Question 1

(1.1) ***Give a Nonlinear Programming (NLP) formulation for the profit maximization problem.***

$$\max z = (100 - 4q)q - 4q - 50$$

(1.2) *Graphically and analitically identify the q value which maximizes the profit.*

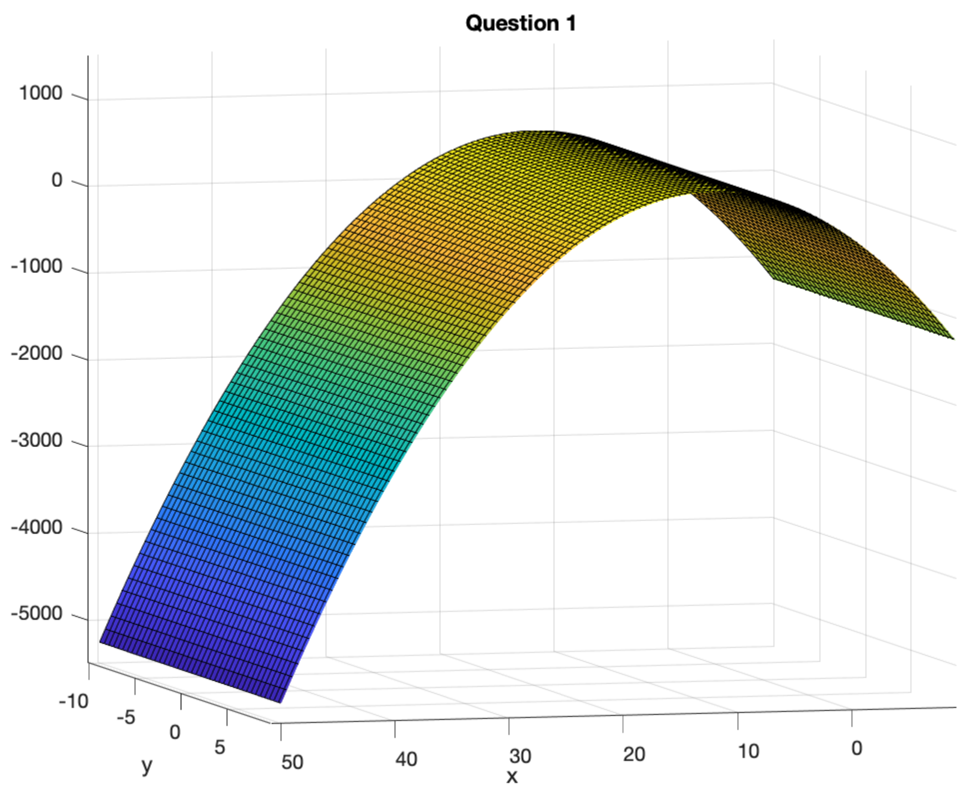
```
[x,y] = meshgrid(-10:0.5:50,-10:0.5:10);

myFunc = 100*x - 4*(x.^2) - 4*x - 50;

surf(x,y,myFunc)

title('Question 1')
xlabel('x')
ylabel('y')

xlim([-9.1 50.9])
ylim([-10.2 9.8])
zlim([-5486 1514])
view([165 6])
```



(1.2) ***Graphically and analitically identify the q value which maximizes the profit.***

- datatip(chart,11.5,6,525,'Location','northeast');
- max Z = 526 and X = 11.5 - 12 (point defined as q in our equation)

(1.3) ***What is the profitable range of production?***

- profitable interval for x is around [23,0.5]

Question 2

Considerthefunction $f(x) = x3 + x2 - 2x + 1$

(2.1) Find the stationary points of f and classify them as local min or local max.

- stationary point = means that function stops chaning and the f(x)' should be equal to zero.
- thus we'll first take first derivative of f(x),

$f(x)' = 3x^2 + 2x - 2$ indicates that $x1, x2 \in R$ and $\Delta > 0$

$\Delta = b^2 - 4ac$ and $\Delta = 24$ satisfies $\Delta > 0$ and means that there should be at least two real roots of this equation. More over $x1 = \frac{-b + \sqrt{\Delta}}{2a}$ and $x2 = \frac{-b - \sqrt{\Delta}}{2a}$ means $x1 = 0.54$ and $x2 = -1.22$
x1 and x2 are stationary points of this function.

- Once we calculated the first derivative of f(x) we need to look at second derivative of f to find whether our function is concave or convex on these given points.

$f(x)'' = 6x + 2$ and $6(0.54) + 2 > 0$ and $6(-1.22) + 2 < 0$ indicates that our function at x1 is strictly convex and at x2 strictly concave.

(2.2) Use bisection method to find the local minimum of f on the interval **[0, 2]**

```
epsilon = 0.001;
a = 0;
b = 2;

while (b - a) > epsilon
    x = (a + b) / 2;
    if evaluateFunc(x) >= evaluateFunc(x + epsilon)
        a = x
    else b = x
    end
end
```

```
b = 1
```

```
a = 0.5000
a = 0.7500
a = 0.8750
a = 0.9375
a = 0.9688
a = 0.9844
a = 0.9922
a = 0.9961
a = 0.9980
a = 0.9990
```

```
(a + b) / 2
```

```
ans = 0.9995
```

***(2.3) Use bisection method to find the local maximum of f on the interval* [−2, 0]**

```
epsilon = 0.001;
a = -2;
b = 0;

while (b - a) > epsilon
    x = (a + b) / 2;
    if evaluateFunc(x) <= evaluateFunc(x + epsilon)
        a = x
    else b = x
    end
end
```

```
b = -1
a = -1.5000
a = -1.2500
a = -1.1250
a = -1.0625
a = -1.0313
a = -1.0156
a = -1.0078
a = -1.0039
a = -1.0020
a = -1.0010
```

```
(a + b) / 2
```

```
ans = -1.0005
```

Question 2 - same question with java code

```
% public class Main {
%     public static void main(String[] args){
%         float b = 10;
%         float a = (float) 0.1; //float is selected to increase precision
%         float e = (float) 0.001;
%         float x = 0;
%         float fx;
%         float fxx;
%         while((b-a) >= e) {
%             x = (a+b)/2;
%             fx = x + 1/x;
%             fxx = (x+e) + 1/(x+e);
%             if(fx >= fxx){
%                  a = x;
%             }
%             else{
%                 b = x;
%             }
%         }
%         System.out.println(x + "is a local minimum of f(x) with " + e + " e approxima
%     }
% }
```

Question 3

Linearize $x \cdot y^k$ for $k \in R, x \geq 0$***is a continuous variable and y*** $\in$ ***{0, 1} is a binary variable.***

- we'll define auxilary variable w. $w = x.y$ then there are only two conditions that we need to consider and these are;

if $y^{\text{any reel number}} = 1$ then $w \leq x$ and $w \geq x - M(1-y)$

if $y^{\text{any reel number}}$ = 0 then $w \geq 0$ and $\leq M(y)$ where M is very big number (big M).

if y^0 means $0'^0$ (undefined)then it should not be available as solution set in LP.

then Linearized Equation will becomes;

$$
\begin{aligned}
&\min w \\
&\text{st} \\
&w \leq x \\
&w \geq x - M(1-y) \\
&w \geq 0 \\
&w \leq \text{My} \\
&y \geq 0 - \text{to avoid } 0^0 \text{ undefined value.} \\
&x \geq 0 \text{ and } y\, \epsilon \{0, 1\}
\end{aligned}
$$

Question 4)

Suppose $S_1, S_2 \subseteq R$ are convex set. Show that $S_1 \oplus S_2 = \{ x_1 + x_2 \subseteq R^n : x_1 \in S_1, x_2 \in S_2 \}$ is convex set.

Let $\lambda \in [0,1]$ Let P_1 and $P_2 \in S \oplus S_2$

$P_1 = x_1 + x$ ad $P_2 = y_1 + y_2$

$$\lambda P_1 + (1-\lambda) P_2 = \lambda(x_1 + x_2) + (1-\lambda)(y_1 + y_2)$$
$$= [\lambda x_1 + (1-\lambda) y_1] + [\lambda x_2 + (1-\lambda) y_2]$$
$$= \underbrace{x_1 + (1-\lambda) y_1}_{P_1} + \underbrace{x_2 + (1-\lambda) y_2}_{P_2}$$

* P_1 is a convex combination of x_1 and y_1 in S_1 domain, and S_1 is convex set ✓
* Any two points in S_1 must belong to its domain so $P_1 = S_1$.
* Same applies to P_2 and $P_1 \in S_1$ and $P_2 \in S_2$

so $P_1 + P_2 \in S_1 \oplus S_2$ so $\lambda P_1 + (1-\lambda) P_2$ is a point in $S_1 \oplus S_2$ domain. That means $S_1 \oplus S_2$ is a convex set.

```
function evalFunc = evaluateFunc(x)
    evalFunc = x + 1/ x;
 end
```