 C)} \tag{32}$$

$$G_{p2} = -\left\{\frac{1}{R_{N1}} + \frac{1}{R_{N2}} + \frac{2R_0}{R(2R + 3R_0 + A_0\omega_0 R^2 C)}\right\} \tag{33}$$

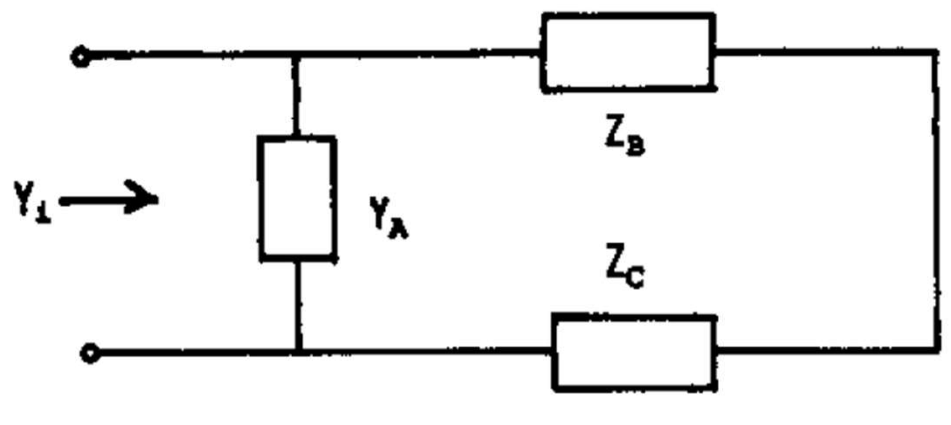
$$\begin{aligned} G_{p3} &= \frac{2\omega^2}{A_0^2\omega_0^2 R}\left\{1 + \frac{4R_0}{R} - \frac{(4RR_0R_I + 2R^2R_0 - 4R_0^2R_I - 2R^3)}{RR_I(2R + 3R_0 + A_0\omega_0R^2C)}\right\} \\ &\simeq \frac{2\omega^2}{A_0^2\omega_0^2 R}\left[1 + \frac{4R_0}{R} + \frac{1}{(2 + A_0\omega_0 RC)} \cdot \left\{-\frac{R_0}{R} + \frac{R}{R_I}\right\}\right] \end{aligned} \tag{34}$$

$$\begin{aligned} C_p &= \frac{1}{A_0\omega_0 R}\left[2 + \frac{R_0}{R} + \frac{1}{RR_I}\left\{\frac{R_0(R_0R_I - R^2)}{2R + 3R_0 + A_0\omega_0 R^2 C}\right\}\right] \\ &\simeq \frac{1}{A_0\omega_0 R}\left[2 + \frac{R_0}{R}\right]. \end{aligned} \tag{35}$$

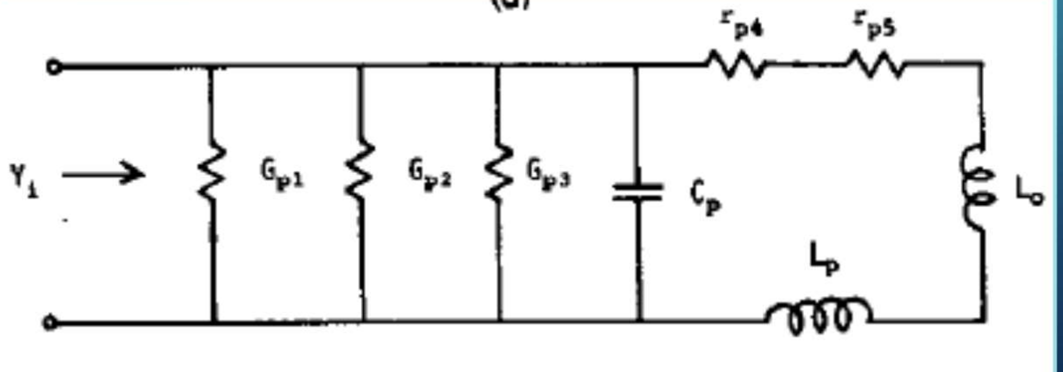
- From (25) (27) and (29),
- $Z_B = L_0 s + L_p s,\ where\ L_0 = R^2 C,\ L_p = \frac{2R}{A_0 \omega_0} + \frac{3R_0}{A_0 \omega_0}$
- Z_B can be considered as a series arrangement of inductances, L_0 and L_p
- Similarly, from (26) (27), (29) and (30),
- $Z_C = r_{p4} + r_{p5}, where\ r_{p4} = \frac{2R}{A_0} + \frac{3R_0}{A_0} + G_L R^2, r_{p5} = \frac{2\omega^2}{A_0^2 \omega_0^2}(R + 6R_0)$
- Z_C can be splited into two series resistances r_{p4} and r_{p5}



(b)



(a)



(b)

INFLUENCE OF OPAMP IMPERFECTIONS

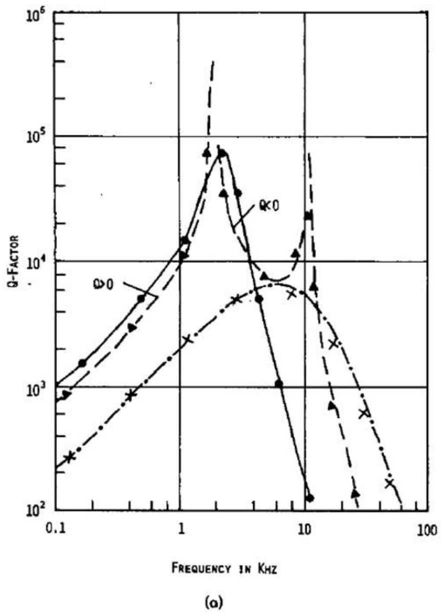
- The influence of amplifier imperfections on the gyrator circuit can now be examined by using the model of Fig. 2(b).
- The loss of the load capacitor introduces a proportional loss in the simulated inductance in the form of a series resistance $G_L R^2$.
- The input resistance R_{p1} at the noninverting input terminal of amplifier A_1 introduces a small loss since, in practice, R_{p1} > 50 M Ohm.
- A finite dc gain gives rise to two sources of loss: a large parallel resistance (=A_0R/2) and a small series resistance (= 2R/ A_0,), according to (32) and (40).

- The conductance G_{p2} is always negative and it gives rise to an effect opposite to that of loss referred to as enhancement.
- Equation (33) shows that the input resistances at the inverting input terminals of the amplifiers R_{N1} and R_{N2} introduce a small quantity of enhancement since R_{N1} , R_{N2} >50 M Ohm. A second source of enhancement is due to $R_0, the\ output\ resistance\ of\ amplifiers$.

- The most significant parameter of the operational amplifier is the gain-bandwidth product $A_0\omega_0$. A noninfinite value of $A_0\omega_0$ gives rise to the following.
- 1) A constant series parasitic inductance L_p.
- 2) A constant parallel parasitic capacitance C_p, (analogous to the winding capacitance in an inductor).
- 3) A frequency-dependent parallel resistance 1/ G_{p3}, which reduces as the frequency is increased. Note that G_{p3} > 0 for the assumed range of amplifier specifications.
- 4) A frequency-dependent series resistance r_{p5} which increases as the frequency is increased (analogous to the winding resistance in an inductor which increases due to the skin effect).
- 5) For a nonzero R_0, enhancement is introduced.

GRAPHICAL REPRESENTATIONS

- The model of Fig. 2(b) has been used for the analysis of the gyrator circuit of Fig. I(a). An exact computer-aided analysis as also been carried out in order to examine the validity of the model. The inductance deviation (ID) is defined as
- $ID = \frac{L - L_0}{L_0} * 100\ percent$
- where L is the actual inductance simulated and L_0 = R^2 *C is the nominal inductance.
- The Q-factor and the ID have been computed for a range of frequencies. The analysis was carried out for a pA741C-type amplifier using the specifications of Table I. The Q-factor of the load capacitance has been assumed to be Q = 10^4. The parallel loss resistance of the load capacitor C was assumed to be independent of the frequency and equal to 10/2piC (i.e. Q = 10^4 at 1 kHz).



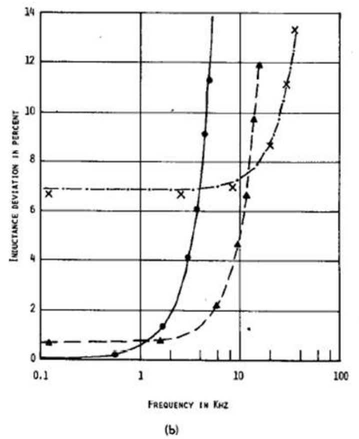


Fig. 3. Frequency responses for different nominal inductances (amplifier type μA741C, $R = 2.5\ \text{k}\Omega$). (a) Q-factor. (b) Inductance deviation.

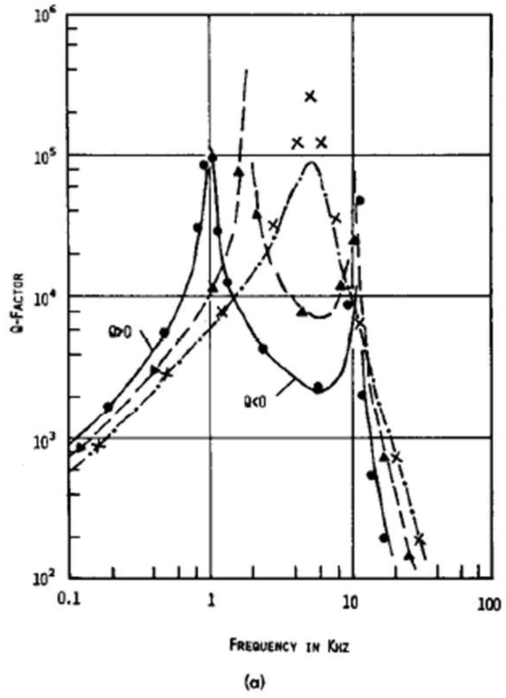
L_0 H	Exact Analysis	Model Analysis
1.0	———————	● ● ●
0.1	– – – – – – –	▲ ▲ ▲
0.01	–·–·–·–·–	× × ×

For L_0 = 0.1 H the enhancement exceeds the total loss in the frequency range 2-10 kHz, and thus a negative Q-factor is obtained.

As $\omega \to 0$, Fig. 2(b) shows that

$$L = L_p + L_0, \quad ID = \frac{L_p}{L_0}*100$$

For a fixed value of R, (38) shows that L_p is invariant. Therefore, as L_0 is increased, the low-frequency ID is reduced. This is evidence Fig. 3(b).



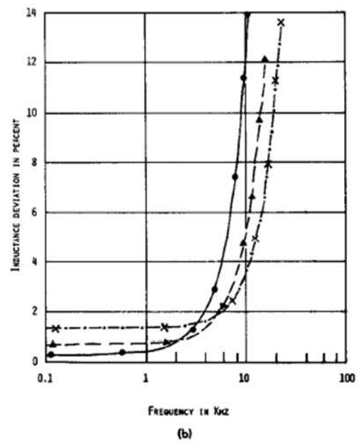
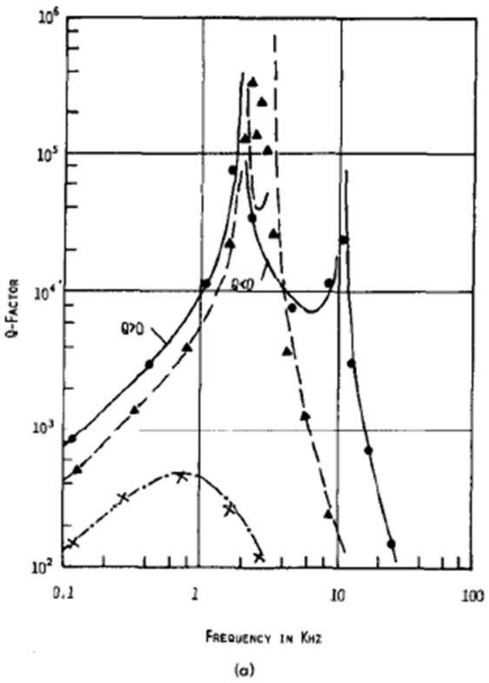


Fig. 4. Frequency responses for different gyration resistances (amplifier type μA741C, $L_0 = 0.1$ H). (a) Q-factor. (b) Inductance deviation.

R kΩ	Exact Analysis	Model Analysis
1.0	————	● ● ●
2.5	- - - - - -	▲ ▲ ▲
5.0	-·-·-·-·-	× × ×

The amount of enhancement in Fig. 4(a) is seen to increase as R is reduced. The low-frequency ID is reduced when R is reduced since L_p is reduced, according to (38), but the useful gyrator bandwidth is also reduced due to an increase in C_p, according to (35).



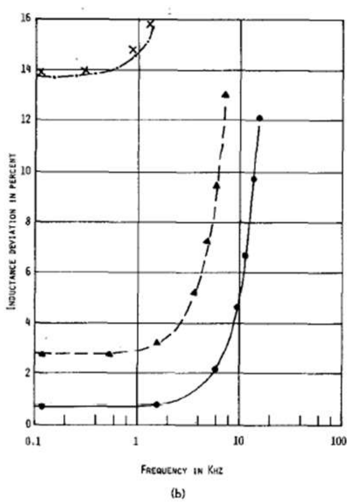


Fig. 5. Frequency responses for different dc gains (amplifier type μA741C, $L_0 = 0.1$ H, $R = 2.5$ kΩ). (a) Q-factor. (b) Inductance deviation.

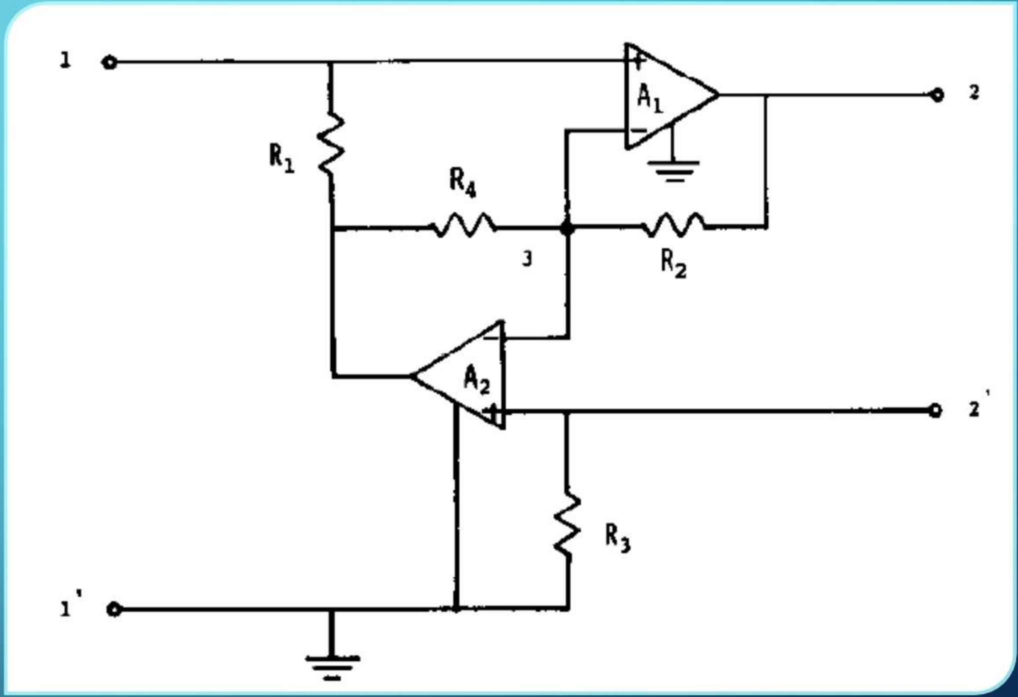
A_0	Exact Analysis	Model Analysis
200×10^3	———	● ● ●
50×10^3	– – – – –	▲ ▲ ▲
10×10^3	–·–·–·–	× × ×

A large dc gain leads to a low loss and, consequently, more pronounced enhancement. The low frequency ID tends to be increased as A0 is reduced, according to (38).

CONCLUSION

- A typical range of amplifier specifications has been assumed and on this basis a model for a capacitively terminated gyrator circuit has been derived. The validity of the model has been confirmed by using an exact computer-aided analysis.
- Although the model was derived on the basis of identical amplifiers, it can, nevertheless, be used for the analysis of gyrator circuits with nonidentical amplifiers.
- The model clearly indicates the influence of each amplifier imperfection on the gyrator circuit. It shows that the useful bandwidth of the gyrator circuit and the stability of the simulated inductance are closely related to the gain-bandwidth product of the amplifiers used. The output resistance of the amplifiers tends to introduce enhancement, and sometimes the Q-factor of the simulated inductance may become negative. The model shows, however, that enhancement can be easily eliminated or increased by using an additional resistor.

- A reduction in the gyration resistance improves the stability of the simulated inductance, but at the same time a larger load capacitance is required. The choice of the gyration resistance also influences the useful bandwidth and the Q-factor of the simulated inductance.



REFERENCES

- A. Antoniou, "New gyrator circuits obtained by using nullors," Electron. Lett., vol. 4, pp. 87-89, Mar. 8, 1968.
- A. Antoniou and K. S. Naidu, "Modeling of a Gyrator Circuit", IEEE Transactions on Circuit Theory, Vol. CT-20, No. 5, September 1973.
- J. J. Kulesz, "A Study of Gyrator Circuits", NPS Archive, December 1969.
- A. Antonlou, "Three terminal **gyrator** circuits using operational amplifiers," Electron. Lett., vol. 4, pp. 591-592, Dec. 27, 1968
- A. Antoniou, "Gyrators using operational amplifiers," Electron. Lett., vol. 3, pp. 350-352, Aug. 1967.
- https://en.wikipedia.org/wiki/Gyrator
- https://www.youtube.com/watch?v=Ls_FvD37yHE
- https://www.electronics-tutorials.ws/opamp/opamp_1.html