Homework Week4 Report

Problem 1

Three types of price returns: $r_t \sim N(0, \sigma^2)$

① Classical Brownian Motion

$r_t = P_t - P_{t-1}$ $\qquad$ $P_t = P_{t-1} + r_t$

$$E(P_t) = E[P_{t-1} + r_t] = E(P_{t-1}) + E(r_t) = P_{t-1}$$

$$Var(P_t) = Var[P_{t-1} + r_t] = Var(r_t) = \sigma^2$$

② Arithmetic Return System

$r_t = \frac{P_t - P_{t-1}}{P_{t-1}}$ $\qquad$ $P_t = P_{t-1}(1 + r_t)$

$$E(P_t) = P_{t-1} \cdot E(1 + r_t) = P_{t-1}$$

$$Var(P_t) = P_{t-1}^2 \, Var(1 + r_t) = P_{t-1}^2 \, \sigma^2$$

③ Log Return or Geometric Brownian Motion

$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right)$ $\qquad$ $P_t = P_{t-1} \times e^{r_t}$

$$E(P_t) = P_{t-1} \times E(e^{r_t}) = P_{t-1} \times e^{\frac{\sigma^2}{2}}$$

$$Var(P_t) = P_{t-1}^2 \cdot Var(e^{r_t}) = P_{t-1}^2 \times (e^{\sigma^2} - 1) \times e^{\sigma^2}$$

To simulate the results: $\begin{cases} \sigma = 1 \\ n = 1000 \\ P_{t-1} = 100 \end{cases}$

Based on the formular above, the expected value of means and standard deviation should be:

	mean	standard deviation
Classical Brownian Motion	100	1
Arithmetic Return	100	100
Log Return	164.87	216.12

Based on the simulation, the expected value of means and standard deviation should be:

	mean	standard deviation
Classical Brownian Motion	100.02	0.98
Arithmetic Return	101.93	97.87
Log Return	168.23	244.49

Thus, the simulation results are broadly consistent with the theoretical expectations.

Problem 2

① Using a normal distribution

$VaR = Z_\alpha \times \sigma$

② Using a normal distribution with EWV

○ put more weight on recent data points to estimate variance, as they considered more relevant for forecasting future volatility

○ $\sigma_t^2 = \lambda \times \sigma_{t-1}^2 + (1-\lambda) \times (r_{t-1} - \mu)^2$

③ Using a MLE fitted T distribution

$VaR = T_{\alpha,v}(\mu, \sigma)$

④ Using a fitted AR(1) model

$\hat{r}_t = \phi \times r_{t-1} + \varepsilon_t$

ϕ is the AR parameter estimated from model

$\hat{r}_t$ is the predicted return

⑤ Using a Historic Simulation:

○ ranks historical returns and takes the quantile corresponding to the confidence level as the VaR estimate.

○ $VaR = Quantile_\alpha$ (historical returns)

Calculate VaR, assuming the confidence level is 95%:

	VaR
normal distribution	0.054184407435059034
normal distribution with an Exponentially Weighted variance (λ = 0. 94)	0.030799203033063743
MLE fitted T distribution	0.043849319518122064
fitted AR(1) model	0.04540614060753156
Historic Simulation	0.043957621172098844

Problem 3

-Calculated using arithmetic method:

$$r_t = \frac{P_t - P_{t-1}}{P_{t-1}}$$

1. Calculate the daily returns of each stock.
2. Calculate the EWMA covariance matrix for the returns using the provided lambda_EW.
3. Calculate the portfolio variance using the weighted sum of the EWMA covariances.

$$\sigma_j{}^2 = W_j^T \cdot \sum \cdot W_j$$

4. Calculate the VaR by applying the normal distribution's quantile function to the portfolio

standard deviation.

$$\mathrm{VaR}_{\alpha,j} = -Z_{\alpha} \cdot \sigma_j$$

Portfolio A VaR: $25991.81
Portfolio B VaR: $10333.36
Portfolio C VaR: $21310.70
Total VaR: $57635.87

-Calculated using log method:
Portfolio A VaR: $26234.16
Portfolio B VaR: $10475.39
Portfolio C VaR: $21449.62
Total VaR: $58159.17

The reason why I chose log method:
1. Stabilizing the Data. By taking the logarithm of returns, extreme values are dampened, leading to a more stable series, which can be advantageous when analyzing long-term data with potentially high volatility.
2. Ease of Data Accumulation. Logarithmic returns are additive. This property simplifies the accumulation of returns over long time horizons, facilitating the calculation of compounded returns or performance metrics.

The total VaR slightly increased from $57,635.87 to $58,159.17. However, the changes in individual portfolio VaR values are relatively small. This indicates that while the choice of return calculation method can affect VaR estimates, the impact may not be significant in all cases.